Chapter I
Design of Composite Members

I1. GENERAL PROVISIONS

Design, detailing, and material properties related to the concrete and steel reinforcing portions of composite members are governed by ACI 318 as modified with composite-specific provisions by the AISC *Specification*.

The available strength of composite sections may be calculated by one of two methods; the plastic stress distribution method, or the strain-compatibility method. The composite design tables in the *Steel Construction Manual* and the Examples are based on the plastic stress distribution method.

Filled composite sections are classified for local buckling according to the slenderness of the compression steel elements as illustrated in AISC *Specification* Table I1.1 and **Examples I.4, I.6 and I.7**. Local buckling effects do not need to be considered for encased composite members.

Terminology used within the Examples for filled composite section geometry is illustrated in Figure I-2.

I2. AXIAL FORCE

The available compressive strength of a composite member is based on a summation of the strengths of all of the components of the column with reductions applied for member slenderness and local buckling effects where applicable.

For tension members, the concrete tensile strength is ignored and only the strength of the steel member and properly connected reinforcing is permitted to be used in the calculation of available tensile strength.

The available compressive strengths given in AISC *Manual* Tables 4-13 through 4-20 reflect the requirements given in AISC *Specification* Sections I1.4 and I2.2. The design of filled composite compression and tension members is presented in **Examples I.4 and I.5.**

The design of encased composite compression and tension members is presented in **Examples I.9 and I.10.** There are no tables in the Manual for the design of these members.

Note that the AISC *Specification* stipulates that the available compressive strength need not be less than that specified for the bare steel member.

I3. FLEXURE

The design of typical composite beams with steel anchors is illustrated in **Examples I.1 and I.2**. AISC *Manual* Table 3-19 provides available flexural strengths for composite beams, Table 3-20 provides lower-bound moments of inertia for plastic composite sections, and Table 3-21 provides shear strengths of steel stud anchors utilized for composite action in composite beams.

The design of filled composite members for flexure is illustrated within **Examples I.6 and I.7**, and the design of encased composite members for flexure is illustrated within **Example I.11**.

I4. SHEAR

For composite beams with formed steel deck, the available shear strength is based upon the properties of the steel section alone in accordance with AISC *Specification* Chapter G as illustrated in **Examples I.1 and I.2**.

For filled and encased composite members, either the shear strength of the steel section alone, the steel section plus the reinforcing steel, or the reinforced concrete alone are permitted to be used in the calculation of available shear strength. The calculation of shear strength for filled composite members is illustrated within **Examples I.6 and I.7** and for encased composite members within **Example I.11**.

I5. COMBINED FLEXURE AND AXIAL FORCE

Design for combined axial force and flexure may be accomplished using either the strain compatibility method or the plastic-distribution method. Several different procedures for employing the plastic-distribution method are outlined in the *Commentary*, and each of these procedures is demonstrated for concrete filled members in **Example I.6** and for concrete encased members in **Example I.11**. Interaction calculations for non-compact and slender concrete filled members are illustrated in **Example I.7**.

To assist in developing the interaction curves illustrated within the design examples, a series of equations is provided in Figure I-1 (Geschwindner, 2010). These equations define selected points on the interaction curve, without consideration of slenderness effects. Figures I-1a through I-1d outline specific cases, and the applicability of the equations to a cross-section that differs should be carefully considered. As an example, the equations in Figure I-1a are appropriate for the case of side bars located at the centerline, but not for other side bar locations. In contrast, these equations are appropriate for any amount of reinforcing at the extreme reinforcing bar location. In Figure I-1b, the equations are appropriate only for the case of 4 reinforcing bars at the corners of the encased section. When design cases deviate from those presented the appropriate interaction equations can be derived from first principles.

I6. LOAD TRANSFER

The AISC *Specification* provides several requirements to ensure that the concrete and steel portions of the section act together. These requirements address both force allocation - how much of the applied loads are resisted by the steel versus the reinforced concrete, and force transfer mechanisms - how the force is transferred between the two materials. These requirements are illustrated in **Example I.3** for concrete filled members and **Example I.8** for encased composite members.

I7. COMPOSITE DIAPHRAGMS AND COLLECTOR BEAMS

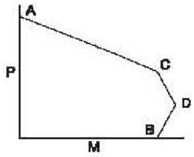
The *Commentary* provides guidance on design methodologies for both composite diaphragms and composite collector beams.

I8. STEEL ANCHORS

AISC *Specification* Section I8 addresses the strength of steel anchors in composite beams and in composite components. **Examples I.1 and I.2** illustrates the design of composite beams with steel headed stud anchors.

The application of steel anchors in composite component provisions have strict limitations as summarized in the User Note provided at the beginning of AISC *Specification* Section I8.3. These provisions do not apply to typical composite beam designs nor do they apply to hybrid construction where the steel and concrete do not resist loads together via composite action such as in embed plates. The most common application for these provisions is for the transfer of longitudinal shear within the load introduction length of composite columns as demonstrated in **Example I.8**. The application of these provisions to an isolated anchor within an applicable composite system is illustrated in **Example I.12**.

Plastic Capacities for Rectangular, Encased W-Shapes Bent About the *X-X* Axis



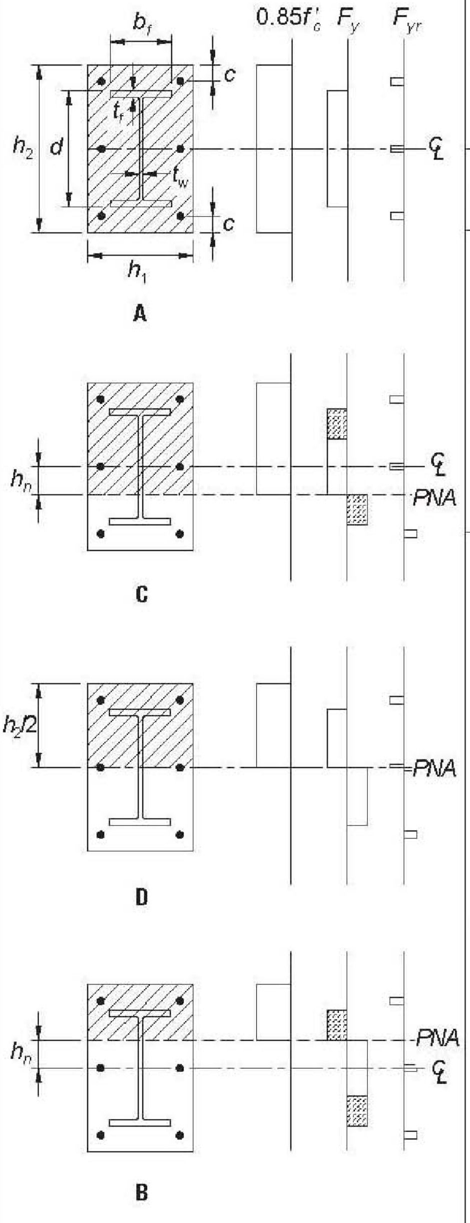
Section / Stress Distribution	Pt.	Defining Equations
	A	$P_A = A_s F_y + A_{sr} F_{yr} + 0.85 f'_c A_c$ $M_A = 0$ A_s = area of steel shape A_{sr} = area of all continuous reinforcing bars $A_c = h_1 h_2 - A_s - A_{sr}$
	C	$P_C = 0.85 f'_c A_c$ $M_C = M_B$
	D	$P_D = \frac{0.85 f'_c A_c}{2}$ $M_D = Z_s F_y + Z_r F_{yr} + \frac{Z_c}{2}(0.85 f'_c)$ Z_s = full *x*-axis plastic section modulus of steel shape A_{srs} = area of continuous reinforcing bars at the centerline $Z_r = (A_{sr} - A_{srs})\left(\frac{h_2}{2} - c\right)$ $Z_c = \frac{h_1 h_2^2}{4} - Z_s - Z_r$
	B	$P_B = 0$ $M_B = M_D - Z_{sn} F_y - \frac{Z_{cn}}{2}(0.85 f'_c)$ $Z_{cn} = h_1 h_n^2 - Z_{sn}$ For h_n below the flange $\left(h_n \le \frac{d}{2} - t_f\right)$ $h_n = \frac{0.85 f'_c (A_c + A_{srs}) - 2F_{yr} A_{srs}}{2\left[0.85 f'_c (h_1 - t_w) + 2F_y t_w\right]}$ $Z_{sn} = t_w h_n^2$ For h_n within the flange $\left(\frac{d}{2} - t_f < h_n \le \frac{d}{2}\right)$ $h_n = \frac{0.85 f'_c (A_c + A_s - d b_f + A_{srs}) - 2F_y (A_s - d b_f) - 2F_{yr} A_{srs}}{2\left[0.85 f'_c (h_1 - b_f) + 2F_y b_f\right]}$ $Z_{sn} = Z_s - b_f\left(\frac{d}{2} - h_n\right)\left(\frac{d}{2} + h_n\right)$ For h_n above the flange $\left(h_n > \frac{d}{2}\right)$ $h_n = \frac{0.85 f'_c (A_c + A_s + A_{srs}) - 2F_y A_s - 2F_{yr} A_{srs}}{2(0.85 f'_c h_1)}$ $Z_{sn} = Z_s$

Fig. I-1a. W-shapes, strong-axis anchor points.

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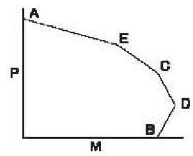
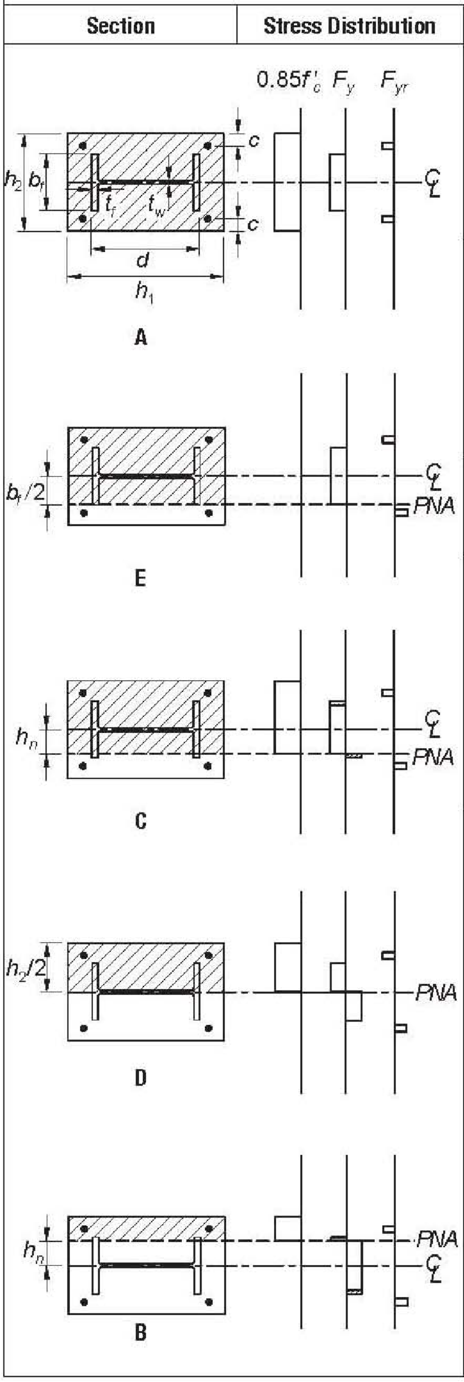
Plastic Capacities for Rectangular, Encased W-Shapes Bent About the *Y-Y* Axis





Pt.	Defining Equations
A	$P_A = A_s F_y + A_{sr} F_{yr} + 0.85 f'_c A_c$ $M_A = 0$ A_s = area of steel shape A_{sr} = area of continuous reinforcing bars $A_c = h_1 h_2 - A_s - A_{sr}$
E	$P_E = A_s F_y + \left(0.85 f'_c\right)\left[A_c - \frac{h_1}{2}\left(h_2 - b_f\right) + \frac{A_{sr}}{2}\right]$ $M_E = M_D - Z_{sE} F_y - \frac{Z_{cE}}{2}\left(0.85 f'_c\right)$ $Z_{sE} = Z_s$ = full y-axis plastic section modulus of steel shape $Z_{cE} = \frac{h_1 b_f^2}{4} - Z_{sE}$
C	$P_C = 0.85 f'_c A_c$ $M_C = M_B$
D	$P_D = \frac{0.85 f'_c A_c}{2}$ $M_D = Z_s F_y + Z_r F_{sr} + \frac{Z_c}{2}\left(0.85 f'_c\right)$ $Z_r = A_{sr}\left(\frac{h_2}{2} - c\right)$ $Z_c = \frac{h_1 h_2^2}{4} - Z_s - Z_r$
B	$P_B = 0$ $M_B = M_D - Z_{sn} F_y - \frac{Z_{cn}}{2}\left(0.85 f'_c\right)$ $Z_{cn} = h_1 h_n^2 - Z_{sn}$ For h_n below the flange $\left(\frac{t_w}{2} < h_n \le \frac{b_f}{2}\right)$ $h_n = \frac{0.85 f'_c\left(A_c + A_s - 2t_f b_f\right) - 2F_y\left(A_s - 2t_f b_f\right)}{2\left[4t_f F_y + \left(h_1 - 2t_f\right)0.85 f'_c\right]}$ $Z_{sn} = Z_s - 2t_f\left(\frac{b_f}{2} + h_n\right)\left(\frac{b_f}{2} - h_n\right)$ For h_n above the flange $\left(h_n > \frac{b_f}{2}\right)$ $h_n = \frac{0.85 f'_c\left(A_c + A_s\right) - 2F_y A_s}{2\left[0.85 f'_c h_1\right]}$ $Z_{sn} = Z_s$

Fig. I-1b. W-shapes, weak-axis anchor points.

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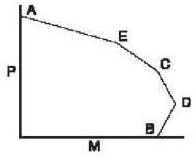
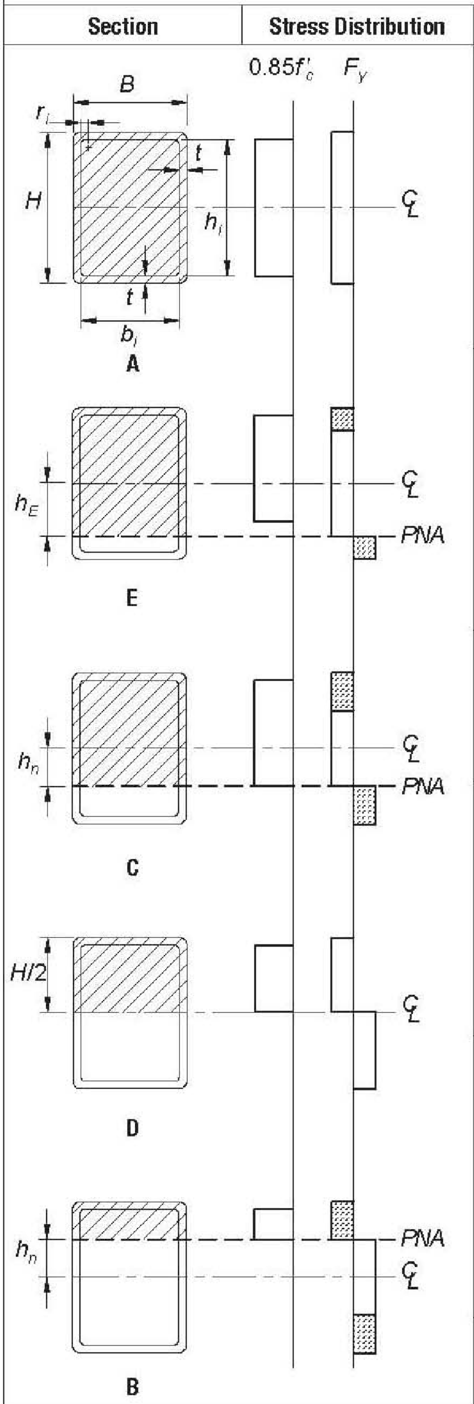
Plastic Capacities for Composite Filled HSS Bent About Either Principal Axis

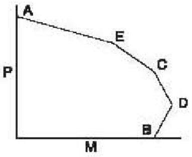




Pt.	Defining Equations
A	$P_A = F_y A_s + 0.85 f'_c A_c$ $M_A = 0$ A_s = area of steel shape $A_c = b_i h_i - 0.858 r_i^2$ $b_i = B - 2t$ $h_i = H - 2t$ $r_i = t$
E	$P_E = \frac{0.85 f'_c A_c}{2} + 0.85 f'_c b_i h_E + 4 F_y t h_E$ $M_E = M_D - F_y Z_{sE} - \frac{0.85 f'_c Z_{cE}}{2}$ $Z_{cE} = b_i h_E^2$ $Z_{sE} = 2 t h_E^2$ $h_E = \frac{h_n}{2} + \frac{H}{4}$
C	$P_C = 0.85 f'_c A_c$ $M_C = M_B$
D	$P_D = \frac{0.85 f'_c A_c}{2}$ $M_D = F_y Z_s + \frac{0.85 f'_c Z_c}{2}$ Z_s = full *x*-axis plastic section modulus of HSS $Z_c = \frac{b_i h_i^2}{4} - 0.192 r_i^3$
B	$P_B = 0$ $M_B = M_D - F_y Z_{sn} - \frac{0.85 f'_c Z_{cn}}{2}$ $Z_{sn} = 2 t h_n^2$ $Z_{cn} = b_i h_n^2$ $h_n = \frac{0.85 f'_c A_c}{2\left[0.85 f'_c b_i + 4 t F_y\right]} \le \frac{h_i}{2}$
	Note: Equations in this table are equally applicable to bending about the shape's *X*-*X* axis (when $H \ge B$) and to bending about the shape's *Y*-*Y* axis (when $B > H$).

Fig. I-1c. Filled rectangular or square HSS, strong-axis anchor points.

Plastic Capacities for Composite Filled Round HSS Bent About Any Axis



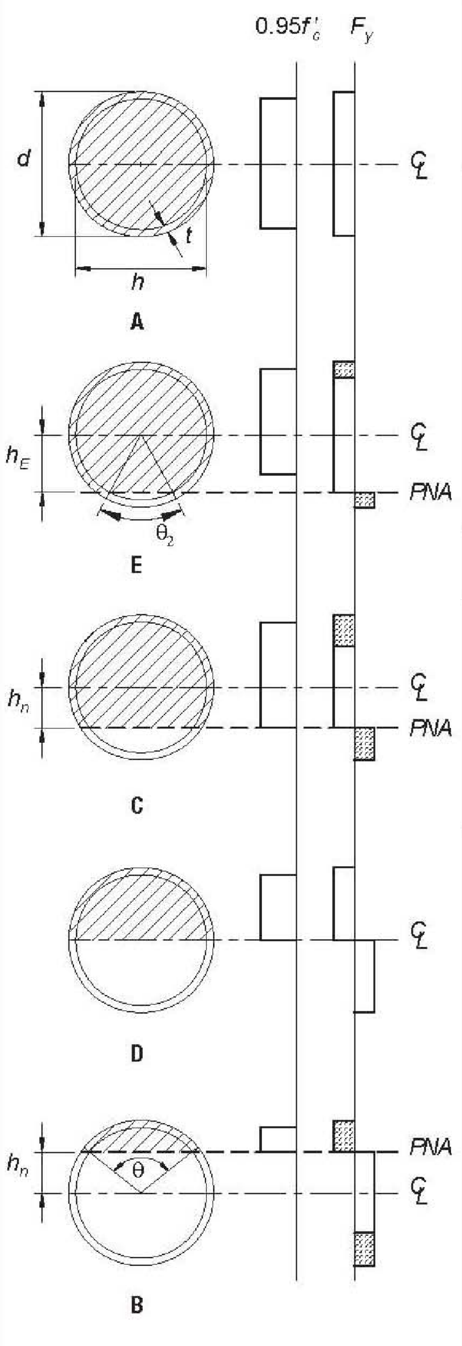
Section	Stress Distribution	Pt.	Defining Equations
		A	$P_A = F_y A_s + 0.95 f'_c A_c$ $M_A = 0$ $A_s = \pi\left(dt - t^2\right)$ $A_c = \dfrac{\pi h^2}{4}$
		E	$P_E = P_A - \tfrac{1}{4}\left[F_y\left(d^2 - h^2\right) + \dfrac{0.95 f'_c}{2} h^2\right]\left(\theta_2 - \sin\theta_2\right)$ $M_E = F_y Z_{sE} + \dfrac{0.95 f'_c Z_{cE}}{2}$ $Z_{cE} = \dfrac{h^3}{6}\sin^3\left(\dfrac{\theta_2}{2}\right)$ $Z_{sE} = \dfrac{\left(d^3 - h^3\right)}{6}\sin\left(\dfrac{\theta_2}{2}\right)$ $h_E = \dfrac{h_n}{2} + \dfrac{h}{4}$ $\theta_2 = \pi - 2\arcsin\left(\dfrac{2h_E}{h}\right)$
		C	$P_C = 0.95 f'_c A_c$ $M_C = M_B$
		D	$P_D = \dfrac{0.95 f'_c A_c}{2}$ $M_D = F_y Z_s + \dfrac{0.95 f'_c Z_c}{2}$ Z_s = plastic section modulus of steel shape $= \dfrac{d^3}{6} - Z_c$ $Z_c = \dfrac{h^3}{6}$
		B	$P_B = 0$ $M_B = F_y Z_{sB} + \dfrac{0.95 f'_c Z_{cB}}{2}$ $Z_{sB} = \dfrac{\left(d^3 - h^3\right)}{6}\sin\left(\dfrac{\theta}{2}\right)$ $Z_{cB} = \dfrac{h^3 \sin^3\left(\dfrac{\theta}{2}\right)}{6}$ $\theta = \dfrac{0.0260K_c - 2K_s}{0.0848K_c} + \dfrac{\sqrt{\left(0.0260K_c + 2K_s\right)^2 + 0.857K_c K_s}}{0.0848K_c}$ (rad) $K_c = f'_c h^2$ $K_s = F_y\left(\dfrac{d-t}{2}\right)t$ ("thin" HSS wall assumed) $h_n = \dfrac{h}{2}\sin\left(\dfrac{\pi - \theta}{2}\right) \le \dfrac{h}{2}$

Fig. I-1d. Filled round HSS anchor points.

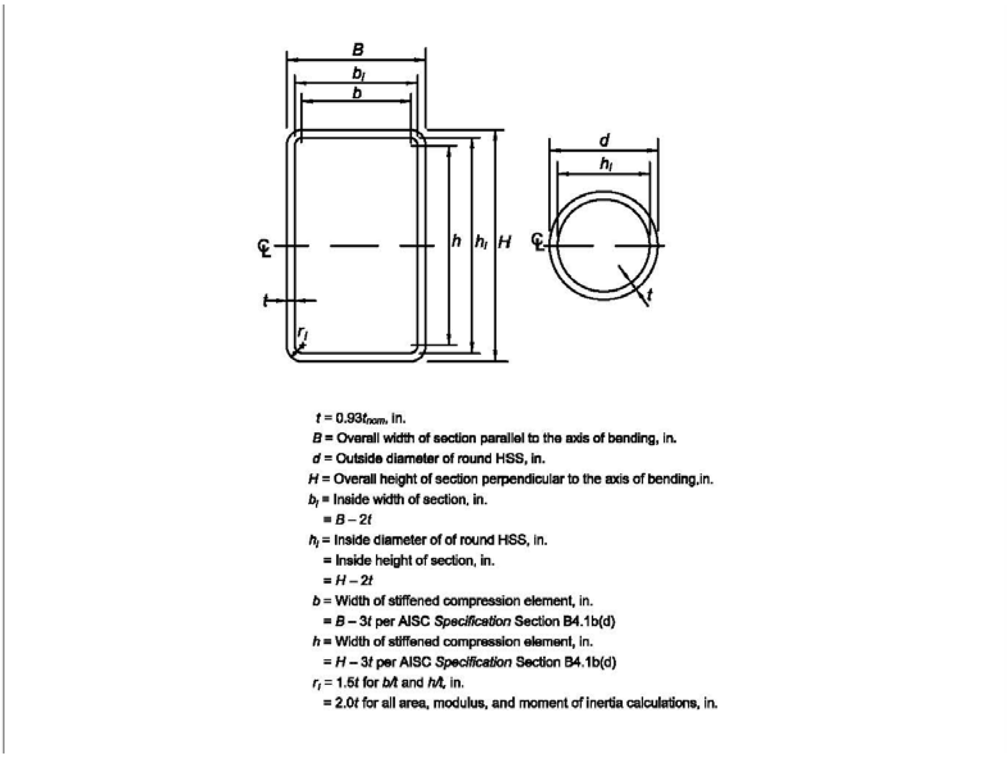


Fig. I-2. Terminology used for filled members.

REFERENCES

Geschwindner, L.F. (2010), "Discussion of Limit State Response of Composite Columns and Beam-Columns Part II: Application of Design Provisions for the 2005 AISC Specification," *Engineering Journal*, AISC, Vol. 47, No. 2, 2nd Quarter, pp. 131–140.

EXAMPLE I.1 COMPOSITE BEAM DESIGN

Given:

A typical bay of a composite floor system is illustrated in Figure I.1-1. Select an appropriate ASTM A992 W-shaped beam and determine the required number of ¾-in.-diameter steel headed stud anchors. The beam will not be shored during construction.

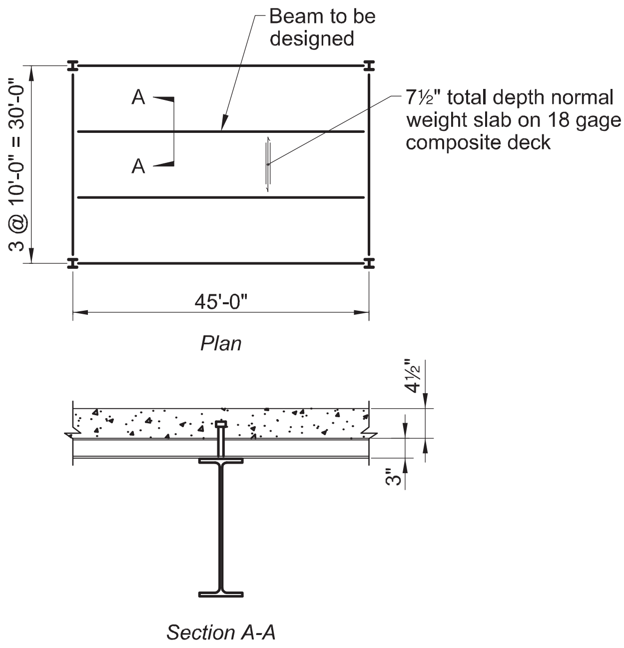


Fig. I.1-1. Composite bay and beam section.

To achieve a two-hour fire rating without the application of spray applied fire protection material to the composite deck, 4½ in. of normal weight (145 lb/ft^3) concrete will be placed above the top of the deck. The concrete has a specified compressive strength, $f_c' = 4$ ksi.

Applied loads are as follows:

Dead Loads:

Pre-composite:

Slab = 75 lb/ft^2 (in accordance with metal deck manufacturer's data)

Self weight = 5 lb/ft^2 (assumed uniform load to account for beam weight)

Composite (applied after composite action has been achieved):

Miscellaneous = 10 lb/ft^2 (HVAC, ceiling, floor covering, etc.)

Live Loads:

Pre-composite:

Construction = 25 lb/ft^2 (temporary loads during concrete placement)

Composite (applied after composite action has been achieved):

Non-reducible = 100 lb/ft^2 (assembly occupancy)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Applied Loads

For slabs that are to be placed at a constant elevation, AISC Design Guide 3 (West and Fisher, 2003) recommends an additional 10% of the nominal slab weight be applied to account for concrete ponding due to deflections resulting from the wet weight of the concrete during placement. For the slab under consideration, this would result in an additional load of 8 lb/ft^2; however, for this design the slab will be placed at a constant thickness thus no additional weight for concrete ponding is required.

For pre-composite construction live loading, 25 lb/ft^2 will be applied in accordance with recommendations from ASCE/SEI 37-02 *Design Loads on Structures During Construction* (ASCE, 2002) for a light duty operational class which includes concrete transport and placement by hose.

Composite Deck and Anchor Requirements

Check composite deck and anchor requirements stipulated in AISC *Specification* Sections I1.3, I3.2c and I8.

(1) Concrete Strength: $3 \text{ ksi} \le f_c' \le 10 \text{ ksi}$

$f_c' = 4 \text{ ksi}$ **o.k.**

(2) Rib height: $h_r \le 3 \text{ in.}$

$h_r = 3 \text{ in.}$ **o.k.**

(3) Average rib width: $w_r \ge 2 \text{ in.}$

$w_r = 6 \text{ in.}$ (from deck manufacturer's literature) **o.k.**

(4) Use steel headed stud anchors ¾ in. or less in diameter.

Use ¾-in.-diameter steel anchors per problem statement **o.k.**

(5) Steel headed stud anchor diameter: $d_{sa} \le 2.5(t_f)$

In accordance with AISC *Specification* Section I8.1, this limit only applies if steel headed stud anchors are not welded to the flange directly over the web. The ¾-in.-diameter anchors will be placed in pairs transverse to the web in some locations, thus this limit must be satisfied. Select a beam size with a minimum flange thickness of 0.30 in., as determined below:

$$t_f \ge \frac{d_{sa}}{2.5}$$

$$\frac{d_{sa}}{2.5} = \frac{\tfrac{3}{4} \text{ in.}}{2.5} = 0.30 \text{ in.}$$

(6) Steel headed stud anchors, after installation, shall extend not less than 1½ in. above the top of the steel deck.

A minimum anchor length of 4½ in. is required to meet this requirement for 3 in. deep deck. From steel headed stud anchor manufacturer's data, a standard stock length of 4⅞ in. is selected. Using a ⅜-in. length reduction to account for burn off during anchor installation through the deck yields a final installed length of 4½ in.

4½ in. = 4½ in. **o.k.**

(7) Minimum length of stud anchors $= 4d_{sa}$

4½ in. > 4(¾ in.) = 3.00 in. **o.k.**

(8) There shall be at least ½ in. of specified concrete cover above the top of the headed stud anchors.

As discussed in AISC *Specification* Commentary to Section I3.2c, it is advisable to provide greater than ½ in. minimum cover to assure anchors are not exposed in the final condition, particularly for intentionally cambered beams.

7½ in. − 4½ in. = 3.00 in. > ½ in. **o.k.**

(9) Slab thickness above steel deck ≥ 2 in.

4½ in. > 2 in. **o.k.**

Design for Pre-Composite Condition

Construction (Pre-Composite) Loads

The beam is uniformly loaded by its tributary width as follows:

$$\begin{aligned} w_D &= \left[(10\text{ ft})\left(75\text{ lb/ft}^2 + 5\text{ lb/ft}^2\right)\right](0.001\text{ kip/lb}) \\ &= 0.800\text{ kip/ft} \end{aligned}$$

$$\begin{aligned} w_L &= \left[(10\text{ ft})\left(25\text{ lb/ft}^2\right)\right](0.001\text{ kip/lb}) \\ &= 0.250\text{ kip/ft} \end{aligned}$$

Construction (Pre-Composite) Flexural Strength

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.800\text{ kip/ft}) + 1.6(0.250\text{ kip/ft})$ $= 1.36\text{ kip/ft}$ $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(1.36\text{ kip/ft})(45\text{ ft})^2}{8}$ $= 344\text{ kip-ft}$	$w_a = 0.800\text{ kip/ft} + 0.250\text{ kip/ft}$ $= 1.05\text{ kip/ft}$ $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(1.05\text{ kip/ft})(45\text{ ft})^2}{8}$ $= 266\text{ kip-ft}$

Beam Selection

Assume that attachment of the deck perpendicular to the beam provides adequate bracing to the compression flange during construction, thus the beam can develop its full plastic moment capacity. The required plastic section modulus, Z_x, is determined as follows, from AISC *Specification* Equation F2-1:

LRFD	ASD
$\phi_b = 0.90$ $Z_{x,min} = \dfrac{M_u}{\phi_b F_y}$ $= \dfrac{(344\text{ kip-ft})(12\text{ in./ft})}{0.90(50\text{ ksi})}$ $= 91.7\text{ in.}^3$	$\Omega_b = 1.67$ $Z_{x,min} = \dfrac{\Omega_b M_a}{F_y}$ $= \dfrac{1.67(266\text{ kip-ft})(12\text{ in./ft})}{50\text{ ksi}}$ $= 107\text{ in.}^3$

From AISC *Manual* Table 3-2 select a W21×50 with a Z_x value of 110 in.^{3}

Note that for the member size chosen, the self weight on a pounds per square foot basis is $50\text{ plf}/10\text{ ft} = 5.00\text{ psf}$; thus the initial self weight assumption is adequate.

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W21×50
$A = 14.7\text{ in.}^2$
$I_x = 984\text{ in.}^4$

Pre-Composite Deflections

AISC Design Guide 3 recommends deflections due to concrete plus self weight not exceed the minimum of L/360 or 1.0 in.

From AISC *Manual* Table 3-23, Case 1:

$$\Delta_{nc} = \frac{5w_D L^4}{384EI}$$

Substituting for the moment of inertia of the non-composite section, $I = 984\text{ in.}^4$, yields a dead load deflection of:

$$\begin{aligned}\Delta_{nc} &= \frac{5\left[\dfrac{(0.800\text{ kip/ft})}{12\text{ in./ft}}\right]\left[(45.0\text{ ft})(12\text{ in./ft})\right]^4}{384(29{,}000\text{ ksi})(984\text{ in.}^4)}\\ &= 2.59\text{ in.}\\ &= L/208 > L/360 \quad \textbf{n.g.}\end{aligned}$$

Pre-composite deflections exceed the recommended limit. One possible solution is to increase the member size. A second solution is to induce camber into the member. For this example, the second solution is selected, and the beam will be cambered to reduce the net pre-composite deflections.

Reducing the estimated simple span deflections to 80% of the calculated value to reflect the partial restraint of the end connections as recommended in AISC Design Guide 3 yields a camber of:

$$\text{Camber} = 0.8(2.59\text{ in.})$$
$$= 2.07\text{ in.}$$

Rounding down to the nearest ¼-in. increment yields a specified camber of 2 in.

Select a W21×50 with 2 in. of camber.

Design for Composite Condition

Required Flexural Strength

Using tributary area calculations, the total uniform loads (including pre-composite dead loads in addition to dead and live loads applied after composite action has been achieved) are determined as:

$$w_D = \left[(10.0\text{ ft})\left(75\text{ lb/ft}^2 + 5\text{ lb/ft}^2 + 10\text{ lb/ft}^2\right)\right](0.001\text{ kip/lb})$$
$$= 0.900\text{ kip/ft}$$
$$w_L = \left[(10.0\text{ ft})\left(100\text{ lb/ft}^2\right)\right](0.001\text{ kip/lb})$$
$$= 1.00\text{ kip/ft}$$

From ASCE/SEI 7 Chapter 2, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.900\text{ kip/ft}) + 1.6(1.00\text{ kip/ft})$ $= 2.68\text{ kip/ft}$ $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(2.68\text{ kip/ft})(45.0\text{ ft})^2}{8}$ $= 678\text{ kip-ft}$	$w_a = 0.900\text{ kip/ft} + 1.00\text{ kip/ft}$ $= 1.90\text{ kip/ft}$ $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(1.90\text{ kip/ft})(45.0\text{ ft})^2}{8}$ $= 481\text{ kip-ft}$

Determine b

The effective width of the concrete slab is the sum of the effective widths to each side of the beam centerline as determined by the minimum value of the three widths set forth in AISC *Specification* Section I3.1a:

(1) one-eighth of the beam span center-to-center of supports

$$\frac{45.0\text{ ft}}{8}(2\text{ sides}) = 11.3\text{ ft}$$

(2) one-half the distance to the centerline of the adjacent beam

$$\frac{10.0\text{ ft}}{2}(2\text{ sides}) = 10.0\text{ ft} \quad \textbf{controls}$$

(3) distance to the edge of the slab

not applicable for an interior member

Available Flexural Strength

According to AISC *Specification* Section I3.2a, the nominal flexural strength shall be determined from the plastic stress distribution on the composite section when $h / t_w \le 3.76\sqrt{E / F_y}$.

From AISC *Manual* Table 1-1, h/t_w for a W21×50 = 49.4.

$$\begin{aligned} 49.4 &\le 3.76\sqrt{29{,}000 \text{ ksi} / 50 \text{ ksi}} \\ &\le 90.6 \end{aligned}$$

Therefore, use the plastic stress distribution to determine the nominal flexural strength.

According to the User Note in AISC *Specification* Section I3.2a, this check is generally unnecessary as all current W-shapes satisfy this limit for $F_y \le 50$ ksi.

Flexural strength can be determined using AISC *Manual* Table 3-19 or calculated directly using the provisions of AISC *Specification* Chapter I. This design example illustrates the use of the *Manual* table only. For an illustration of the direct calculation procedure, refer to Design Example I.2.

To utilize AISC *Manual* Table 3-19, the distance from the compressive concrete flange force to beam top flange, $Y2$, must first be determined as illustrated by *Manual* Figure 3-3. Fifty percent composite action [$\Sigma Q_n \approx 0.50(A_sF_y)$] is used to calculate a trial value of the compression block depth, a_{trial}, for determining $Y2$ as follows:

$$\begin{aligned} a_{trial} &= \frac{\Sigma Q_n}{0.85 f_c' b} \\ &= \frac{0.50\left(A_s F_y\right)}{0.85 f_c' b} \\ &= \frac{0.50\left(14.7 \text{ in.}^2\right)\left(50 \text{ ksi}\right)}{0.85\left(4 \text{ ksi}\right)\left(10.0 \text{ ft}\right)\left(12 \text{ in./ft}\right)} \\ &= 0.90 \text{ in.} \rightarrow \text{say } 1.0 \text{ in.} \end{aligned}$$ (from *Manual*. Eq. 3-7)

Note that a trial value of $a = 1.0$ in. is a common starting point in many design problems.

$$Y2 = Y_{con} - \frac{a_{trial}}{2}$$ (from *Manual*. Eq. 3-6)

where

Y_{con} = distance from top of steel beam to top of slab, in.
= 7.50 in.

$$\begin{aligned} Y2 &= 7.50 \text{ in.} - \frac{1.0 \text{ in.}}{2} \\ &= 7.00 \text{ in.} \end{aligned}$$

Enter AISC *Manual* Table 3-19 with the required strength and $Y2$ = 7.00 in. to select a plastic neutral axis location for the W21×50 that provides sufficient available strength.

Selecting PNA location 5 (BFL) with $\Sigma Q_n = 386$ kips provides a flexural strength of:

LRFD	ASD
$\phi_b M_n \geq M_u$ $\phi_b M_n = 769 \text{ kip-ft} \geq 678 \text{ kip-ft}$ **o.k.**	$M_n / \Omega_b \geq M_a$ $M_n / \Omega_b = 512 \text{ kip-ft} \geq 481 \text{ kip-ft}$ **o.k.**

Based on the available flexural strength provided in Table 3-19, the required PNA location for ASD and LRFD design methodologies differ. This discrepancy is due to the live to dead load ratio in this example, which is not equal to the ratio of 3 at which ASD and LRFD design methodologies produce equivalent results as discussed in AISC *Specification* Commentary Section B3.4. The selected PNA location 5 is acceptable for ASD design, and more conservative for LRFD design.

The actual value for the compression block depth, a, is determined as follows:

$$a = \frac{\Sigma Q_n}{0.85 f_c' b} \qquad \text{(\textit{Manual} Eq. 3-7)}$$

$$= \frac{386 \text{ kips}}{0.85(4 \text{ ksi})(10 \text{ ft})(12 \text{ in./ft})}$$

$$= 0.946 \text{ in.}$$

$$a = 0.946 \text{ in.} < a_{trial} = 1.0 \text{ in.} \quad \textbf{o.k.}$$

Live Load Deflection

Deflections due to live load applied after composite action has been achieved will be limited to $L/360$ under the design live load as required by Table 1604.3 of the 2009 *International Building Code* (IBC) (ICC, 2009), or 1 in. using a 50% reduction in design live load as recommended by AISC Design Guide 3.

Deflections for composite members may be determined using the lower bound moment of inertia provided by *Specification* Commentary Equation C-I3-1 and tabulated in AISC *Manual* Table 3-20. The *Specification* Commentary also provides an alternate method for determining deflections of a composite member through the calculation of an effective moment of inertia. This design example illustrates the use of the *Manual* table. For an illustration of the direct calculation procedure for each method, refer to Design Example I.2.

