

CE 415

DESIGN OF STEEL STRUCTURES

LECTURE 11

COMPRESSION MEMBER

SEMESTER: SPRING 2021

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OUTLINE

- COLUMN BUCKLING
- Short and long column
- Euler buckling
- Column capacity calculation (AISC/LRFD)

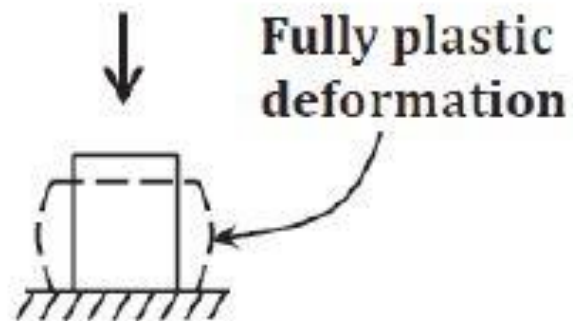
Column:

**Stanchion, Post, Strut, Pillar, Prop,
Buttress, Pier, Pilaster, Baluster**

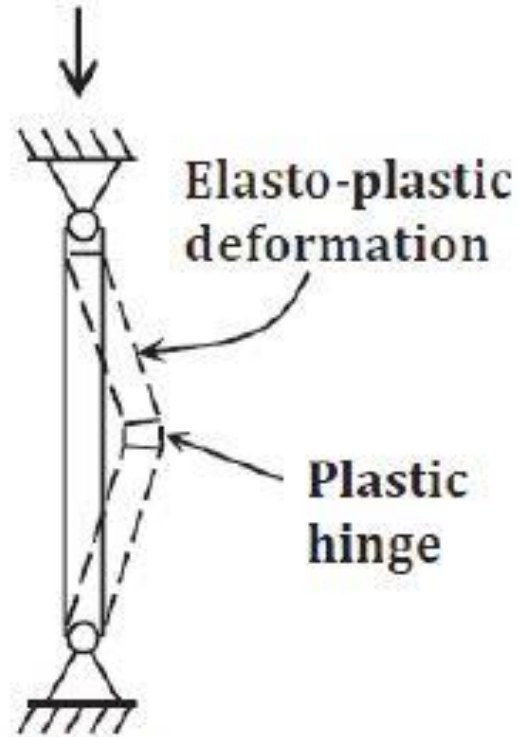
- Members loaded under axial load accompanied by negligible bending if any.**
- Only very short columns can be loaded to their yield stress.**
- The usual situation is that buckling, or sudden bending as a result of instability, occurs prior to developing the full material strength of the member.**



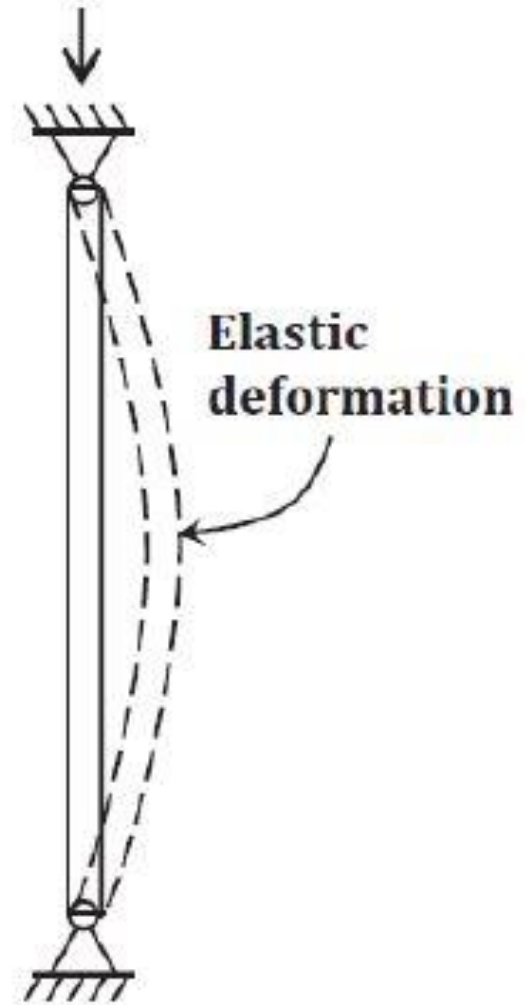
Behavior of a prismatic member under compression



