CE 415
DESIGN OF STEEL STRUCTURES

LECTURE 11
COMPRESSION MEMBER

SEMESTER: SUMMER 2021

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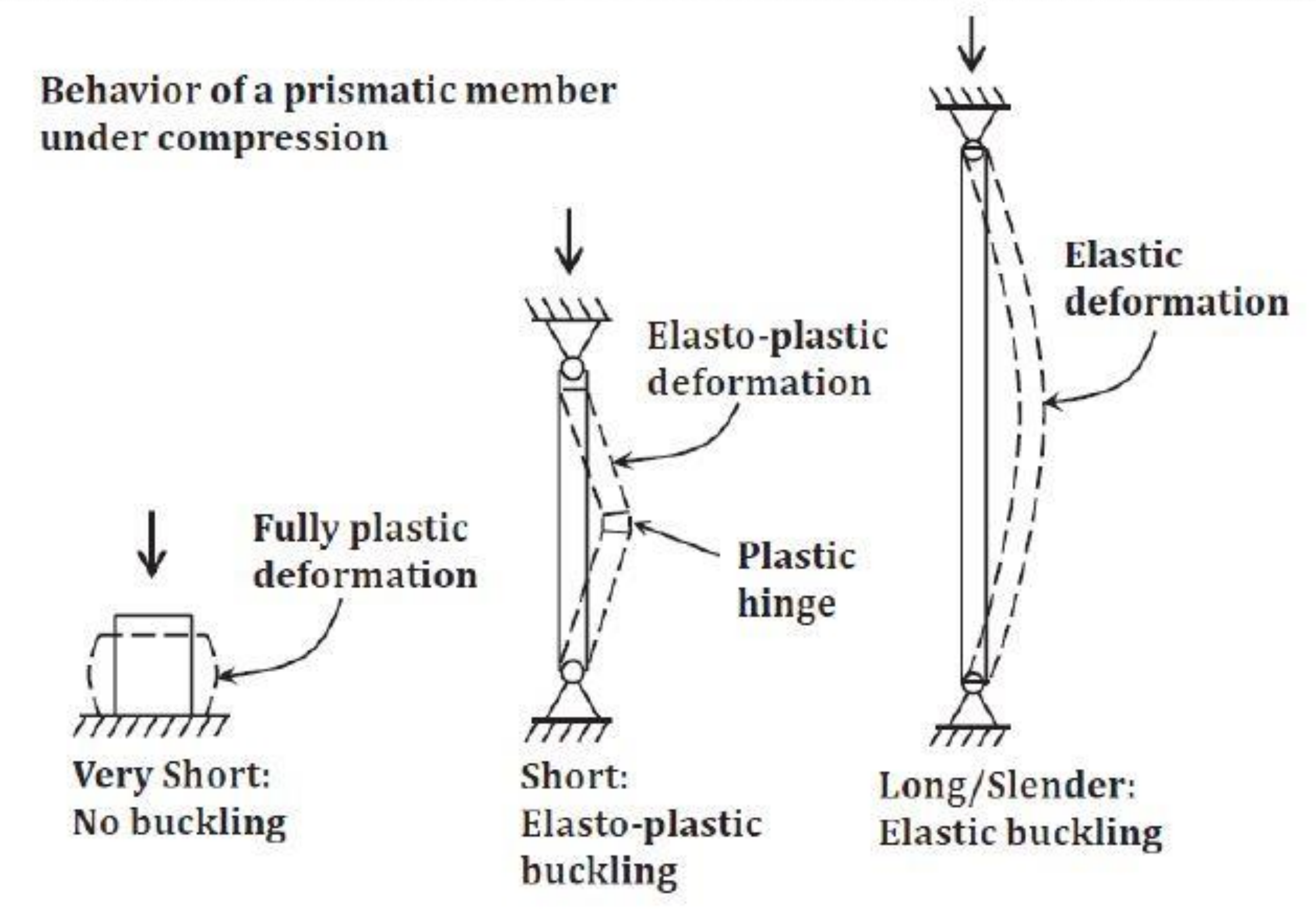
OUTLINE

- COLUMN BUCKLING
- Short and long column
- Euler buckling
- Column capacity calculation (AISC/LRFD)