Entering Table 3-20, for a W21×50 with PNA location 5 and $Y2$ = 7.00 in., provides a lower bound moment of inertia of $I_{LB} = 2{,}520 \text{ in.}^4$

Inserting I_{LB} into AISC *Manual* Table 3-23, Case 1, to determine the live load deflection under the full design live load for comparison to the IBC limit yields:

$$\Delta_c = \frac{5 w_L L^4}{384 E I_{LB}}$$

$$= \frac{5\left[\dfrac{(1.00 \text{ kip/ft})}{12 \text{ in./ft}}\right]\left[(45.0 \text{ ft})(12 \text{ in./ft})\right]^4}{384(29{,}000 \text{ ksi})(2{,}520 \text{ in.}^4)}$$

$$= 1.26 \text{ in.}$$

$$= L/429 < L/360 \quad \textbf{o.k.}$$

Performing the same check with 50% of the design live load for comparison to the AISC Design Guide 3 limit yields:

$$\Delta_c = 0.50(1.26 \text{ in.})$$
$$= 0.630 \text{ in.} < 1.0 \text{ in.} \quad \textbf{o.k.}$$

Steel Anchor Strength

Steel headed stud anchor strengths are tabulated in AISC *Manual* Table 3-21 for typical conditions. Conservatively assuming that all anchors are placed in the weak position, the strength for ¾-in.-diameter anchors in normal weight concrete with $f_c' = 4$ ksi and deck oriented perpendicular to the beam is:

1 anchor per rib: $Q_n = 17.2$ kips/anchor
2 anchors per rib: $Q_n = 14.6$ kips/anchor

Number and Spacing of Anchors

Deck flutes are spaced at 12 in. on center according to the deck manufacturer's literature. The minimum number of deck flutes along each half of the 45-ft-long beam, assuming the first flute begins a maximum of 12 in. from the support line at each end, is:

$$n_{flutes} = n_{spaces} + 1$$
$$= \frac{45.0 \text{ ft} - 2(12 \text{ in.})(1 \text{ ft/12 in.})}{2(1 \text{ ft per space})} + 1$$
$$= 22.5 \rightarrow \text{say 22 flutes}$$

According to AISC *Specification* Section I8.2c, the number of steel headed stud anchors required between the section of maximum bending moment and the nearest point of zero moment is determined by dividing the required horizontal shear, $\sum Q_n$, by the nominal shear strength per anchor, Q_n. Assuming one anchor per flute:

$$n_{anchors} = \frac{\sum Q_n}{Q_n}$$
$$= \frac{386 \text{ kips}}{17.2 \text{ kips/anchor}}$$
$$= 22.4 \rightarrow \text{place 23 anchors on each side of the beam centerline}$$

As the number of anchors exceeds the number of available flutes by one, place two anchors in the first flute. The revised horizontal shear capacity of the anchors taking into account the reduced strength for two anchors in one flute is:

$$\sum Q_n = 2(14.6 \text{ kips}) + 21(17.2 \text{ kips})$$
$$= 390 \text{ kips} \geq 386 \text{ kips} \quad \textbf{o.k.}$$

The final anchor pattern chosen is illustrated in Figure I.1-2.

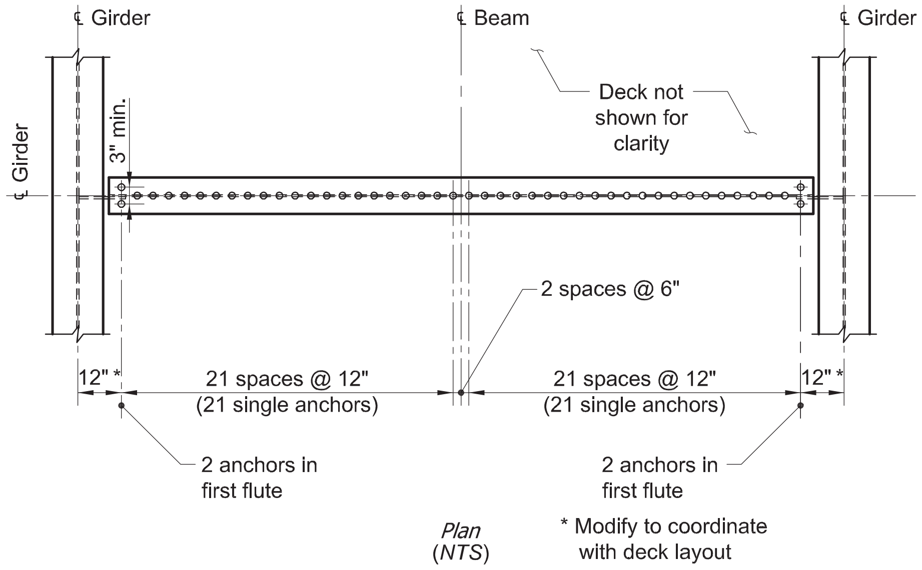


Fig. I.1-2. Steel headed stud anchor layout.

Review steel headed stud anchor spacing requirements of AISC Specification Sections I8.2d and I3.2c.

(1) Maximum anchor spacing along beam: $8t_{slab} = 8(7.5\text{ in.}) = 60.0\text{ in. or } 36\text{ in.}$

$12\text{ in.} < 36\text{ in.}$ **o.k.**

(2) Minimum anchor spacing along beam: $6d_{sa} = 6(3/4\text{ in.}) = 4.50\text{ in.}$

$12\text{ in.} > 4.50\text{ in.}$ **o.k.**

(3) Minimum transverse spacing between anchor pairs: $4d_{sa} = 4(3/4\text{ in.}) = 3.00\text{ in.}$

$3.00\text{ in.} = 3.00\text{ in.}$ **o.k.**

(4) Minimum distance to free edge in the direction of the horizontal shear force:

AISC *Specification* Section I8.2d requires that the distance from the center of an anchor to a free edge in the direction of the shear force be a minimum of 8 in. for normal weight concrete slabs.

(5) Maximum spacing of deck attachment:

AISC *Specification* Section I3.2c(4) requires that steel deck be anchored to all supporting members at a maximum spacing of 18 in. The stud anchors are welded through the metal deck at a maximum spacing of 12 inches in this example, thus this limit is met without the need for additional puddle welds or mechanical fasteners.

Available Shear Strength

According to AISC *Specification* Section I4.2, the beam should be assessed for available shear strength as a bare steel beam using the provisions of Chapter G.

Applying the loads previously determined for the governing ASCE/SEI 7-10 load combinations and using available shear strengths from AISC *Manual* Table 3-2 for a W21×50 yields the following:

LRFD	ASD
$V_u = \dfrac{w_u L}{2}$ $= \dfrac{(2.68\text{ kips/ft})(45.0\text{ ft})}{2}$ $= 60.3\text{ kips}$ $\phi_v V_n > V_u$ $\phi_v V_n = 237\text{ kips} \geq 60.3\text{ kips}$ **o.k.**	$V_a = \dfrac{w_a L}{2}$ $= \dfrac{(1.90\text{ kips/ft})(45.0\text{ ft})}{2}$ $= 42.8\text{ kips}$ $V_n / \Omega_v \geq V_a$ $V_n / \Omega_v = 158\text{ kips} > 42.8\text{ kips}$ **o.k.**

Serviceability

Depending on the intended use of this bay, vibrations might need to be considered. See AISC Design Guide 11 (Murray et al., 1997) for additional information.

Summary

From Figure I.1-2, the total number of stud anchors used is equal to $(2)(2+21) = 46$. A plan layout illustrating the final beam design is provided in Figure I.1-3:

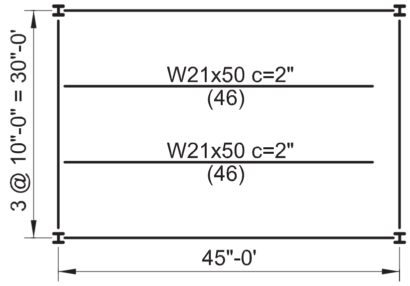


Fig. I.1-3. Revised plan.

A W21×50 with 2 in. of camber and 46, ¾-in.-diameter by 4⅞-in.-long steel headed stud anchors is adequate to resist the imposed loads.

EXAMPLE I.2 COMPOSITE GIRDER DESIGN

Given:

Two typical bays of a composite floor system are illustrated in Figure I.2-1. Select an appropriate ASTM A992 W-shaped girder and determine the required number of steel headed stud anchors. The girder will not be shored during construction.

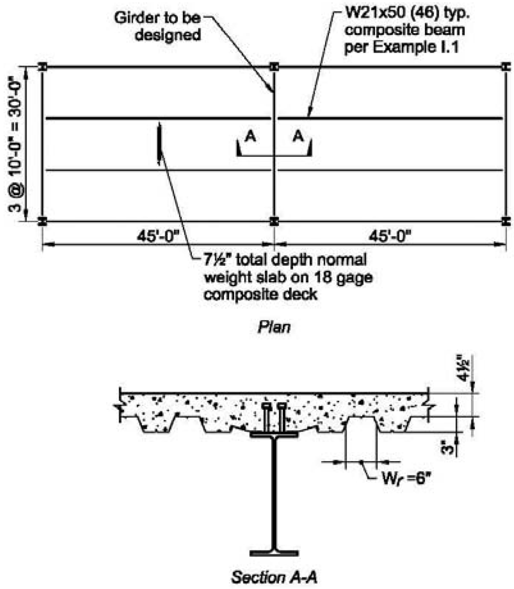


Fig. I.2-1. Composite bay and girder section.

To achieve a two-hour fire rating without the application of spray applied fire protection material to the composite deck, 4½ in. of normal weight (145 lb/ft^3) concrete will be placed above the top of the deck. The concrete has a specified compressive strength, f_c' = 4 ksi.

Applied loads are as follows:

Dead Loads:
- Pre-composite:
  - Slab = 75 lb/ft^2 (in accordance with metal deck manufacturer's data)
  - Self weight = 80 lb/ft (trial girder weight)
  - = 50 lb/ft (beam weight from Design Example I.1)
- Composite (applied after composite action has been achieved):
  - Miscellaneous = 10 lb/ft^2 (HVAC, ceiling, floor covering, etc.)

Live Loads:
- Pre-composite:
  - Construction = 25 lb/ft^2 (temporary loads during concrete placement)
- Composite (applied after composite action has been achieved):
  - Non-reducible = 100 lb/ft^2 (assembly occupancy)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Applied Loads

For slabs that are to be placed at a constant elevation, AISC Design Guide 3 (West and Fisher, 2003) recommends an additional 10% of the nominal slab weight be applied to account for concrete ponding due to deflections resulting from the wet weight of the concrete during placement. For the slab under consideration, this would result in an additional load of 8 lb/ft^2; however, for this design the slab will be placed at a constant thickness thus no additional weight for concrete ponding is required.

For pre-composite construction live loading, 25 lb/ft^2 will be applied in accordance with recommendations from ASCE/SEI 37-02 *Design Loads on Structures During Construction* (ASCE, 2002) for a light duty operational class which includes concrete transport and placement by hose.

Composite Deck and Anchor Requirements

Check composite deck and anchor requirements stipulated in AISC *Specification* Sections I1.3, I3.2c and I8.

(1) Concrete Strength: $3 \text{ ksi} \le f'_c \le 10 \text{ ksi}$

$f'_c = 4 \text{ ksi}$ **o.k.**

(2) Rib height: $h_r \le 3 \text{ in.}$

$h_r = 3 \text{ in.}$ **o.k.**

(3) Average rib width: $w_r \ge 2 \text{ in.}$

$w_r = 6 \text{ in.}$ (See Figure I.2-1) **o.k.**

(4) Use steel headed stud anchors ¾ in. or less in diameter.

Select ¾-in.-diameter steel anchors **o.k.**

(5) Steel headed stud anchor diameter: $d_{sa} \le 2.5(t_f)$

In accordance with AISC *Specification* Section I8.1, this limit only applies if steel headed stud anchors are not welded to the flange directly over the web. The ¾-in.-diameter anchors will be attached in a staggered pattern, thus this limit must be satisfied. Select a girder size with a minimum flange thickness of 0.30 in., as determined below:

$$t_f \ge \frac{d_{sa}}{2.5}$$

$$\frac{d_{sa}}{2.5} = \frac{\text{¾ in.}}{2.5}$$

$$= 0.30 \text{ in.}$$

(6) Steel headed stud anchors, after installation, shall extend not less than 1½ in. above the top of the steel deck.

A minimum anchor length of 4½ in. is required to meet this requirement for 3 in. deep deck. From steel headed stud anchor manufacturer's data, a standard stock length of 4⅞ in. is selected. Using a 3⁄16-in. length reduction to account for burn off during anchor installation directly to the girder flange yields a final installed length of 4 11⁄16 in.

4 11⁄16 in. > 4½ in. **o.k.**

(7) Minimum length of stud anchors = $4d_{sa}$

4 11⁄16 in. > 4(¾ in.) = 3.00 in. **o.k.**

(8) There shall be at least ½ in. of specified concrete cover above the top of the headed stud anchors.

As discussed in the *Specification* Commentary to Section I3.2c, it is advisable to provide greater than ½ in. minimum cover to assure anchors are not exposed in the final condition.

7½ in. − 4 11⁄16 in. = 2 13⁄16 in. > ½ in. **o.k.**

(9) Slab thickness above steel deck ≥ 2 in.

4½ in. > 2 in. **o.k.**

Design for Pre-Composite Condition

Construction (Pre-Composite) Loads

The girder will be loaded at third points by the supported beams. Determine point loads using tributary areas.

$$P_D = \left[(45.0\text{ ft})(10.0\text{ ft})\left(75\text{ lb/ft}^2\right) + (45.0\text{ ft})(50\text{ lb/ft})\right](0.001\text{ kip/lb})$$
$$= 36.0\text{ kips}$$
$$P_L = \left[(45.0\text{ ft})(10.0\text{ ft})\left(25\text{ lb/ft}^2\right)\right](0.001\text{ kip/lb})$$
$$= 11.3\text{ kips}$$

Construction (Pre-Composite) Flexural Strength

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$P_u = 1.2(36.0\text{ kips}) + 1.6(11.3\text{ kips})$ $= 61.3\text{ kips}$ $w_u = 1.2(80\text{ lb/ft})(0.001\text{ kip/lb})$ $= 0.0960\text{ kip/ft}$ $M_u = P_u a + \dfrac{w_u L^2}{8}$ $= (61.3\text{ kips})(10.0\text{ ft}) + \dfrac{(0.0960\text{ kip/ft})(30.0\text{ ft})^2}{8}$ $= 624\text{ kip-ft}$	$P_a = 36.0\text{ kips} + 11.3\text{ kips}$ $= 47.3\text{ kips}$ $w_a = (80\text{ lb/ft})(0.001\text{ kip/lb})$ $= 0.0800\text{ kip/ft}$ $M_a = P_a a + \dfrac{w_a L^2}{8}$ $= (47.3\text{ kips})(10.0\text{ ft}) + \dfrac{(0.0800\text{ kip/ft})(30.0\text{ ft})^2}{8}$ $= 482\text{ kip-ft}$

Girder Selection

Based on the required flexural strength under construction loading, a trial member can be selected utilizing AISC *Manual* Table 3-2. For the purposes of this example, the unbraced length of the girder prior to hardening of the concrete is taken as the distance between supported beams (one third of the girder length).

Try a W24×76

$L_b = 10.0$ ft
$L_p = 6.78$ ft
$L_r = 19.5$ ft

LRFD	ASD
$\phi_b BF = 22.6$ kips $\phi_b M_{px} = 750$ kip-ft $\phi_b M_{rx} = 462$ kip-ft	$BF / \Omega_b = 15.1$ kips $M_{px} / \Omega_b = 499$ kip-ft $M_{rx} / \Omega_b = 307$ kip-ft

Because L_p<L_b<L_r, use AISC *Manual* Equations 3-4a and 3-4b with $C_b = 1.0$ within the center girder segment in accordance with *Manual* Table 3-1:

LRFD	ASD
$\phi_b M_n = C_b \left[\phi_b M_{px} - \phi_b BF(L_b - L_p) \right] \le \phi_b M_{px}$ $= 1.0[750 \text{ kip-ft} - 22.6 \text{ kips}(10.0 \text{ ft} - 6.78 \text{ ft})]$ $= 677 \text{ kip-ft} \le 750 \text{ kip-ft}$ $\phi_b M_n \ge M_u$ $677 \text{ kip-ft} > 624 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = C_b \left[\dfrac{M_{px}}{\Omega_b} - \dfrac{BF}{\Omega_b}(L_b - L_p) \right] \le \dfrac{M_{px}}{\Omega_b}$ $= 1.0[499 \text{ kip-ft} - 15.1 \text{ kips}(10.0 \text{ ft} - 6.78 \text{ ft})]$ $= 450 \text{ kip-ft} \le 499 \text{ kip-ft}$ $\dfrac{M_n}{\Omega_b} \ge M_a$ $450 \text{ kip-ft} < 482 \text{ kip-ft}$ **n.g.**

For this example, the relatively low live load to dead load ratio results in a lighter member when LRFD methodology is employed. When ASD methodology is employed, a heavier member is required, and it can be shown that a W24×84 is adequate for pre-composite flexural strength. This example uses a W24×76 member to illustrate the determination of flexural strength of the composite section using both LRFD and ASD methodologies; however, this is done for comparison purposes only, and calculations for a W24×84 would be required to provide a satisfactory ASD design. Calculations for the heavier section are not shown as they would essentially be a duplication of the calculations provided for the W24×76 member.

Note that for the member size chosen, $76 \text{ lb/ft} \le 80 \text{ lb/ft}$, thus the initial weight assumption is adequate.

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W24×76
$A = 22.4 \text{ in.}^2$
$I_x = 2{,}100 \text{ in.}^4$
$b_f = 8.99$ in.
$t_f = 0.680$ in.
$d = 23.9$ in.

Pre-Composite Deflections

AISC Design Guide 3 recommends deflections due to concrete plus self weight not exceed the minimum of L/360 or 1.0 in.

From the superposition of AISC *Manual* Table 3-23, Cases 1 and 9:

$$\Delta_{nc} = \frac{P_D L^3}{28EI} + \frac{5w_D L^4}{384EI}$$

Substituting for the moment of inertia of the non-composite section, $I = 2{,}100\text{ in.}^4$, yields a dead load deflection of:

$$\begin{aligned}\Delta_{nc} &= \frac{36.0\text{ kips}\left[(30.0\text{ ft})(12\text{ in./ft})\right]^3}{28(29{,}000\text{ ksi})(2{,}100\text{ in.}^4)} + \frac{5\left[\dfrac{(0.0760\text{ kip/ft})}{12\text{ in./ft}}\right]\left[(30.0\text{ ft})(12\text{ in./ft})\right]^4}{384(29{,}000\text{ ksi})(2{,}100\text{ in.}^4)}\\ &= 1.01\text{ in.}\\ &= L/356 > L/360 \quad \textbf{n.g.}\end{aligned}$$

Pre-composite deflections slightly exceed the recommended value. One possible solution is to increase the member size. A second solution is to induce camber into the member. For this example, the second solution is selected, and the girder will be cambered to reduce pre-composite deflections.

Reducing the estimated simple span deflections to 80% of the calculated value to reflect the partial restraint of the end connections as recommended in AISC Design Guide 3 yields a camber of:

$$\begin{aligned}\text{Camber} &= 0.8(1.01\text{ in.})\\ &= 0.808\text{ in.}\end{aligned}$$

Rounding down to the nearest ¼-in. increment yields a specified camber of ¾ in.

Select a W24×76 with ¾ in. of camber.

Design for Composite Flexural Strength

Required Flexural Strength

Using tributary area calculations, the total applied point loads (including pre-composite dead loads in addition to dead and live loads applied after composite action has been achieved) are determined as:

$$\begin{aligned}P_D &= \left[(45.0\text{ ft})(10.0\text{ ft})(75\text{ lb/ft}^2 + 10\text{ lb/ft}^2) + (45.0\text{ ft})(50\text{ lb/ft})\right](0.001\text{ kip/lb})\\ &= 40.5\text{ kips}\\ P_L &= \left[(45.0\text{ ft})(10.0\text{ ft})(100\text{ lb/ft}^2)\right](0.001\text{ kip/lb})\\ &= 45.0\text{ kips}\end{aligned}$$

The required flexural strength diagram is illustrated by Figure I.2-2:

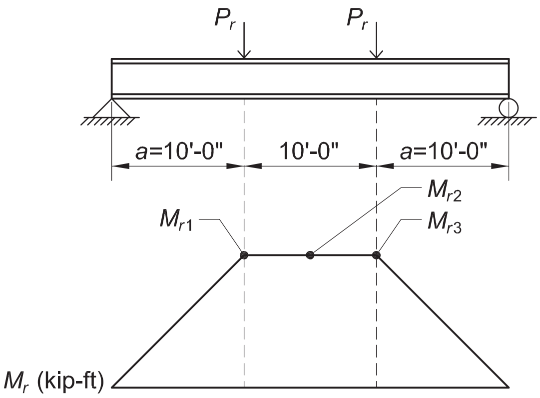


Fig. I.2-2. Required flexural strength.

From ASCE/SEI 7-10 Chapter 2, the required flexural strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(40.5 \text{ kips}) + 1.6(45.0 \text{ kips})$ $= 121 \text{ kips}$ $w_u = 1.2(0.0760 \text{ kip/ft})$ $= 0.0912 \text{ kip/ft}$ from self weight of W24×76 From AISC *Manual* Table 3-23, Case 1 and 9. $M_{u1} = M_{u3}$ $= P_u a + \frac{w_u a}{2}(L-a)$ $= (121 \text{ kips})(10.0 \text{ ft}) + \frac{(0.0912 \text{ kip/ft})(10.0 \text{ ft})}{2}(30.0 \text{ ft} - 10.0 \text{ ft})$ $= 1{,}220 \text{ kip-ft}$ $M_{u2} = P_u a + \frac{w_u L^2}{8}$ $= (121 \text{ kips})(10.0 \text{ ft}) + \frac{(0.0912 \text{ kip/ft})(30.0 \text{ ft})^2}{8}$ $= 1{,}220 \text{ kip-ft}$	$P_r = P_a$ $= 40.5 \text{ kips} + 45.0 \text{ kips}$ $= 85.5 \text{ kips}$ $w_a = 0.0760 \text{ kip/ft}$ from self weight of W24×76 From AISC *Manual* Table 3-23, Case 1 and 9. $M_{a1} = M_{a3}$ $= P_a a + \frac{w_a a}{2}(L-a)$ $= (85.5 \text{ kips})(10.0 \text{ ft}) + \frac{(0.0760 \text{ kip/ft})(10.0 \text{ ft})}{2}(30.0 \text{ ft} - 10.0 \text{ ft})$ $= 863 \text{ kip-ft}$ $M_{a2} = P_a a + \frac{w_a L^2}{8}$ $= (85.5 \text{ kips})(10.0 \text{ ft}) + \frac{(0.0760 \text{ kip/ft})(30.0 \text{ ft})^2}{8}$ $= 864 \text{ kip-ft}$

Determine b

The effective width of the concrete slab is the sum of the effective widths to each side of the beam centerline as determined by the minimum value of the three conditions set forth in AISC *Specification* Section I3.1a:

(1) one-eighth of the girder span center-to-center of supports

$$\frac{30.0 \text{ ft}}{8}(2 \text{ sides}) = 7.50 \text{ ft} \quad \textbf{controls}$$

(2) one-half the distance to the centerline of the adjacent girder

$$\frac{45\text{ ft}}{2}(2\text{ sides}) = 45.0\text{ ft}$$

(3) distance to the edge of the slab

not applicable for an interior member

Available Flexural Strength

According to AISC *Specification* Section I3.2a, the nominal flexural strength shall be determined from the plastic stress distribution on the composite section when $h/t_w \le 3.76\sqrt{E/F_y}$.

From AISC *Manual* Table 1-1, h/t_w for a W24×76 = 49.0.

$$\begin{aligned} 49.0 &\le 3.76\sqrt{29{,}000\text{ ksi}/50\text{ ksi}} \\ &\le 90.6 \end{aligned}$$

Therefore, use the plastic stress distribution to determine the nominal flexural strength.

According to the User Note in AISC *Specification* Section I3.2a, this check is generally unnecessary as all current W-shapes satisfy this limit for $F_y \le 50$ ksi.

AISC *Manual* Table 3-19 can be used to facilitate the calculation of flexural strength for composite beams. Alternately, the available flexural strength can be determined directly using the provisions of AISC *Specification* Chapter I. Both methods will be illustrated for comparison in the following calculations.

Method 1: AISC *Manual*

To utilize AISC *Manual* Table 3-19, the distance from the compressive concrete flange force to beam top flange, $Y2$, must first be determined as illustrated by *Manual* Figure 3-3. Fifty percent composite action [$\Sigma Q_n \approx 0.50(A_sF_y)$] is used to calculate a trial value of the compression block depth, a_{trial}, for determining $Y2$ as follows:

$$\begin{aligned} a_{trial} &= \frac{\Sigma Q_n}{0.85 f'_c b} \\ &= \frac{0.50(A_s F_y)}{0.85 f'_c b} \\ &= \frac{0.50(22.4\text{ in.}^2)(50\text{ ksi})}{0.85(4\text{ ksi})(7.50\text{ ft})(12\text{ in./ft})} \\ &= 1.83\text{ in.} \end{aligned}$$

(from *Manual*. Eq. 3-7)

$$Y2 = Y_{con} - \frac{a_{trial}}{2}$$

(from *Manual*. Eq. 3-6)

where

Y_{con} = distance from top of steel beam to top of slab
= 7.50 in.

$$Y2 = 7.50 \text{ in.} - \frac{1.83 \text{ in.}}{2}$$
$$= 6.59 \text{ in.}$$

Enter AISC *Manual* Table 3-19 with the required strength and $Y2 = 6.59$ in. to select a plastic neutral axis location for the W24×76 that provides sufficient available strength. Based on the available flexural strength provided in Table 3-19, the required PNA location for ASD and LRFD design methodologies differ. This discrepancy is due to the live to dead load ratio in this example, which is not equal to the ratio of 3 at which ASD and LRFD design methodologies produce equivalent results as discussed in AISC *Specification* Commentary Section B3.4.

Selecting PNA location 5 (BFL) with $\sum Q_n = 509$ kips provides a flexural strength of:

LRFD	ASD
$\phi_b M_n \geq M_u$ $\phi_b M_n = 1{,}240 \text{ kip-ft} > 1{,}220 \text{ kip-ft}$ **o.k.**	$M_n / \Omega_b \geq M_a$ $M_n / \Omega_b = 823 \text{ kip-ft} < 864 \text{ kip-ft}$ **n.g.**

The selected PNA location 5 is acceptable for LRFD design, but inadequate for ASD design. For ASD design, it can be shown that a W24×76 is adequate if a higher composite percentage of approximately 60% is employed. However, as discussed previously, this beam size is not adequate for construction loading and a larger section is necessary when designing utilizing ASD.

The actual value for the compression block depth, *a*, for the chosen PNA location is determined as follows:

$$a = \frac{\sum Q_n}{0.85 f_c' b} \qquad (\textit{Manual} \text{ Eq. 3-7})$$
$$= \frac{509 \text{ kips}}{0.85(4 \text{ ksi})(7.50 \text{ ft})(12 \text{ in./ft})}$$
$$= 1.66 \text{ in.}$$
$$a = 1.66 \text{ in.} < a_{trial} = 1.83 \text{ in.} \quad \textbf{o.k. for LRFD design}$$

Method 2: Direct Calculation

According to AISC *Specification* Commentary Section I3.2a, the number and strength of steel headed stud anchors will govern the compressive force, *C*, for a partially composite beam. The composite percentage is based on the minimum of the limit states of concrete crushing and steel yielding as follows:

(1) Concrete crushing

A_c = Area of concrete slab within effective width. Assume that the deck profile is 50% void and 50% concrete fill.

$$= b_{eff}(4\tfrac{1}{2} \text{ in.}) + (b_{eff}/2)(3.00 \text{ in.})$$
$$= (7.50 \text{ ft})(12 \text{ in./ft})(4\tfrac{1}{2} \text{ in.}) + \left[\frac{(7.50 \text{ ft})(12 \text{ in./ft})}{2}\right](3.00 \text{ in.})$$
$$= 540 \text{ in.}^2$$

$$C = 0.85 f_c' A_c \qquad (\textit{Comm}. \text{ Eq. C-I3-7})$$
$$= 0.85(4 \text{ ksi})(540 \text{ in.}^2)$$
$$= 1{,}840 \text{ kips}$$

(2) Steel yielding

$$C = A_s F_y \qquad \text{(from } \textit{Comm.} \text{ Eq. C-I3-6)}$$
$$= (22.4 \text{ in.}^2)(50 \text{ ksi})$$
$$= 1{,}120 \text{ kips}$$

(3) Shear transfer

Fifty percent is used as a trial percentage of composite action as follows:

$$C = \Sigma Q_n \qquad (\textit{Comm.} \text{ Eq. C-I3-8})$$
$$= 50\%\left(\text{Min}\begin{Bmatrix}1{,}840 \text{ kips}\\ 1{,}120 \text{ kips}\end{Bmatrix}\right)$$
$$= 560 \text{ kips to achieve 50\% composite action}$$

Location of the Plastic Neutral Axis

The plastic neutral axis (PNA) is located by determining the axis above and below which the sum of horizontal forces is equal. This concept is illustrated in Figure I.2-3, assuming the trial PNA location is within the top flange of the girder.

$$\Sigma F_{above\ PNA} = \Sigma F_{below\ PNA}$$
$$C + xb_f F_y = (A_s - b_f x) F_y$$

Solving for x:

$$x = \frac{A_s F_y - C}{2 b_f F_y}$$
$$= \frac{(22.4 \text{ in.}^2)(50 \text{ ksi}) - 560 \text{ kips}}{2(8.99 \text{ in.})(50 \text{ ksi})}$$
$$= 0.623 \text{ in.}$$
$$x = 0.623 \text{ in.} \le t_f = 0.680 \text{ in.} \quad \text{PNA in flange}$$

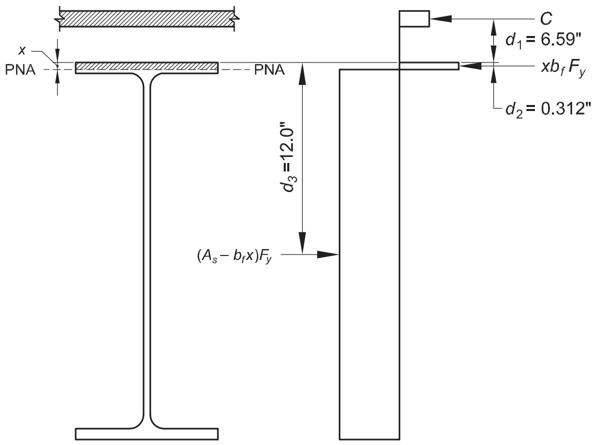


Fig. I.2-3. Plastic neutral axis location.

Determine the nominal moment resistance of the composite section following the procedure in *Specification* Commentary Section I3.2a as illustrated in Figure C-I3.3.

$$M_n = C\left(d_1 + d_2\right) + P_y\left(d_3 - d_2\right) \quad (Comm.\ \text{Eq. C-I3-10})$$

$$\begin{aligned} a &= \frac{C}{0.85 f'_c b} \quad (Comm.\ \text{Eq. C-I3-9}) \\ &= \frac{560\text{ kips}}{0.85\left(4\text{ ksi}\right)\left(7.50\text{ ft}\right)\left(12\text{ in./ft}\right)} \\ &= 1.83\text{ in.} < 4.5\text{ in.} \quad \textbf{Above top of deck} \end{aligned}$$

$$\begin{aligned} d_1 &= t_{slab} - a/2 \\ &= 7.50\text{ in.} - 1.83\text{ in./2} \\ &= 6.59\text{ in.} \end{aligned}$$

$$\begin{aligned} d_2 &= x/2 \\ &= 0.623\text{ in./2} \\ &= 0.312\text{ in.} \end{aligned}$$

$$\begin{aligned} d_3 &= d/2 \\ &= 23.9\text{ in./2} \\ &= 12.0\text{ in.} \end{aligned}$$

$$\begin{aligned} P_y &= A_s F_y \\ &= 22.4\text{ in.}^2\left(50\text{ ksi}\right) \\ &= 1{,}120\text{ kips} \end{aligned}$$

$$\begin{aligned} M_n &= \left[\left(560\text{ kips}\right)\left(6.59\text{ in.} + 0.312\text{ in.}\right) + \left(1{,}120\text{ kips}\right)\left(12.0\text{ in.} - 0.312\text{ in.}\right)\right] \big/ 12\text{ in./ft} \\ &= \frac{17{,}000\text{ kip-in.}}{12\text{ in./ft}} \\ &= 1{,}420\text{ kip-ft} \end{aligned}$$

Note that Equation C-I3-10 is based on summation of moments about the centroid of the compression force in the steel; however, the same answer may be obtained by summing moments about any arbitrary point.

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n \ge M_u$ $\phi_b M_n = 0.90\left(1{,}420\text{ kip-ft}\right)$ $= 1{,}280\text{ kip-ft} > 1{,}220\text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $M_n / \Omega_b \ge M_a$ $M_n / \Omega_b = \dfrac{1{,}420\text{ kip-ft}}{1.67}$ $= 850\text{ kip-ft} < 864\text{ kip-ft}$ **n.g.**

As was determined previously using the Manual Tables, a W24×76 with 50% composite action is acceptable when LRFD methodology is employed, while for ASD design the beam is inadequate at this level of composite action.

Continue with the design using a W24×76 with 50% composite action.

Steel Anchor Strength

Steel headed stud anchor strengths are tabulated in AISC *Manual* Table 3-21 for typical conditions and may be calculated according to AISC *Specification* Section I8.2a as follows:

$$Q_n = 0.5A_{sa}\sqrt{f_c'E_c} \le R_g R_p A_{sa} F_u$$ (*Spec*. Eq. I8-1)

$$A_{sa} = \pi d_{sa}^2 / 4$$
$$= \pi\left(\text{¾ in.}\right)^2 / 4$$
$$= 0.442 \text{ in.}^2$$
$$f_c' = 4 \text{ ksi}$$
$$E_c = w_c^{1.5}\sqrt{f_c'}$$
$$= \left(145 \text{ lb/ft}^3\right)^{1.5}\sqrt{4 \text{ ksi}}$$
$$= 3{,}490 \text{ ksi}$$

$R_g = 1.0$ Stud anchors welded directly to the steel shape within the slab haunch

$R_p = 0.75$ Stud anchors welded directly to the steel shape

$F_u = 65$ ksi From AISC *Manual* Table 2-6 for ASTM A108 steel anchors

$$Q_n = (0.5)\left(0.442 \text{ in.}^2\right)\sqrt{(4 \text{ ksi})(3{,}490 \text{ ksi})} \le (1.0)(0.75)\left(0.442 \text{ in.}^2\right)(65 \text{ ksi})$$
$$= 26.1 \text{ kips} > 21.5 \text{ kips}$$

use $Q_n = 21.5$ kips

Number and Spacing of Anchors

According to AISC *Specification* Section I8.2c, the number of steel headed stud anchors required between any concentrated load and the nearest point of zero moment shall be sufficient to develop the maximum moment required at the concentrated load point.

From Figure I.2-2 the moment at the concentrated load points, M_{r1} and M_{r3}, is approximately equal to the maximum beam moment, M_{r2} . The number of anchors between the beam ends and the point loads should therefore be adequate to develop the required compressive force associated with the maximum moment, C, previously determined to be 560 kips.

$$N_{anchors} = \frac{\sum Q_n}{Q_n}$$
$$= \frac{C}{Q_n}$$
$$= \frac{560 \text{ kips}}{21.5 \text{ kips/anchor}}$$
$$= 26 \text{ anchors from each end to concentrated load points}$$

In accordance with AISC *Specification* Section I8.2d, anchors between point loads should be spaced at a maximum of:

$8t_{slab} = 60.0$ in.

or 36 in. **controls**

For beams with deck running parallel to the span such as the one under consideration, spacing of the stud anchors is independent of the flute spacing of the deck. Single anchors can therefore be spaced as needed along the beam length provided a minimum longitudinal spacing of six anchor diameters in accordance with AISC *Specification* Section I8.2d is maintained. Anchors can also be placed in aligned or staggered pairs provided a minimum transverse spacing of four stud diameters = 3 in. is maintained. For this design, it was chosen to use pairs of anchors along each end of the girder to meet strength requirements and single anchors along the center section of the girder to meet maximum spacing requirements as illustrated in Figure I.2-4.

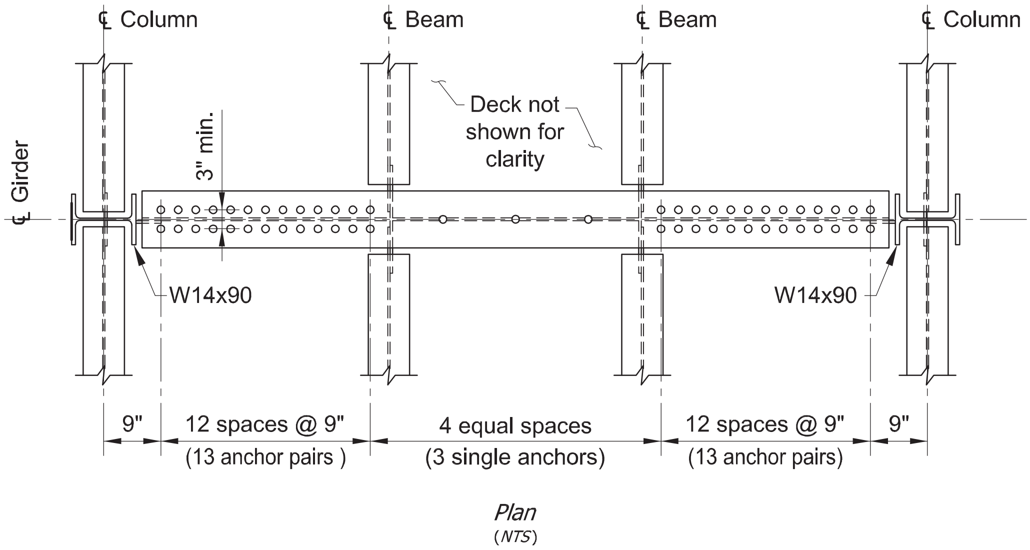


Fig. I.2-4. Steel headed stud anchor layout.

AISC *Specification* Section I8.2d requires that the distance from the center of an anchor to a free edge in the direction of the shear force be a minimum of 8 in. for normal weight concrete slabs. For simply-supported composite beams this provision could apply to the distance between the slab edge and the first anchor at each end of the beam. Assuming the slab edge is coincident to the centerline of support, Figure I.2-4 illustrates an acceptable edge distance of 9 in., though in this case the column flange would prevent breakout and negate the need for this check. The slab edge is often uniformly supported by a column flange or pour stop in typical composite construction thus preventing the possibility of a concrete breakout failure and nullifying the edge distance requirement as discussed in AISC *Specification* Commentary Section I8.3.

For this example, the minimum number of headed stud anchors required to meet the maximum spacing limit previously calculated is used within the middle third of the girder span. Note also that AISC *Specification* Section I3.2c(1)(4) requires that steel deck be anchored to all supporting members at a maximum spacing of 18 in. Additionally, ANSI/SDI C1.0-2006, *Standard for Composite Steel Floor Deck* (SDI, 2006), requires deck attachment at an average of 12 in. but no more than 18 in.

From the previous discussion and Figure I.2-4, the total number of stud anchors used is equal to $(13)(2)+3+(13)(2)=55$. A plan layout illustrating the final girder design is provided in Figure I.2-5.

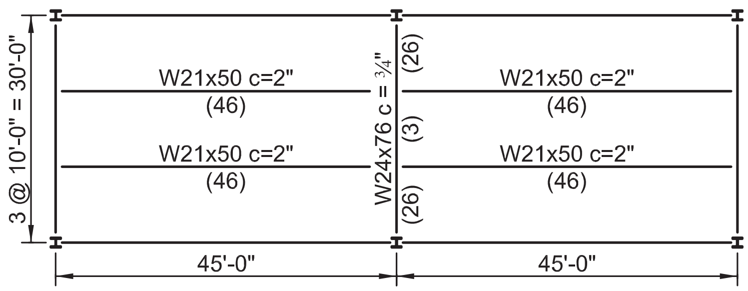


Fig. I.2-5. Revised plan.

Live Load Deflection Criteria

Deflections due to live load applied after composite action has been achieved will be limited to $L/360$ under the design live load as required by Table 1604.3 of the 2009 *International Building Code* (IBC) (ICC, 2009), or 1 in. using a 50% reduction in design live load as recommended by AISC Design Guide 3.