**Very Short:
No buckling**



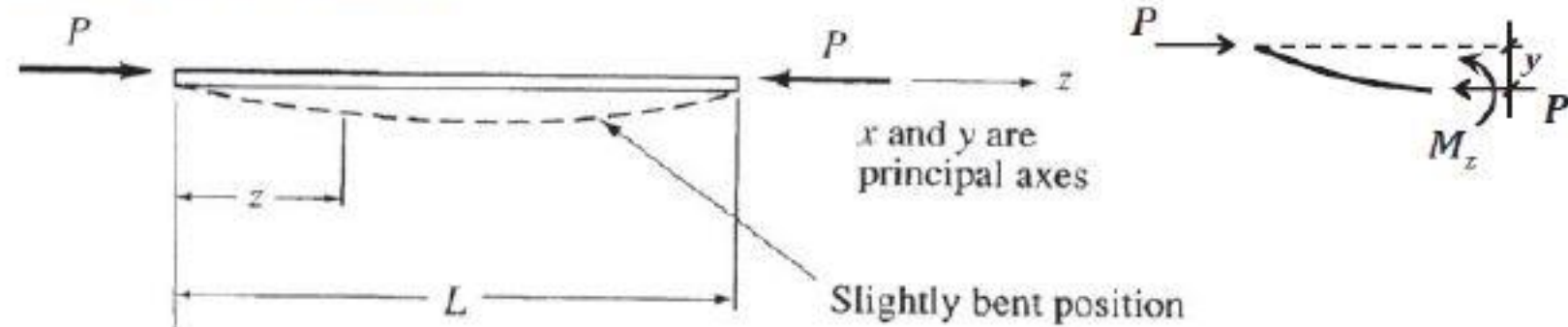
**Short:
Elasto-plastic
buckling**



**Long/Slender:
Elastic buckling**

Euler Elastic Buckling Formula Derivation

Euler Elastic Buckling



$$M_z = Py$$

$$\text{But } \frac{d^2y}{dz^2} = -\frac{M_z}{EI}$$

$$\therefore \frac{d^2y}{dz^2} + \frac{P}{EI}y = 0$$

$$\text{letting } k^2 = P/EI$$

Solution

$$y = A \sin kz + B \cos kz$$

Boundary cond. (a): $y=0$ at $z=0$

$$\Rightarrow 0 = A \sin 0 + B \cos 0 = B$$

$$\therefore y = A \sin kz$$

Boundary cond. (b): $y=0$ at $z=L$

$$\Rightarrow 0 = A \sin kL$$

$$\Rightarrow \sin kL = 0$$

Euler Elastic Buckling Formula Derivation

$$\therefore kL = n\pi \quad \Rightarrow k^2 L^2 = n^2 \pi^2 \quad \Rightarrow \frac{P}{EI} = \frac{n^2 \pi^2}{L^2}$$

$$\therefore P = \frac{n^2 \pi^2 EI}{L^2}$$

The fundamental buckling mode will occur when $n=1$, which is defined as The Euler Critical Load:

$$\therefore P_{cr} = \frac{\pi^2 EI}{L^2}$$

Dividing both sides by gross x-section area, A

$$\Rightarrow \frac{P_{cr}}{A} = \frac{\pi^2 E(I/A)}{L^2} \quad \Rightarrow F_{cr} = \frac{\pi^2 Er^2}{L^2}$$

$$\Rightarrow F_{cr} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$$

Determine Column Capacity Using AISC LRFD

Ques. A steel column of 25 ft length is made of W 14 × 61 shape which is supported by a fixed-hinge joint. Determine the axial capacity of the section. Steel is A992.

Soltuion.

$$K = 0.80 \quad (\text{for fixed-hinge joint})$$

$$L = 25 \text{ ft}$$

$$F_y = 50 \text{ ksi}$$

From [Table 1-7](#) of AISC Manual, $A_g = 17.90 \text{ in}^2$ and $r_y = 2.45 \text{ in}$.

Check Failure Mode

$$KL = 0.8 \times 25 = 20 \text{ ft}$$

$$\frac{KL}{r} = \frac{20 \times 12}{2.45} = 97.9$$

$$C_c = 4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29000}{50}} = 113.4$$

Since, $KL/r < C_c$, failure is by crushing.

Determine Capacity

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 \times 29000}{97.9^2} = 29.83 \text{ ksi}$$

$$F_{cr} = 0.658^{F_y/F_e} F_y = 0.658^{50/29.83} \times 50 = 24.79 \text{ ksi}$$

$$\Phi_c P_n = \Phi_c F_{cr} A_g = 0.9 \times 24.79 \times 17.9 = 399.3 \text{ kip}$$

Ans. 399.3 kip