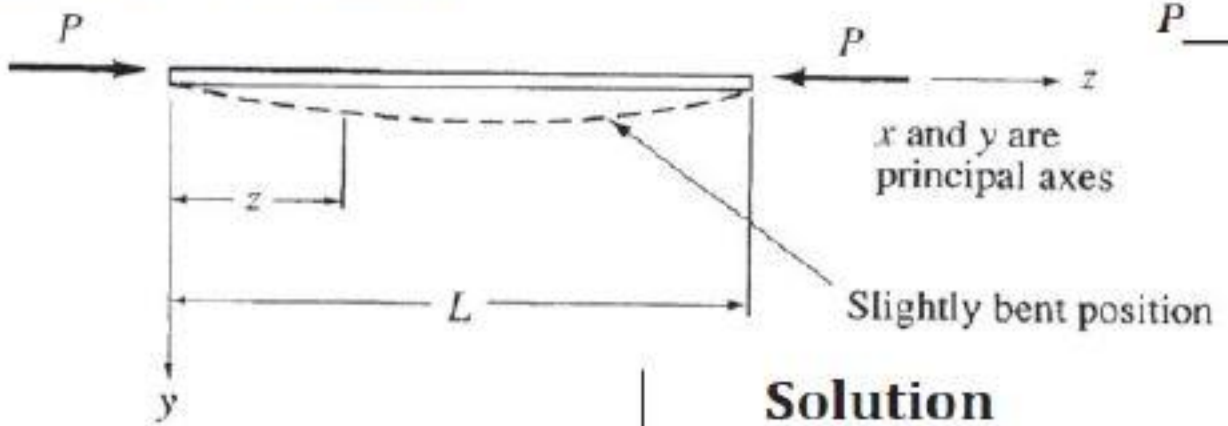
Column:
Stanchion, Post, Strut, Pillar, Prop, Buttress, Pier, Pilaster, Baluster

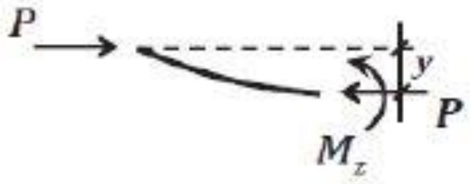
- Members loaded under axial load accompanied by negligible bending if any.
- Only very short columns can be loaded to their yield stress.
- The usual situation is that buckling, or sudden bending as a result of instability, occurs prior to developing the full material strength of the member.


Behavior of a prismatic member
under compression
Fully plastic
deformation
Very Short:
No buckling
Elasto-plastic
deformation
Plastic
hinge
Short:
Elasto-plastic
buckling
Elastic
deformation
Long/Slender:
Elastic buckling

Euler Elastic Buckling Formula Derivation

Euler Elastic Buckling





$$M_z = Py$$

But $$\frac{d^2y}{dz^2} = -\frac{M_z}{EI}$$

$$\therefore \quad \frac{d^2y}{dz^2} + \frac{P}{EI}y = 0$$

letting $k^2 = P/EI$

Solution

$$y = A\sin kz + B\cos kz$$

Boundary cond. (a): *y*=0 at *z*=0

$\Rightarrow$ **0 = *A* sin 0 + *B* cos 0 = B**

$$\therefore \ y = A\sin kz$$

Boundary cond. (b): *y*=0 at *z*=*L*

$\Rightarrow$ **0 = *A* sin *kL***

$\Rightarrow$ **sin *kL* = 0**

Euler Elastic Buckling Formula Derivation

$$\therefore kL = n\pi \qquad \Rightarrow k^2L^2 = n^2\pi^2 \qquad \Rightarrow \frac{P}{EI} = \frac{n^2\pi^2}{L^2}$$

$$\therefore P = \frac{n^2\pi^2 EI}{L^2}$$

The fundamental buckling mode will occur when *n*=1, which is defined as The Euler Critical Load:

$$\therefore P_{cr} = \frac{\pi^2 EI}{L^2}$$

Dividing both sides by gross x-section area, *A*

$$\Rightarrow \frac{P_{cr}}{A} = \frac{\pi^2 E(I/A)}{L^2} \qquad \Rightarrow F_{cr} = \frac{\pi^2 E r^2}{L^2}$$

$$\Rightarrow F_{cr} = \frac{\pi^2 E}{\left(L/r\right)^2}$$

Determine Column Capacity Using AISC LRFD

Ques. A steel column of 25 ft length is made of W 14 × 61 shape which is supported by a fixed-hinge joint. Determine the axial capacity of the section. Steel is A992.

Soltuion.

$$K = 0.80 \quad \text{(for fixed-hinge joint)}$$

$$L = 25 \text{ ft}$$

$$F_y = 50 \text{ ksi}$$

From Table 1-7 of AISC Manual, $A_g = 17.90 \text{ in}^2$ and $r_y = 2.45$ in.

Check Failure Mode

$$KL = 0.8 \times 25 = 20 \text{ ft}$$

$$\frac{KL}{r} = \frac{20 \times 12}{2.45} = 97.9$$

$$C_c = 4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{50}} = 113.4$$

Since, $KL/r < C_c$, failure is by crushing.

Determine Capacity

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 \times 29000}{97.9^2} = 29.83 \text{ ksi}$$

$$F_{cr} = 0.658^{F_y/F_e} F_y = 0.658^{50/29.83} \times 50 = 24.79 \text{ ksi}$$

$$\Phi_c P_n = \Phi_c F_{cr} A_g = 0.9 \times 24.79 \times 17.9 = 399.3 \text{ kip}$$

Ans. 399.3 kip