Deflections for composite members may be determined using the lower bound moment of inertia provided in AISC *Specification* Commentary Equation C-I3-1 and tabulated in AISC *Manual* Table 3-20. The *Specification* Commentary also provides an alternate method for determining deflections through the calculation of an effective moment of inertia. Both methods are acceptable and are illustrated in the following calculations for comparison purposes:

Method 1: Calculation of the lower bound moment of inertia, I_{LB}

$$I_{LB} = I_s + A_s\left(Y_{ENA} - d_3\right)^2 + \left(\frac{\Sigma Q_n}{F_y}\right)\left(2d_3 + d_1 - Y_{ENA}\right)^2 \qquad (\textit{Comm}.\ \text{Eq. C-I3-1})$$

Variables d_1, d_2 and d_3 in AISC *Specification* Commentary Equation C-I3-1 are determined using the same procedure previously illustrated for calculating nominal moment resistance. However, for the determination of I_{LB} the nominal strength of steel anchors is calculated between the point of maximum positive moment and the point of zero moment as opposed to between the concentrated load and point of zero moment used previously. The maximum moment is located at the center of the span and it can be seen from Figure I.2-4 that 27 anchors are located between the midpoint of the beam and each end.

$$\begin{aligned}\Sigma Q_n &= (27\text{ anchors})(21.5\text{ kips/anchor})\\ &= 581\text{ kips}\end{aligned}$$

$$\begin{aligned}a &= \frac{C}{0.85 f'_c b} \qquad (\textit{Comm}.\ \text{Eq. C-I3-9})\\ &= \frac{\Sigma Q_n}{0.85 f'_c b}\\ &= \frac{581\text{ kips}}{0.85(4\text{ ksi})(7.50\text{ ft})(12\text{ in./ft})}\\ &= 1.90\text{ in.}\end{aligned}$$

$$\begin{aligned}d_1 &= t_{slab} - a/2\\ &= 7.50\text{ in.} - 1.90\text{ in./2}\\ &= 6.55\text{ in.}\end{aligned}$$

$$\begin{aligned}x &= \frac{A_s F_y - \Sigma Q_n}{2 b_f F_y}\\ &= \frac{(22.4\text{ in.}^2)(50\text{ ksi}) - 581\text{ kips}}{2(8.99\text{ in.})(50\text{ ksi})}\\ &= 0.600\text{ in.} < \text{t}_f = 0.680\text{ in.} \quad \text{(PNA within flange)}\end{aligned}$$

$$\begin{aligned}d_2 &= x/2\\ &= 0.600\text{ in.}/2\\ &= 0.300\text{ in.}\end{aligned}$$

$$\begin{aligned}d_3 &= d/2\\ &= 23.9\text{ in./2}\\ &= 12.0\text{ in.}\end{aligned}$$

The distance from the top of the steel section to the elastic neutral axis, Y_{ENA}, for use in Equation C-I3-1 is calculated using the procedure provided in AISC *Specification* Commentary Section I3.2 as follows:

$$Y_{ENA} = \frac{A_s d_3 + \left(\frac{\Sigma Q_n}{F_y}\right)(2d_3 + d_1)}{A_s + \left(\frac{\Sigma Q_n}{F_y}\right)} \qquad (Comm.\ \text{Eq. C-I3-2})$$

$$= \frac{(22.4 \text{ in.}^2)(12.0 \text{ in.}) + \left(\frac{581 \text{ kips}}{50 \text{ ksi}}\right)\left[2(12.0 \text{ in.}) + 6.55 \text{ in.}\right]}{22.4 \text{ in.}^2 + \left(\frac{581 \text{ kips}}{50 \text{ ksi}}\right)}$$

$$= 18.3 \text{ in.}$$

Substituting these values into AISC *Specification* Commentary Equation C-I3-1 yields the following lower bound moment of inertia:

$$I_{LB} = 2{,}100 \text{ in.}^4 + (22.4 \text{ in.})(18.3 \text{ in.} - 12.0 \text{ in.})^2 + \left(\frac{581 \text{ kips}}{50 \text{ ksi}}\right)\left[2(12.0 \text{ in.}) + 6.55 \text{ in.} - 18.3 \text{ in.}\right]^2$$

$$= 4{,}730 \text{ in.}^4$$

Alternately, this value can be determined directly from AISC *Manual* Table 3-20 as illustrated in Design Example I.1.

Method 2: Calculation of the effective moment of inertia, I_{eff}

An alternate procedure for determining a moment of inertia for deflections of the composite section is presented in AISC *Specification* Commentary Section I3.2 as follows:

Transformed Moment of Inertia, I_{tr}

The effective width of the concrete below the top of the deck may be approximated with the deck profile resulting in a 50% effective width as depicted in Figure I.2-6. The effective width, b_{eff} = 7.50 ft(12 in./ft) = 90.0 in.

Transformed slab widths are calculated as follows:

$$\begin{aligned} n &= E_s / E_c \\ &= 29{,}000 \text{ ksi}/3{,}490 \text{ ksi} \\ &= 8.31 \\ b_{tr1} &= b_{eff} / n \\ &= 90.0 \text{ in.}/8.31 \\ &= 10.8 \text{ in.} \\ b_{tr2} &= 0.5 b_{eff} / n \\ &= 0.5(90.0 \text{ in.})/8.31 \\ &= 5.42 \text{ in.} \end{aligned}$$

The transformed model is illustrated in Figure I.2-7.

Determine the elastic neutral axis of the transformed section (assuming fully composite action) and calculate the transformed moment of inertia using the information provided in Table I.2-1 and Figure I.2-7. For this problem, a trial location for the elastic neutral axis (ENA) is assumed to be within the depth of the composite deck.

Table I.2-1. Properties for Elastic Neutral Axis Determination of Transformed Section

Part	A (in.^{2})	y (in.)	I (in.^{4})
A_1	48.6	$2.25+x$	82.0
A_2	$5.42x$	$x/2$	$0.452x^3$
W24×76	22.4	$x - 15.0$	2,100

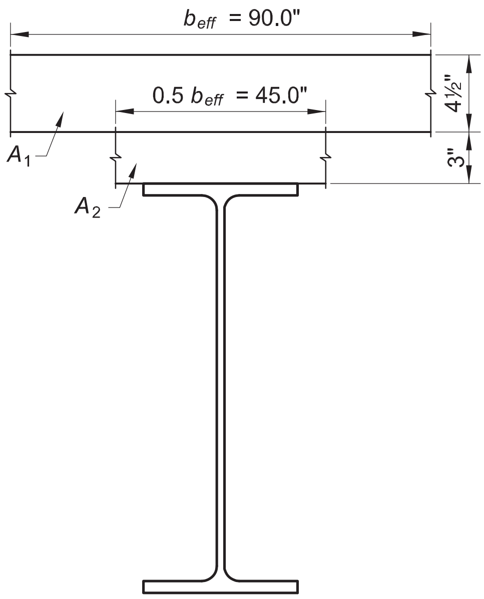


Fig. I.2-6. Effective concrete width.

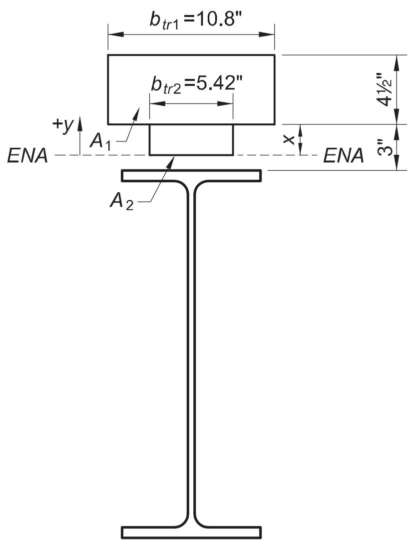


Fig. I.2-7. Transformed area model.

$\sum Ay$ about Elastic Neutral Axis = 0

$$\left(48.6 \text{ in.}^2\right)\left(2.25 \text{ in.}+x\right)+\left(5.42 \text{ in.}\right)\left(\frac{x^2}{2}\right)+\left(22.4 \text{ in.}^2\right)\left(x-15 \text{ in.}\right)=0$$

solve for $x \rightarrow x = 2.88$ in.

Verify trial location:

$2.88 \text{ in.} < h_r = 3 \text{ in.}$ **Elastic Neutral Axis within composite deck**

Utilizing the parallel axis theorem and substituting for x yields:

$$\begin{aligned} I_{tr} &= \Sigma I + \Sigma A y^2 \\ &= 82.0 \text{ in.}^4 + \left(0.452 \text{ in.}\right)\left(2.88 \text{ in.}\right)^3 + 2{,}100 \text{ in.}^4 + \left(48.6 \text{ in.}^2\right)\left(2.25 \text{ in.}+2.88 \text{ in.}\right)^2 \\ &\quad + \frac{\left(5.42 \text{ in.}\right)\left(2.88 \text{ in.}\right)^3}{4} + \left(22.4 \text{ in.}^2\right)\left(2.88 \text{ in.}-15.0 \text{ in.}\right)^2 \\ &= 6{,}800 \text{ in.}^4 \end{aligned}$$

Determine the equivalent moment of inertia, I_{equiv}

$$I_{equiv} = I_s + \sqrt{\left(\Sigma Q_n / C_f\right)}\left(I_{tr} - I_s\right) \qquad (\textit{Comm}.\ \text{Eq. C-I3-4})$$

ΣQ_n = 581 kips (previously determined in Method 1)

C_f = Compression force for fully composite beam previously determined to be controlled by A_sF_y = 1,120 kips

$$\begin{aligned} I_{equiv} &= 2{,}100 \text{ in.}^4 + \sqrt{\left(581 \text{ kips}/1{,}120 \text{ kips}\right)}\left(6{,}800 \text{ in.}^4 - 2{,}100 \text{ in.}^4\right) \\ &= 5{,}490 \text{ in.}^4 \end{aligned}$$

According to *Specification* Commentary Section I3.2:

$$\begin{aligned} I_{eff} &= 0.75 I_{equiv} \\ &= 0.75\left(5{,}490 \text{ in.}^4\right) \\ &= 4{,}120 \text{ in.}^4 \end{aligned}$$

Comparison of Methods and Final Deflection Calculation

I_{LB} was determined to be 4,730 in.^{4} and I_{eff} was determined to be 4,120 in.^{4} I_{LB} will be used for the remainder of this example.

From AISC *Manual* Table 3-23, Case 9:

$$\begin{aligned} \Delta_{LL} &= \frac{P_L L^3}{28 E I_{LB}} \\ &= \frac{\left(45.0 \text{ kips}\right)\left[\left(30.0 \text{ ft}\right)\left(12 \text{ in./ft}\right)\right]^3}{28\left(29{,}000 \text{ ksi}\right)\left(4{,}730 \text{ in.}^4\right)} \end{aligned}$$

$= 0.547 \text{ in.} < 1.00 \text{ in.}$ **o.k. for AISC Design Guide 3 limit**

(50% reduction in design live load as allowed by Design Guide 3 was not necessary to meet this limit)

$= L / 658 < L / 360$ **o.k. for IBC 2009 Table 1604.3 limit**

Available Shear Strength

According to AISC *Specification* Section I4.2, the girder should be assessed for available shear strength as a bare steel beam using the provisions of Chapter G.

Applying the loads previously determined for the governing load combination of ASCE/SEI 7-10 and obtaining available shear strengths from AISC *Manual* Table 3-2 for a W24×76 yields the following:

LRFD	ASD
$V_u = 121 \text{ kips} + (0.0912 \text{ kip/ft})(30.0 \text{ ft/2})$ $= 122 \text{ kips}$ $\phi_v V_n \geq V_u$ $\phi_v V_n = 315 \text{ kips} > 122 \text{ kips}$ **o.k.**	$V_a = 85.5 \text{ kips} + (0.0760 \text{ kip/ft})(30.0 \text{ ft/2})$ $= 86.6 \text{ kips}$ $V_n / \Omega_v \geq V_a$ $V_n / \Omega_v = 210 \text{ kips} > 86.6 \text{ kips}$ **o.k.**

Serviceability

Depending on the intended use of this bay, vibrations might need to be considered. See AISC Design Guide 11 (Murray et al., 1997) for additional information.

It has been observed that cracking of composite slabs can occur over girder lines. The addition of top reinforcing steel transverse to the girder span will aid in mitigating this effect.

Summary

Using LRFD design methodology, it has been determined that a W24×76 with ¾ in. of camber and 55, ¾-in.-diameter by 4⅞-in.-long steel headed stud anchors as depicted in Figure I.2-4, is adequate for the imposed loads and deflection criteria. Using ASD design methodology, a W24×84 with a steel headed stud anchor layout determined using a procedure analogous to the one demonstrated in this example would be required.

EXAMPLE I.3 FILLED COMPOSITE MEMBER FORCE ALLOCATION AND LOAD TRANSFER

Given:

Refer to Figure I.3-1.

Part I: For each loading condition (a) through (c) determine the required longitudinal shear force, V_r', to be transferred between the steel section and concrete fill.

Part II: For loading condition (a), investigate the force transfer mechanisms of direct bearing, shear connection, and direct bond interaction.

The composite member consists of an ASTM A500 Grade B HSS with normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, $f_c' = 5$ ksi. Use ASTM A36 material for the bearing plate.

Applied loading, P_r, for each condition illustrated in Figure I.3-1 is composed of the following nominal loads:

$P_D = 32.0$ kips
$P_L = 84.0$ kips

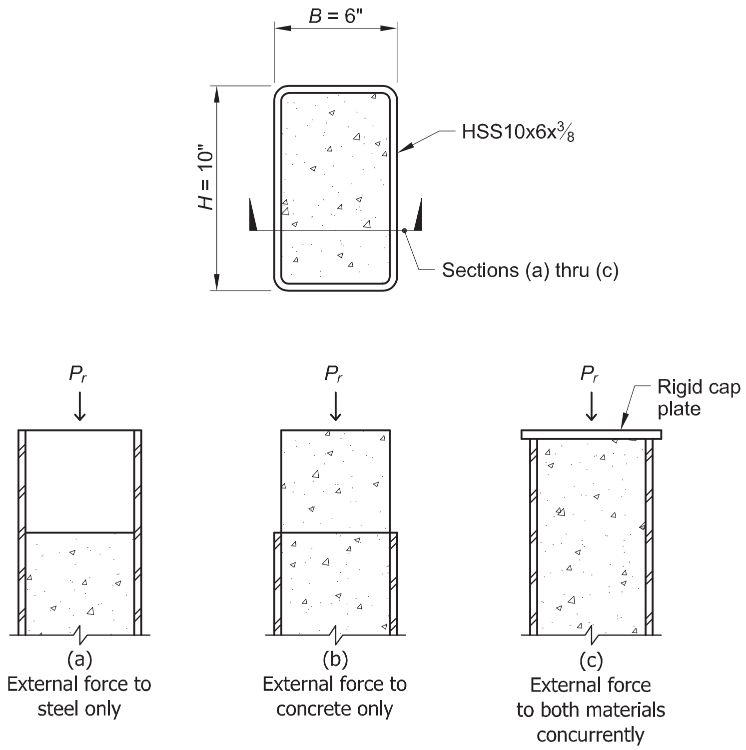


Fig. I.3-1. Concrete filled member in compression.

Solution:

Part I—Force Allocation

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B
$F_y = 46$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-11 and Figure I.3-1, the geometric properties are as follows:

HSS10×6×⅜
A_s = 10.4 in.^{2}
H = 10.0 in.
B = 6.00 in.
t_{nom} = ⅜ in. (nominal wall thickness)
t = 0.349 in. (design wall thickness in accordance with AISC *Specification* Section B4.2)
h/t = 25.7
b/t = 14.2

Calculate the concrete area using geometry compatible with that used in the calculation of the steel area in AISC *Manual* Table 1-11 (taking into account the design wall thickness and a corner radii of two times the design wall thickness in accordance with AISC *Manual* Part 1), as follows:

$$
\begin{aligned}
h_i &= H - 2t \\
&= 10.0 \text{ in.} - 2(0.349 \text{ in.}) \\
&= 9.30 \text{ in.} \\
b_i &= B - 2t \\
&= 6.00 \text{ in.} - 2(0.349 \text{ in.}) \\
&= 5.30 \text{ in.} \\
A_c &= b_i h_i - t^2(4 - \pi) \\
&= (5.30 \text{ in.})(9.30 \text{ in.}) - (0.349)^2(4 - \pi) \\
&= 49.2 \text{ in.}^2
\end{aligned}
$$

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(32.0 \text{ kips}) + 1.6(84.0 \text{ kips})$ $= 173 \text{ kips}$	$P_r = P_a$ $= 32.0 \text{ kips} + 84.0 \text{ kips}$ $= 116 \text{ kips}$

Composite Section Strength for Force Allocation

In order to determine the composite section strength, the member is first classified as compact, noncompact or slender in accordance with AISC *Specification* Table I1.1a. However, the results of this check do not affect force allocation calculations as *Specification* Section I6.2 requires the use of Equation I2-9a regardless of the local buckling classification, thus this calculation is omitted for this example. The nominal axial compressive strength without consideration of length effects, P_{no}, used for force allocation calculations is therefore determined as:

$$P_{no} = P_p \qquad \text{(Spec. Eq. I2-9a)}$$

$$= F_y A_s + C_2 f_c' \left(A_c + A_{sr} \frac{E_s}{E_c} \right) \qquad \text{(Spec. Eq. I2-9b)}$$

where
C_2 = 0.85 for rectangular sections
A_{sr} = 0 when no reinforcing steel is present within the HSS

$$P_{no} = (46 \text{ ksi})(10.4 \text{ in.}^2) + 0.85(5 \text{ ksi})(49.2 \text{ in.}^2 + 0.0 \text{ in.}^2)$$
$$= 688 \text{ kips}$$

Transfer Force for Condition (a)

Refer to Figure I.3-1(a). For this condition, the entire external force is applied to the steel section only, and the provisions of AISC *Specification* Section I6.2a apply.

$$V_r' = P_r \left(1 - \frac{F_y A_s}{P_{no}} \right) \qquad \text{(Spec. Eq. I6-1)}$$

$$= P_r \left[1 - \frac{(46 \text{ ksi})(10.4 \text{ in.}^2)}{688 \text{ kips}} \right]$$

$$= 0.305 P_r$$

LRFD	ASD
$V_r' = 0.305(173 \text{ kips})$ $= 52.8 \text{ kips}$	$V_r' = 0.305(116 \text{ kips})$ $= 35.4 \text{ kips}$

Transfer Force for Condition (b)

Refer to Figure I.3-1(b). For this condition, the entire external force is applied to the concrete fill only, and the provisions of AISC *Specification* Section I6.2b apply.

$$V_r' = P_r \left(\frac{F_y A_s}{P_{no}} \right) \qquad \text{(Spec. Eq. I6-2)}$$

$$= P_r \left[\frac{(46 \text{ ksi})(10.4 \text{ in.}^2)}{688 \text{ kips}} \right]$$

$$= 0.695 P_r$$

LRFD	ASD
$V_r' = 0.695(173 \text{ kips})$ $= 120 \text{ kips}$	$V_r' = 0.695(116 \text{ kips})$ $= 80.6 \text{ kips}$

Transfer Force for Condition (c)

Refer to Figure I.3-1(c). For this condition, external force is applied to the steel section and concrete fill concurrently, and the provisions of AISC *Specification* Section I6.2c apply.

AISC *Specification* Commentary Section I6.2 states that when loads are applied to both the steel section and concrete fill concurrently, V_r' can be taken as the difference in magnitudes between the portion of the external force applied directly to the steel section and that required by Equation I6-2. Using the plastic distribution approach employed in *Specification* Equations I6-1 and I6-2, this concept can be written in equation form as follows:

$$V_r' = \left| P_{rs} - P_r\left(\frac{A_s F_y}{P_{no}}\right) \right| \quad \text{(Eq. 1)}$$

where

P_{rs} = portion of external force applied directly to the steel section, kips

Currently the *Specification* provides no specific requirements for determining the distribution of the applied force for the determination of P_{rs}, so it is left to engineering judgment. For a bearing plate condition such as the one represented in Figure I.3-1(c), one possible method for determining the distribution of applied forces is to use an elastic distribution based on the material axial stiffness ratios as follows:

$$\begin{aligned} E_c &= w_c^{1.5}\sqrt{f_c'} \\ &= \left(145 \text{ lb/ft}^3\right)^{1.5}\sqrt{5 \text{ ksi}} \\ &= 3{,}900 \text{ ksi} \end{aligned}$$

$$\begin{aligned} P_{rs} &= \left(\frac{E_s A_s}{E_s A_s + E_c A_c}\right)\left(P_r\right) \\ &= \left[\frac{(29{,}000 \text{ ksi})(10.4 \text{ in.}^2)}{(29{,}000 \text{ ksi})(10.4 \text{ in.}^2) + (3{,}900 \text{ ksi})(49.2 \text{ in.}^2)}\right]\left(P_r\right) \\ &= 0.611P_r \end{aligned}$$

Substituting the results into Equation 1 yields:

$$\begin{aligned} V_r' &= \left| 0.611P_r - P_r\left(\frac{A_s F_y}{P_{no}}\right) \right| \\ &= \left| 0.611P_r - P_r\left[\frac{(10.4 \text{ in.}^2)(46 \text{ ksi})}{688 \text{ kips}}\right] \right| \\ &= 0.0843P_r \end{aligned}$$

LRFD	ASD
$V_r' = 0.0843(173 \text{ kips})$ $= 14.6 \text{ kips}$	$V_r' = 0.0843(116 \text{ kips})$ $= 9.78 \text{ kips}$

An alternate approach would be the use of a plastic distribution method whereby the load is partitioned to each material in accordance with their contribution to the composite section strength given in Equation I2-9b. This method eliminates the need for longitudinal shear transfer provided the local bearing strength of the concrete and steel are adequate to resist the forces resulting from this distribution.

Additional Discussion

- The design and detailing of the connections required to deliver external forces to the composite member should be performed according to the applicable sections of AISC *Specification* Chapters J and K. Note that for checking bearing strength on concrete confined by a steel HSS or box member, the $\sqrt{A_2 / A_1}$ term in Equation J8-2 may be taken as 2.0 according to the User Note in *Specification* Section I6.2.

- The connection cases illustrated by Figure I.3-1 are idealized conditions representative of the mechanics of actual connections. For instance, a standard shear connection welded to the face of an HSS column is an example of a condition where all external force is applied directly to the steel section only. Note that the connection configuration can also impact the strength of the force transfer mechanism as illustrated in Part II of this example.

Solution:

Part II—Load Transfer

The required longitudinal force to be transferred, V_r', determined in Part I condition (a) will be used to investigate the three applicable force transfer mechanisms of AISC *Specification* Section I6.3: direct bearing, shear connection, and direct bond interaction. As indicated in the *Specification*, these force transfer mechanisms may not be superimposed; however, the mechanism providing the greatest nominal strength may be used.

Direct Bearing

Trial Layout of Bearing Plate

For investigating the direct bearing load transfer mechanism, the external force is delivered directly to the HSS section by standard shear connections on each side of the member as illustrated in Figure I.3-2. One method for utilizing direct bearing in this instance is through the use of an internal bearing plate. Given the small clearance within the HSS section under consideration, internal access for welding is limited to the open ends of the HSS; therefore, the HSS section will be spliced at the bearing plate location. Additionally, it is a practical consideration that no more than 50% of the internal width of the HSS section be obstructed by the bearing plate in order to facilitate concrete placement. It is essential that concrete mix proportions and installation of concrete fill produce full bearing above and below the projecting plate. Based on these considerations, the trial bearing plate layout depicted in Figure I.3-2 was selected using an internal plate protrusion, L_p, of 1.0 in.

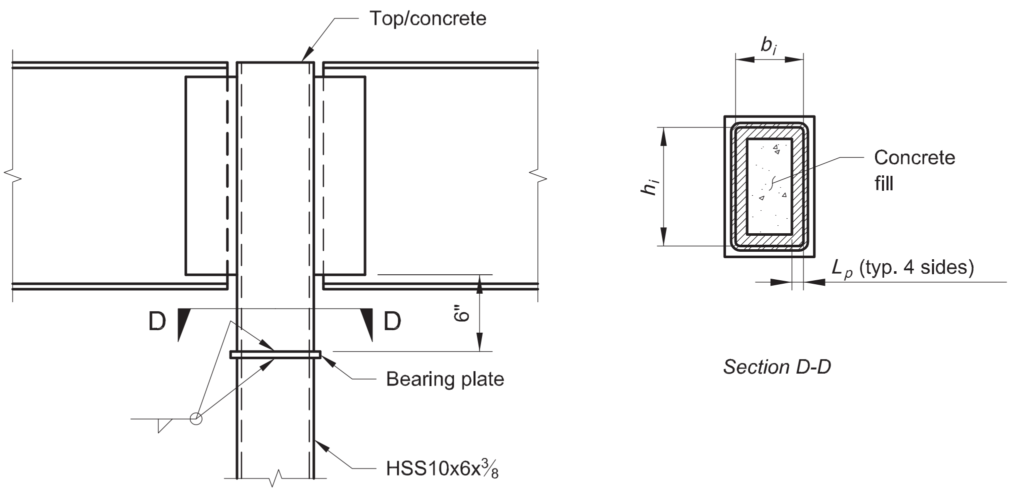


Fig. I.3-2. Internal bearing plate configuration.

Location of Bearing Plate

The bearing plate is placed within the load introduction length discussed in AISC *Specification* Section I6.4b. The load introduction length is defined as two times the minimum transverse dimension of the HSS both above and below the load transfer region. The load transfer region is defined in *Specification* Commentary Section I6.4 as the depth of the connection. For the configuration under consideration, the bearing plate should be located within 2(B = 6 in.) = 12 in. of the bottom of the shear connection. From Figure I.3-2, the location of the bearing plate is 6 in. from the bottom of the shear connection and is therefore adequate.

Available Strength for the Limit State of Direct Bearing

The contact area between the bearing plate and concrete, A_1, may be determined as follows:

$$A_1 = A_c - (b_i - 2L_p)(h_i - 2L_p) \quad \text{(Eq. 2)}$$

where

L_p = typical protrusion of bearing plate inside HSS
= 1.0 in.

Substituting for the appropriate geometric properties previously determined in Part I into Equation 2 yields:

$$\begin{aligned} A_1 &= 49.2 \text{ in.}^2 - \left[5.30 \text{ in.} - 2(1.0 \text{ in.})\right]\left[9.30 \text{ in.} - 2(1.0 \text{ in.})\right] \\ &= 25.1 \text{ in.}^2 \end{aligned}$$

The available strength for the direct bearing force transfer mechanism is:

$$R_n = 1.7 f_c' A_1 \quad (\textit{Spec}. \text{ Eq. I6-3})$$

LRFD	ASD
$\phi_B = 0.65$ $\phi_B R_n \geq V_r'$ $\phi_B R_n = 0.65(1.7)(5 \text{ ksi})(25.1 \text{ in.}^2)$ $= 139 \text{ kips} > 52.8 \text{ kips}$ **o.k.**	$\Omega_B = 2.31$ $R_n / \Omega_B \geq V_r'$ $R_n / \Omega_B = \dfrac{1.7(5 \text{ ksi})(25.1 \text{ in.}^2)}{2.31}$ $= 92.4 \text{ kips} > 35.4 \text{ kips}$ **o.k.**

Required Thickness of Internal Bearing Plate

There are several methods available for determining the bearing plate thickness. For round HSS sections with circular bearing plate openings, a closed-form elastic solution such as those found in *Roark's Formulas for Stress and Strain* (Young and Budynas, 2002) may be used. Alternately, the use of computational methods such as finite element analysis may be employed.

For this example, yield line theory can be employed to determine a plastic collapse mechanism of the plate. In this case, the walls of the HSS lack sufficient stiffness and strength to develop plastic hinges at the perimeter of the bearing plate. Utilizing only the plate material located within the HSS walls, and ignoring the HSS corner radii, the yield line pattern is as depicted in Figure I.3-3.

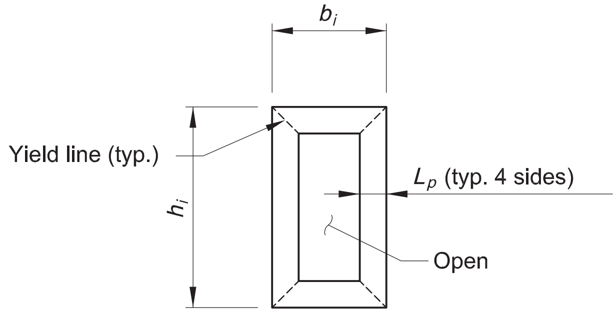


Fig. I.3-3. Yield line pattern.

Utilizing the results of the yield line analysis with $F_y = 36$ ksi plate material, the plate thickness may be determined as follows:

LRFD	ASD
$\phi = 0.90$ $t_p = \sqrt{\dfrac{w_u}{2\phi F_y}\left[L_p\left(b_i + h_i\right) - \dfrac{8L_p^2}{3}\right]}$ where w_u = bearing pressure on plate determined using LRFD load combinations $= \dfrac{V_r'}{A_1}$ $= \dfrac{52.8 \text{ kips}}{25.1 \text{ in.}^2}$ $= 2.10$ ksi $t_p = \sqrt{\dfrac{(2.10\text{ ksi})}{2(0.9)(36\text{ ksi})}\left[(1.0\text{ in.})(5.30\text{ in.} + 9.30\text{ in.}) - \dfrac{8(1.0\text{ in.})^2}{3}\right]}$ $= 0.622$ in.	$\Omega = 1.67$ $t_p = \sqrt{\dfrac{\Omega w_a}{2F_y}\left[L_p\left(b_i + h_i\right) - \dfrac{8L_p^2}{3}\right]}$ where w_a = bearing pressure on plate determined using ASD load combinations $= \dfrac{V_r'}{A_1}$ $= \dfrac{35.4 \text{ kips}}{25.1 \text{ in.}^2}$ $= 1.41$ ksi $t_p = \sqrt{\dfrac{(1.67)(1.41\text{ ksi})}{2(36\text{ ksi})}\left[(1.0\text{ in.})(5.30\text{ in.} + 9.30\text{ in.}) - \dfrac{8(1.0\text{ in.})^2}{3}\right]}$ $= 0.625$ in.

Thus, select a ¾-in.-thick bearing plate.

Splice Weld

The HSS is in compression due to the imposed loads, therefore the splice weld indicated in Figure I.3-2 is sized according to the minimum weld size requirements of Chapter J. Should uplift or flexure be applied in other loading conditions, the splice should be designed to resist these forces using the applicable provisions of AISC *Specification* Chapters J and K.

Shear Connection

Shear connection involves the use of steel headed stud or channel anchors placed within the HSS section to transfer the required longitudinal shear force. The use of the shear connection mechanism for force transfer in filled HSS is usually limited to large HSS sections and built-up box shapes, and is not practical for the composite member in question. Consultation with the fabricator regarding their specific capabilities is recommended to determine the feasibility of shear connection for HSS and box members. Should shear connection be a feasible load transfer mechanism, AISC *Specification* Section I6.3b in conjunction with the steel anchors in composite component provisions of Section I8.3 apply.

Direct Bond Interaction

The use of direct bond interaction for load transfer is limited to filled HSS and depends upon the location of the load transfer point within the length of the member being considered (end or interior) as well as the number of faces to which load is being transferred.

From AISC *Specification* Section I6.3c, the nominal bond strength for a rectangular section is:

$$R_n = B^2 C_{in} F_{in} \qquad \text{(}\textit{Spec}\text{. Eq. I6-5)}$$

where

B = overall width of rectangular steel section along face transferring load, in.

C_{in} = 2 if the filled composite member extends to one side of the point of force transfer

= 4 if the filled composite member extends to both sides of the point of force transfer

F_{in} = 0.06 ksi

For the design of this load transfer mechanism, two possible cases will be considered:

Case 1: End Condition – Load Transferred to Member from Four Sides Simultaneously

For this case the member is loaded at an end condition (the composite member only extends to one side of the point of force transfer). Force is applied to all four sides of the section simultaneously thus allowing the full perimeter of the section to be mobilized for bond strength.

From AISC *Specification* Equation I6-5:

LRFD	ASD
$\phi = 0.45$ $\phi R_n \geq V_r'$ $\phi R_n = 0.45\left[2(6.00\text{ in.})^2 + 2(10.0\text{ in.})^2\right](2)(0.06\text{ ksi})$ $= 14.7\text{ kips} < 52.8\text{ kips}$ **n.g.**	$\Omega = 3.33$ $R_n / \Omega \geq V_r'$ $R_n / \Omega = \dfrac{\left[2(6.00\text{ in.})^2 + 2(10.0\text{ in.})^2\right](2)(0.06\text{ ksi})}{3.33}$ $= 9.80\text{ kips} < 35.4\text{ kips}$ **n.g.**

Bond strength is inadequate and another force transfer mechanism such as direct bearing must be used to meet the load transfer provisions of AISC *Specification* Section I6.

Alternately, the detail could be revised so that the external force is applied to both the steel section and concrete fill concurrently as schematically illustrated in Figure I.3-1(c). Comparing bond strength to the load transfer requirements for concurrent loading determined in Part I of this example yields:

LRFD	ASD
$\phi = 0.45$ $\phi R_n \geq V_r'$ $\phi R_n = 14.7\text{ kips} > 14.6\text{ kips}$ **o.k.**	$\Omega = 3.33$ $R_n / \Omega \geq V_r'$ $R_n / \Omega = 9.80\text{ kips} > 9.78\text{ kips}$ **o.k.**

Case 2: Interior Condition – Load Transferred to Three Faces

For this case the composite member is loaded from three sides away from the end of the member (the composite member extends to both sides of the point of load transfer) as indicated in Figure I.3-4.

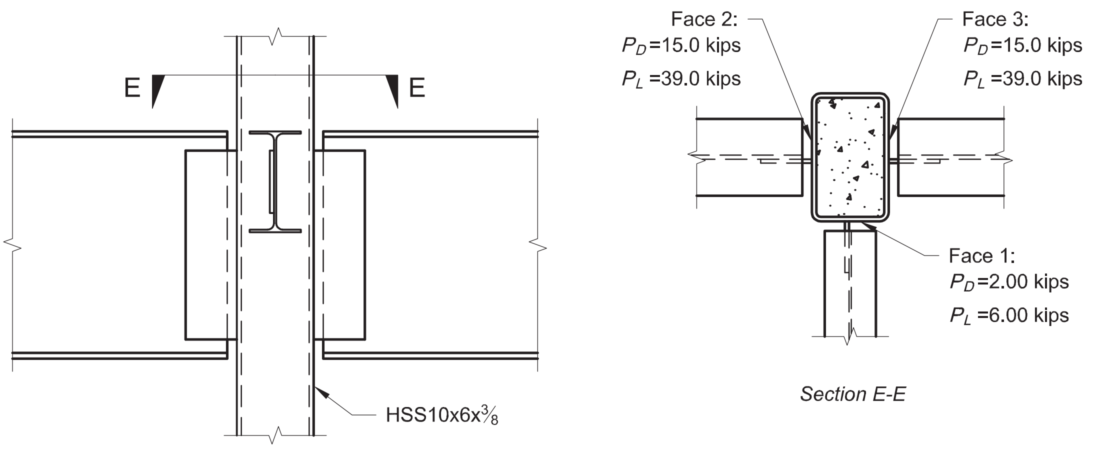


Fig. I.3-4. Case 2 load transfer.

Longitudinal shear forces to be transferred at each face of the HSS are calculated using the relationship to external forces determined in Part I of the example for condition (a) shown in Figure I.3-1, and the applicable ASCE/SEI 7-10 load combinations as follows:

LRFD	ASD
Face 1:	Face 1:
$P_{r1} = P_u$	$P_{r1} = P_a$
$= 1.2(2.00 \text{ kips}) + 1.6(6.00 \text{ kips})$	$= 2.00 \text{ kips} + 6.00 \text{ kips}$
$= 12.0 \text{ kips}$	$= 8.00 \text{ kips}$
$V'_{r1} = 0.305P_{r1}$	$V'_{r1} = 0.305P_{r1}$
$= 0.305(12.0 \text{ kips})$	$= 0.305(8.00 \text{ kips})$
$= 3.66 \text{ kips}$	$= 2.44 \text{ kips}$
Faces 2 and 3:	Faces 2 and 3:
$P_{r2-3} = P_u$	$P_{r2-3} = P_u$
$= 1.2(15.0 \text{ kips}) + 1.6(39.0 \text{ kips})$	$= 15.0 \text{ kips} + 39.0 \text{ kips}$
$= 80.4 \text{ kips}$	$= 54.0 \text{ kips}$
$V'_{r2-3} = 0.305P_{r2-3}$	$V'_{r2-3} = 0.305P_{r2-3}$
$= 0.305(80.4 \text{ kips})$	$= 0.305(54.0 \text{ kips})$
$= 24.5 \text{ kips}$	$= 16.5 \text{ kips}$

Load transfer at each face of the section is checked separately for the longitudinal shear at that face using Equation I6-5 as follows:

LRFD	ASD
$\phi = 0.45$	$\Omega = 3.33$
Face 1:	Face 1:
$\phi R_{n1} \geq V'_{r1}$	$R_{n1} / \Omega \geq V'_{r1}$
$\phi R_{n1} = 0.45(6.00 \text{ in.})^2(4)(0.06 \text{ ksi})$	$R_{n1} / \Omega = \dfrac{(6.00 \text{ in.})^2(4)(0.06 \text{ ksi})}{3.33}$
$= 3.89 \text{ kips} > 3.66 \text{ kips}$ **o.k.**	$= 2.59 \text{ kips} > 2.44 \text{ kips}$ **o.k.**

LRFD	ASD
Faces 2 and 3: $\phi R_{n2-3} \geq V'_{r2-3}$ $\phi R_{n2-3} = 0.45(10.0\text{ in.})^2(4)(0.06\text{ ksi})$ $= 10.8\text{ kips} < 24.5\text{ kips}$ **n.g.**	Faces 2 and 3: $R_{n2-3} / \Omega \geq V'_{r2-3}$ $R_{n2-3} / \Omega = \dfrac{(10.0\text{ in.})^2(4)(0.06\text{ ksi})}{3.33}$ $= 7.21\text{ kips} < 16.5\text{ kips}$ **n.g.**

The calculations indicate that the bond strength is inadequate for two of the three loaded faces, thus an alternate means of load transfer such as the use of internal bearing plates as demonstrated previously in this example is necessary.

As demonstrated by this example, direct bond interaction provides limited available strength for transfer of longitudinal shears and is generally only acceptable for lightly loaded columns or columns with low shear transfer requirements such as those with loads applied to both concrete fill and steel encasement simultaneously.

EXAMPLE I.4 FILLED COMPOSITE MEMBER IN AXIAL COMPRESSION

Given:

Determine if the 14 ft long, filled composite member illustrated in Figure I.4-1 is adequate for the indicated dead and live loads.

The composite member consists of an ASTM A500 Grade B HSS with normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, $f_c' = 5$ ksi.

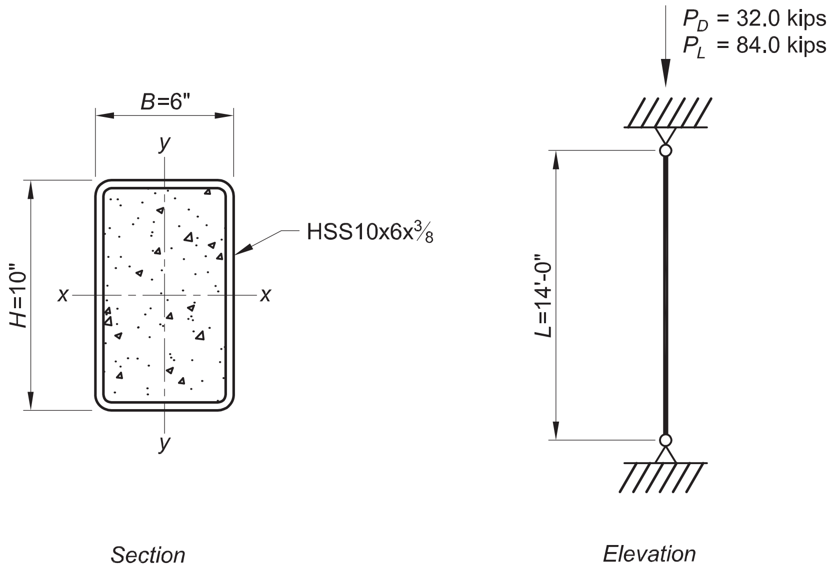


Fig. I.4-1. Concrete filled member section and applied loading.

Solution:

From AISC *Manual* Table 2-4, the material properties are:

ASTM A500 Grade B
$F_y = 46$ ksi
$F_u = 58$ ksi

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(32.0 \text{ kips}) + 1.6(84.0 \text{ kips})$ $= 173 \text{ kips}$	$P_r = P_a$ $= 32.0 \text{ kips} + 84.0 \text{ kips}$ $= 116 \text{ kips}$

Method 1: AISC Manual Tables

The most direct method of calculating the available compressive strength is through the use of AISC *Manual* Table 4-14. A K factor of 1.0 is used for a pin-ended member. Because the unbraced length is the same in both the x-x and y-y directions, and I_x exceeds I_y, y-y axis buckling will govern.

Entering Table 4-14 with KL_y = 14 ft yields:

LRFD	ASD
$\phi_c P_n = 354 \text{ kips}$ $\phi_c P_n \geq P_u$ $354 \text{ kips} > 173 \text{ kips}$ **o.k.**	$P_n / \Omega_c = 236 \text{ kips}$ $P_n / \Omega_c \geq P_a$ $236 \text{ kips} > 116 \text{ kips}$ **o.k.**

Method 2: AISC Specification Calculations

As an alternate to the AISC *Manual* tables, the available compressive strength can be calculated directly using the provisions of AISC *Specification* Chapter I.

From AISC *Manual* Table 1-11 and Figure I.4-1, the geometric properties of an HSS10×6×⅜ are as follows:

A_s = 10.4 in.^{2}
H = 10.0 in.
B = 6.00 in.
t_{nom} = ⅜ in. (nominal wall thickness)
t = 0.349 in. (design wall thickness in accordance with AISC *Specification* Section B4.2)
h/t = 25.7
b/t = 14.2
I_{sx} = 137 in.^{4}
I_{sy} = 61.8 in.^{4}

Internal clear distances are determined as:

$$\begin{aligned} h_i &= H - 2t \\ &= 10.0 \text{ in.} - 2(0.349 \text{ in.}) \\ &= 9.30 \text{ in.} \\ b_i &= B - 2t \\ &= 6.0 \text{ in.} - 2(0.349 \text{ in.}) \\ &= 5.30 \text{ in.} \end{aligned}$$

From Design Example I.3, the area of concrete, A_c, equals 49.2 in.^{2} The steel and concrete areas can be used to calculate the gross cross-sectional area as follows:

$$\begin{aligned} A_g &= A_s + A_c \\ &= 10.4 \text{ in.}^2 + 49.2 \text{ in.}^2 \\ &= 59.6 \text{ in.}^2 \end{aligned}$$

Calculate the concrete moment of inertia using geometry compatible with that used in the calculation of the steel area in AISC *Manual* Table 1-11 (taking into account the design wall thickness and corner radii of two times the design wall thickness in accordance with AISC *Manual* Part 1), the following equations may be used, based on the terminology given in Figure I-2 of the introduction to these examples:

For bending about the *x-x* axis:

$$I_{cx} = \frac{(B-4t)h_i^3}{12} + \frac{t(H-4t)^3}{6} + \frac{(9\pi^2 - 64)t^4}{36\pi} + \pi t^2 \left(\frac{H-4t}{2} + \frac{4t}{3\pi}\right)^2$$

$$= \frac{\left[6.00 \text{ in.} - 4(0.349 \text{ in.})\right](9.30 \text{ in.})^3}{12} + \frac{(0.349 \text{ in.})\left[10.0 \text{ in.} - 4(0.349 \text{ in.})\right]^3}{6} + \frac{(9\pi^2 - 64)(0.349 \text{ in.})^4}{36\pi}$$

$$+ \pi(0.349 \text{ in.})^2 \left(\frac{10.0 \text{ in.} - 4(0.349 \text{ in.})}{2} + \frac{4(0.349 \text{ in.})}{3\pi}\right)^2$$

$$= 353 \text{ in.}^4$$

For bending about the *y-y* axis:

$$I_{cy} = \frac{(H - 4t)b_i^3}{12} + \frac{t(B - 4t)^3}{6} + \frac{(9\pi^2 - 64)t^4}{36\pi} + \pi t^2 \left(\frac{B - 4t}{2} + \frac{4t}{3\pi}\right)^2$$

$$= \frac{\left[10.0 \text{ in.} - 4(0.349 \text{ in.})\right](5.30 \text{ in.})^3}{12} + \frac{(0.349 \text{ in.})\left[6.00 \text{ in.} - 4(0.349 \text{ in.})\right]^3}{6} + \frac{(9\pi^2 - 64)(0.349 \text{ in.})^4}{36\pi}$$

$$+ \pi(0.349 \text{ in.})^2 \left(\frac{6.00 \text{ in.} - 4(0.349 \text{ in.})}{2} + \frac{4(0.349 \text{ in.})}{3\pi}\right)^2$$

$$= 115 \text{ in.}^4$$

Limitations of AISC Specification Sections I1.3 and I2.2a

(1) Concrete Strength: $3 \text{ ksi} \le f_c' \le 10 \text{ ksi}$

$f_c' = 5 \text{ ksi}$ **o.k.**

(2) Specified minimum yield stress of structural steel: $F_y \le 75 \text{ ksi}$

$F_y = 46 \text{ ksi}$ **o.k.**

(3) Cross-sectional area of steel section: $A_s \ge 0.01 A_g$

$$10.4 \text{ in.}^2 \ge (0.01)(59.6 \text{ in.}^2)$$

$$> 0.596 \text{ in.}^2 \quad \textbf{o.k.}$$

There are no minimum longitudinal reinforcement requirements in the AISC *Specification* within filled composite members; therefore, the area of reinforcing bars, A_{sr}, for this example is zero.

Classify Section for Local Buckling

In order to determine the strength of the composite section subject to axial compression, the member is first classified as compact, noncompact or slender in accordance with AISC *Specification* Table I1.1A.

$$\lambda_p = 2.26\sqrt{\frac{E}{F_y}}$$

$$= 2.26\sqrt{\frac{29{,}000 \text{ ksi}}{46 \text{ ksi}}}$$

$$= 56.7$$

$$\lambda_{controlling} = \max\begin{pmatrix} h/t = 25.7 \\ b/t = 14.2 \end{pmatrix}$$

$$= 25.7$$

$\lambda_{controlling} \le \lambda_p$ **section is compact**

Available Compressive Strength

The nominal axial compressive strength for compact sections without consideration of length effects, P_{no}, is determined from AISC *Specification* Section I2.2b as:

$$P_{no} = P_p \quad \text{(Spec. Eq. I2-9a)}$$

$$= F_y A_s + C_2 f_c' \left(A_c + A_{sr} \frac{E_s}{E_c} \right) \quad \text{(Spec. Eq. I2-9b)}$$

where $C_2 = 0.85$ for rectangular sections

$$P_{no} = (46 \text{ ksi})(10.4 \text{ in.}^2) + 0.85(5 \text{ ksi})(49.2 \text{ in.}^2 + 0.0 \text{ in.}^2)$$
$$= 688 \text{ kips}$$

Because the unbraced length is the same in both the *x-x* and *y-y* directions, the column will buckle about the weaker *y-y* axis (the axis having the lower moment of inertia). I_{cy} and I_{sy} will therefore be used for calculation of length effects in accordance with AISC *Specification* Sections I2.2b and I2.1b as follows:

$$C_3 = 0.6 + 2\left(\frac{A_s}{A_c + A_s} \right) \leq 0.9 \quad \text{(Spec. Eq. I2-13)}$$

$$= 0.6 + 2\left(\frac{10.4 \text{ in.}^2}{49.2 \text{ in.}^2 + 10.4 \text{ in.}^2} \right) \leq 0.9$$

$$= 0.949 > 0.9 \qquad \textbf{0.9 controls}$$

$$E_c = w_c^{1.5} \sqrt{f_c'}$$
$$= (145 \text{ lb/ft}^3)^{1.5} \sqrt{5 \text{ ksi}}$$
$$= 3{,}900 \text{ ksi}$$

$$EI_{eff} = E_s I_{sy} + E_s I_{sr} + C_3 E_c I_{cy} \quad \text{(Spec. Eq. I2-12)}$$
$$= (29{,}000 \text{ ksi})(61.8 \text{ in.}^4) + 0 + 0.9(3{,}900 \text{ ksi})(115 \text{ in.}^4)$$
$$= 2{,}200{,}000 \text{ kip-in.}^2$$

$$P_e = \pi^2 (EI_{eff}) / (KL)^2 \quad \text{(from Spec. Eq. I2-5)}$$

where K=1.0 for a pin-ended member

$$P_e = \frac{\pi^2 (2{,}200{,}000 \text{ kip-in.}^2)}{\left[(1.0)(14.0 \text{ ft})(12 \text{ in./ft})\right]^2}$$
$$= 769 \text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{688 \text{ kips}}{769 \text{ kips}}$$
$$= 0.895 < 2.25$$

Therefore, use AISC Specification Equation I2-2.

$$P_n = P_{no} \left[0.658^{\frac{P_{no}}{P_e}} \right] \quad \text{(Spec. Eq. I2-2)}$$

$$= (688 \text{ kips})(0.658)^{0.895}$$
$$= 473 \text{ kips}$$

Check adequacy of the composite column for the required axial compressive strength:

LRFD	ASD
$\phi_c = 0.75$ $\phi_c P_n \geq P_u$ $\phi_c P_n = 0.75(473\text{ kips})$ $= 355\text{ kips} > 173\text{ kips}$ **o.k.**	$\Omega_c = 2.00$ $P_n / \Omega_c \geq P_a$ $P_n / \Omega_c = \dfrac{473\text{ kips}}{2.00}$ $= 237\text{ kips} > 116\text{ kips}$ **o.k.**

The slight differences between these values and those tabulated in the AISC *Manual* are due to the number of significant digits carried through the calculations.

Available Compressive Strength of Bare Steel Section

Due to the differences in resistance and safety factors between composite and noncomposite column provisions, it is possible to calculate a lower available compressive strength for a composite column than one would calculate for the corresponding bare steel section. However, in accordance with AISC *Specification* Section I2.1b, the available compressive strength need not be less than that calculated for the bare steel member in accordance with Chapter E.

From AISC *Manual* Table 4-3, for an HSS10×6×⅜, $KL_y = 14.0$ ft:

LRFD	ASD
$\phi_c P_n = 313\text{ kips}$ $313\text{ kips} < 355\text{ kips}$	$P_n / \Omega_c = 208\text{ kips}$ $208\text{ kips} < 237\text{ kips}$

Thus, the composite section strength controls and is adequate for the required axial compressive strength as previously demonstrated.

Force Allocation and Load Transfer

Load transfer calculations for external axial forces should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions is provided in Design Example I.3.

EXAMPLE I.5 FILLED COMPOSITE MEMBER IN AXIAL TENSION

Given:

Determine if the 14 ft long, filled composite member illustrated in Figure I.5-1 is adequate for the indicated dead load compression and wind load tension. The entire load is applied to the steel section.

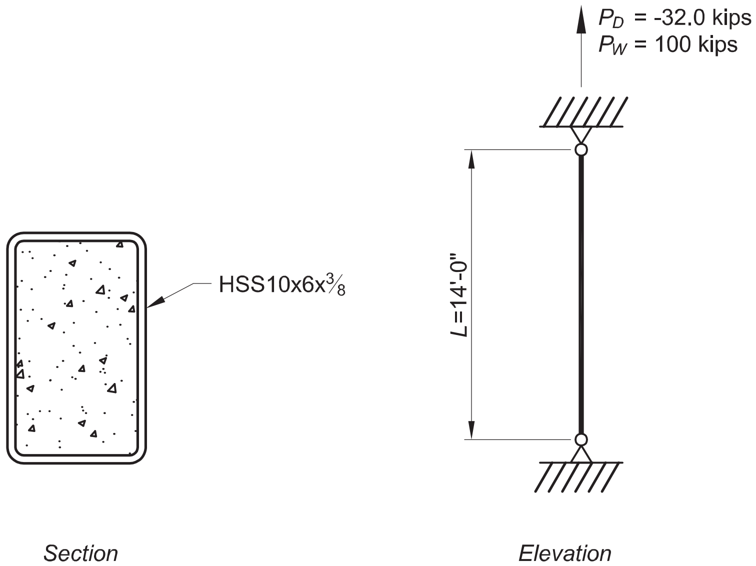


Fig. I.5-1. Concrete filled member section and applied loading.

The composite member consists of an ASTM A500 Grade B HSS with normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, $f_c' = 5$ ksi.

Solution:

From AISC *Manual* Table 2-4, the material properties are:

ASTM A500 Grade B
$F_y = 46$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS10×6×3⁄8
$A_s = 10.4$ in.^{2}

There are no minimum requirements for longitudinal reinforcement in the AISC *Specification*; therefore it is common industry practice to use filled shapes without longitudinal reinforcement, thus A_{sr} =0.

From Chapter 2 of ASCE/SEI 7, the required compressive strength is (taking compression as negative and tension as positive):

LRFD	ASD
Governing Uplift Load Combination $= 0.9D + 1.0W$ $P_r = P_u$ $= 0.9(-32.0\text{ kips}) + 1.0(100\text{ kips})$ $= 71.2\text{ kips}$	Governing Uplift Load Combination $= 0.6D + 0.6W$ $P_r = P_a$ $= 0.6(-32.0\text{ kips}) + 0.6(100\text{ kips})$ $= 40.8\text{ kips}$

Available Tensile Strength

Available tensile strength for a filled composite member is determined in accordance with AISC *Specification* Section I2.2c.

$$P_n = A_s F_y + A_{sr} F_{ysr}$$ (*Spec*. Eq. I2-14)

$$= (10.4\text{ in.}^2)(46\text{ ksi}) + (0.0\text{ in.}^2)(60\text{ ksi})$$

$$= 478\text{ kips}$$

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n \geq P_u$ $\phi_t P_n = 0.90(478\text{ kips})$ $= 430\text{ kips} > 71.2\text{ kips}$ **o.k.**	$\Omega_t = 1.67$ $P_n / \Omega_t \geq P_a$ $P_n / \Omega_t = \dfrac{478\text{ kips}}{1.67}$ $= 286\text{ kips} > 40.8\text{ kips}$ **o.k.**

For concrete filled HSS members with no internal longitudinal reinforcing, the values for available tensile strength may also be taken directly from AISC *Manual* Table 5-4.

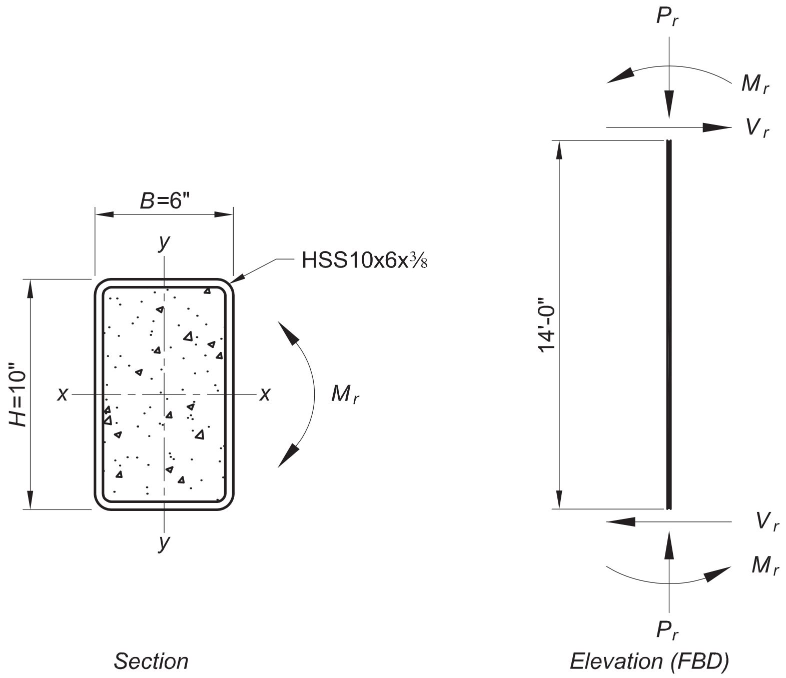
Force Allocation and Load Transfer

Load transfer calculations are not required for concrete filled members in axial tension that do not contain longitudinal reinforcement, such as the one under investigation, as only the steel section resists tension.

EXAMPLE I.6 FILLED COMPOSITE MEMBER IN COMBINED AXIAL COMPRESSION, FLEXURE AND SHEAR

Given:

Determine if the 14 ft long, filled composite member illustrated in Figure I.6-1 is adequate for the indicated axial forces, shears and moments that have been determined in accordance with the direct analysis method of AISC *Specification* Chapter C for the controlling ASCE/SEI 7-10 load combinations.



	LRFD	ASD
P_r (kips)	129	98.2
M_r (kip-ft)	120	54.0
V_r (kips)	17.1	10.3

Fig. I.6-1. Concrete filled member section and member forces.

The composite member consists of an ASTM A500 Grade B HSS with normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, f_c' = 5 ksi.

Solution:

From AISC *Manual* Table 2-4, the material properties are:

ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-11 and Figure I.6-1, the geometric properties are as follows:

HSS10×6×⅜
H = 10.0 in.
B = 6.00 in.

t_{nom} = ⅜ in. (nominal wall thickness)
t = 0.349 in. (design wall thickness)
h/t = 25.7
b/t = 14.2
A_s = 10.4 in.^{2}
I_{sx} = 137 in.^{4}
I_{sy} = 61.8 in.^{4}
Z_{sx} = 33.8 in.^{3}

Additional geometric properties used for composite design are determined in Design Examples I.3 and I.4 as follows:

h_i = 9.30 in. clear distance between HSS walls (longer side)
b_i = 5.30 in. clear distance between HSS walls (shorter side)
A_c = 49.2 in.^{2} cross-sectional area of concrete fill
A_g = 59.6 in.^{2} gross cross-sectional area of composite member
A_{sr} = 0 in.^{2} area of longitudinal reinforcement
E_c = 3,900 ksi modulus of elasticity of concrete
I_{cx} = 353 in.^{4} moment of inertia of concrete fill about the x-x axis
I_{cy} = 115 in.^{4} moment of inertia of concrete fill about the y-y axis

Limitations of AISC Specification Sections I1.3 and I2.2a

(1) Concrete Strength: $3 \text{ ksi} \le f_c' \le 10 \text{ ksi}$

$f_c' = 5 \text{ ksi}$ **o.k.**

(2) Specified minimum yield stress of structural steel: $F_y \le 75 \text{ ksi}$

$F_y = 46 \text{ ksi}$ **o.k.**

(3) Cross-sectional area of steel section: $A_s \ge 0.01 A_g$

$$10.4 \text{ in.}^2 \ge (0.01)(59.6 \text{ in.}^2)$$
$$> 0.596 \text{ in.}^2 \quad \textbf{o.k.}$$

Classify Section for Local Buckling

The composite member in question was shown to be compact for pure compression in Design Example I.4 in accordance with AISC *Specification* Table I1.1a. The section must also be classified for local buckling due to flexure in accordance with *Specification* Table I1.1b; however, since the limits for members subject to flexure are equal to or less stringent than those for members subject to compression, the member is compact for flexure.

Interaction of Axial Force and Flexure

The interaction between axial forces and flexure in composite members is governed by AISC *Specification* Section I5 which, for compact members, permits the use of a strain compatibility method or plastic stress distribution method, with the option to use the interaction equations of Section H1.1.

The strain compatibility method is a generalized approach that allows for the construction of an interaction diagram based upon the same concepts used for reinforced concrete design. Application of the strain compatibility method is required for irregular/nonsymmetrical sections, and its general application may be found in reinforced concrete design texts and will not be discussed further here.

Plastic stress distribution methods are discussed in AISC *Specification* Commentary Section I5 which provides three acceptable procedures for filled members. The first procedure, Method 1, invokes the interaction equations of

Section H1. This is the only method applicable to sections with noncompact or slender elements. The second procedure, Method 2, involves the construction of a piecewise-linear interaction curve using the plastic strength equations provided in Figure I.1c located within the front matter of the Chapter I Design Examples. The third procedure, Method 2 – Simplified, is a reduction of the piecewise-linear interaction curve that allows for the use of less conservative interaction equations than those presented in Chapter H.

For this design example, each of the three applicable plastic stress distribution procedures are reviewed and compared.

Method 1: Interaction Equations of Section H1

The most direct and conservative method of assessing interaction effects is through the use of the interaction equations of AISC *Specification* Section H1. For HSS shapes, both the available compressive and flexural strengths can be determined from *Manual* Table 4-14. In accordance with the direct analysis method, a K factor of 1 is used. Because the unbraced length is the same in both the x-x and y-y directions, and I_x exceeds I_y, y-y axis buckling will govern for the compressive strength. Flexural strength is determined for the x-x axis to resist the applied moment about this axis indicated in Figure I.6-1.

Entering Table 4-14 with KL_y = 14 ft yields:

LRFD	ASD
$\phi_c P_n = 354\text{ kips}$ $\phi_b M_{nx} = 130\text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{129\text{ kips}}{354\text{ kips}}$ $= 0.364 \ge 0.2$ Therefore, use AISC *Specification* Equation H1-1a. $\dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_u}{\phi_b M_n}\right) \le 1.0$ $\dfrac{129\text{ kips}}{354\text{ kips}} + \dfrac{8}{9}\left(\dfrac{120\text{ kip-ft}}{130\text{ kip-ft}}\right) \le 1.0$ $1.18 > 1.0$ **n.g.**	$P_n / \Omega_c = 236\text{ kips}$ $M_{nx} / \Omega_c = 86.6\text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_c}$ $= \dfrac{98.2\text{ kips}}{236\text{ kips}}$ $= 0.416 \ge 0.2$ Therefore, use AISC *Specification* Equation H1-1a. $\dfrac{P_a}{P_n / \Omega_c} + \dfrac{8}{9}\left(\dfrac{M_a}{M_n / \Omega_b}\right) \le 1.0$ $\dfrac{98.2\text{ kips}}{236\text{ kips}} + \dfrac{8}{9}\left(\dfrac{54.0\text{ kip-ft}}{86.6\text{ kip-ft}}\right) \le 1.0$ $0.97 < 1.0$ **o.k.**

Using LRFD methodology, Method 1 indicates that the section is inadequate for the applied loads. The designer can elect to choose a new section that passes the interaction check or re-analyze the current section using a less conservative design method such as Method 2. The use of Method 2 is illustrated in the following section.

Method 2: Interaction Curves from the Plastic Stress Distribution Model

The procedure for creating an interaction curve using the plastic stress distribution model is illustrated graphically in Figure I.6-2.

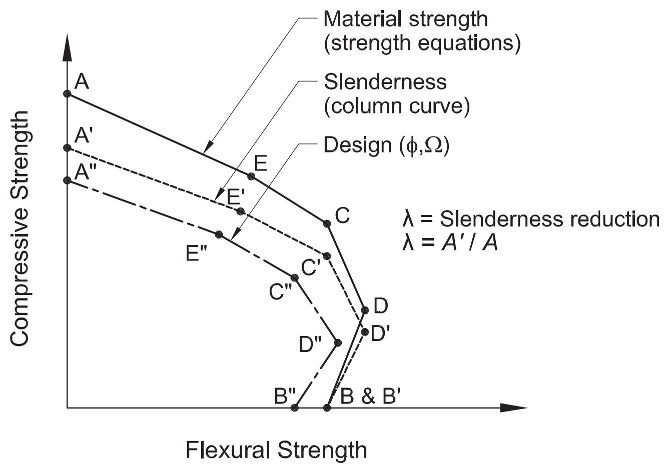


Fig. I.6-2. Interaction diagram for composite beam-column —Method 2.

Referencing Figure I.6-2, the nominal strength interaction surface A,B,C,D,E is first determined using the equations of Figure I-1c found in the introduction of the Chapter I Design Examples. This curve is representative of the short column member strength without consideration of length effects. A slenderness reduction factor, λ, is then calculated and applied to each point to create surface A′, B′, C′, D′, E′. The appropriate resistance or safety factors are then applied to create the design surface A″, B″, C″, D″, E″. Finally, the required axial and flexural strengths from the applicable load combinations of ASCE/SEI 7-10 are plotted on the design surface, and the member is acceptable for the applied loading if all points fall within the design surface. These steps are illustrated in detail by the following calculations.

Step 1: Construct nominal strength interaction surface A, B, C, D, E without length effects

Using the equations provided in Figure I-1c for bending about the *x-x* axis yields:

Point A (pure axial compression):

$$\begin{aligned}
P_A &= F_y A_s + 0.85 f_c' A_c \\
&= (46\text{ ksi})(10.4\text{ in.}^2) + 0.85(5\text{ ksi})(49.2\text{ in.}^2) \\
&= 688\text{ kips} \\
M_A &= 0\text{ kip-ft}
\end{aligned}$$

Point D (maximum nominal moment strength):

$$\begin{aligned}
P_D &= \frac{0.85 f_c' A_c}{2} \\
&= \frac{0.85(5\text{ ksi})(49.2\text{ in.}^2)}{2} \\
&= 105\text{ kips} \\
Z_{sx} &= 33.8\text{ in.}^3
\end{aligned}$$

$$Z_c = \frac{b_i h_i^2}{4} - 0.192 r_i^3 \quad \text{where } r_i = t$$

$$= \frac{(5.30 \text{ in.})(9.30 \text{ in.})^2}{4} - 0.192(0.349 \text{ in.})^3$$

$$= 115 \text{ in.}^3$$

$$M_D = F_y Z_{sx} + \frac{0.85 f'_c Z_c}{2}$$

$$= (46 \text{ ksi})(33.8 \text{ in.}^3) + \frac{0.85(5 \text{ ksi})(115 \text{ in.}^3)}{2}$$

$$= \frac{1{,}800 \text{ kip-in.}}{12 \text{ in./ft}}$$

$$= 150 \text{ kip-ft}$$

Point B (pure flexure):

$$P_B = 0 \text{ kips}$$

$$h_n = \frac{0.85 f'_c A_c}{2(0.85 f'_c b_i + 4tF_y)} \le \frac{h_i}{2}$$

$$= \frac{0.85(5 \text{ ksi})(49.2 \text{ in.}^2)}{2[0.85(5 \text{ ksi})(5.30 \text{ in.}) + 4(0.349 \text{ in.})(46 \text{ ksi})]} \le \frac{9.30 \text{ in.}}{2}$$

$$= 1.21 \text{ in.} \le 4.65 \text{ in.}$$

$$= 1.21 \text{ in.}$$

$$Z_{sn} = 2th_n^2$$

$$= 2(0.349 \text{ in.})(1.21 \text{ in.})^2$$

$$= 1.02 \text{ in.}^3$$

$$Z_{cn} = b_i h_n^2$$

$$= (5.30 \text{ in.})(1.21 \text{ in.})^2$$

$$= 7.76 \text{ in.}^3$$

$$M_B = M_D - F_y Z_{sn} - \frac{0.85 f'_c Z_{cn}}{2}$$

$$= 1{,}800 \text{ kip-in.} - (46 \text{ ksi})(1.02 \text{ in.}^3) - \frac{0.85(5 \text{ ksi})(7.76 \text{ in.}^3)}{2}$$

$$= \frac{1{,}740 \text{ kip-in.}}{12 \text{ in./ft}}$$

$$= 145 \text{ kip-ft}$$

Point C (intermediate point):

$$P_C = 0.85 f'_c A_c$$

$$= 0.85(5 \text{ ksi})(49.2 \text{ in.}^2)$$

$$= 209 \text{ kips}$$

$$M_C = M_B$$

$$= 145 \text{ kip-ft}$$

Point E (optional):

Point E is an optional point that helps better define the interaction curve.

$$h_E = \frac{h_n}{2} + \frac{H}{4} \quad \text{where } h_n = 1.21 \text{ in. from Point B}$$

$$= \frac{1.21 \text{ in.}}{2} + \frac{10.0 \text{ in.}}{4}$$

$$= 3.11 \text{ in.}$$

$$P_E = \frac{0.85 f'_c A_c}{2} + 0.85 f'_c b_i h_E + 4 F_y t h_E$$

$$= \frac{0.85(5 \text{ ksi})(49.2 \text{ in.}^2)}{2} + 0.85(5 \text{ ksi})(5.30 \text{ in.})(3.11 \text{ in.}) + 4(46 \text{ ksi})(0.349 \text{ in.})(3.11 \text{ in.})$$

$$= 374 \text{ kips}$$

$$Z_{cE} = b_i h_E^2$$

$$= (5.30 \text{ in.})(3.11 \text{ in.})^2$$

$$= 51.3 \text{ in.}^3$$

$$Z_{sE} = 2 t h_E^2$$

$$= 2(0.349 \text{ in.})(3.11 \text{ in.})^2$$

$$= 6.75 \text{ in.}^3$$

$$M_E = M_D - F_y Z_{sE} - \frac{0.85 f'_c Z_{cE}}{2}$$

$$= 1{,}800 \text{ kip-in.} - (46 \text{ ksi})(6.75 \text{ in.}^3) - \frac{0.85(5 \text{ ksi})(51.3 \text{ in.}^3)}{2}$$

$$= \frac{1{,}380 \text{ kip-in.}}{12 \text{ in./ft}}$$

$$= 115 \text{ kip-ft}$$

The calculated points are plotted to construct the nominal strength interaction surface without length effects as depicted in Figure I.6-3.

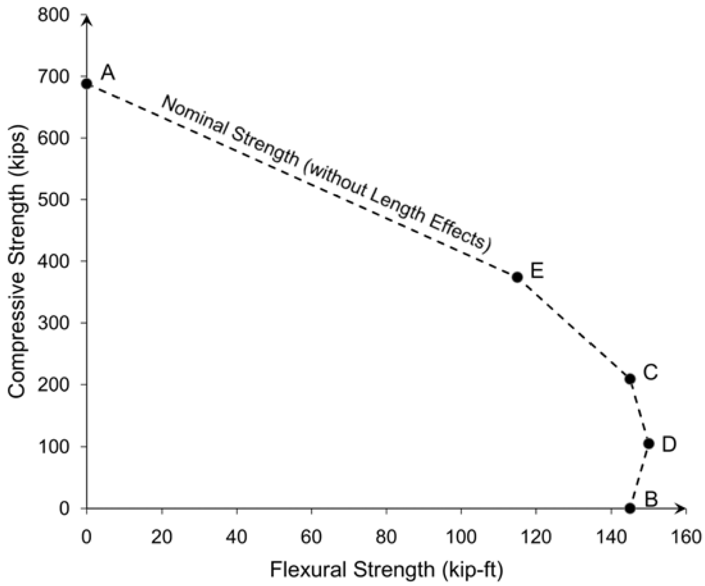


Fig. I.6-3. Nominal strength interaction surface without length effects.

Step 2: Construct nominal strength interaction surface A′, B′, C′, D′, E′ with length effects

The slenderness reduction factor, λ, is calculated for Point A using AISC *Specification* Section I2.2 in accordance with *Specification* Commentary Section I5.

$$P_{no} = P_A$$
$$= 688 \text{ kips}$$

$$C_3 = 0.6 + 2\left(\frac{A_s}{A_c + A_s}\right) \leq 0.9 \qquad \text{(Spec. Eq. I2-13)}$$

$$= 0.6 + 2\left(\frac{10.4 \text{ in.}^2}{49.2 \text{ in.}^2 + 10.4 \text{ in.}^2}\right) \leq 0.9$$

$$= 0.949 > 0.9 \qquad \textbf{0.9 controls}$$

$$EI_{eff} = E_s I_{sy} + E_s I_{sr} + C_3 E_c I_{cy} \qquad \text{(from Spec. Eq. I2-12)}$$

$$= (29{,}000 \text{ ksi})(61.8 \text{ in.}^4) + 0 + 0.9(3{,}900 \text{ ksi})(115 \text{ in.}^4)$$

$$= 2{,}200{,}000 \text{ ksi}$$

$$P_e = \pi^2 (EI_{eff}) / (KL)^2 \text{ where } K = 1.0 \text{ in accordance with the direct analysis method} \qquad \text{(Spec. Eq. I2-5)}$$

$$= \frac{\pi^2 (2{,}200{,}000 \text{ ksi})}{\left[(14.0 \text{ ft})(12 \text{ in./ft})\right]^2}$$

$$= 769 \text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{688 \text{ kips}}{769 \text{ kips}}$$

$$= 0.895 < 2.25$$

Use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left[0.658^{\frac{P_{no}}{P_e}}\right] \qquad \text{(\textit{Spec}. Eq. I2-2)}$$

$$= 688 \text{ kips}(0.658)^{0.895}$$
$$= 473 \text{ kips}$$

$$\lambda = \frac{P_n}{P_{no}}$$
$$= \frac{473 \text{ kips}}{688 \text{ kips}}$$
$$= 0.688$$

In accordance with AISC *Specification* Commentary Section I5, the same slenderness reduction is applied to each of the remaining points on the interaction surface as follows:

$$P_{A'} = \lambda P_A$$
$$= 0.688(688 \text{ kips})$$
$$= 473 \text{ kips}$$
$$P_{B'} = \lambda P_B$$
$$= 0.688(0 \text{ kips})$$
$$= 0 \text{ kips}$$
$$P_{C'} = \lambda P_C$$
$$= 0.688(209 \text{ kips})$$
$$= 144 \text{ kips}$$
$$P_{D'} = \lambda P_D$$
$$= 0.688(105 \text{ kips})$$
$$= 72.2 \text{ kips}$$
$$P_{E'} = \lambda P_E$$
$$= 0.688(374 \text{ kips})$$
$$= 257 \text{ kips}$$

The modified axial strength values are plotted with the flexural strength values previously calculated to construct the nominal strength interaction surface including length effects. These values are superimposed on the nominal strength surface not including length effects for comparison purposes in Figure I.6-4.

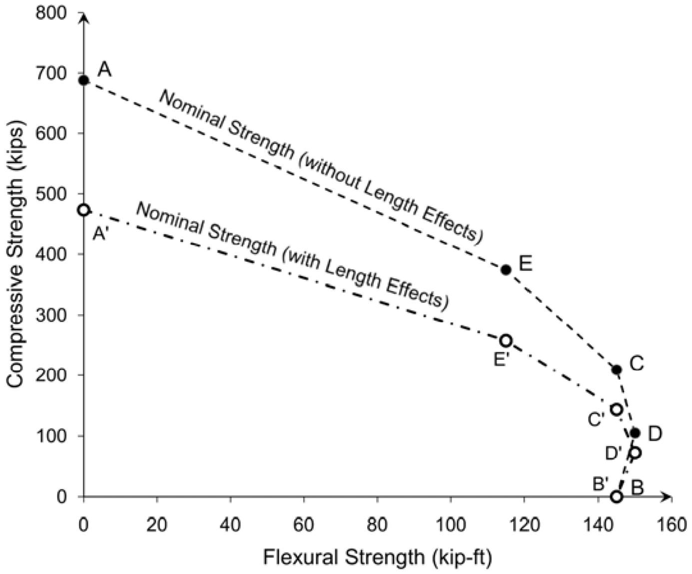


Fig. I.6-4. Nominal strength interaction surfaces (with and without length effects).

Step 3: Construct design interaction surface A″, B″, C″, D″, E″ and verify member adequacy

The final step in the Method 2 procedure is to reduce the interaction surface for design using the appropriate resistance or safety factors.

LRFD	ASD
Design compressive strength: $\phi_c = 0.75$ $P_{X''} = \phi_c P_{X'}$ where X = A, B, C, D or E $P_{A''} = 0.75(473 \text{ kips})$ $= 355 \text{ kips}$ $P_{B''} = 0.75(0 \text{ kips})$ $= 0 \text{ kips}$ $P_{C''} = 0.75(144 \text{ kips})$ $= 108 \text{ kips}$ $P_{D''} = 0.75(72.2 \text{ kips})$ $= 54.2 \text{ kips}$ $P_{E''} = 0.75(257 \text{ kips})$ $= 193 \text{ kips}$ Design flexural strength: $\phi_b = 0.90$ $M_{X''} = \phi_b M_{X'}$ where X = A, B, C, D or E	Allowable compressive strength: $\Omega_c = 2.00$ $P_{X''} = P_{X'} / \Omega_c$ where X = A, B, C, D or E $P_{A''} = 473 \text{ kips} / 2.00$ $= 237 \text{ kips}$ $P_{B''} = 0 \text{ kips} / 2.00$ $= 0 \text{ kips}$ $P_{C''} = 144 \text{ kips} / 2.00$ $= 72 \text{ kips}$ $P_{D''} = 72.2 \text{ kips} / 2.00$ $= 36.1 \text{ kips}$ $P_{E''} = 257 \text{ kips} / 2.00$ $= 129 \text{ kips}$ Allowable flexural strength: $\Omega_b = 1.67$ $M_{X''} = M_{X'} / \Omega_b$ where X = A, B, C, D or E

LRFD	ASD
$M_{A''} = 0.90(0 \text{ kip-ft})$ $= 0 \text{ kip-ft}$ $M_{B''} = 0.90(145 \text{ kip-ft})$ $= 131 \text{ kip-ft}$ $M_{C''} = 0.90(145 \text{ kip-ft})$ $= 131 \text{ kip-ft}$ $M_{D''} = 0.90(150 \text{ kip-ft})$ $= 135 \text{ kip-ft}$ $M_{E''} = 0.90(115 \text{ kip-ft})$ $= 104 \text{ kip-ft}$	$M_{A''} = 0 \text{ kip-ft} / 1.67$ $= 0 \text{ kip-ft}$ $M_{B''} = 145 \text{ kip-ft} / 1.67$ $= 86.8 \text{ kip-ft}$ $M_{C''} = 145 \text{ kip-ft} / 1.67$ $= 86.8 \text{ kip-ft}$ $M_{D''} = 150 \text{ kip-ft} / 1.67$ $= 89.8 \text{ kip-ft}$ $M_{E''} = 115 \text{ kip-ft} / 1.67$ $= 68.9 \text{ kip-ft}$

The available strength values for each design method can now be plotted. These values are superimposed on the nominal strength surfaces (with and without length effects) previously calculated for comparison purposes in Figure I.6-5.

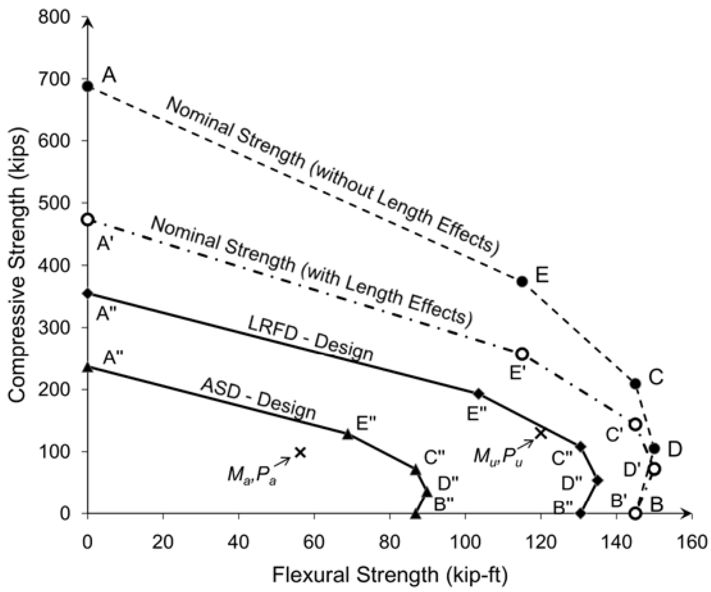


Fig. I.6-5. Available and nominal interaction surfaces.

By plotting the required axial and flexural strength values determined for the governing load combinations on the available strength surfaces indicated in Figure I.6-5, it can be seen that both ASD (M_a, P_a) and LRFD (M_u, P_u) points lie within their respective design surfaces. The member in question is therefore adequate for the applied loads.

Designers should carefully review the proximity of the available strength values in relation to point D'' on Figure I.6-5 as it is possible for point D'' to fall outside of the nominal strength curve, thus resulting in an unsafe design. This possibility is discussed further in AISC Commentary Section I5 and is avoided through the use of Method 2 – Simplified as illustrated in the following section.

Method 2: Simplified

The simplified version of Method 2 involves the removal of points D'' and E'' from the Method 2 interaction surface leaving only points A'', B'' and C'' as illustrated in the comparison of the two methods in Figure I.6-6.

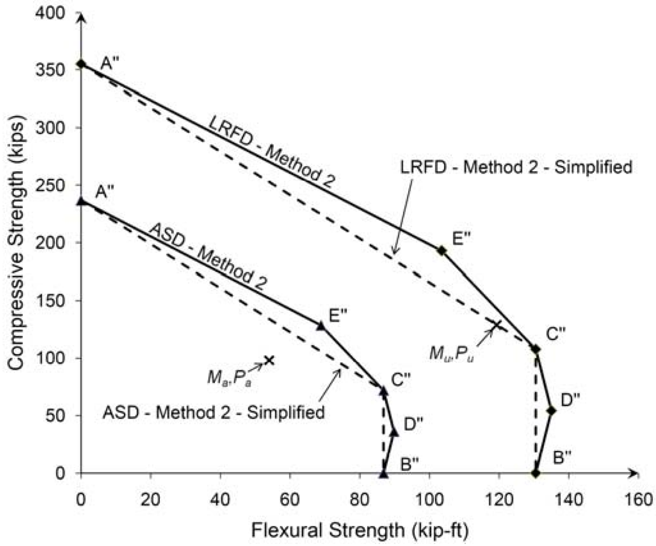


Fig. I.6-6. Comparison of Method 2 and Method 2 – Simplified.

Reducing the number of interaction points allows for a bilinear interaction check defined by AISC *Specification* Commentary Equations C-I5-1a and C-I5-1b to be performed. Using the available strength values previously calculated in conjunction with the Commentary equations, interaction ratios are determined as follows:

LRFD	ASD
$P_r = P_u$ $= 129 \text{ kips}$ $P_r \geq P_{C''}$ $\geq 108 \text{ kips}$	$P_r = P_a$ $= 98.2 \text{ kips}$ $P_r \geq P_{C''}$ $\geq 72 \text{ kips}$
Use AISC *Specification* Commentary Equation C-I5-1b.	Use AISC *Specification* Commentary Equation C-I5-1b.
$\dfrac{P_r - P_C}{P_A - P_C} + \dfrac{M_r}{M_C} \leq 1.0$	$\dfrac{P_r - P_C}{P_A - P_C} + \dfrac{M_r}{M_C} \leq 1.0$
which for LRFD equals: $\dfrac{P_u - P_{C''}}{P_{A''} - P_{C''}} + \dfrac{M_u}{M_{C''}} \leq 1.0$ $\dfrac{129 \text{ kips} - 108 \text{ kips}}{355 \text{ kips} - 108 \text{ kips}} + \dfrac{120 \text{ kip-ft}}{131 \text{ kip-ft}} \leq 1.0$ $1.00 = 1.0$ **o.k.**	which for ASD equals: $\dfrac{P_a - P_{C''}}{P_{A''} - P_{C''}} + \dfrac{M_a}{M_{C''}} \leq 1.0$ $\dfrac{98.2 \text{ kips} - 72.0 \text{ kips}}{237 \text{ kips} - 72.0 \text{ kips}} + \dfrac{54.0 \text{ kip-ft}}{86.8 \text{ kip-ft}} \leq 1.0$ $0.781 < 1.0$ **o.k.**

Thus, the member is adequate for the applied loads.

Comparison of Methods

The composite member was found to be inadequate using Method 1—Chapter H interaction equations, but was found to be adequate using both Method 2 and Method 2—Simplified procedures. A comparison between the methods is most easily made by overlaying the design curves from each method as illustrated in Figure I.6-7 for LRFD design.

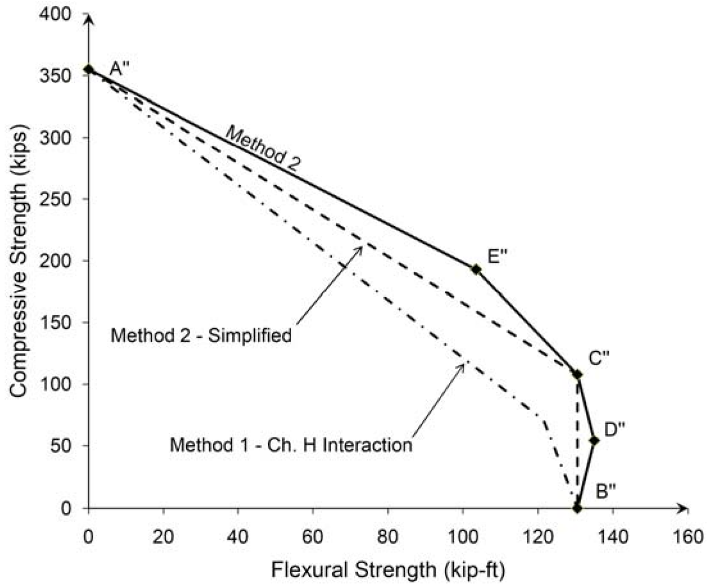


Fig. I.6-7. Comparison of interaction methods (LRFD).

From Figure I.6-7, the conservative nature of the Chapter H interaction equations can be seen. Method 2 provides the highest available strength; however, the Method 2—Simplified procedure also provides a good representation of the complete design curve. By using Part 4 of the AISC *Manual* to determine the available strength of the composite member in compression and flexure (Points A'' and B'' respectively), the modest additional effort required to calculate the available compressive strength at Point C'' can result in appreciable gains in member strength when using Method 2—Simplified as opposed to Method 1.

Available Shear Strength

AISC *Specification* Section I4.1 provides three methods for determining the available shear strength of a filled member: available shear strength of the steel section alone in accordance with Chapter G, available shear strength of the reinforced concrete portion alone per ACI 318, or available shear strength of the steel section plus the reinforcing steel ignoring the contribution of the concrete.

Available Shear Strength of Steel Section

From AISC *Specification* Section G5, the nominal shear strength, V_n, of HSS members is determined using the provisions of Section G2.1(b) with $k_v = 5$. The provisions define the width of web resisting the shear force, h, as the outside dimension minus three times the design wall thickness.

$$
\begin{aligned}
h &= H - 3t \\
&= 10.0 \text{ in.} - 3(0.349 \text{ in.}) \\
&= 8.95 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
A_w &= 2ht \\
&= 2(8.95 \text{ in.})(0.349 \text{ in.}) \\
&= 6.25 \text{ in.}^2
\end{aligned}
$$

The slenderness value, h/t_w, used to determine the web shear coefficient, C_v, is provided in AISC *Manual* Table 1-11 as 25.7.

$$\frac{h}{t_w} \le 1.10\sqrt{k_v E/F_y}$$

$$\le 1.10\sqrt{5\left(\frac{29{,}000 \text{ ksi}}{46 \text{ ksi}}\right)}$$

$$25.7 < 61.8$$

Use AISC *Specification* Equation G2-3.

(*Spec*. Eq. G2-3)

$$C_v = 1.0$$

The nominal shear strength is calculated as:

$$V_n = 0.6F_y A_w C_v \quad \text{(Spec. Eq. G2-1)}$$
$$= 0.6(46 \text{ ksi})(6.25 \text{ in.}^2)(1.0)$$
$$= 173 \text{ kips}$$

The available shear strength of the steel section is:

LRFD	ASD
$V_u = 17.1$ kips $\phi_v = 0.90$ $\phi_v V_n \ge V_u$ $\phi_v V_n = 0.90(173 \text{ kips})$ $= 156 \text{ kips} > 17.1 \text{ kips}$ **o.k.**	$V_a = 10.3$ kips $\Omega_v = 1.67$ $V_n / \Omega_v \ge V_a$ $V_n / \Omega_v = \dfrac{173 \text{ kips}}{1.67}$ $= 104 \text{ kips} > 10.3 \text{ kips}$ **o.k.**

Available Shear Strength of the Reinforced Concrete

The available shear strength of the steel section alone has been shown to be sufficient, but the available shear strength of the concrete will be calculated for demonstration purposes. Considering that the member does not have longitudinal reinforcing, the method of shear strength calculation involving reinforced concrete is not valid; however, the design shear strength of the plain concrete using Chapter 22 of ACI 318 can be determined as follows:

ϕ = 0.60 for plain concrete design from ACI 318 Section 9.3.5
λ = 1.0 for normal weight concrete from ACI 318 Section 8.6.1

$$V_n = \left(\frac{4}{3}\right)\lambda\sqrt{f'_c}\,b_w h \quad \text{(ACI 318 Eq. 22-9)}$$

$$b_w = b_i$$
$$h = h_i$$

$$V_n = \left(\frac{4}{3}\right)(1.0)\sqrt{5{,}000 \text{ psi}}\,(5.30 \text{ in.})(9.30 \text{ in.})\left(\frac{1 \text{ kip}}{1{,}000 \text{ lb}}\right)$$
$$= 4.65 \text{ kips}$$

$$\phi V_n = 0.60(4.65 \text{ kips})$$
$$= 2.79 \text{ kips}$$

$$\phi V_n \ge V_u \quad \text{(ACI 318 Eq. 22-8)}$$

$2.79 \text{ kips} < 17.1 \text{ kips}$ **n.g.**

As can be seen from this calculation, the shear resistance provided by plain concrete is small and the strength of the steel section alone is generally sufficient.

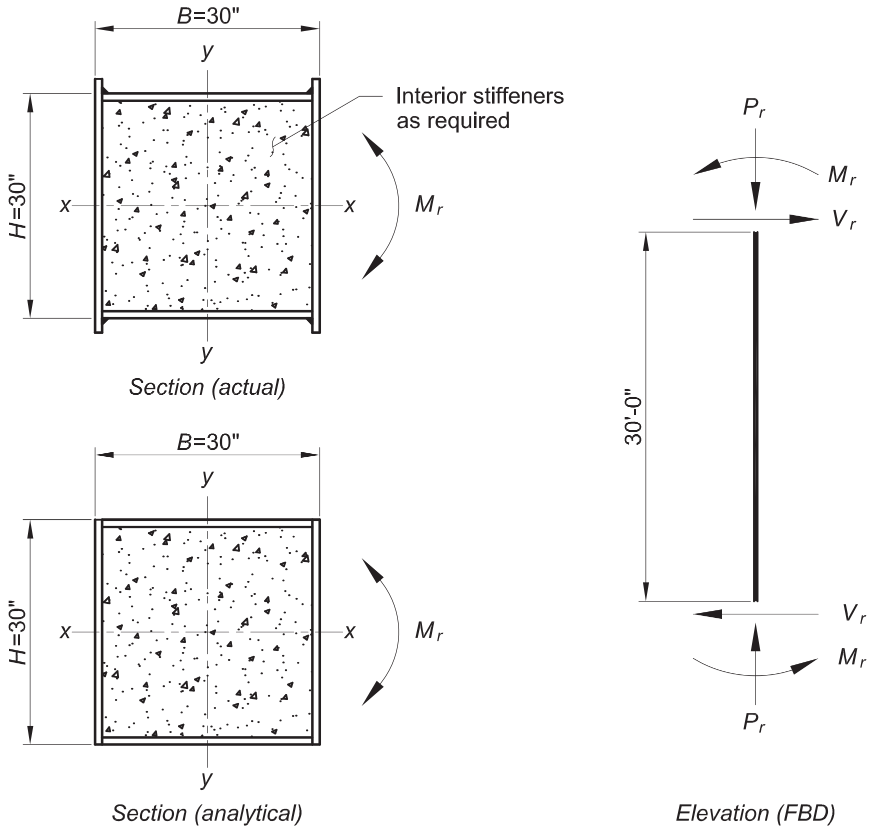
Force Allocation and Load Transfer

Load transfer calculations for applied axial forces should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions is provided in Design Example I.3.

EXAMPLE I.7 FILLED BOX COLUMN WITH NONCOMPACT/SLENDER ELEMENTS

Given:

Determine the required ASTM A36 plate thickness of the 30 ft long, composite box column illustrated in Figure I.7-1 to resist the indicated axial forces, shears and moments that have been determined in accordance with the direct analysis method of AISC *Specification* Chapter C for the controlling ASCE/SEI 7-10 load combinations. The core is composed of normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, $f_c' = 7$ ksi.



	LRFD	ASD
P_r (kips)	1,310	1,370
M_r (kip-ft)	552	248
V_r (kips)	36.8	22.1

Fig. I.7-1. Composite box column section and member forces.

Solution:

From AISC *Manual* Table 2-4, the material properties are:

ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

Trial Size 1 (Noncompact)

For ease of calculation the contribution of the plate extensions to the member strength will be ignored as illustrated by the analytical model in Figure I.7-1.

Trial Plate Thickness and Geometric Section Properties of the Composite Member

Select a trial plate thickness, t, of ⅜ in. Note that the design wall thickness reduction of AISC *Specification* Section B4.2 applies only to electric-resistance-welded HSS members and does not apply to built-up sections such as the one under consideration.

The calculated geometric properties of the 30 in. by 30 in. steel box column are:

$$B = 30.0 \text{ in.}$$
$$H = 30.0 \text{ in.}$$
$$b_i = B - 2t = 29.25 \text{ in.}$$
$$h_i = H - 2t = 29.25 \text{ in.}$$

$$A_g = 900 \text{ in.}^2$$
$$A_c = 856 \text{ in.}^2$$
$$A_s = 44.4 \text{ in.}^2$$

$$\begin{aligned} E_c &= w_c^{1.5}\sqrt{f_c'} \\ &= \left(145 \text{ lb/ft}^3\right)^{1.5}\sqrt{7 \text{ ksi}} \\ &= 4{,}620 \text{ ksi} \end{aligned}$$

$$\begin{aligned} I_{gx} &= BH^3/12 \\ &= 67{,}500 \text{ in.}^4 \end{aligned}$$

$$\begin{aligned} I_{cx} &= b_i h_i^3/12 \\ &= 61{,}000 \text{ in.}^4 \end{aligned}$$

$$\begin{aligned} I_{sx} &= I_{gx} - I_{cx} \\ &= 6{,}500 \text{ in.}^4 \end{aligned}$$

Limitations of AISC Specification Sections I1.3 and I2.2a

(1) Concrete Strength: $3 \text{ ksi} \le f_c' \le 10 \text{ ksi}$

$f_c' = 7 \text{ ksi}$ **o.k.**

(2) Specified minimum yield stress of structural steel: $F_y \le 75 \text{ ksi}$

$F_y = 36 \text{ ksi}$ **o.k.**

(3) Cross-sectional area of steel section: $A_s \ge 0.01A_g$

$$\begin{aligned} 44.4 \text{ in.}^2 &\ge (0.01)\left(900 \text{ in.}^2\right) \\ &> 9.00 \text{ in.}^2 \quad \textbf{o.k.} \end{aligned}$$

Classify Section for Local Buckling

Classification of the section for local buckling is performed in accordance with AISC *Specification* Table I1.1A for compression and Table I1.1B for flexure. As noted in *Specification* Section I1.4, the definitions of width, depth and thickness used in the evaluation of slenderness are provided in Tables B4.1a and B4.1b.

For box columns, the widths of the stiffened compression elements used for slenderness checks, b and h, are equal to the clear distances between the column walls, b_i and h_i. The slenderness ratios are determined as follows:

$$\begin{aligned} \lambda &= \frac{b_i}{t} \\ &= \frac{h_i}{t} \\ &= \frac{29.25 \text{ in.}}{3/8 \text{ in.}} \\ &= 78.0 \end{aligned}$$

Classify section for local buckling in steel elements subject to axial compression from AISC *Specification* Table I1.1A:

$$\lambda_p = 2.26\sqrt{\frac{E}{F_y}}$$
$$= 2.26\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 64.1$$
$$\lambda_r = 3.00\sqrt{\frac{E}{F_y}}$$
$$= 3.00\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 85.1$$

$\lambda_p \le \lambda \le \lambda_r$
$64.1 \le 78.0 \le 85.1$; therefore, the section is noncompact for compression.

According to AISC *Specification* Section I1.4, if any side of the section in question is noncompact or slender, then the entire section is treated as noncompact or slender. For the square section under investigation; however, this distinction is unnecessary as all sides are equal in length.

Classification of the section for local buckling in elements subject to flexure is performed in accordance with AISC *Specification* Table I1.1B. Note that flanges and webs are treated separately; however, for the case of a square section only the most stringent limitations, those of the flange, need be applied. Noting that the flange limitations for bending are the same as those for compression,

$\lambda_p \le \lambda \le \lambda_r$
$64.1 \le 78.0 \le 85.1$; therefore, the section is noncompact for flexure

Available Compressive Strength

Compressive strength for noncompact filled members is determined in accordance with AISC *Specification* Section I2.2b(b).

$$P_p = F_y A_s + C_2 f_c'\left(A_c + A_{sr}\frac{E_s}{E_c}\right) \text{ where } C_2 = 0.85 \text{ for rectangular sections} \quad (\textit{Spec}. \text{ Eq. I2-9b})$$
$$= (36 \text{ ksi})(44.4 \text{ in.}^2) + 0.85(7 \text{ ksi})(856 \text{ in.}^2 + 0)$$
$$= 6{,}690 \text{ kips}$$
$$P_y = F_y A_s + 0.7 f_c'\left(A_c + A_{sr}\frac{E_s}{E_c}\right) \quad (\textit{Spec}. \text{ Eq. I2-9d})$$
$$= (36 \text{ ksi})(44.4 \text{ in.}^2) + 0.7(7 \text{ ksi})(856 \text{ in.}^2 + 0)$$
$$= 5{,}790 \text{ kips}$$
$$P_{no} = P_p - \frac{P_p - P_y}{(\lambda_r - \lambda_p)^2}(\lambda - \lambda_p)^2 \quad (\textit{Spec}. \text{ Eq. I2-9c})$$
$$= 6{,}690 \text{ kips} - \frac{6{,}690 \text{ kips} - 5{,}790 \text{ kips}}{(85.1 - 64.1)^2}(78.0 - 64.1)^2$$
$$= 6{,}300 \text{ kips}$$

$$C_3 = 0.6 + 2\left(\frac{A_s}{A_c + A_s}\right) \leq 0.9$$ (*Spec*. Eq. I2-13)

$$= 0.6 + 2\left(\frac{44.4 \text{ in.}^2}{856 \text{ in.}^2 + 44.4 \text{ in.}^2}\right) \leq 0.9$$

$$= 0.699 \leq 0.9$$

$$EI_{eff} = E_s I_s + E_s I_{sr} + C_3 E_c I_c$$ (*Spec*. Eq. I2-12)

$$= (29{,}000 \text{ ksi})(6{,}500 \text{ in.}^4) + 0.0 + 0.699(4{,}620 \text{ ksi})(61{,}000 \text{ in.}^4)$$

$$= 385{,}000{,}000 \text{ ksi}$$

$$P_e = \pi^2 (EI_{eff})/(KL)^2 \text{ where } K\text{=1.0 in accordance with the direct analysis method}$$ (*Spec*. Eq. I2-5)

$$= \frac{\pi^2 (385{,}000{,}000 \text{ ksi})}{\left[(30.0 \text{ ft})(12 \text{ in./ft})\right]^2}$$

$$= 29{,}300 \text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{6{,}300 \text{ kips}}{29{,}300 \text{ kips}}$$

$$= 0.215 < 2.25$$

Therefore, use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left[0.658^{\frac{P_{no}}{P_e}}\right]$$ (*Spec*. Eq. I2-2)

$$= 6{,}300 \text{ kips}(0.658)^{0.215}$$

$$= 5{,}760 \text{ kips}$$

According to AISC *Specification* Section I2.2b, the compression strength need not be less than that specified for the bare steel member as determined by *Specification* Chapter E. It can be shown that the compression strength of the bare steel for this section is equal to 955 kips, thus the strength of the composite section controls.

The available compressive strength is:

LRFD	ASD
$\phi_c = 0.75$ $\phi_c P_n = 0.75(5{,}760 \text{ kips})$ $= 4{,}320 \text{ kips}$	$\Omega_c = 2.00$ $P_n / \Omega_c = 5{,}760 \text{ kips/2.00}$ $= 2{,}880 \text{ kips}$

Available Flexural Strength

Flexural strength of noncompact filled composite members is determined in accordance with AISC *Specification* Section I3.4b(b):

$$M_n = M_p - (M_p - M_y)\frac{(\lambda - \lambda_p)}{(\lambda_r - \lambda_p)}$$ (*Spec*. Eq. I3-3b)

In order to utilize Equation I3-3b, both the plastic moment strength of the section, M_p, and the yield moment strength of the section, M_y, must be calculated.

Plastic Moment Strength

The first step in determining the available flexural strength of a noncompact section is to calculate the moment corresponding to the plastic stress distribution over the composite cross section. This concept is illustrated graphically in AISC *Specification* Commentary Figure C-I3.7(a) and follows the force distribution depicted in Figure I.7-2 and detailed in Table I.7-1.

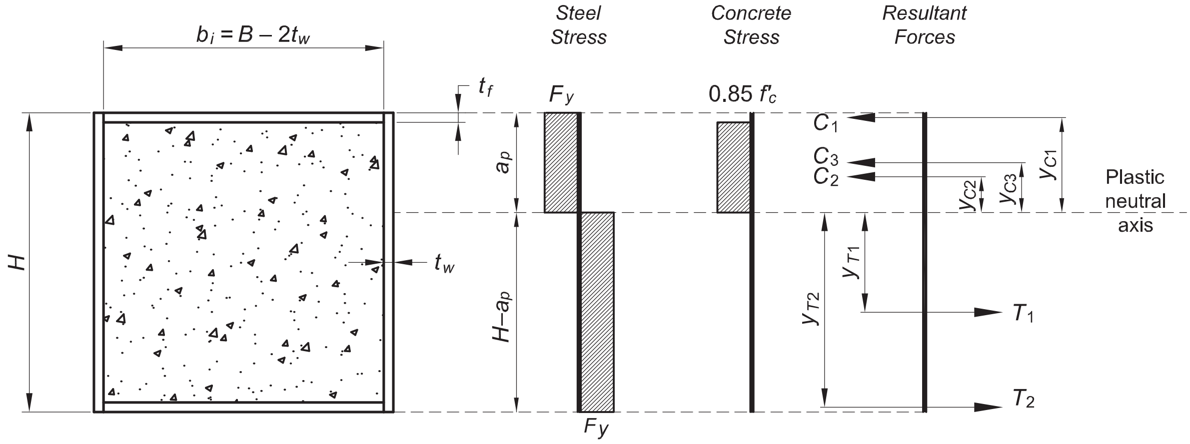


Figure I.7-2. Plastic moment stress blocks and force distribution.

Table I.7-1. Plastic Moment Equations

Component	Force	Moment Arm
Compression in steel flange	$C_1 = b_i t_f F_y$	$y_{C1} = a_p - \dfrac{t_f}{2}$
Compression in concrete	$C_2 = 0.85 f_c'\left(a_p - t_f\right)b_i$	$y_{C2} = \dfrac{a_p - t_f}{2}$
Compression in steel web	$C_3 = a_p 2t_w F_y$	$y_{C3} = \dfrac{a_p}{2}$
Tension in steel web	$T_1 = \left(H - a_p\right)2t_w F_y$	$y_{T1} = \dfrac{H - a_p}{2}$
Tension in steel flange	$T_2 = b_i t_f F_y$	$y_{T2} = H - a_p - \dfrac{t_f}{2}$

where:

$$a_p = \frac{2F_y H t_w + 0.85 f_c' b_i t_f}{4t_w F_y + 0.85 f_c' b_i}$$

$$M_p = \sum\left(\text{force}\right)\left(\text{moment arm}\right)$$

Using the equations provided in Table I.7-1 for the section in question results in the following:

$$a_p = \frac{2\left(36\text{ ksi}\right)\left(30.0\text{ in.}\right)\left(\tfrac{3}{8}\text{ in.}\right) + 0.85\left(7\text{ ksi}\right)\left(29.25\text{ in.}\right)\left(\tfrac{3}{8}\text{ in.}\right)}{4\left(\tfrac{3}{8}\text{ in.}\right)\left(36\text{ ksi}\right) + 0.85\left(7\text{ ksi}\right)\left(29.25\text{ in.}\right)}$$

$$= 3.84\text{ in.}$$

Force	Moment Arm	Force × Moment Arm
$C_1 = (29.25\text{ in.})(\tfrac{3}{8}\text{ in.})(36\text{ ksi})$ $= 395\text{ kips}$	$y_{C1} = 3.84\text{ in.} - \dfrac{\tfrac{3}{8}\text{ in.}}{2}$ $= 3.65\text{ in.}$	$C_1 y_{C1} = 1{,}440\text{ kip-in.}$
$C_2 = 0.85(7\text{ ksi})(3.84\text{ in.} - \tfrac{3}{8}\text{ in.})(29.25\text{ in.})$ $= 603\text{ kips}$	$y_{C2} = \dfrac{3.84\text{ in.} - \tfrac{3}{8}\text{ in.}}{2}$ $= 1.73\text{ in.}$	$C_2 y_{C2} = 1{,}040\text{ kip-in.}$
$C_3 = (3.84\text{ in.})(2)(\tfrac{3}{8}\text{ in.})(36\text{ ksi})$ $= 104\text{ kips}$	$y_{C3} = \dfrac{3.84\text{ in.}}{2}$ $= 1.92\text{ in.}$	$C_3 y_{C3} = 200\text{ kip-in.}$
$T_1 = (30.0\text{ in.} - 3.84\text{ in.})(2)(\tfrac{3}{8}\text{ in.})(36\text{ ksi})$ $= 706\text{ kips}$	$y_{T1} = \dfrac{30.0\text{ in.} - 3.84\text{ in.}}{2}$ $= 13.1\text{ in.}$	$T_1 y_{T1} = 9{,}250\text{ kip-in.}$
$T_2 = (29.25\text{ in.})(\tfrac{3}{8}\text{ in.})(36\text{ ksi})$ $= 395\text{ kips}$	$y_{T2} = 30.0\text{ in.} - 3.84\text{ in.} - \dfrac{\tfrac{3}{8}\text{ in.}}{2}$ $= 26.0\text{ in.}$	$T_2 y_{T2} = 10{,}300\text{ kip-in.}$

$$
\begin{aligned}
M_p &= \sum(\text{force})(\text{moment arm}) \\
&= \frac{1{,}440\text{ kip-in.} + 1{,}040\text{ kip-in.} + 200\text{ kip-in.} + 9{,}250\text{ kip-in.} + 10{,}300\text{ kip-in.}}{12\text{ in./ft}} \\
&= 1{,}850\text{ kip-ft}
\end{aligned}
$$

Yield Moment Strength

The next step in determining the available flexural strength of a noncompact filled member is to determine the yield moment strength. The yield moment is defined in AISC *Specification* Section I3.4b(b) as the moment corresponding to first yield of the compression flange calculated using a linear elastic stress distribution with a maximum concrete compressive stress of $0.7 f_c'$. This concept is illustrated diagrammatically in *Specification* Commentary Figure C-I3.7(b) and follows the force distribution depicted in Figure I.7-3 and detailed in Table I.7-2.

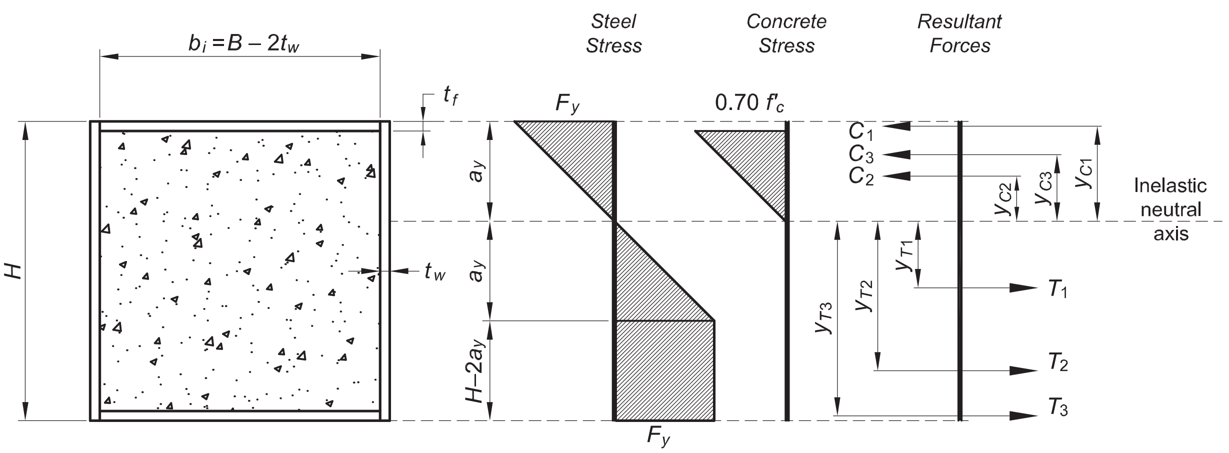


Figure I.7-3. Yield moment stress blocks and force distribution.

Table I.7-2. Yield Moment Equations

Component	Force	Moment Arm
Compression in steel flange	$C_1 = b_i t_f F_y$	$y_{C1} = a_y - \dfrac{t_f}{2}$
Compression in concrete	$C_2 = 0.35 f_c' \left(a_y - t_f\right) b_i$	$y_{C2} = \dfrac{2\left(a_y - t_f\right)}{3}$
Compression in steel web	$C_3 = a_y 2t_w 0.5F_y$	$y_{C3} = \dfrac{2a_y}{3}$
Tension in steel web	$T_1 = a_y 2t_w 0.5F_y$ $T_2 = \left(H - 2a_y\right) 2t_w F_y$	$y_{T1} = \dfrac{2a_y}{3}$ $y_{T2} = \dfrac{H}{2}$
Tension in steel flange	$T_3 = b_i t_f F_y$	$y_{T3} = H - a_y - \dfrac{t_f}{2}$

where:

$$a_y = \frac{2F_y H t_w + 0.35 f_c' b_i t_f}{4t_w F_y + 0.35 f_c' b_i}$$

$$M_y = \sum(\text{force})(\text{moment arm})$$

Using the equations provided in Table I.7-2 for the section in question results in the following:

$$\begin{aligned} a_y &= \frac{2(36\text{ ksi})(30.0\text{ in.})(\tfrac{3}{8}\text{ in.}) + 0.35(7\text{ ksi})(29.25\text{ in.})(\tfrac{3}{8}\text{ in.})}{4(\tfrac{3}{8}\text{ in.})(36\text{ ksi}) + 0.35(7\text{ ksi})(29.25\text{ in.})} \\ &= 6.66\text{ in.} \end{aligned}$$

Force	Moment Arm	Force × Moment Arm
$C_1 = (29.25\text{ in.})(\tfrac{3}{8}\text{ in.})(36\text{ ksi})$ $= 395\text{ kips}$	$y_{C1} = 6.66\text{ in.} - \dfrac{\tfrac{3}{8}\text{ in.}}{2}$ $= 6.47\text{ in.}$	$C_1 y_{C1} = 2{,}560\text{ kip-in.}$
$C_2 = 0.35(7\text{ ksi})(6.66\text{ in.} - \tfrac{3}{8}\text{ in.})(29.25\text{ in.})$ $= 450\text{ kips}$	$y_{C2} = \dfrac{2(6.66\text{ in.} - \tfrac{3}{8}\text{ in.})}{3}$ $= 4.19\text{ in.}$	$C_2 y_{C2} = 1{,}890\text{ kip-in.}$
$C_3 = (6.66\text{ in.})(2)(\tfrac{3}{8}\text{ in.})(0.5)(36\text{ ksi})$ $= 89.9\text{ kips}$	$y_{C3} = \dfrac{2(6.66\text{ in.})}{3}$ $= 4.44\text{ in.}$	$C_3 y_{C3} = 399\text{ kip-in.}$
$T_1 = (6.66\text{ in.})(2)(\tfrac{3}{8}\text{ in.})(0.5)(36\text{ ksi})$ $= 89.9\text{ kips}$	$y_{T1} = \dfrac{2(6.66\text{ in.})}{3}$ $= 4.44\text{ in.}$	$T_1 y_{T1} = 399\text{ kip-in.}$
$T_2 = \left[30.0 - 2(6.66\text{ in.})\right](2)(\tfrac{3}{8}\text{ in.})(36\text{ ksi})$ $= 450\text{ kips}$	$y_{T2} = \dfrac{30.0\text{ in.}}{2}$ $= 15.0\text{ in.}$	$T_2 y_{T2} = 6{,}750\text{ kip-in.}$
$T_3 = (29.25\text{ in.})(\tfrac{3}{8}\text{ in.})(36\text{ ksi})$ $= 395\text{ kips}$	$y_{T3} = 30.0\text{ in.} - 6.66\text{ in.} - \dfrac{\tfrac{3}{8}\text{ in.}}{2}$ $= 23.2\text{ in.}$	$T_3 y_{T3} = 9{,}160\text{ kip-in.}$

$$M_y = \sum(\text{force})(\text{moment arm})$$
$$= \frac{2{,}560 \text{ kip-in.} + 1{,}890 \text{ kip-in.} + 399 \text{ kip-in.} + 399 \text{ kip-in.} + 6{,}750 \text{ kip-in.} + 9{,}160 \text{ kip-in.}}{12 \text{ in./ft}}$$
$$= 1{,}760 \text{ kip-ft}$$

Now that both M_p and M_y have been determined, Equation I3-3b may be used in conjunction with the flexural slenderness values previously calculated to determine the nominal flexural strength of the composite section as follows:

$$M_n = M_p - (M_p - M_y)\frac{(\lambda - \lambda_p)}{(\lambda_r - \lambda_p)}$$ (*Spec*. Eq. I3-3b)

$$M_n = 1{,}850 \text{ kip-ft} - (1{,}850 \text{ kip-ft} - 1{,}760 \text{ kip-ft})\frac{(78.0 - 64.1)}{(85.1 - 64.1)}$$
$$= 1{,}790 \text{ kip-ft}$$

The available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(1{,}790 \text{ kip-ft})$ $= 1{,}610 \text{ kip-ft}$	$\Omega_b = 1.67$ $M_n/\Omega_b = 1{,}790 \text{ kip-ft}/1.67$ $= 1{,}070 \text{ kip-ft}$

Interaction of Flexure and Compression

Design of members for combined forces is performed in accordance with AISC *Specification* Section I5. For filled composite members with noncompact or slender sections, interaction is determined in accordance with Section H1.1 as follows:

LRFD	ASD
$P_u = 1{,}310 \text{ kips}$ $M_u = 552 \text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{1{,}310 \text{ kips}}{4{,}320 \text{ kips}}$ $= 0.303 \geq 0.2$ Use *Specification* Equation H1-1a. $\dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_u}{\phi_b M_n}\right) \leq 1.0$ $\dfrac{1{,}310 \text{ kips}}{4{,}320 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{552 \text{ kip-ft}}{1{,}610 \text{ kip-ft}}\right) \leq 1.0$ $0.608 < 1.0$ **o.k.**	$P_a = 1{,}370 \text{ kips}$ $M_a = 248 \text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_c}$ $= \dfrac{1{,}370 \text{ kips}}{2{,}880 \text{ kips}}$ $= 0.476 \geq 0.2$ Use *Specification* Equation H1-1a. $\dfrac{P_a}{P_n / \Omega_c} + \dfrac{8}{9}\left(\dfrac{M_a}{M_n / \Omega_b}\right) \leq 1.0$ $\dfrac{1{,}370 \text{ kips}}{2{,}880 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{248 \text{ kip-ft}}{1{,}070 \text{ kip-ft}}\right) \leq 1.0$ $0.682 < 1.0$ **o.k.**

The composite section is adequate; however, as there is available strength remaining for the trial plate thickness chosen, re-analyze the section to determine the adequacy of a reduced plate thickness.

Trial Size 2 (Slender)

The calculated geometric section properties using a reduced plate thickness of t = ¼ in. are:

$$B = 30.0\text{ in.}$$
$$H = 30.0\text{ in.}$$
$$b_i = B - 2t = 29.50\text{ in.}$$
$$h_i = H - 2t = 29.50\text{ in.}$$

$$A_g = 900\text{ in.}^2$$
$$A_c = 870\text{ in.}^2$$
$$A_s = 29.8\text{ in.}^2$$

$$\begin{aligned} E_c &= w_c^{1.5}\sqrt{f_c'} \\ &= \left(145\text{ lb/ft}^3\right)^{1.5}\sqrt{7\text{ ksi}} \\ &= 4{,}620\text{ ksi} \end{aligned}$$

$$\begin{aligned} I_{gx} &= BH^3/12 \\ &= 67{,}500\text{ in.}^4 \end{aligned}$$

$$\begin{aligned} I_{cx} &= b_i h_i^3/12 \\ &= 63{,}100\text{ in.}^4 \end{aligned}$$

$$\begin{aligned} I_{sx} &= I_{gx} - I_{cx} \\ &= 4{,}400\text{ in.}^4 \end{aligned}$$

Limitations of AISC Specification Sections I1.3 and I2.2a

(1) Concrete Strength: $3\text{ ksi} \le f_c' \le 10\text{ ksi}$

$f_c' = 7\text{ ksi}$ **o.k.**

(2) Specified minimum yield stress of structural steel: $F_y \le 75\text{ ksi}$

$F_y = 36\text{ ksi}$ **o.k.**

(3) Cross sectional area of steel section: $A_s \ge 0.01A_g$

$$\begin{aligned} 29.8\text{ in.}^2 &\ge (0.01)\left(900\text{ in.}^2\right) \\ &> 9.00\text{ in.}^2 \quad \textbf{o.k.} \end{aligned}$$

Classify Section for Local Buckling

As noted previously, the definitions of width, depth and thickness used in the evaluation of slenderness are provided in AISC *Specification* Tables B4.1a and B4.1b.

For a box column, the slenderness ratio is determined as the ratio of clear distance to wall thickness:

$$\begin{aligned} \lambda &= \frac{b_i}{t} \\ &= \frac{h_i}{t} \\ &= \frac{29.5\text{ in.}}{\text{¼ in.}} \\ &= 118 \end{aligned}$$

Classify section for local buckling in steel elements subject to axial compression from AISC *Specification* Table I1.1A. As determined previously, λ_r = 85.1.

$$\lambda_{max} = 5.00\sqrt{\frac{E}{F_y}}$$
$$= 5.00\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 142$$

$\lambda_r \quad \le \lambda \le \lambda_{max}$

$85.1 \le 118 \le 142$; therefore, the section is slender for compression

Classification of the section for local buckling in elements subject to flexure occurs separately per AISC *Specification* Table I1.1B. Because the flange limitations for bending are the same as those for compression,

$\lambda_r \le \lambda \le \lambda_{max}$

$85.1 \le 118 \le 142$; therefore, the section is slender for flexure

Available Compressive Strength

Compressive strength for a slender filled member is determined in accordance with AISC *Specification* Section I2.2b(c).

$$F_{cr} = \frac{9E_s}{\left(\frac{b}{t}\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. I2-10)}$$
$$= \frac{9(29{,}000 \text{ ksi})}{(118)^2}$$
$$= 18.7 \text{ ksi}$$

$$P_{no} = F_{cr}A_s + 0.7f_c'\left(A_c + A_{sr}\frac{E_s}{E_c}\right) \qquad \text{(}\textit{Spec}\text{. Eq. I2-9e)}$$
$$= (18.7 \text{ ksi})(29.8 \text{ in.}^2) + 0.7(7 \text{ ksi})(870 \text{ in.}^2 + 0)$$
$$= 4{,}820 \text{ kips}$$

$$C_3 = 0.6 + 2\left(\frac{A_s}{A_c + A_s}\right) \le 0.9 \qquad \text{(}\textit{Spec}\text{. Eq. I2-13)}$$
$$= 0.6 + 2\left(\frac{29.8 \text{ in.}^2}{870 \text{ in.}^2 + 29.8 \text{ in.}^2}\right) \le 0.9$$
$$= 0.666 < 0.9$$

$$EI_{eff} = E_sI_s + E_sI_{sr} + C_3E_cI_c \qquad \text{(}\textit{Spec}\text{. Eq. I2-12)}$$
$$= (29{,}000 \text{ ksi})(4{,}400 \text{ in.}^4) + 0 + 0.666(4{,}620 \text{ ksi})(63{,}100 \text{ in.}^4)$$
$$= 322{,}000{,}000 \text{ ksi}$$

$$P_e = \pi^2(EI_{eff})/(KL)^2 \text{ where } K = 1.0 \text{ in accordance with the direct analysis method} \qquad \text{(}\textit{Spec}\text{. Eq. I2-5)}$$
$$= \frac{\pi^2(322{,}000{,}000 \text{ ksi})}{\left[(30.0 \text{ ft})(12 \text{ in./ft})\right]^2}$$
$$= 24{,}500 \text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{4{,}820 \text{ kips}}{24{,}500 \text{ kips}} = 0.197 < 2.25$$

Therefore, use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left[0.658^{\frac{P_{no}}{P_e}}\right] \qquad (\textit{Spec.}\text{ Eq. I2-2})$$

$$= 4{,}820 \text{ kips}(0.658)^{0.197}$$
$$= 4{,}440 \text{ kips}$$

According to AISC *Specification* Section I2.2b the compression strength need not be less than that determined for the bare steel member using *Specification* Chapter E. It can be shown that the compression strength of the bare steel for this section is equal to 450 kips, thus the strength of the composite section controls.

The available compressive strength is:

LRFD	ASD
$\phi_c = 0.75$ $\phi_c P_n = 0.75(4{,}440 \text{ kips})$ $= 3{,}330 \text{ kips}$	$\Omega_c = 2.00$ $P_n/\Omega_c = 4{,}440 \text{ kips}/2.00$ $= 2{,}220 \text{ kips}$

Available Flexural Strength

Flexural strength of slender filled composite members is determined in accordance with AISC *Specification* Section I3.4b(c). The nominal flexural strength is determined as the first yield moment, M_{cr}, corresponding to a flange compression stress of F_{cr} using a linear elastic stress distribution with a maximum concrete compressive stress of $0.7f_c'$. This concept is illustrated diagrammatically in *Specification* Commentary Figure C-I3.7(c) and follows the force distribution depicted in Figure I.7-4 and detailed in Table I.7-3.

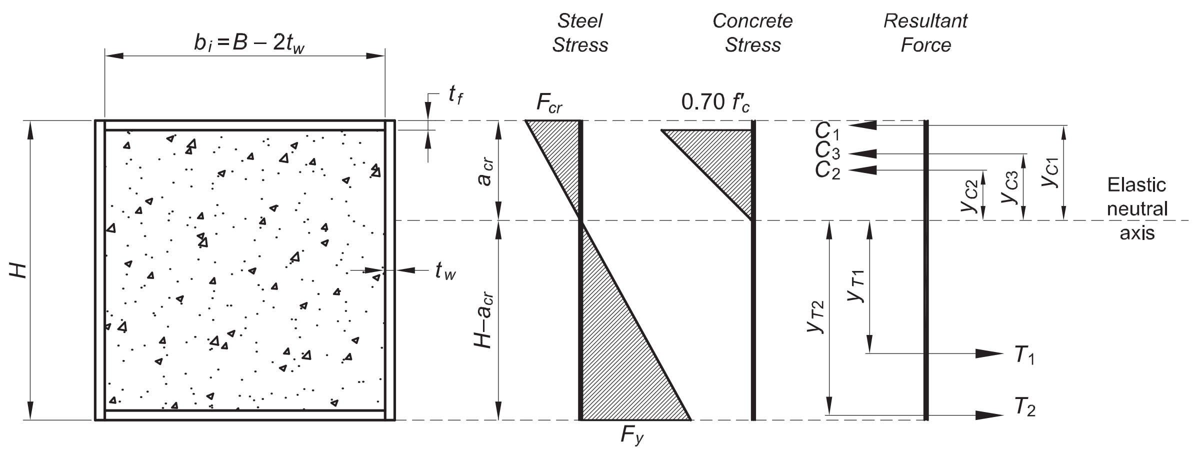


Figure I.7-4. First yield moment stress blocks and force distribution.

Table I.7-3. First Yield Moment Equations

Component	Force	Moment arm
Compression in steel flange	$C_1 = b_i t_f F_{cr}$	$y_{C1} = a_{cr} - \dfrac{t_f}{2}$
Compression in concrete	$C_2 = 0.35 f_c'\left(a_{cr} - t_f\right) b_i$	$y_{C2} = \dfrac{2\left(a_{cr} - t_f\right)}{3}$
Compression in steel web	$C_3 = a_{cr} 2t_w 0.5F_{cr}$	$y_{C3} = \dfrac{2a_{cr}}{3}$
Tension in steel web	$T_1 = \left(H - a_{cr}\right) 2t_w 0.5F_y$	$y_{T1} = \dfrac{2\left(H - a_{cr}\right)}{3}$
Tension in steel flange	$T_2 = b_i t_f F_y$	$y_{T2} = H - a_{cr} - \dfrac{t_f}{2}$
where: $a_{cr} = \dfrac{F_y H t_w + \left(0.35 f_c' + F_y - F_{cr}\right) b_i t_f}{t_w\left(F_{cr} + F_y\right) + 0.35 f_c' b_i}$ $M_{cr} = \sum(\text{force})(\text{moment arm})$		

Using the equations provided in Table I.7-3 for the section in question results in the following:

$$a_{cr} = \frac{(36\text{ ksi})(30.0\text{ in.})(¼\text{ in.}) + \left[0.35(7\text{ ksi}) + 36\text{ ksi} - 18.7\text{ ksi}\right](29.5\text{ in.})(¼\text{ in.})}{(¼\text{ in.})(18.7\text{ ksi} + 36\text{ ksi}) + 0.35(7\text{ ksi})(29.5\text{ in.})}$$
$$= 4.84\text{ in.}$$

Force	Moment Arm	Force × Moment Arm
$C_1 = (29.5\text{ in.})(¼\text{ in.})(18.7\text{ ksi})$ $= 138\text{ kips}$	$y_{C1} = 4.84\text{ in.} - \dfrac{¼\text{ in.}}{2}$ $= 4.72\text{ in.}$	$C_1 y_{C1} = 651\text{ kip-in.}$
$C_2 = 0.35(7\text{ ksi})(4.84\text{ in.} - ¼\text{ in.})(29.5\text{ in.})$ $= 332\text{ kips}$	$y_{C2} = \dfrac{2(4.84\text{ in.} - ¼\text{ in.})}{3}$ $= 3.06\text{ in.}$	$C_2 y_{C2} = 1{,}020\text{ kip-in.}$
$C_3 = (4.84\text{ in.})(2)(¼\text{ in.})(0.5)(18.7\text{ ksi})$ $= 22.6\text{ kips}$	$y_{C3} = \dfrac{2(4.84\text{ in.})}{3}$ $= 3.23\text{ in.}$	$C_3 y_{C3} = 73.0\text{ kip-in.}$
$T_1 = (30.0\text{ in.} - 4.84\text{ in.})(2)(¼\text{ in.})(0.5)(36\text{ ksi})$ $= 226\text{ kips}$	$y_{T1} = \dfrac{2(30.0\text{ in.} - 4.84\text{ in.})}{3}$ $= 16.8\text{ in.}$	$T_1 y_{T1} = 3{,}800\text{ kip-in.}$
$T_2 = (29.5\text{ in.})(¼\text{ in.})(36\text{ ksi})$ $= 266\text{ kips}$	$y_{T2} = 30.0\text{ in.} - 4.84\text{ in.} - \dfrac{¼\text{ in}}{2}$ $= 25.0\text{ in.}$	$T_2 y_{T2} = 6{,}650\text{ kip-in.}$

$$M_{cr} = \sum(\text{force component})(\text{moment arm})$$
$$= \frac{651\text{ kip-in.} + 1{,}020\text{ kip-in.} + 73.0\text{ kip-in.} + 3{,}800\text{ kip-in.} + 6{,}650\text{ kip-in.}}{12\text{ in./ft}}$$
$$= 1{,}020\text{ kip-ft}$$

The available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $M_n = 0.90(1{,}020 \text{ kip-ft})$ $= 918 \text{ kip-ft}$	$\Omega_b = 1.67$ $M_n/\Omega_b = 1{,}020 \text{ kip-ft}/1.67$ $= 611 \text{ kip-ft}$

Interaction of Flexure and Compression

The interaction of flexure and compression is determined in accordance with AISC *Specification* Section H1.1 as follows:

LRFD	ASD
$P_u = 1{,}310 \text{ kips}$ $M_u = 552 \text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{1{,}310 \text{ kips}}{3{,}330 \text{ kips}}$ $= 0.393 \geq 0.2$ Use AISC *Specification* Equation H1-1a. $\dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_u}{\phi_b M_n}\right) \leq 1.0$ $\dfrac{1{,}310 \text{ kips}}{3{,}330 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{552 \text{ kip-ft}}{918 \text{ kip-ft}}\right) \leq 1.0$ $0.928 < 1.0$ **o.k.**	$P_a = 1{,}370 \text{ kips}$ $M_a = 248 \text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_c}$ $= \dfrac{1{,}370 \text{ kips}}{2{,}220 \text{ kips}}$ $= 0.617 \geq 0.2$ Use AISC *Specification* Equation H1-1a. $\dfrac{P_a}{P_n / \Omega_c} + \dfrac{8}{9}\left(\dfrac{M_a}{M_n / \Omega_c}\right) \leq 1.0$ $\dfrac{1{,}370 \text{ kips}}{2{,}220 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{248 \text{ kip-ft}}{611 \text{ kip-ft}}\right) \leq 1.0$ $0.978 < 1.0$ **o.k.**

Thus, a plate thickness of ¼ in. is adequate.

Note that in addition to the design checks performed for the composite condition, design checks for other load stages should be performed as required by AISC *Specification* Section I1. These checks should take into account the effect of hydrostatic loads from concrete placement as well as the strength of the steel section alone prior to composite action.

Available Shear Strength

According to AISC *Specification* Section I4.1 there are three acceptable methods for determining the available shear strength of the member: available shear strength of the steel section alone in accordance with Chapter G, available shear strength of the reinforced concrete portion alone per ACI 318, or available shear strength of the steel section in addition to the reinforcing steel ignoring the contribution of the concrete. Considering that the member in question does not have longitudinal reinforcing, it is determined by inspection that the shear strength will be controlled by the steel section alone using the provisions of Chapter G.

From AISC *Specification* Section G5 the nominal shear strength, V_n, of box members is determined using the provisions of Section G2.1 with $k_v = 5$. As opposed to HSS sections which require the use of a reduced web area to take into account the corner radii, the full web area of a box section may be used as follows:

$A_w = 2dt_w$ where d = full depth of section parallel to the required shear force

$= 2(30.0 \text{ in.})(\tfrac{1}{4} \text{ in.})$

$= 15.0 \text{ in.}^2$

The slenderness value, h/t_w, for the web used in *Specification* Section G2.1(b) is the same as that calculated previously for use in local buckling classification, $\lambda = 118$.

$$\frac{h}{t_w} > 1.37\sqrt{k_v E/F_y}$$

$$\frac{h}{t_w} > 1.37\sqrt{5\left(\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}\right)}$$

$$118 > 86.9$$

Therefore, use AISC *Specification* Equation G2-5.

The web shear coefficient and nominal shear strength are calculated as:

$$C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} \quad \text{(Spec. Eq. G2-5)}$$

$$= \frac{1.51(5)(29{,}000 \text{ ksi})}{(118)^2 (36 \text{ ksi})}$$

$$= 0.437$$

$$V_n = 0.6 F_y A_w C_v \quad \text{(Spec. Eq. G2-1)}$$

$$= 0.6(36 \text{ ksi})(15.0 \text{ in.}^2)(0.437)$$

$$= 142 \text{ kips}$$

The available shear strength is checked as follows:

LRFD	ASD
$V_u = 36.8$ kips $\phi_v = 0.9$ $\phi_v V_n \geq V_u$ $\phi_v V_n = 0.9(142 \text{ kips})$ $= 128 \text{ kips} > 36.8 \text{ kips}$ **o.k.**	$V_a = 22.1$ kips $\Omega_v = 1.67$ $V_n / \Omega_v \geq V_a$ $V_n / \Omega_v = \dfrac{142 \text{ kips}}{1.67}$ $= 85.0 \text{ kips} > 22.1 \text{ kips}$ **o.k.**

Force allocation and load transfer

Load transfer calculations for applied axial forces should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions is provided in Design Example I.3.

Summary

It has been determined that a 30 in. × 30 in. composite box column composed of ¼-in.-thick plate is adequate for the imposed loads.

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EXAMPLE I.8 ENCASED COMPOSITE MEMBER FORCE ALLOCATION AND LOAD TRANSFER

Given:

Refer to Figure I.8-1.

Part I: For each loading condition (a) through (c) determine the required longitudinal shear force, V_r', to be transferred between the embedded steel section and concrete encasement.

Part II: For loading condition (b), investigate the force transfer mechanisms of direct bearing and shear connection.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, f_c' = 5 ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, F_{yr}, of 60 ksi.

Applied loading, P_r, for each condition illustrated in Figure I.8-1 is composed of the following loads:

P_D = 260 kips
P_L = 780 kips

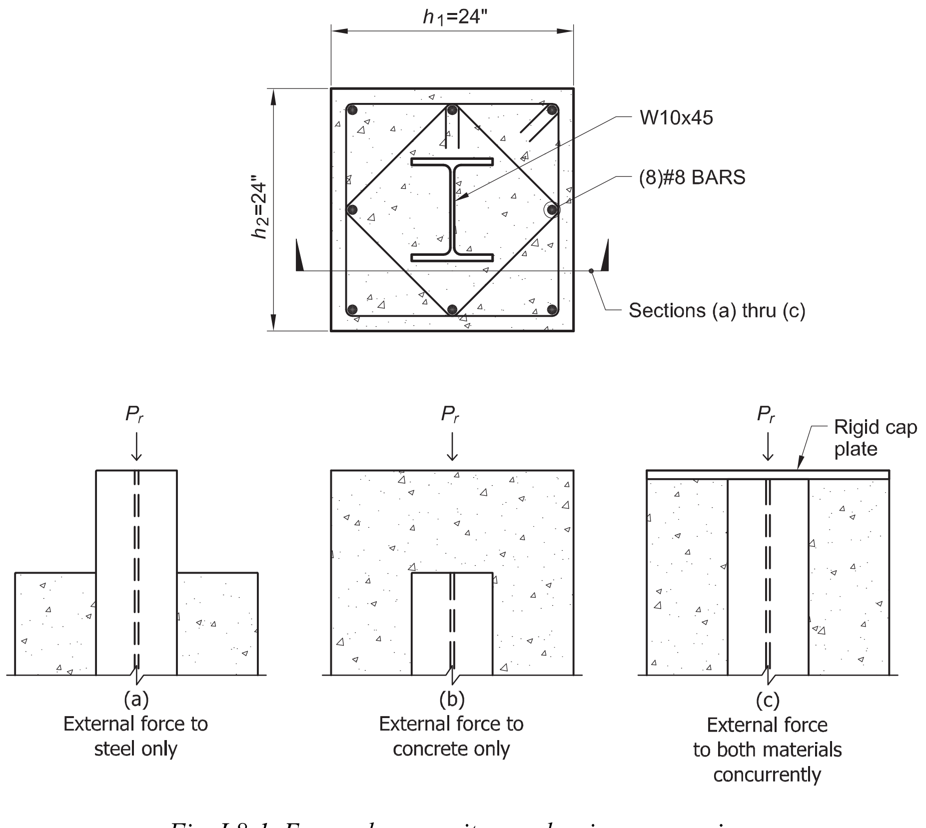

Fig. I.8-1. Encased composite member in compression.

Solution:

Part I—Force Allocation

From AISC *Manual* Table 2-4, the steel material properties are:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1 and Figure I.8-1, the geometric properties of the encased W10×45 are as follows:

$A_s = 13.3 \text{ in.}^2$ $\quad t_w = 0.350 \text{ in.}$ $\quad h_1 = 24.0 \text{ in.}$
$b_f = 8.02 \text{ in.}$ $\quad d = 10.1 \text{ in.}$ $\quad h_2 = 24.0 \text{ in.}$
$t_f = 0.620 \text{ in.}$

Additional geometric properties of the composite section used for force allocation and load transfer are calculated as follows:

$$\begin{aligned} A_g &= h_1 h_2 \\ &= (24.0 \text{ in.})(24.0 \text{ in.}) \\ &= 576 \text{ in.}^2 \end{aligned}$$

$$A_{sri} = 0.79 \text{ in.}^2 \text{ for a No. 8 bar}$$

$$\begin{aligned} A_{sr} &= \sum_{i=1}^{n} A_{sri} \\ &= 8(0.79 \text{ in.}^2) \\ &= 6.32 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_c &= A_g - A_s - A_{sr} \\ &= 576 \text{ in.}^2 - 13.3 \text{ in.}^2 - 6.32 \text{ in.}^2 \\ &= 556 \text{ in.}^2 \end{aligned}$$

where
A_c = cross-sectional area of concrete encasement, in.^{2}
A_g = gross cross-sectional area of composite section, in.^{2}
A_{sri} = cross-sectional area of reinforcing bar *i*, in.^{2}
A_{sr} = cross-sectional area of continuous reinforcing bars, in.^{2}
n = number of continuous reinforcing bars in composite section

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(260 \text{ kips}) + 1.6(780 \text{ kips})$ $= 1{,}560 \text{ kips}$	$P_r = P_a$ $= 260 \text{ kips} + 780 \text{ kips}$ $= 1{,}040 \text{ kips}$

Composite Section Strength for Force Allocation

In accordance with AISC *Specification* Section I6, force allocation calculations are based on the nominal axial compressive strength of the encased composite member without length effects, P_{no}. This section strength is defined in Section I2.1b as:

$$P_{no} = F_y A_s + F_{ysr} A_{sr} + 0.85 f_c' A_c \qquad (Spec. \text{ Eq. I2-4})$$

$$\begin{aligned} &= (50 \text{ ksi})(13.3 \text{ in.}^2) + (60 \text{ ksi})(6.32 \text{ in.}^2) + 0.85(5 \text{ ksi})(556 \text{ in.}^2) \\ &= 3{,}410 \text{ kips} \end{aligned}$$

Transfer Force for Condition (a)

Refer to Figure I.8-1(a). For this condition, the entire external force is applied to the steel section only, and the provisions of AISC *Specification* Section I6.2a apply.

$$V_r' = P_r\left(1 - \frac{F_y A_s}{P_{no}}\right) \qquad (Spec.\ \text{Eq. I6-1})$$

$$= P_r\left[1 - \frac{(50\text{ ksi})(13.3\text{ in.}^2)}{3{,}410\text{ kips}}\right]$$

$$= 0.805P_r$$

LRFD	ASD
$V_r' = 0.805(1{,}560\text{ kips})$ $= 1{,}260\text{ kips}$	$V_r' = 0.805(1{,}040\text{ kips})$ $= 837\text{ kips}$

Transfer Force for Condition (b)

Refer to Figure I.8-1(b). For this condition, the entire external force is applied to the concrete encasement only, and the provisions of AISC *Specification* Section I6.2b apply.

$$V_r' = P_r\left(\frac{F_y A_s}{P_{no}}\right) \qquad (Spec.\ \text{Eq. I6-2})$$

$$= P_r\left[\frac{(50\text{ ksi})(13.3\text{ in.}^2)}{3{,}410\text{ kips}}\right]$$

$$= 0.195P_r$$

LRFD	ASD
$V_r' = 0.195(1{,}560\text{ kips})$ $= 304\text{ kips}$	$V_r' = 0.195(1{,}040\text{ kips})$ $= 203\text{ kips}$

Transfer Force for Condition (c)

Refer to Figure I.8-1(c). For this condition, external force is applied to the steel section and concrete encasement concurrently, and the provisions of AISC *Specification* Section I6.2c apply.

AISC *Specification Commentary* Section I6.2 states that when loads are applied to both the steel section and concrete encasement concurrently, V_r' can be taken as the difference in magnitudes between the portion of the external force applied directly to the steel section and that required by Equation I6-2. This concept can be written in equation form as follows:

$$V_r' = \left|P_{rs} - P_r\left(\frac{F_y A_s}{P_{no}}\right)\right| \qquad \text{(Eq. 1)}$$

where

P_{rs} = portion of external force applied directly to the steel section (kips)

Currently the *Specification* provides no specific requirements for determining the distribution of the applied force for the determination of P_{rs}, so it is left to engineering judgment. For a bearing plate condition such as the one represented in Figure I.8-1(c), one possible method for determining the distribution of applied forces is to use an elastic distribution based on the material axial stiffness ratios as follows:

$$E_c = w_c^{1.5}\sqrt{f_c'}$$
$$= \left(145 \text{ lb/ft}^3\right)^{1.5}\sqrt{5 \text{ ksi}}$$
$$= 3,900 \text{ ksi}$$

$$P_{rs} = \left(\frac{E_s A_s}{E_s A_s + E_c A_c + E_{sr} A_{sr}}\right)P_r$$
$$= \left[\frac{(29,000 \text{ ksi})(13.3 \text{ in.}^2)}{(29,000 \text{ ksi})(13.3 \text{ in.}^2) + (3,900 \text{ ksi})(556 \text{ in.}^2) + (29,000 \text{ ksi})(6.32 \text{ in.}^2)}\right]P_r$$
$$= 0.141P_r$$

Substituting the results into Equation 1 yields:

$$V_r' = \left|0.141P_r - P_r\left(\frac{F_y A_s}{P_{no}}\right)\right|$$
$$= \left|0.141P_r - P_r\left[\frac{(50 \text{ ksi})(13.3 \text{ in.}^2)}{3,410 \text{ kips}}\right]\right|$$
$$= 0.0540P_r$$

LRFD	ASD
$V_r' = 0.0540(1,560 \text{ kips})$ $= 84.2 \text{ kips}$	$V_r' = 0.0540(1,040 \text{ kips})$ $= 56.2 \text{ kips}$

An alternate approach would be use of a plastic distribution method whereby the load is partitioned to each material in accordance with their contribution to the composite section strength given in Equation I2-4. This method eliminates the need for longitudinal shear transfer provided the local bearing strength of the concrete and steel are adequate to resist the forces resulting from this distribution.

Additional Discussion

- The design and detailing of the connections required to deliver external forces to the composite member should be performed according to the applicable sections of AISC *Specification* Chapters J and K.

- The connection cases illustrated by Figure I.8-1 are idealized conditions representative of the mechanics of actual connections. For instance, an extended single plate connection welded to the flange of the W10 and extending out beyond the face of concrete to attach to a steel beam is an example of a condition where it may be assumed that all external force is applied directly to the steel section only.

Solution:

Part II—Load Transfer

The required longitudinal force to be transferred, V_r', determined in Part I condition (b) is used to investigate the applicable force transfer mechanisms of AISC *Specification* Section I6.3: direct bearing and shear connection. As indicated in the *Specification*, these force transfer mechanisms may not be superimposed; however, the mechanism providing the greatest nominal strength may be used. Note that direct bond interaction is not applicable to encased composite members as the variability of column sections and connection configurations makes confinement and bond strength more difficult to quantify than in filled HSS.

Direct Bearing

Determine Layout of Bearing Plates

One method of utilizing direct bearing as a load transfer mechanism is through the use of internal bearing plates welded between the flanges of the encased W-shape as indicated in Figure I.8-2.

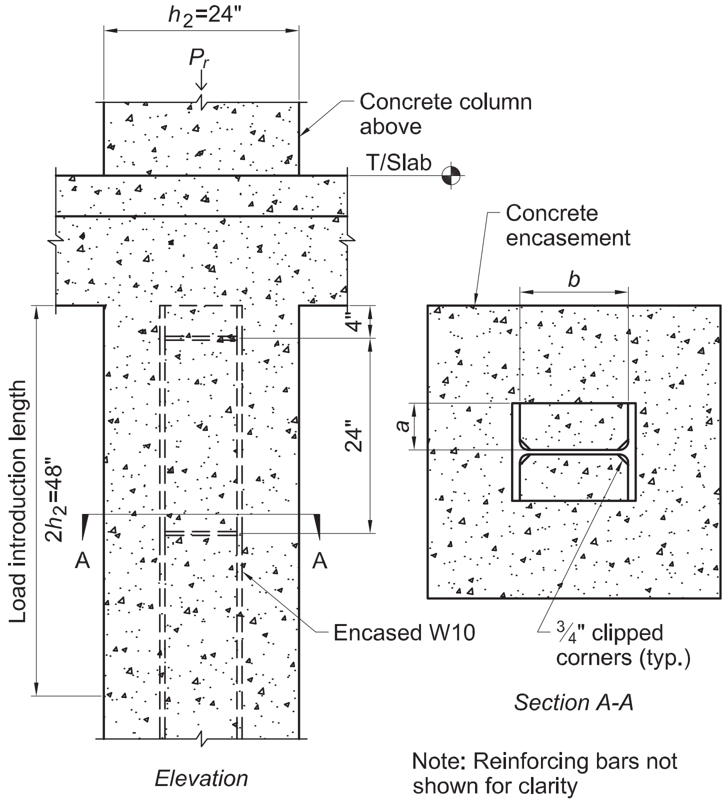

Fig. I.8-2. Composite member with internal bearing plates.

When using bearing plates in this manner, it is essential that concrete mix proportions and installation techniques produce full bearing at the plates. Where multiple sets of bearing plates are used as illustrated in Figure I.8-2, it is recommended that the minimum spacing between plates be equal to the depth of the encased steel member to enhance constructability and concrete consolidation. For the configuration under consideration, this guideline is met with a plate spacing of $24 \text{ in.} \geq d = 10.1 \text{ in.}$

Bearing plates should be located within the load introduction length given in AISC *Specification* Section I6.4a. The load introduction length is defined as two times the minimum transverse dimension of the composite member both above and below the load transfer region. The load transfer region is defined in *Specification* Commentary Section I6.4 as the depth of the connection. For the connection configuration under consideration, where the majority of the required force is being applied from the concrete column above, the depth of connection is conservatively taken as zero. Because the composite member only extends to one side of the point of force transfer, the bearing plates should be located within $2h_2$ = 48 in. of the top of the composite member as indicated in Figure I.8-2.

Available Strength for the Limit State of Direct Bearing

Assuming two sets of bearing plates are to be used as indicated in Figure I.8-2, the total contact area between the bearing plates and the concrete, A_1, may be determined as follows:

$$a = \frac{b_f - t_w}{2}$$
$$= \frac{8.02 \text{ in.} - 0.350 \text{ in.}}{2}$$
$$= 3.84 \text{ in.}$$

$$b = d - 2t_f$$
$$= 10.1 \text{ in.} - 2(0.620 \text{ in.})$$
$$= 8.86 \text{ in.}$$

c = width of clipped corners
= ¾ in.

$$A_1 = \left(2ab - 2c^2\right)\left(\text{number of bearing plate sets}\right)$$
$$= \left[2\left(3.84 \text{ in.}\right)\left(8.86 \text{ in.}\right) - 2\left(\tfrac{3}{4} \text{ in.}\right)^2\right]\left(2\right)$$
$$= 134 \text{ in.}^2$$

The available strength for the direct bearing force transfer mechanism is:

$$R_n = 1.7 f_c' A_1 \qquad (\textit{Spec}. \text{ Eq. I6-3})$$
$$= 1.7\left(5 \text{ ksi}\right)\left(134 \text{ in.}^2\right)$$
$$= 1{,}140 \text{ kips}$$

LRFD	ASD
$\phi_B = 0.65$ $\phi_B R_n \geq V_r'$ $\phi_B R_n = 0.65\left(1{,}140 \text{ kips}\right)$ $= 741 \text{ kips} > 304 \text{ kips}$ **o.k.**	$\Omega_B = 2.31$ $R_n / \Omega_B \geq V_r'$ $R_n / \Omega_B = \dfrac{1{,}140 \text{ kips}}{2.31}$ $= 494 \text{ kips} > 203 \text{ kips}$ **o.k.**

Thus two sets of bearing plates are adequate. From these calculations it can be seen that one set of bearing plates are adequate for force transfer purposes; however, the use of two sets of bearing plates serves to reduce the bearing plate thickness calculated in the following section.

Required Bearing Plate Thickness

There are several methods available for determining the bearing plate thickness. For rectangular plates supported on three sides, elastic solutions for plate stresses such as those found in *Roark's Formulas for Stress and Strain* (Young and Budynas, 2002) may be used in conjunction with AISC *Specification* Section F12 for thickness calculations. Alternately, yield line theory or computational methods such as finite element analysis may be employed.

For this example, yield line theory is employed. Results of the yield line analysis depend on an assumption of column flange strength versus bearing plate strength in order to estimate the fixity of the bearing plate to column flange connection. In general, if the thickness of the bearing plate is less than the column flange thickness, fixity and plastic hinging can occur at this interface; otherwise, the use of a pinned condition is conservative. Ignoring the fillets of the W-shape and clipped corners of the bearing plate, the yield line pattern chosen for the fixed condition is depicted in Figure I.8-3. Note that the simplifying assumption of 45° yield lines illustrated in Figure I.8-3 has been shown to provide reasonably accurate results (Park and Gamble, 2000), and that this yield line pattern is only valid where $b \geq 2a$.

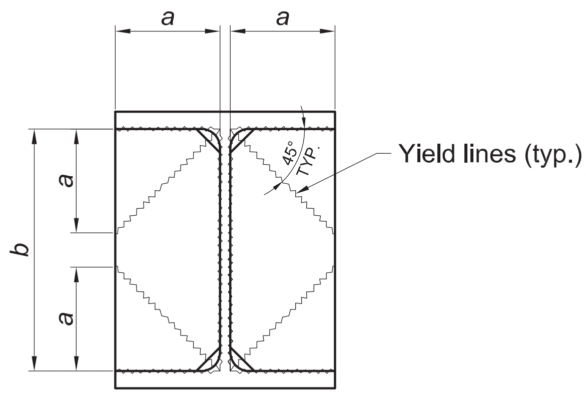

Fig. I.8-3. Internal bearing plate yield line pattern (fixed condition).

The plate thickness using $F_y = 36$ ksi material may be determined as:

LRFD	ASD
$\phi = 0.90$ If $t_p \geq t_f$: $t_p = \sqrt{\dfrac{2a^2 w_u (3b-2a)}{3\phi F_y (4a+b)}}$ If $t_p < t_f$: $t_p = \sqrt{\dfrac{2a^2 w_u (3b-2a)}{3\phi F_y (6a+b)}}$ where w_u = bearing pressure on plate determined using LRFD load combinations $= \dfrac{V_r'}{A_1}$ $= \dfrac{304 \text{ kips}}{134 \text{ in.}^2}$ $= 2.27$ ksi Assuming $t_p \geq t_f$ $t_p = \sqrt{\dfrac{2(3.84 \text{ in.})^2 (2.27 \text{ ksi})[3(8.86 \text{ in.}) - 2(3.84 \text{ in.})]}{3(0.90)(36 \text{ ksi})[4(3.84 \text{ in.}) + 8.86 \text{ in.}]}}$ $= 0.733$ in. Select ¾-in. plate. $t_p = ¾$ in. $> t_f = 0.620$ in. **assumption o.k.**	$\Omega = 1.67$ If $t_p \geq t_f$: $t_p = \sqrt{\left(\dfrac{2\Omega}{3F_y}\right)\left[\dfrac{a^2 w_u (3b-2a)}{(4a+b)}\right]}$ If $t_p < t_f$: $t_p = \sqrt{\left(\dfrac{2\Omega}{3F_y}\right)\left[\dfrac{a^2 w_u (3b-2a)}{(6a+b)}\right]}$ where w_u = bearing pressure on plate determined using ASD load combinations $= \dfrac{V_r'}{A_1}$ $= \dfrac{203 \text{ kips}}{134 \text{ in.}^2}$ $= 1.51$ ksi Assuming $t_p \geq t_f$ $t_p = \sqrt{\dfrac{2(1.67)(3.84 \text{ in.})^2 (1.51 \text{ ksi})[3(8.86 \text{ in.}) - 2(3.84 \text{ in.})]}{3(36 \text{ ksi})[4(3.84 \text{ in.}) + 8.86 \text{ in.}]}}$ $= 0.733$ in. Select ¾-in. plate $t_p = ¾$ in. $> t_f = 0.620$ in. **assumption o.k.**

Thus, select ¾-in.-thick bearing plates.

Bearing Plate to Encased Steel Member Weld

The bearing plates should be connected to the encased steel member using welds designed in accordance with AISC *Specification* Chapter J to develop the full strength of the plate. For fillet welds, a weld size of $\frac{5}{8}t_p$ will serve to develop the strength of either a 36- or 50-ksi plate as discussed in AISC *Manual* Part 10.

Shear Connection

Shear connection involves the use of steel headed stud or channel anchors placed on at least two faces of the steel shape in a generally symmetric configuration to transfer the required longitudinal shear force. For this example, ¾-in.-diameter × 4$\frac{3}{16}$-in.-long steel headed stud anchors composed of ASTM A108 material are selected. From AISC *Manual* Table 2-6, the specified minimum tensile strength, F_u, of ASTM A108 material is 65 ksi.

Available Shear Strength of Steel Headed Stud Anchors

The available shear strength of an individual steel headed stud anchor is determined in accordance with the composite component provisions of AISC *Specification* Section I8.3 as directed by Section I6.3b.

$$Q_{nv} = F_u A_{sa} \qquad \text{(Spec. Eq. I8-3)}$$

$$A_{sa} = \frac{\pi(\text{3/4 in.})^2}{4} = 0.442 \text{ in.}^2$$

LRFD	ASD
$\phi_v = 0.65$ $\phi_v Q_{nv} = 0.65(65 \text{ ksi})(0.442 \text{ in.}^2)$ $= 18.7$ kips per steel headed stud anchor	$\Omega_v = 2.31$ $Q_{nv}/\Omega_v = \frac{(65 \text{ ksi})(0.442 \text{ in.}^2)}{2.31}$ $= 12.4$ kips per steel headed stud anchor

Required Number of Steel Headed Stud Anchors

The number of steel headed stud anchors required to transfer the longitudinal shear is calculated as follows:

LRFD	ASD
$n_{anchors} = \frac{V_r'}{\phi_v Q_{nv}}$ $= \frac{304 \text{ kips}}{18.7 \text{ kips}}$ $= 16.3$ steel headed stud anchors	$n_{anchors} = \frac{V_r'}{Q_{nv}/\Omega_v}$ $= \frac{203 \text{ kips}}{12.4 \text{ kips}}$ $= 16.4$ steel headed stud anchors

With anchors placed in pairs on each flange, select 20 anchors to satisfy the symmetry provisions of AISC *Specification* Section I6.4a.

Placement of Steel Headed Stud Anchors

Steel headed stud anchors are placed within the load introduction length in accordance with AISC *Specification* Section I6.4a. Since the composite member only extends to one side of the point of force transfer, the steel anchors are located within $2h_2 = 48$ in. of the top of the composite member.

Placing two anchors on each flange provides four anchors per group, and maximum stud spacing within the load introduction length is determined as:

$$s_{max} = \frac{\text{load introduction length} - \text{distance to first anchor group from upper end of encased shape}}{\left[\left(\text{total number of anchors}\right) \big/ \left(\text{number of anchors per group}\right)\right] - 1}$$

$$= \frac{48 \text{ in.} - 6 \text{ in.}}{\left[\left(20 \text{ anchors}\right) \big/ \left(4 \text{ anchors per group}\right)\right] - 1}$$

$$= 10.5 \text{ in.}$$

Use 10.0 in. spacing beginning 6 in. from top of encased member.

In addition to anchors placed within the load introduction length, anchors must also be placed along the remainder of the composite member at a maximum spacing of 32 times the anchor shank diameter = 24 in. in accordance with AISC *Specification* Sections I6.4a and I8.3e.

The chosen anchor layout and spacing is illustrated in Figure I.8-4.

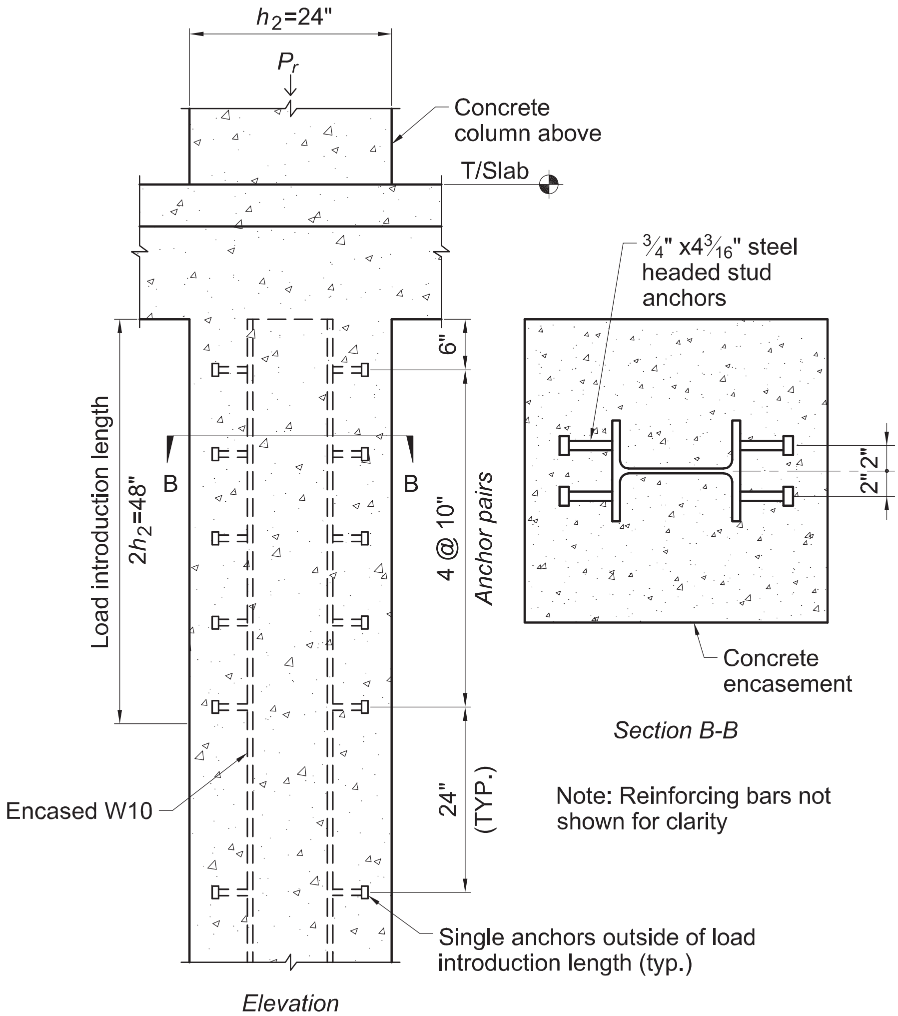

Fig. I.8-4. Composite member with steel anchors.

Steel Headed Stud Anchor Detailing Limitations of AISC *Specification* Sections I6.4a, I8.1 and I8.3

Steel headed stud anchor detailing limitations are reviewed in this section with reference to the anchor configuration provided in Figure I.8-4 for anchors having a shank diameter, d_{sa}, of ¾ in. Note that these provisions are specific to the detailing of the anchors themselves and that additional limitations for the structural steel, concrete and reinforcing components of composite members should be reviewed as demonstrated in Design Example I.9.

(1) Anchors must be placed on at least two faces of the steel shape in a generally symmetric configuration:

Anchors are located in pairs on both faces. **o.k.**

(2) Maximum anchor diameter: $d_{sa} \leq 2.5\left(t_f\right)$

$$\text{¾ in.} < 2.5\left(0.620\text{ in.}\right) = 1.55\text{ in.} \quad \textbf{o.k.}$$

(3) Minimum steel headed stud anchor height-to-diameter ratio: $h / d_{sa} \geq 5$

The minimum ratio of installed anchor height (base to top of head), h, to shank diameter, d_{sa}, must meet the provisions of AISC *Specification* Section I8.3 as summarized in the User Note table at the end of the section. For shear in normal weight concrete the limiting ratio is five. As previously discussed, a 4³⁄₁₆-in.-long anchor was selected from anchor manufacturer's data. As the h/d_{sa} ratio is based on the installed length, a length reduction for burn off during installation of ³⁄₁₆ in. is taken to yield the final installed length of 4 in.

$$\frac{h}{d_{sa}} = \frac{4\text{ in.}}{\text{¾ in.}} = 5.33 > 5 \quad \textbf{o.k.}$$

(4) Minimum lateral clear concrete cover = 1 in.

From AWS D1.1 Figure 7.1, the head diameter of a ¾-in.-diameter stud anchor is equal to 1.25 in.

$$\begin{aligned}\text{lateral clear cover} &= \left(\frac{h_1}{2}\right) - \left(\frac{\text{lateral spacing between anchor centerlines}}{2}\right) - \left(\frac{\text{anchor head diameter}}{2}\right)\\ &= \left(\frac{24\text{ in.}}{2}\right) - \left(\frac{4\text{ in.}}{2}\right) - \left(\frac{1.25\text{ in.}}{2}\right)\\ &= 9.38\text{ in.} > 1.0\text{ in.} \quad \textbf{o.k.}\end{aligned}$$

(5) Minimum anchor spacing:

$$\begin{aligned}s_{min} &= 4d_{sa}\\ &= 4\left(\text{¾ in.}\right)\\ &= 3.00\text{ in.}\end{aligned}$$

In accordance with AISC *Specification* Section I8.3e, this spacing limit applies in any direction.

$$s_{transverse} = 4\text{ in.} \geq s_{min} \quad \textbf{o.k.}$$

$$s_{longitudinal} = 10\text{ in.} \geq s_{min} \quad \textbf{o.k.}$$

(6) Maximum anchor spacing:

$$\begin{aligned}s_{max} &= 32d_{sa}\\ &= 32\left(\text{¾ in.}\right)\\ &= 24.0\text{ in.}\end{aligned}$$

In accordance with AISC *Specification* Section I6.4a, the spacing limits of Section I8.1 apply to steel anchor spacing both within and outside of the load introduction region.

$s = 24.0 \text{ in.} \leq s_{max}$ **o.k.**

(7) Clear cover above the top of the steel headed stud anchors:

Minimum clear cover over the top of the steel headed stud anchors is not explicitly specified for steel anchors in composite components; however, in keeping with the intent of AISC *Specification* Section I1.1, it is recommended that the clear cover over the top of the anchor head follow the cover requirements of ACI 318 Section 7.7. For concrete columns, ACI 318 specifies a clear cover of 1½ in.

$$\begin{aligned}\text{clear cover above anchor} &= \frac{h_2}{2} - \frac{d}{2} - \text{installed anchor length} \\ &= \frac{24 \text{ in.}}{2} - \frac{10.1 \text{ in.}}{2} - 4 \text{ in.} \\ &= 2.95 \text{ in.} > 1\tfrac{1}{2} \text{ in.} \quad \textbf{o.k.}\end{aligned}$$

Concrete Breakout

AISC *Specification* Section I8.3a states that in order to use Equation I8-3 for shear strength calculations as previously demonstrated, concrete breakout strength in shear must not be an applicable limit state. If concrete breakout is deemed to be an applicable limit state, the *Specification* provides two alternatives: either the concrete breakout strength can be determine explicitly using ACI 318 Appendix D in accordance with *Specification* Section I8.3a(2), or anchor reinforcement can be provided to resist the breakout force as discussed in *Specification* Section I8.3a(1).

Determining whether concrete breakout is a viable failure mode is left to the engineer. According to AISC *Specification* Commentary Section I8.3, "it is important that it be deemed by the engineer that a concrete breakout failure mode in shear is directly avoided through having the edges perpendicular to the line of force supported, and the edges parallel to the line of force sufficiently distant that concrete breakout through a side edge is not deemed viable."

For the composite member being designed, no free edge exists in the direction of shear transfer along the length of the column, and concrete breakout in this direction is not an applicable limit state. However, it is still incumbent upon the engineer to review the possibility of concrete breakout through a side edge parallel to the line of force.

One method for explicitly performing this check is through the use of the provisions of ACI 318 Appendix D as follows:

ACI 318 Section D.6.2.1(c) specifies that concrete breakout shall be checked for shear force parallel to the edge of a group of anchors using twice the value for the nominal breakout strength provided by ACI 318 Equation D-22 when the shear force in question acts perpendicular to the edge.

For the composite member being designed, symmetrical concrete breakout planes form to each side of the encased shape, one of which is illustrated in Figure I.8-5.

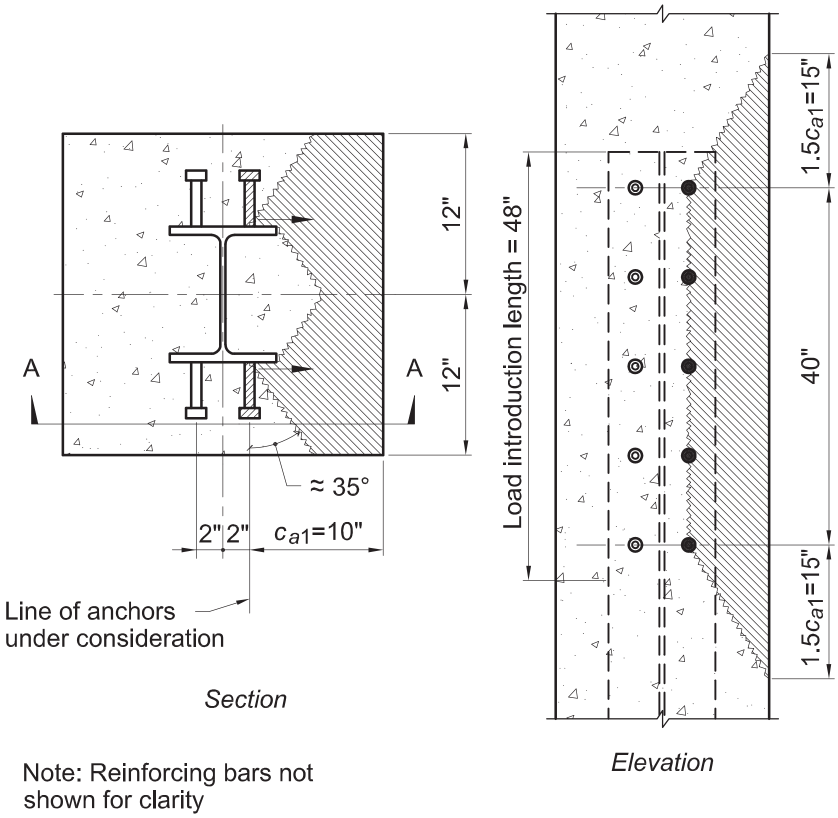

Fig. I.8-5. Concrete breakout check for shear force parallel to an edge.

ϕ = 0.75 for anchors governed by concrete breakout with supplemental reinforcement (provided by tie reinforcement) in accordance with ACI 318 Section D.4.4(c).

$$V_{cbg} = 2\left[\frac{A_{Vc}}{A_{Vco}}\Psi_{ec,V}\Psi_{ed,V}\Psi_{c,V}\Psi_{h,V}V_b\right] \text{ for shear force parallel to an edge} \quad \text{(ACI 318 Eq. D-22)}$$

$$A_{Vco} = 4.5\left(c_{a1}\right)^2 \quad \text{(ACI 318 Eq. D-23)}$$
$$= 4.5\left(10 \text{ in.}\right)^2$$
$$= 450 \text{ in.}^2$$

$$A_{Vc} = \left(15 \text{ in.} + 40 \text{ in.} + 15 \text{ in.}\right)\left(24 \text{ in.}\right) \text{ from Figure I.8-5}$$
$$= 1{,}680 \text{ in.}^2$$

$\Psi_{ec,V}$ = 1.0 no eccentricity

$\Psi_{ed,V}$ = 1.0 in accordance with ACI 318 Section D.6.2.1(c)

$\Psi_{c,V}$ = 1.4 compression-only member assumed uncracked

$\Psi_{h,V}$ = 1.0

$$V_b = \left[8\left(\frac{l_e}{d_{sa}}\right)^{0.2}\sqrt{d_{sa}}\right]\lambda\sqrt{f_c'}\left(c_{a1}\right)^{1.5} \quad \text{(ACI 318 Eq. D-25)}$$

where

l_e = 4 in. − ⅜-in. anchor head thickness from AWS D1.1, Figure 7.1
= 3.63 in.

d_{sa} = ¾-in. anchor diameter

λ = 1.0 from ACI 318 Section 8.6.1 for normal weight concrete

$$V_b = \left[8\left(\frac{3.63\text{ in.}}{¾\text{ in.}}\right)^{0.2}\sqrt{¾\text{ in.}}\right](1.0)\frac{\sqrt{5,000\text{ psi}}}{1,000\text{ lb/kip}}(10\text{ in.})^{1.5}$$
$$= 21.2\text{ kips}$$

$$V_{cbg} = 2\left[\frac{1,680\text{ in.}^2}{450\text{ in.}^2}(1.0)(1.0)(1.4)(1.0)(21.2\text{ kips})\right]$$
$$= 222\text{ kips}$$

$$\phi V_{cbg} = 0.75(222\text{ kips})$$
$$= 167\text{ kips per breakout plane}$$

$$\phi V_{cbg} = (2\text{ breakout planes})(167\text{ kips/plane})$$
$$= 334\text{ kips}$$

$\phi V_{cbg} \geq V_r' = 304$ kips **o.k.**

Thus, concrete breakout along an edge parallel to the direction of the longitudinal shear transfer is not a controlling limit state, and Equation I8-3 is appropriate for determining available anchor strength.

Encased beam-column members with reinforcing detailed in accordance with the AISC *Specification* have demonstrated adequate confinement in tests to prevent concrete breakout along a parallel edge from occurring; however, it is still incumbent upon the engineer to review the project-specific detailing used for susceptibility to this limit state.

If concrete breakout was determined to be a controlling limit state, transverse reinforcing ties could be analyzed as anchor reinforcement in accordance with AISC *Specification* Section I8.3a(1), and tie spacing through the load introduction length adjusted as required to prevent breakout. Alternately, the steel headed stud anchors could be relocated to the web of the encased member where breakout is prevented by confinement between the column flanges.

EXAMPLE I.9 ENCASED COMPOSITE MEMBER IN AXIAL COMPRESSION

Given:

Determine if the 14 ft long, encased composite member illustrated in Figure I.9-1 is adequate for the indicated dead and live loads.

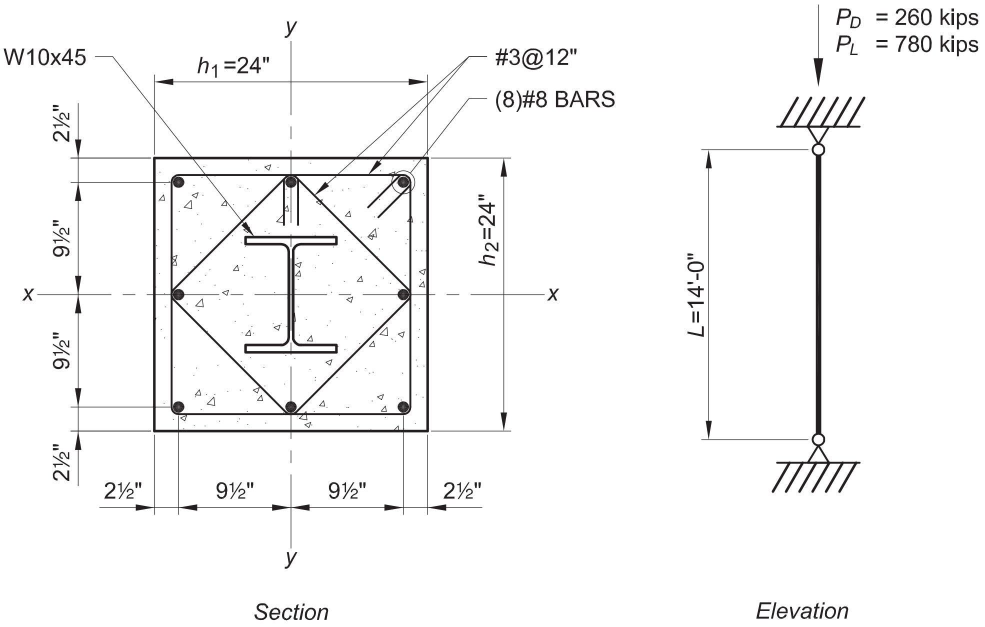

Fig. I.9-1. Encased composite member section and applied loading.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, $f_c' = 5$ ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, F_{yr}, of 60 ksi.

Solution:

From AISC *Manual* Table 2-4, the steel material properties are:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, Figure I.9-1, and Design Example I.8, geometric and material properties of the composite section are:

$$A_s = 13.3 \text{ in.}^2 \qquad I_{sx} = 248 \text{ in.}^4 \qquad A_g = 576 \text{ in.}^2$$
$$b_f = 8.02 \text{ in.} \qquad I_{sy} = 53.4 \text{ in.}^4 \qquad A_{sri} = 0.79 \text{ in.}^2$$
$$t_f = 0.620 \text{ in.} \qquad h_1 = 24.0 \text{ in.} \qquad A_{sr} = 6.32 \text{ in.}^2$$
$$t_w = 0.350 \text{ in.} \qquad h_2 = 24.0 \text{ in.} \qquad A_c = 556 \text{ in.}^2$$
$$E_c = 3{,}900 \text{ ksi}$$

The moment of inertia of the reinforcing bars about the elastic neutral axis of the composite section, I_{sr}, is required for composite member design and is calculated as follows:

$$d_b = 1 \text{ in. for the diameter of a No. 8 bar}$$

$$I_{sri} = \frac{\pi d_b^4}{64}$$
$$= \frac{\pi(1 \text{ in.})^4}{64}$$
$$= 0.0491 \text{ in.}^4$$

$$I_{sr} = \sum_{i=1}^{n} I_{sri} + \sum_{i=1}^{n} A_{sri} e_i^2$$
$$= 8\left(0.0491 \text{ in.}^4\right) + 6\left(0.79 \text{ in.}^2\right)\left(9.50 \text{ in.}\right)^2 + 2\left(0.79 \text{ in.}^2\right)\left(0 \text{ in.}\right)^2$$
$$= 428 \text{ in.}^4$$

where

A_{sri} = cross-sectional area of reinforcing bar i, in.^{2}
I_{sri} = moment of inertia of reinforcing bar i about its elastic neutral axis, in.^{4}
I_{sr} = moment of inertia of the reinforcing bars about the elastic neutral axis of the composite section, in.^{4}
d_b = nominal diameter of reinforcing bar, in.
e_i = eccentricity of reinforcing bar i with respect to the elastic neutral axis of the composite section, in.
n = number of reinforcing bars in composite section

Note that the elastic neutral axis for each direction of the section in question is located at the *x-x* and *y-y* axes illustrated in Figure I.9-1, and that the moment of inertia calculated for the longitudinal reinforcement is valid about either axis due to symmetry.

The moment of inertia values for the concrete about each axis are determined as:

$$I_{cx} = I_{gx} - I_{sx} - I_{srx}$$
$$= \frac{(24.0 \text{ in.})^4}{12} - 248 \text{ in.}^4 - 428 \text{ in.}^4$$
$$= 27{,}000 \text{ in.}^4$$

$$I_{cy} = I_{gy} - I_{sy} - I_{sry}$$
$$= \frac{(24.0 \text{ in.})^4}{12} - 53.4 \text{ in.}^4 - 428 \text{ in.}^4$$
$$= 27{,}200 \text{ in.}^4$$

Classify Section for Local Buckling

In accordance with AISC *Specification* Section I1.2, local buckling effects need not be considered for encased composite members, thus all encased sections are treated as compact sections for strength calculations.

Material and Detailing Limitations

According to the User Note at the end of AISC *Specification* Section I1.1, the intent of the *Specification* is to implement the noncomposite detailing provisions of ACI 318 in conjunction with the composite-specific provisions of *Specification* Chapter I. Detailing provisions may be grouped into material related limits, transverse reinforcement provisions, and longitudinal and structural steel reinforcement provisions as illustrated in the following discussion.

Material limits are provided in AISC *Specification* Sections I1.1(2) and I1.3 as follows:

(1) Concrete strength: $3 \text{ ksi} \le f_c' \le 10 \text{ ksi}$

$f_c' = 5 \text{ ksi}$ **o.k.**

(2) Specified minimum yield stress of structural steel: $F_y \le 75 \text{ ksi}$

$F_y = 50 \text{ ksi}$ **o.k.**

(3) Specified minimum yield stress of reinforcing bars: $F_{yr} \le 75 \text{ ksi}$

$F_{yr} = 60 \text{ ksi}$ **o.k.**

Transverse reinforcement limitations are provided in AISC *Specification* Section I1.1(3), I2.1a(2) and ACI 318 as follows:

(1) Tie size and spacing limitations:

The AISC *Specification* requires that either lateral ties or spirals be used for transverse reinforcement. Where lateral ties are used, a minimum of either No. 3 bars spaced at a maximum of 12 in. on center or No. 4 bars or larger spaced at a maximum of 16 in. on center are required.

No. 3 lateral ties at 12 in. o.c. are provided. **o.k.**

Note that AISC *Specification* Section I1.1(1) specifically excludes the composite column provisions of ACI 318 Section 10.13, so it is unnecessary to meet the tie reinforcement provisions of ACI 318 Section 10.13.8 when designing composite columns using the provisions of AISC *Specification* Chapter I.

If spirals are used, the requirements of ACI 318 Sections 7.10 and 10.9.3 should be met according to the User Note at the end of AISC *Specification* Section I2.1a.

(2) Additional tie size limitation:

No. 4 ties or larger are required where No. 11 or larger bars are used as longitudinal reinforcement in accordance with ACI 318 Section 7.10.5.1.

No. 3 lateral ties are provided for No. 8 longitudinal bars. **o.k.**

(3) Maximum tie spacing should not exceed 0.5 times the least column dimension:

$$s_{max} = 0.5 \min \begin{Bmatrix} h_1 = 24.0 \text{ in.} \\ h_2 = 24.0 \text{ in.} \end{Bmatrix}$$

$$= 12.0 \text{ in.}$$

$$s = 12.0 \text{ in.} \le s_{max} \quad \textbf{o.k.}$$

(4) Concrete cover:

ACI 318 Section 7.7 contains concrete cover requirements. For concrete not exposed to weather or in contact with ground, the required cover for column ties is 1½ in.

$$\text{cover} = 2.5 \text{ in.} - \frac{d_b}{2} - \text{diameter of No. 3 tie}$$

$$= 2.5 \text{ in.} - \tfrac{1}{2} \text{ in.} - \tfrac{3}{8} \text{ in.}$$

$$= 1.63 \text{ in.} > 1\tfrac{1}{2} \text{ in.} \quad \textbf{o.k.}$$

(5) Provide ties as required for lateral support of longitudinal bars:

AISC *Specification* Commentary Section I2.1a references Chapter 7 of ACI 318 for additional transverse tie requirements. In accordance with ACI 318 Section 7.10.5.3 and Figure R7.10.5, ties are required to support longitudinal bars located farther than 6 in. clear on each side from a laterally supported bar. For corner bars, support is typically provided by the main perimeter ties. For intermediate bars, Figure I.9-1 illustrates one method for providing support through the use of a diamond-shaped tie.

Longitudinal and structural steel reinforcement limits are provided in AISC *Specification* Sections I1.1(4), I2.1 and ACI 318 as follows:

(1) Structural steel minimum reinforcement ratio: $A_s/A_g \geq 0.01$

$$\frac{13.3 \text{ in.}^2}{576 \text{ in.}^2} = 0.0231 \quad \textbf{o.k.}$$

An explicit maximum reinforcement ratio for the encased steel shape is not provided in the AISC *Specification*; however, a range of 8 to 12% has been noted in the literature to result in economic composite members for the resistance of gravity loads (Leon and Hajjar, 2008).

(2) Minimum longitudinal reinforcement ratio: $A_{sr}/A_g \geq 0.004$

$$\frac{6.32 \text{ in.}^2}{576 \text{ in.}^2} = 0.0110 \quad \textbf{o.k.}$$

As discussed in AISC *Specification Commentary* Section I2.1a(3), only continuously developed longitudinal reinforcement is included in the minimum reinforcement ratio, so longitudinal restraining bars and other discontinuous longitudinal reinforcement is excluded. Note that this limitation is used in lieu of the minimum ratio provided in ACI 318 as discussed in *Specification* Commentary Section I1.1(4).

(3) Maximum longitudinal reinforcement ratio: $A_{sr}/A_g \leq 0.08$

$$\frac{6.32 \text{ in.}^2}{576 \text{ in.}^2} = 0.0110 \quad \textbf{o.k.}$$

This longitudinal reinforcement limitation is provided in ACI 318 Section 10.9.1. It is recommended that all longitudinal reinforcement, including discontinuous reinforcement not used in strength calculations, be included in this ratio as it is considered a practical limitation to mitigate congestion of reinforcement. If longitudinal reinforcement is lap spliced as opposed to mechanically coupled, this limit is effectively reduced to 4% in areas away from the splice location.

(4) Minimum number of longitudinal bars:

ACI 318 Section 10.9.2 requires a minimum of four longitudinal bars within rectangular or circular members with ties and six bars for columns utilizing spiral ties. The intent for rectangular sections is to provide a minimum of one bar in each corner, so irregular geometries with multiple corners require additional longitudinal bars.

8 bars provided. **o.k.**

(5) Clear spacing between longitudinal bars:

ACI 318 Section 7.6.3 requires a clear distance between bars of $1.5d_b$ or 1½ in.

$$s_{min} = \max \begin{Bmatrix} 1.5d_b = 1\tfrac{1}{2} \text{ in.} \\ 1\tfrac{1}{2} \text{ in.} \end{Bmatrix}$$
$$= 1\tfrac{1}{2} \text{ in. clear}$$
$$s = 9.5 \text{ in.} - 1.0 \text{ in.}$$
$$= 8.5 \text{ in.} > 1\tfrac{1}{2} \text{ in.} \quad \textbf{o.k.}$$

(6) Clear spacing between longitudinal bars and the steel core:

AISC *Specification* Section I2.1e requires a minimum clear spacing between the steel core and longitudinal reinforcement of 1.5 reinforcing bar diameters, but not less than 1½ in.

$$s_{min} = \max \begin{Bmatrix} 1.5d_b = 1\tfrac{1}{2} \text{ in.} \\ 1\tfrac{1}{2} \text{ in.} \end{Bmatrix}$$
$$= 1\tfrac{1}{2} \text{ in. clear}$$

Closest reinforcing bars to the encased section are the center bars adjacent to each flange:

$$s = \frac{h_2}{2} - \frac{d}{2} - 2.5 \text{ in.} - \frac{d_b}{2}$$
$$= \frac{24 \text{ in.}}{2} - \frac{10.1 \text{ in.}}{2} - 2.5 \text{ in.} - \frac{1 \text{ in.}}{2}$$
$$= 3.95 \text{ in.}$$
$$s = 3.95 \text{ in.} \geq s_{min} = 1\tfrac{1}{2} \text{ in.} \quad \textbf{o.k.}$$

(7) Concrete cover for longitudinal reinforcement:

ACI 318 Section 7.7 provides concrete cover requirements for reinforcement. The cover requirements for column ties and primary reinforcement are the same, and the tie cover was previously determined to be acceptable, thus the longitudinal reinforcement cover is acceptable by inspection.

From Chapter 2 of ASCE/SEI, the required compressive strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(260 \text{ kips}) + 1.6(780 \text{ kips})$ $= 1{,}560 \text{ kips}$	$P_r = P_a$ $= 260 \text{ kips} + 780 \text{ kips}$ $= 1{,}040 \text{ kips}$

Available Compressive Strength

The nominal axial compressive strength without consideration of length effects, P_{no}, is determined from AISC *Specification* Section I2.1b as:

$$P_{no} = F_y A_s + F_{ysr} A_{sr} + 0.85 f_c' A_c \qquad (Spec.\text{ Eq. I2-4})$$
$$= (50 \text{ ksi})(13.3 \text{ in.}^2) + (60 \text{ ksi})(6.32 \text{ in.}^2) + 0.85(5 \text{ ksi})(556 \text{ in.}^2)$$
$$= 3{,}410 \text{ kips}$$

Because the unbraced length is the same in both the *x-x* and *y-y* directions, the column will buckle about the axis having the smaller effective composite section stiffness, EI_{eff}. Noting the moment of inertia values determined previously for the concrete and reinforcing steel are similar about each axis, the column will buckle about the weak axis of the steel shape by inspection. I_{cy}, I_{sy} and I_{sry} are therefore used for calculation of length effects in accordance with AISC *Specification* Section I2.1b as follows:

$$C_1 = 0.1 + 2\left(\frac{A_s}{A_c + A_s}\right) \leq 0.3 \qquad \text{(}Spec.\text{ Eq. I2-7)}$$

$$= 0.1 + 2\left(\frac{13.3 \text{ in.}^2}{556 \text{ in.}^2 + 13.3 \text{ in.}^2}\right) \leq 0.3$$

$$= 0.147 < 0.3 \qquad \textbf{0.147 controls}$$

$$EI_{eff} = E_s I_{sy} + 0.5 E_s I_{sry} + C_1 E_c I_{cy} \qquad \text{(from }Spec.\text{ Eq. I2-6)}$$

$$= (29{,}000 \text{ ksi})(53.4 \text{ in.}^4) + 0.5(29{,}000 \text{ ksi})(428 \text{ in.}^4)$$

$$+ 0.147(3{,}900 \text{ ksi})(27{,}200 \text{ in.}^4)$$

$$= 23{,}300{,}000 \text{ ksi}$$

$$P_e = \pi^2 (EI_{eff}) / (KL)^2 \text{ where } K = 1.0 \text{ for a pin-ended member} \qquad \text{(}Spec.\text{ Eq. I2-5)}$$

$$= \frac{\pi^2 (23{,}300{,}000 \text{ ksi})}{\left[(1.0)(14 \text{ ft})(12 \text{ in./ft})\right]^2}$$

$$= 8{,}150 \text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{3{,}410 \text{ kips}}{8{,}150 \text{ kips}}$$

$$= 0.418 < 2.25$$

Therefore, use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left[0.658^{\frac{P_{no}}{P_e}}\right] \qquad \text{(}Spec.\text{ Eq. I2-2)}$$

$$= (3{,}410 \text{ kips})(0.658)^{0.418}$$

$$= 2{,}860 \text{ kips}$$

Check adequacy of the composite column for the required axial compressive strength:

LRFD	ASD
$\phi_c = 0.75$ $\phi_c P_n \geq P_u$ $\phi_c P_n = 0.75(2{,}860 \text{ kips})$ $= 2{,}150 \text{ kips} > 1{,}560 \text{ kips}$ **o.k.**	$\Omega_c = 2.00$ $P_n / \Omega_c \geq P_a$ $P_n / \Omega_c = \dfrac{2{,}860 \text{ kips}}{2.00}$ $= 1{,}430 \text{ kips} > 1{,}040 \text{ kips}$ **o.k.**

Available Compressive Strength of Composite Section Versus Bare Steel Section

Due to the differences in resistance and safety factors between composite and noncomposite column provisions, it is possible in rare instances to calculate a lower available compressive strength for an encased composite column than one would calculate for the corresponding bare steel section. However, in accordance with AISC *Specification* Section I2.1b, the available compressive strength need not be less than that calculated for the bare steel member in accordance with Chapter E.

From AISC *Manual* Table 4-1:

LRFD	ASD
$\phi_c P_n = 359 \text{ kips}$ $359 \text{ kips} < 2,150 \text{ kips}$	$P_n / \Omega_c = 239 \text{ kips}$ $239 \text{ kips} < 1,430 \text{ kips}$

Thus, the composite section strength controls and is adequate for the required axial compressive strength as previously demonstrated.

Force Allocation and Load Transfer

Load transfer calculations for external axial forces should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions for encased composite members is provided in Design Example I.8.

Typical Detailing Convention

Designers are directed to AISC Design Guide 6 (Griffis, 1992) for additional discussion and typical details of encased composite columns not explicitly covered in this example.

EXAMPLE I.10 ENCASED COMPOSITE MEMBER IN AXIAL TENSION

Given:

Determine if the 14 ft long, encased composite member illustrated in Figure I.10-1 is adequate for the indicated dead load compression and wind load tension. The entire load is applied to the encased steel section.

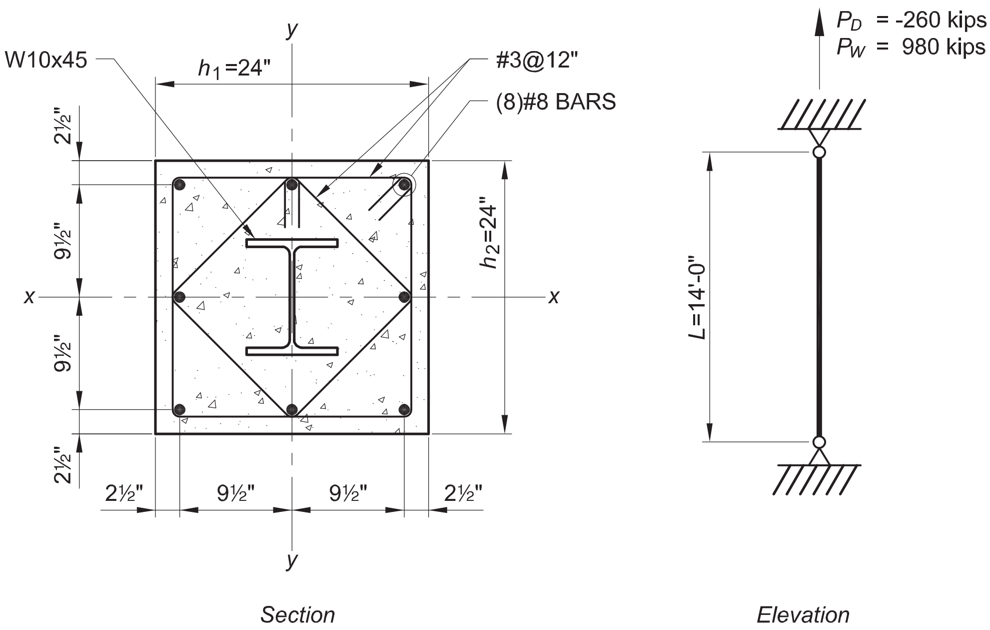

Fig. I.10-1. Encased composite member section and applied loading.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, $f_c' = 5$ ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, F_{yr}, of 60 ksi.

Solution:

From AISC *Manual* Table 2-4, the steel material properties are:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1 and Figure I.10-1, the relevant properties of the composite section are:

$A_s = 13.3$ in.^{2}

$A_{sr} = 6.32$ in.^{2} (area of eight No. 8 bars)

Material and Detailing Limitations

Refer to Design Example I.9 for a check of material and detailing limitations specified in AISC *Specification* Chapter I for encased composite members.

Taking compression as negative and tension as positive, from Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
Governing Uplift Load Combination $= 0.9D + 1.0W$ $P_r = P_u$ $= 0.9(-260 \text{ kips}) + 1.0(980 \text{ kips})$ $= 746 \text{ kips}$	Governing Uplift Load Combination $= 0.6D + 0.6W$ $P_r = P_a$ $= 0.6(-260 \text{ kips}) + 0.6(980 \text{ kips})$ $= 432 \text{ kips}$

Available Tensile Strength

Available tensile strength for an encased composite member is determined in accordance with AISC *Specification* Section I2.1c.

$$
\begin{aligned}
P_n &= F_y A_s + F_{ysr} A_{sr} \qquad \text{(}\textit{Spec}\text{. Eq. I2-8)} \\
&= (50 \text{ ksi})(13.3 \text{ in.}^2) + (60 \text{ ksi})(6.32 \text{ in.}^2) \\
&= 1{,}040 \text{ kips}
\end{aligned}
$$

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n \geq P_u$ $\phi_t P_n = 0.90(1{,}040 \text{ kips})$ $= 936 \text{ kips} > 746 \text{ kips}$ **o.k.**	$\Omega_t = 1.67$ $P_n / \Omega_t \geq P_a$ $P_n / \Omega_t = \dfrac{1{,}040 \text{ kips}}{1.67}$ $= 623 \text{ kips} > 432 \text{ kips}$ **o.k.**

Force Allocation and Load Transfer

In cases where all of the tension is applied to either the reinforcing steel or the encased steel shape, and the available strength of the reinforcing steel or encased steel shape by itself is adequate, no additional load transfer calculations are required.

In cases such as the one under consideration, where the available strength of both the reinforcing steel and the encased steel shape are needed to provide adequate tension resistance, AISC *Specification* Section I6 can be modified for tensile load transfer requirements by replacing the P_{no} term in Equations I6-1 and I6-2 with the nominal tensile strength, P_n, determined from Equation I2-8.

For external tensile force applied to the encased steel section:

$$V_r' = P_r\left(1 - \frac{F_y A_s}{P_n}\right) \qquad \text{(Eq. 1)}$$

For external tensile force applied to the longitudinal reinforcement of the concrete encasement:

$$V_r' = P_r\left(\frac{F_y A_s}{P_n}\right) \qquad \text{(Eq. 2)}$$

where

P_r = required external tensile force applied to the composite member, kips

P_n = nominal tensile strength of encased composite member from Equation I2-8, kips

Per the problem statement, the entire external force is applied to the encased steel section, thus Equation 1 is used as follows:

$$V_r' = P_r\left[1-\frac{(50\text{ ksi})(13.3\text{ in.}^2)}{1{,}040\text{ kips}}\right]$$
$$= 0.361P_r$$

LRFD	ASD
$V_r' = 0.361(746\text{ kips})$ $= 269\text{ kips}$	$V_r' = 0.361(432\text{ kips})$ $= 156\text{ kips}$

The longitudinal shear force must be transferred between the encased steel shape and longitudinal reinforcing using the force transfer mechanisms of direct bearing or shear connection in accordance with AISC *Specification* Section I6.3 as illustrated in Design Example I.8.

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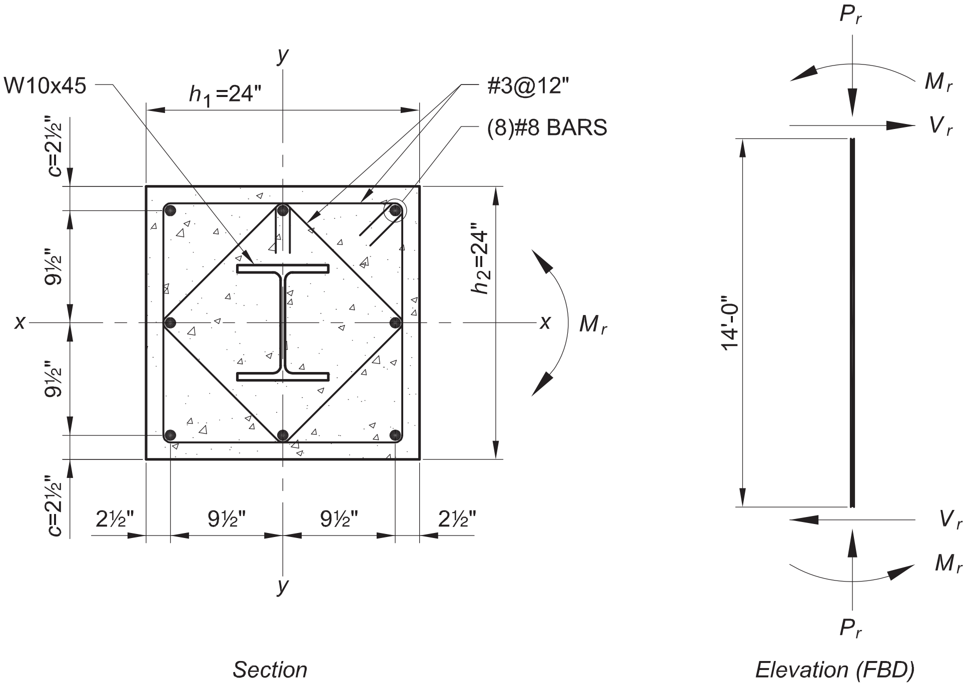
EXAMPLE I.11 ENCASED COMPOSITE MEMBER IN COMBINED AXIAL COMPRESSION, FLEXURE AND SHEAR

Given:

Determine if the 14 ft long, encased composite member illustrated in Figure I.11-1 is adequate for the indicated axial forces, shears and moments that have been determined in accordance with the direct analysis method of AISC *Specification* Chapter C for the controlling ASCE/SEI 7-10 load combinations.



	LRFD	ASD
P_r (kips)	1,170	879
M_r (kip-ft)	670	302
V_r (kips)	95.7	57.4

Fig. I.11-1. Encased composite member section and member forces.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, f'_c = 5 ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, F_{yr}, of 60 ksi.

Solution:

From AISC *Manual* Table 2-4, the steel material properties are:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, Figure I.11-1, and Design Examples I.8 and I.9, the geometric and material properties of the composite section are:

$A_s = 13.3 \text{ in.}^2$	$I_{sy} = 53.4 \text{ in.}^4$	$A_g = 576 \text{ in.}^2$	$h_1 = 24.0 \text{ in.}$
$d = 10.1 \text{ in.}$	$Z_{sx} = 54.9 \text{ in.}^3$	$A_{sr} = 6.32 \text{ in.}^2$	$h_2 = 24.0 \text{ in.}$
$b_f = 8.02 \text{ in.}$	$S_{sx} = 49.1 \text{ in.}^3$	$A_c = 556 \text{ in.}^2$	$c = 2\tfrac{1}{2} \text{ in.}$
$t_f = 0.620 \text{ in.}$	$E_c = 3{,}900 \text{ ksi}$	$I_{sr} = 428 \text{ in.}^4$	$I_{cx} = 27{,}000 \text{ in.}^4$
$t_w = 0.350 \text{ in.}$			$I_{cy} = 27{,}200 \text{ in.}^4$

The area of continuous reinforcing located at the centerline of the composite section, A_{srs}, is determined from Figure I.11-1 as follows:

$$\begin{aligned} A_{srs} &= 2\left(A_{srsi}\right) \\ &= 2\left(0.79 \text{ in.}^2\right) \\ &= 1.58 \text{ in.}^2 \end{aligned}$$

where

A_{srsi} = area of reinforcing bar i at centerline of composite section
= 0.79 in.^2 for a No. 8 bar

For the section under consideration, A_{srs} is equal about both the x-x and y-y axis.

Classify Section for Local Buckling

In accordance with AISC *Specification* Section I1.2, local buckling effects need not be considered for encased composite members, thus all encased sections are treated as compact sections for strength calculations.

Material and Detailing Limitations

Refer to Design Example I.9 for a check of material and detailing limitations.

Interaction of Axial Force and Flexure

Interaction between axial forces and flexure in composite members is governed by AISC *Specification* Section I5 which permits the use of a strain compatibility method or plastic stress distribution method.

The strain compatibility method is a generalized approach that allows for the construction of an interaction diagram based upon the same concepts used for reinforced concrete design. Application of the strain compatibility method is required for irregular/nonsymmetrical sections, and its general implementation may be found in reinforced concrete design texts and will not be discussed further here.

Plastic stress distribution methods are discussed in AISC *Specification* Commentary Section I5 which provides four procedures. The first procedure, Method 1, invokes the interaction equations of Section H1. The second procedure, Method 2, involves the construction of a piecewise-linear interaction curve using the plastic strength equations provided in Figure I-1 located within the front matter of the Chapter I Design Examples. The third procedure, Method 2—Simplified, is a reduction of the piecewise-linear interaction curve that allows for the use of less conservative interaction equations than those presented in Chapter H. The fourth and final procedure, Method 3, utilizes AISC *Design Guide 6* (Griffis, 1992).

For this design example, three of the available plastic stress distribution procedures are reviewed and compared. Method 3 is not demonstrated as it is not applicable to the section under consideration due to the area of the encased steel section being smaller than the minimum limit of 4% of the gross area of the composite section provided in the earlier *Specification* upon which Design Guide 6 is based.

Method 1—Interaction Equations of Section H1

The most direct and conservative method of assessing interaction effects is through the use of the interaction equations of AISC *Specification* Section H1. Unlike concrete filled HSS shapes, the available compressive and flexural strengths of encased members are not tabulated in the AISC *Manual* due to the large variety of possible combinations. Calculations must therefore be performed explicitly using the provisions of Chapter I.

Available Compressive Strength

The available compressive strength is calculated as illustrated in Design Example I.9.

LRFD	ASD
$\phi_c P_n = 2{,}150 \text{ kips}$	$P_n / \Omega_c = 1{,}430 \text{ kips}$

Nominal Flexural Strength

The applied moment illustrated in Figure I.11-1 is resisted by the flexural strength of the composite section about its strong (*x-x*) axis. The strength of the section in pure flexure is calculated using the equations of Figure I-1a found in the front matter of the Chapter I Design Examples for Point B. Note that the calculation of the flexural strength at Point B first requires calculation of the flexural strength at Point D as follows:

$$
\begin{aligned}
Z_r &= \left(A_{sr} - A_{srs}\right)\left(\frac{h_2}{2} - c\right) \\
&= \left(6.32 \text{ in.}^2 - 1.58 \text{ in.}^2\right)\left(\frac{24.0 \text{ in.}}{2} - 2.5 \text{ in.}\right) \\
&= 45.0 \text{ in.}^3
\end{aligned}
$$

$$
\begin{aligned}
Z_c &= \frac{h_1 h_2^2}{4} - Z_s - Z_r \\
&= \frac{(24.0 \text{ in.})(24.0 \text{ in.})^2}{4} - 54.9 \text{ in.}^3 - 45.0 \text{ in.}^3 \\
&= 3{,}360 \text{ in.}^3
\end{aligned}
$$

$$
\begin{aligned}
M_D &= Z_s F_y + Z_r F_{yr} + \frac{Z_c}{2}\left(0.85 f_c'\right) \\
&= \left(54.9 \text{ in.}^3\right)(50 \text{ ksi}) + \left(45.0 \text{ in.}^3\right)(60 \text{ ksi}) + \frac{3{,}360 \text{ in.}^3}{2}(0.85)(5 \text{ ksi}) \\
&= \frac{12{,}600 \text{ kip-in.}}{12 \text{ in./ft}} \\
&= 1{,}050 \text{ kip-ft}
\end{aligned}
$$

Assuming h_n is within the flange $\left(\frac{d}{2} - t_f < h_n \leq \frac{d}{2}\right)$:

$$
\begin{aligned}
h_n &= \frac{0.85 f_c'\left(A_c + A_s - d b_f + A_{srs}\right) - 2F_y\left(A_s - d b_f\right) - 2F_{yr} A_{srs}}{2\left[0.85 f_c'\left(h_1 - b_f\right) + 2F_y b_f\right]} \\
&= \Big\{0.85(5 \text{ ksi})\left[556 \text{ in.}^2 + 13.3 \text{ in.}^2 - (10.1 \text{ in.})(8.02 \text{ in.}) + 1.58 \text{ in.}^2\right] - 2(50 \text{ ksi})\left[13.3 \text{ in.}^2 - (10.1 \text{ in.})(8.02 \text{ in.})\right] \\
&\quad - 2(60 \text{ ksi})\left(1.58 \text{ in.}^2\right)\Big\} \Big/ 2\left[0.85(5 \text{ ksi})(24.0 \text{ in.} - 8.02 \text{ in.}) + 2(50 \text{ ksi})(8.02 \text{ in.})\right] \\
&= 4.98 \text{ in.}
\end{aligned}
$$

Check assumption:

$$\left(\frac{10.1\text{ in.}}{2} - 0.620\text{ in.}\right) \le h_n \le \frac{10.1\text{ in.}}{2}$$

$$4.43\text{ in.} < h_n = 4.98\text{ in.} < 5.05\text{ in.} \quad \text{assumption } \textbf{o.k.}$$

$$\begin{aligned}
Z_{sn} &= Z_s - b_f\left(\frac{d}{2} - h_n\right)\left(\frac{d}{2} + h_n\right) \\
&= 54.9\text{ in.}^3 - (8.02\text{ in.})\left(\frac{10.1\text{ in.}}{2} - 4.98\text{ in.}\right)\left(\frac{10.1\text{ in.}}{2} + 4.98\text{ in.}\right) \\
&= 49.3\text{ in.}^3
\end{aligned}$$

$$\begin{aligned}
Z_{cn} &= h_1 h_n^2 - Z_{sn} \\
&= (24.0\text{ in.})(4.98\text{ in.})^2 - 49.3\text{ in.}^3 \\
&= 546\text{ in.}^3
\end{aligned}$$

$$\begin{aligned}
M_B &= M_D - Z_{sn}F_y - \frac{Z_{cn}(0.85 f_c')}{2} \\
&= 12{,}600\text{ kip-in.} - (49.3\text{ in.}^3)(50\text{ ksi}) - \frac{(546\text{ in.}^3)(0.85)(5\text{ ksi})}{2} \\
&= \frac{8{,}970\text{ kip-in.}}{12\text{ in./ft}} \\
&= 748\text{ kip-ft}
\end{aligned}$$

Available Flexural Strength

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(748\text{ kip-ft})$ $= 673\text{ kip-ft}$	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{748\text{ kip-ft}}{1.67}$ $= 448\text{ kip-ft}$

Interaction of Axial Compression and Flexure

LRFD	ASD
$\phi_c P_n = 2{,}150\text{ kips}$ $\phi_b M_n = 673\text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{1{,}170\text{ kips}}{2{,}150\text{ kips}}$ $= 0.544 > 0.2$ Use AISC *Specification* Equation H1-1a. $\dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_u}{\phi_b M_n}\right) \le 1.0$	$P_n / \Omega_c = 1{,}430\text{ kips}$ $M_n / \Omega_c = 448\text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_c}$ $= \dfrac{879\text{ kips}}{1{,}430\text{ kips}}$ $= 0.615 > 0.2$ Use AISC *Specification* Equation H1-1a. $\dfrac{P_a}{P_n / \Omega_c} + \dfrac{8}{9}\left(\dfrac{M_a}{M_n / \Omega_b}\right) \le 1.0$

LRFD	ASD
$\dfrac{1{,}170 \text{ kips}}{2{,}150 \text{ kips}}+\dfrac{8}{9}\left(\dfrac{670 \text{ kip-ft}}{673 \text{ kip-ft}}\right)\leq 1.0$ $1.43>1.0$ **n.g.**	$\dfrac{879 \text{ kips}}{1{,}430 \text{ kips}}+\dfrac{8}{9}\left(\dfrac{302 \text{ kip-ft}}{448 \text{ kip-ft}}\right)\leq 1.0$ $1.21>1.0$ **n.g.**

Method 1 indicates that the section is inadequate for the applied loads. The designer can elect to choose a new section that passes the interaction check or re-analyze the current section using a less conservative design method such as Method 2. The use of Method 2 is illustrated in the following section.

Method 2—Interaction Curves from the Plastic Stress Distribution Model

The procedure for creating an interaction curve using the plastic stress distribution model is illustrated graphically in AISC *Specification* Commentary Figure C-I5.2, and repeated here.

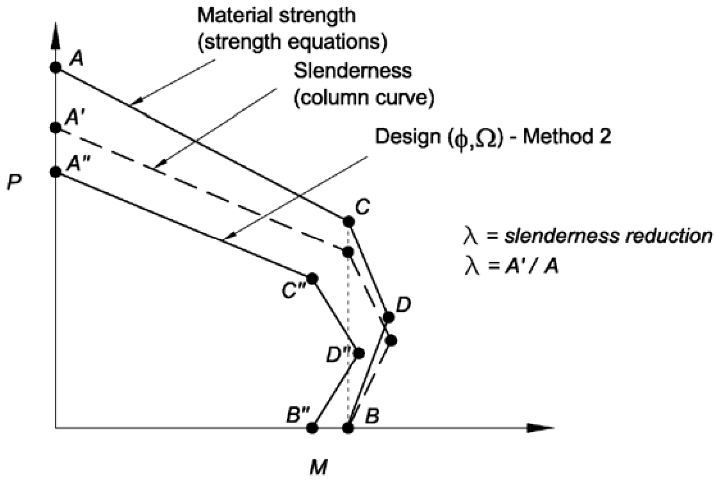


Fig. C-I5.2. Interaction diagram for composite beam-columns – Method 2.

Referencing Figure C.I5.2, the nominal strength interaction surface A, B, C, D is first determined using the equations of Figure I-1a found in the front matter of the Chapter I Design Examples. This curve is representative of the short column member strength without consideration of length effects. A slenderness reduction factor, λ, is then calculated and applied to each point to create surface A′, B′, C′, D′ . The appropriate resistance or safety factors are then applied to create the design surface A″ , B″ , C″ , D″ . Finally, the required axial and flexural strengths from the applicable load combinations of ASCE/SEI 7-10 are plotted on the design surface. The member is then deemed acceptable for the applied loading if all points fall within the design surface. These steps are illustrated in detail by the following calculations.

Step 1: Construct nominal strength interaction surface A, B, C, D without length effects

Using the equations provided in Figure I-1a for bending about the *x-x* axis yields:

Point A (pure axial compression):

$$\begin{aligned} P_A &= A_sF_y + A_{sr}F_{yr} + 0.85f_c'A_c \\ &= \left(13.3 \text{ in.}^2\right)\left(50 \text{ ksi}\right)+\left(6.32 \text{ in.}^2\right)\left(60 \text{ ksi}\right)+0.85\left(5 \text{ ksi}\right)\left(556 \text{ in.}^2\right) \\ &= 3{,}410 \text{ kips} \end{aligned}$$

$$M_A = 0 \text{ kip-ft}$$

Point D (maximum nominal moment strength):

$$P_D = \frac{0.85 f_c' A_c}{2}$$
$$= \frac{0.85(5 \text{ ksi})(556 \text{ in.}^2)}{2}$$
$$= 1{,}180 \text{ kips}$$

Calculation of M_D was demonstrated previously in Method 1.

$$M_D = 1{,}050 \text{ kip-ft}$$

Point B (pure flexure):

$$P_B = 0 \text{ kips}$$

Calculation of M_B was demonstrated previously in Method 1.

$$M_B = 748 \text{ kip-ft}$$

Point C (intermediate point):

$$P_C = 0.85 f_c' A_c$$
$$= 0.85(5 \text{ ksi})(556 \text{ in.}^2)$$
$$= 2{,}360 \text{ kips}$$
$$M_C = M_B$$
$$= 748 \text{ kip-ft}$$

The calculated points are plotted to construct the nominal strength interaction surface without length effects as depicted in Figure I.11-2.

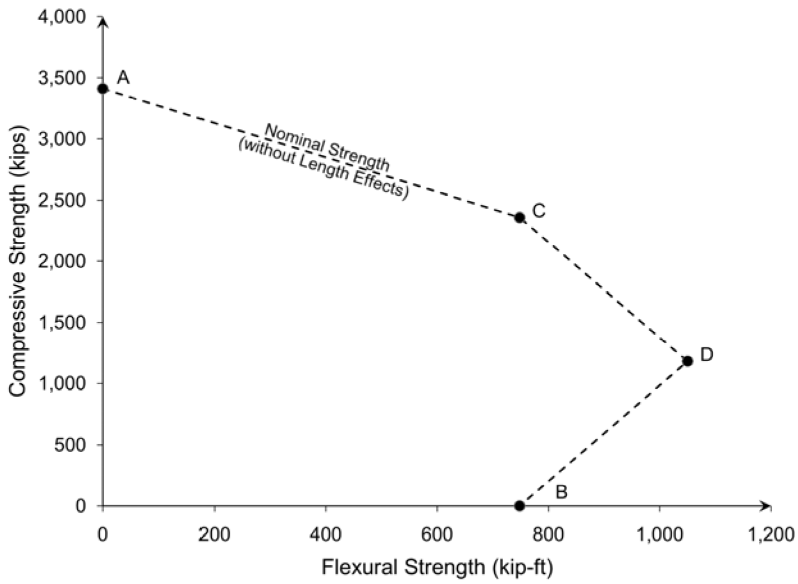


Fig. I.11-2. Nominal strength interaction surface without length effects.

Step 2: Construct nominal strength interaction surface A′, B′, C′, D′ with length effects

The slenderness reduction factor, λ, is calculated for Point A using AISC *Specification* Section I2.1 in accordance with *Specification* Commentary Section I5.

Because the unbraced length is the same in both the *x-x* and *y-y* directions, the column will buckle about the axis having the smaller effective composite section stiffness, EI_{eff}. Noting the moment of inertia values for the concrete and reinforcing steel are similar about each axis, the column will buckle about the weak axis of the steel shape by inspection. I_{cy}, I_{sy} and I_{sry} are therefore used for calculation of length effects in accordance with AISC *Specification* Section I2.1b.

$$\begin{aligned} P_{no} &= P_A \\ &= 3{,}410 \text{ kips} \end{aligned}$$

$$\begin{aligned} C_1 &= 0.1 + 2\left(\frac{A_s}{A_c + A_s}\right) \le 0.3 \qquad (\textit{Spec}.\text{ Eq. I2-7}) \\ &= 0.1 + 2\left(\frac{13.3 \text{ in.}^2}{556 \text{ in.}^2 + 13.3 \text{ in.}^2}\right) \le 0.3 \\ &= 0.147 < 0.3; \text{ therefore } C_1 = 0.147. \end{aligned}$$

$$\begin{aligned} EI_{eff} &= E_s I_{sy} + 0.5 E_s I_{sry} + C_1 E_c I_{cy} \qquad (\textit{Spec}.\text{ Eq. I2-6}) \\ &= (29{,}000 \text{ ksi})(53.4 \text{ in.}^4) + 0.5(29{,}000 \text{ ksi})(428 \text{ in.}^4) \\ &\quad + 0.147(3{,}900 \text{ ksi})(27{,}200 \text{ in.}^4) \\ &= 23{,}300{,}000 \text{ ksi} \end{aligned}$$

$$\begin{aligned} P_e &= \pi^2 (EI_{eff})/(KL)^2 \text{ where } K = 1.0 \text{ in accordance with the direct analysis method} \qquad (\textit{Spec}.\text{ Eq. I2-5}) \\ &= \frac{\pi^2 (23{,}300{,}000 \text{ ksi})}{\left[(1.0)(14 \text{ ft})(12 \text{ in./ft})\right]^2} \\ &= 8{,}150 \text{ kips} \end{aligned}$$

$$\begin{aligned} \frac{P_{no}}{P_e} &= \frac{3{,}410 \text{ kips}}{8{,}150 \text{ kips}} \\ &= 0.418 < 2.25 \end{aligned}$$

Therefore, use AISC *Specification* Equation I2-2.

$$\begin{aligned} P_n &= P_{no}\left[0.658^{\frac{P_{no}}{P_e}}\right] \qquad (\textit{Spec}.\text{ Eq. I2-2}) \\ &= 3{,}410 \text{ kips}(0.658)^{0.418} \\ &= 2{,}860 \text{ kips} \end{aligned}$$

$$\begin{aligned} \lambda &= \frac{P_n}{P_{no}} \\ &= \frac{2{,}860 \text{ kips}}{3{,}410 \text{ kips}} \\ &= 0.839 \end{aligned}$$

In accordance with AISC *Specification* Commentary Section I5, the same slenderness reduction is applied to each of the remaining points on the interaction surface as follows:

$$P_{A'} = \lambda P_A$$
$$= 0.839(3,410 \text{ kips})$$
$$= 2,860 \text{ kips}$$

$$P_{B'} = \lambda P_B$$
$$= 0.839(0 \text{ kips})$$
$$= 0 \text{ kips}$$

$$P_{C'} = \lambda P_C$$
$$= 0.839(2,360 \text{ kips})$$
$$= 1,980 \text{ kips}$$

$$P_{D'} = \lambda P_D$$
$$= 0.839(1,180 \text{ kips})$$
$$= 990 \text{ kips}$$

The modified axial strength values are plotted with the flexural strength values previously calculated to construct the nominal strength interaction surface including length effects. These values are superimposed on the nominal strength surface not including length effects for comparison purposes in Figure I.11-3.

The consideration of length effects results in a vertical reduction of the nominal strength curve as illustrated by Figure I.11-3. This vertical movement creates an unsafe zone within the shaded area of the figure where flexural capacities of the nominal strength (with length effects) curve exceed the section capacity. Application of resistance or safety factors reduces this unsafe zone as illustrated in the following step; however, designers should be cognizant of the potential for unsafe designs with loads approaching the predicted flexural capacity of the section. Alternately, the use of Method 2—Simplified eliminates this possibility altogether.

Step 3: Construct design interaction surface A″, B″, C″, D″ and verify member adequacy

The final step in the Method 2 procedure is to reduce the interaction surface for design using the appropriate resistance or safety factors.

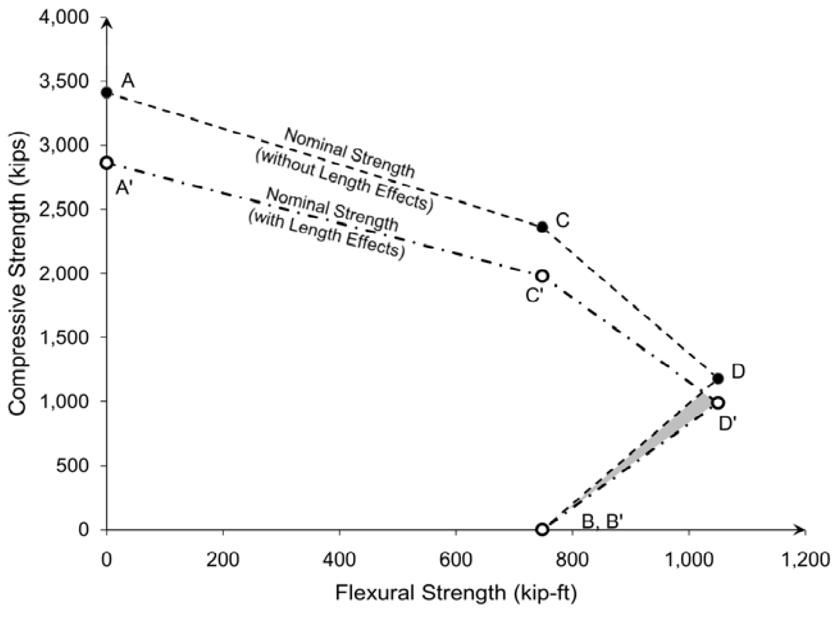


Fig. I.11-3. Nominal strength interaction surfaces (with and without length effects).

The available compressive and flexural strengths are determined as follows:

LRFD	ASD
Design compressive strength:	Allowable compressive strength:
$\phi_c = 0.75$	$\Omega_c = 2.00$
$P_{X''} = \phi_c P_{X'}$	$P_{X''} = P_{X'} / \Omega_c$
where $X = A,\ B,\ C$ or D	where $X = A,\ B,\ C$ or D
$P_{A''} = 0.75(2{,}860\text{ kips})$	$P_{A''} = 2{,}860\text{ kips} / 2.00$
$= 2{,}150\text{ kips}$	$= 1{,}430\text{ kips}$
$P_{B''} = 0.75(0\text{ kips})$	$P_{B''} = 0\text{ kips} / 2.00$
$= 0\text{ kips}$	$= 0\text{ kips}$
$P_{C''} = 0.75(1{,}980\text{ kips})$	$P_{C''} = 1{,}980\text{ kips} / 2.00$
$= 1{,}490\text{ kips}$	$= 990\text{ kips}$
$P_{D''} = 0.75(990\text{ kips})$	$P_{D''} = 990\text{ kips} / 2.00$
$= 743\text{ kips}$	$= 495\text{ kips}$
Design flexural strength:	Allowable flexural strength:
$\phi_b = 0.90$	$\Omega_b = 1.67$
$M_{X''} = \phi_b M_{X'}$	$M_{X''} = M_{X'} / \Omega_b$
where $X = A,\ B,\ C$ or D	where $X = A,\ B,\ C$ or D
$M_{A''} = 0.90(0\text{ kip-ft})$	$M_{A''} = 0\text{ kip-ft} / 1.67$
$= 0\text{ kip-ft}$	$= 0\text{ kip-ft}$
$M_{B''} = 0.90(748\text{ kip-ft})$	$M_{B''} = 748\text{ kip-ft} / 1.67$
$= 673\text{ kip-ft}$	$= 448\text{ kip-ft}$
$M_{C''} = 0.90(748\text{ kip-ft})$	$M_{C''} = 748\text{ kip-ft} / 1.67$
$= 673\text{ kip-ft}$	$= 448\text{ kip-ft}$
$M_{D''} = 0.90(1{,}050\text{ kip-ft})$	$M_{D''} = 1{,}050\text{ kip-ft} / 1.67$
$= 945\text{ kip-ft}$	$= 629\text{ kip-ft}$

The available strength values for each design method can now be plotted. These values are superimposed on the nominal strength surfaces (with and without length effects) previously calculated for comparison purposes in Figure I.11-4.

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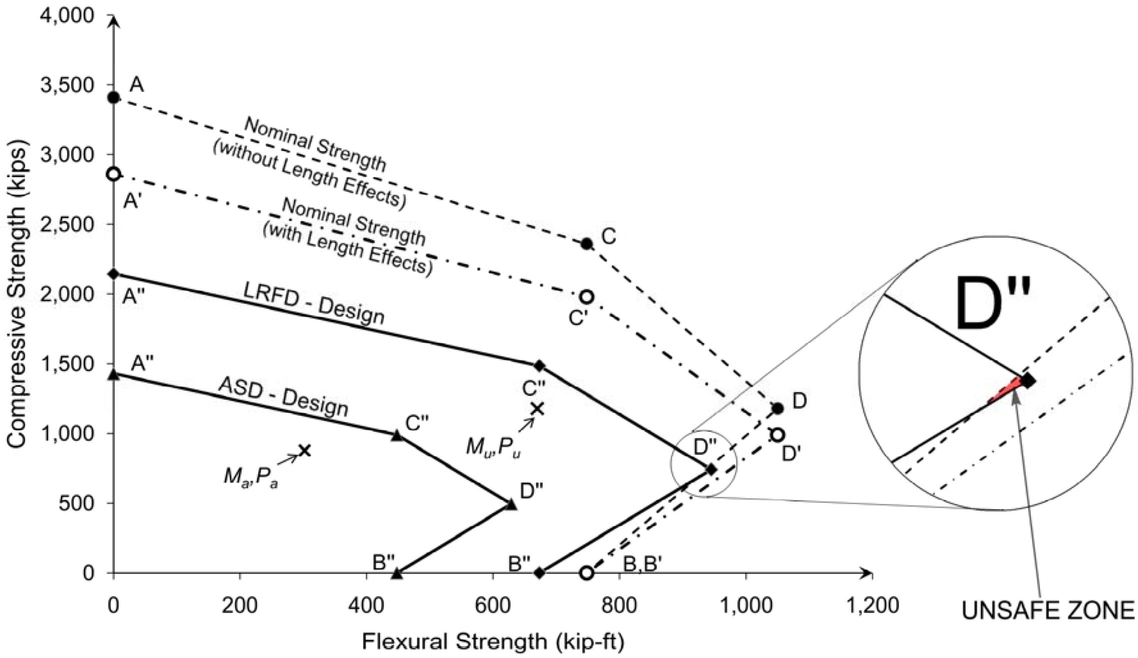

Fig. I.11-4. Available and nominal interaction surfaces.

By plotting the required axial and flexural strength values on the available strength surfaces indicated in Figure I.11-4, it can be seen that both ASD (M_a,P_a) and LRFD (M_u,P_u) points lie within their respective design surfaces. The member in question is therefore adequate for the applied loads.

As discussed previously in Step 2 as well as in AISC *Specification* Commentary Section I5, when reducing the flexural strength of Point D for length effects and resistance or safety factors, an unsafe situation could result whereby additional flexural strength is permitted at a lower axial compressive strength than predicted by the cross section strength of the member. This effect is highlighted by the magnified portion of Figure I.11-4, where LRFD design point D″ falls slightly below the nominal strength curve. Designs falling within this zone are unsafe and not permitted.

Method 2—Simplified

The unsafe zone discussed in the previous section for Method 2 is avoided in the Method 2—Simplified procedure by the removal of Point D″ from the Method 2 interaction surface leaving only points A″, B″ and C″ as illustrated in Figure I.11-5. Reducing the number of interaction points also allows for a bilinear interaction check defined by AISC *Specification* Commentary Equations C-I5-1a and C-I5-1b to be performed.

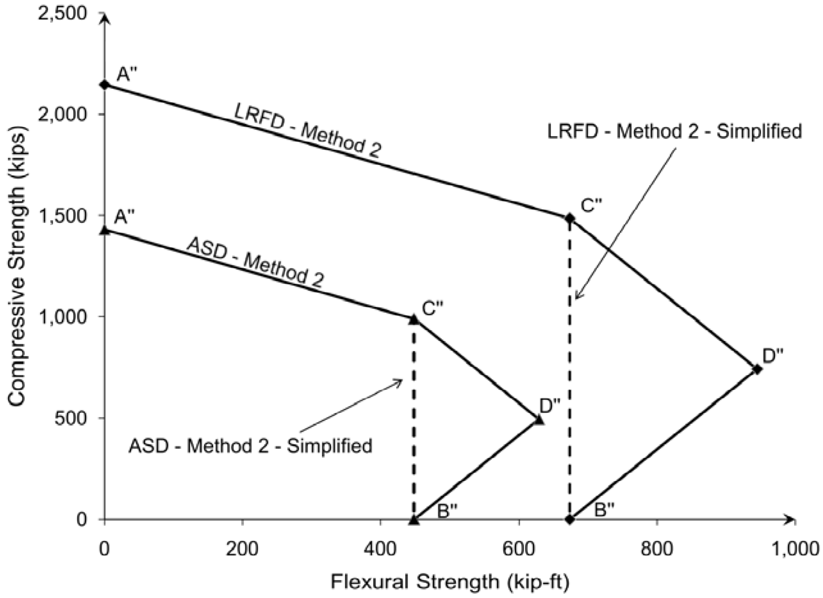

Fig. I.11-5. Comparison of Method 2 and Method 2 —Simplified.

Using the available strength values previously calculated in conjunction with the Commentary equations, interaction ratios are determined as follows:

LRFD	ASD
$P_r = P_u$ $= 1{,}170$ kips $P_r < P_{C''}$ $< 1{,}490$ kips Therefore, use Commentary Equation C-I5-1a. $\frac{M_r}{M_C} = \frac{M_u}{M_{C''}} \leq 1.0$ $\frac{670 \text{ kip-ft}}{673 \text{ kip-ft}} \leq 1.0$ $1.0 = 1.0$ **o.k.**	$P_r = P_a$ $= 879$ kips $P_r < P_{C''}$ < 990 kips Therefore, use Commentary Equation C-I5-1a. $\frac{M_r}{M_C} = \frac{M_a}{M_{C''}} \leq 1.0$ $\frac{302 \text{ kip-ft}}{448 \text{ kip-ft}} \leq 1.0$ $0.67 < 1.0$ **o.k.**

Thus, the member is adequate for the applied loads.

Comparison of Methods

The composite member was found to be inadequate using Method 1—Chapter H interaction equations, but was found to be adequate using both Method 2 and Method 2—Simplified procedures. A comparison between the methods is most easily made by overlaying the design curves from each method as illustrated in Figure I.11-6 for LRFD design.

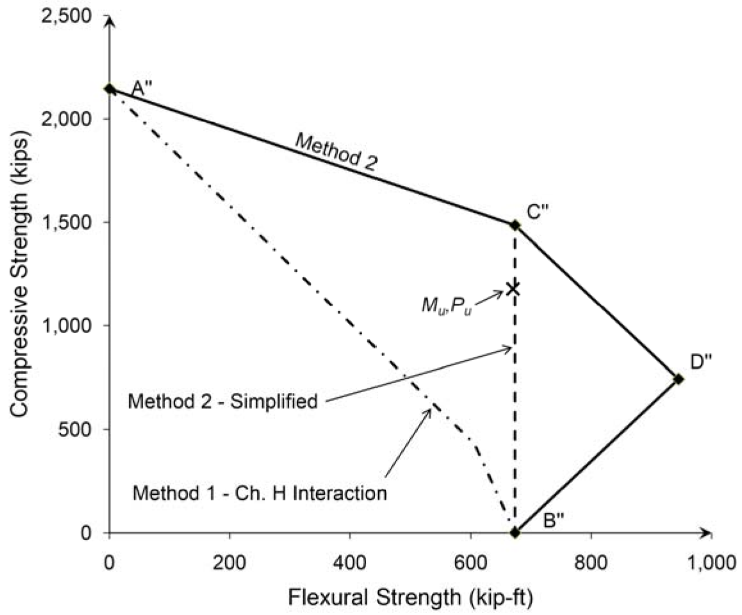

Fig. I.11-6. Comparison of interaction methods (LRFD).

From Figure I.11-6, the conservative nature of the Chapter H interaction equations can be seen. Method 2 provides the highest available strength; however, the Method 2—Simplified procedure also provides a good representation of the design curve. The procedure in Figure I-1 for calculating the flexural strength of Point C″ first requires the calculation of the flexural strength for Point D″. The design effort required for the Method 2—Simplified procedure, which utilizes Point C″, is therefore not greatly reduced from Method 2.

Available Shear Strength

According to AISC *Specification* Section I4.1, there are three acceptable options for determining the available shear strength of an encased composite member:

- Option 1—Available shear strength of the steel section alone in accordance with AISC *Specification* Chapter G.
- Option 2—Available shear strength of the reinforced concrete portion alone per ACI 318.
- Option 3—Available shear strength of the steel section in addition to the reinforcing steel ignoring the contribution of the concrete.

Option 1—Available Shear Strength of Steel Section

A W10×45 member meets the criteria of AISC *Specification* Section G2.1(a) according to the User Note at the end of the section. As demonstrated in Design Example I.9, No. 3 ties at 12 in. on center as illustrated in Figure I.11-1 satisfy the minimum detailing requirements of the *Specification*. The nominal shear strength may therefore be determined as:

$$C_v = 1.0 \qquad (Spec.\ \text{Eq. G2-2})$$

$$A_w = dt_w$$
$$= (10.1 \text{ in.})(0.350 \text{ in.})$$
$$= 3.54 \text{ in.}^2$$

$$V_n = 0.6F_yA_wC_v \qquad (Spec.\ \text{Eq. G2-1})$$
$$= 0.6(50 \text{ ksi})(3.54 \text{ in.}^2)(1.0)$$
$$= 106 \text{ kips}$$

The available shear strength of the steel section is:

LRFD	ASD
$V_u = 95.7$ kips $\phi_v = 1.0$ $\phi_v V_n \geq V_u$ $\phi_v V_n = 1.0(106 \text{ kips})$ $= 106 \text{ kips} > 95.7 \text{ kips}$ **o.k.**	$V_a = 57.4$ kips $\Omega_v = 1.50$ $V_n / \Omega_v \geq V_a$ $V_n / \Omega_v = \dfrac{106 \text{ kips}}{1.50}$ $= 70.7 \text{ kips} > 57.4 \text{ kips}$ **o.k.**

Option 2—Available Shear Strength of the Reinforced Concrete (Concrete and Transverse Steel Reinforcement)

The available shear strength of the steel section alone has been shown to be sufficient; however, the amount of transverse reinforcement required for shear resistance in accordance with AISC *Specification* Section I4.1(b) will be determined for demonstration purposes.

Tie Requirements for Shear Resistance

The nominal concrete shear strength is:

$$V_c = 2\lambda\sqrt{f_c'}b_w d \qquad \text{(ACI 318 Eq. 11-3)}$$

where

$\lambda = 1.0$ for normal weight concrete from ACI 318 Section 8.6.1

$b_w = h_1$

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement

$$= 24 \text{ in.} - 2\tfrac{1}{2} \text{ in.}$$
$$= 21.5 \text{ in.}$$

$$V_c = 2(1.0)\sqrt{5{,}000 \text{ psi}}\,(24 \text{ in.})(21.5 \text{ in.})\left(\frac{1 \text{ kip}}{1{,}000 \text{ lb}}\right)$$
$$= 73.0 \text{ kips}$$

The tie requirements for shear resistance are determined from ACI 318 Chapter 11 and AISC *Specification* Section I4.1(b), as follows:

LRFD	ASD
$V_u = 95.7$ kips $\phi_v = 0.75$ $\dfrac{A_v}{s} = \dfrac{V_u - \phi_v V_c}{\phi_v f_{yr} d}$ $= \dfrac{95.7 \text{ kips} - 0.75(73.0 \text{ kips})}{0.75(60 \text{ ksi})(21.5 \text{ in.})}$ $= 0.0423$ in. Using two legs of No. 3 ties with $A_v = 0.11$ in.^{2} from ACI 318 Appendix E: $\dfrac{2(0.11 \text{ in.}^2)}{s} = 0.0423$ in. $s = 5.20$ in. Using two legs of the No. 4 ties with $A_v = 0.20$ in.^{2}: $\dfrac{2(0.20 \text{ in.}^2)}{s} = 0.0423$ in. $s = 9.46$ in. From ACI 318 Section 11.4.5.1, the maximum spacing is: $s_{max} = \dfrac{d}{2}$ $= \dfrac{21.5 \text{ in.}}{2}$ $= 10.8$ in. **Use No. 3 ties at 5 in. o.c. or No. 4 ties at 9 in. o.c.**	$V_a = 57.4$ kips $\Omega_v = 2.00$ $\dfrac{A_v}{s} = \dfrac{V_a - (V_c/\Omega_v)}{f_{yr} d/\Omega_v}$ $= \dfrac{57.4 \text{ kips} - \left(\dfrac{73.0 \text{ kips}}{2.00}\right)}{\dfrac{(60 \text{ ksi})(21.5 \text{ in.})}{2.00}}$ $= 0.0324$ in. Using two legs of No. 3 ties with $A_v = 0.11$ in.^{2} from ACI 318 Appendix E: $\dfrac{2(0.11 \text{ in.}^2)}{s} = 0.0324$ in. $s = 6.79$ in. Using two legs of the No. 4 ties with $A_v = 0.20$ in.^{2}: $\dfrac{2(0.20 \text{ in.}^2)}{s} = 0.0324$ in. $s = 12.3$ in. From ACI 318 Section 11.4.5.1, the maximum spacing is: $s_{max} = \dfrac{d}{2}$ $= \dfrac{21.5 \text{ in.}}{2}$ $= 10.8$ in. **Use No. 3 ties at 6 in. o.c. or No. 4 ties at 10 in. o.c.**

Minimum Reinforcing Limits

Check that the minimum shear reinforcement is provided as required by ACI 318, Section 11.4.6.3.

$$A_{v,min} = 0.75\sqrt{f'_c}\left(\frac{b_w s}{f_{yr}}\right) \ge \frac{50 b_w s}{f_{yr}} \quad \text{(from ACI 318 Eq. 11-13)}$$

$$\frac{A_{v,min}}{s} = \frac{0.75\sqrt{5{,}000 \text{ psi}}(24 \text{ in.})}{60{,}000 \text{ psi}} \ge \frac{50(24 \text{ in.})}{60{,}000 \text{ psi}}$$

$$= 0.0212 \ge 0.0200$$

LRFD	ASD
$\dfrac{A_v}{s} = 0.0423 \text{ in.} > 0.0212$ **o.k.**	$\dfrac{A_v}{s} = 0.0324 \text{ in.} > 0.0212$ **o.k.**

Maximum Reinforcing Limits

From ACI 318 Section 11.4.5.3, maximum stirrup spacing is reduced to $d/4$ if $V_s \geq 4\sqrt{f_c'}b_w d$. If No. 4 ties at 9 in. on center are selected:

$$V_s = \frac{A_v f_{yr} d}{s} \qquad \text{(ACI 318 Eq. 11-15)}$$

$$= \frac{2\left(0.20 \text{ in.}^2\right)\left(60 \text{ ksi}\right)\left(21.5 \text{ in.}\right)}{9 \text{ in.}}$$

$$= 57.3 \text{ kips}$$

$$V_{s,max} = 4\sqrt{f_c'}b_w d$$

$$= 4\sqrt{5{,}000 \text{ psi}}\left(24 \text{ in.}\right)\left(21.5 \text{ in.}\right)\left(\frac{1 \text{ kip}}{1{,}000 \text{ lb}}\right)$$

$$= 146 \text{ kips} > 57.3 \text{ kips}$$

Therefore, the stirrup spacing is acceptable.

Option 3—Determine Available Shear Strength of the Steel Section Plus Reinforcing Steel

The third procedure combines the shear strength of the reinforcing steel with that of the encased steel section, ignoring the contribution of the concrete. AISC *Specification* Section I4.1(c) provides a combined resistance and safety factor for this procedure. Note that the combined resistance and safety factor takes precedence over the factors in Chapter G used for the encased steel section alone in Option 1. The amount of transverse reinforcement required for shear resistance is determined as follows:

Tie Requirements for Shear Resistance

The nominal shear strength of the encased steel section was previously determined to be:

$$V_{n,steel} = 106 \text{ kips}$$

The tie requirements for shear resistance are determined from ACI 318 Chapter 11 and AISC *Specification* Section I4.1(c), as follows:

LRFD	ASD
$V_u = 95.7$ kips as	$V_a = 57.4$ kips
$\phi_v = 0.75$	$\Omega_v = 2.00$
$\frac{A_v}{s} = \frac{V_u - \phi_v V_{n,steel}}{\phi_v f_{yr} d}$	$\frac{A_v}{s} = \frac{V_a - \left(V_{n,steel}/\Omega_v\right)}{f_{yr} d/\Omega_v}$
$= \frac{95.7 \text{ kips} - 0.75\left(106 \text{ kips}\right)}{0.75\left(60 \text{ ksi}\right)\left(21.5 \text{ in.}\right)}$	$= \frac{57.4 \text{ kips} - \left(106 \text{ kips}/2.00\right)}{\left[\frac{\left(60 \text{ ksi}\right)\left(21.5 \text{ in.}\right)}{2.00}\right]}$
$= 0.0167$ in.	$= 0.00682$ in.

As determined in Option 2, the minimum value of $A_v/s = 0.0212$, and the maximum tie spacing for shear resistance is 10.8 in. Using two legs of No. 3 ties for A_v:

$$\frac{2\left(0.11\text{ in.}^2\right)}{s} = 0.0212\text{ in.}$$
$$s = 10.4\text{ in.} < s_{max} = 10.8\text{ in.}$$

Use No. 3 ties at 10 in. o.c.

Summary and Comparison of Available Shear Strength Calculations

The use of the steel section alone is the most expedient method for calculating available shear strength and allows the use of a tie spacing which may be greater than that required for shear resistance by ACI 318. Where the strength of the steel section alone is not adequate, Option 3 will generally result in reduced tie reinforcement requirements as compared to Option 2.

Force Allocation and Load Transfer

Load transfer calculations should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions for encased composite members is provided in Design Example I.8 and AISC Design Guide 6.

EXAMPLE I.12 STEEL ANCHORS IN COMPOSITE COMPONENTS

Given:

Select an appropriate ¾-in.-diameter, Type B steel headed stud anchor to resist the dead and live loads indicated in Figure I.12-1. The anchor is part of a composite system that may be designed using the steel anchor in composite components provisions of AISC *Specification* Section I8.3.

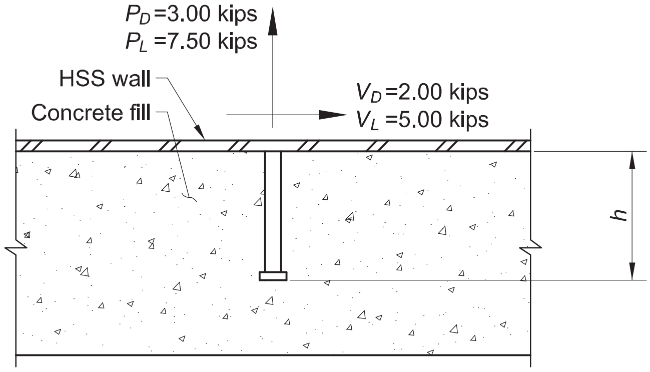

Fig. I.12-1. Steel headed stud anchor and applied loading.

The steel headed stud anchor is encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, f_c' = 5 ksi. In accordance with AWS D1.1, steel headed stud anchors shall be made from material conforming to the requirements of ASTM A108. From AISC *Manual* Table 2-6, the specified minimum tensile stress, F_u, for ASTM A108 material is 65 ksi.

The anchor is located away from edges such that concrete breakout in shear is not a viable limit state, and the nearest anchor is located 24 in. away. The concrete is considered to be uncracked.

Solution:

Minimum Anchor Length

AISC *Specification* Section I8.3 provides minimum length to shank diameter ratios for anchors subjected to shear, tension, and interaction of shear and tension in both normal weight and lightweight concrete. These ratios are also summarized in the User Note provided within Section I8.3. For normal weight concrete subject to shear and tension, $h/d \geq 8$, thus:

$$\begin{aligned} h &\geq 8d \\ &\geq 8(\text{¾ in.}) \\ &\geq 6.00 \text{ in.} \end{aligned}$$

This length is measured from the base of the steel headed stud anchor to the top of the head after installation. From anchor manufacturer's data, a standard stock length of 6³⁄16 in. is selected. Using a ³⁄16-in. length reduction to account for burn off during installation yields a final installed length of 6.00 in.

$$6.00 \text{ in.} = 6.00 \text{ in.} \quad \textbf{o.k.}$$

Select a ¾-in.-diameter × 6³⁄16-in.-long headed stud anchor.

Required Shear and Tensile Strength

From Chapter 2 of ASCE/SEI 7, the required shear and tensile strengths are:

LRFD	ASD
Governing Load Combination for interaction: $= 1.2D + 1.6L$ $Q_{uv} = 1.2(2.00 \text{ kips}) + 1.6(5.00 \text{ kips})$ $= 10.4 \text{ kips (shear)}$ $Q_{ut} = 1.2(3.00 \text{ kips}) + 1.6(7.50 \text{ kips})$ $= 15.6 \text{ kips (tension)}$	Governing Load Combination for interaction: $= D + L$ $Q_{av} = 2.00 \text{ kips} + 5.00 \text{ kips}$ $= 7.00 \text{ kips (shear)}$ $Q_{at} = 3.00 \text{ kips} + 7.50 \text{ kips}$ $= 10.5 \text{ kips (tension)}$

Available Shear Strength

Per the problem statement, concrete breakout is not considered to be an applicable limit state. AISC Equation I8-3 may therefore be used to determine the available shear strength of the steel headed stud anchor as follows:

$$Q_{nv} = F_u A_{sa} \qquad (\textit{Spec}. \text{ Eq. I8-3})$$

where

A_{sa} = cross-sectional area of steel headed stud anchor

$$= \frac{\pi(\frac{3}{4} \text{ in.})^2}{4}$$

$$= 0.442 \text{ in.}^2$$

$$Q_{nv} = (65 \text{ ksi})(0.442 \text{ in.}^2)$$

$$= 28.7 \text{ kips}$$

LRFD	ASD
$\phi_v = 0.65$ $\phi_v Q_{nv} = 0.65(28.7 \text{ kips})$ $= 18.7 \text{ kips}$	$\Omega_v = 2.31$ $Q_{nv} / \Omega_v = \dfrac{28.7 \text{ kips}}{2.31}$ $= 12.4 \text{ kips}$

Alternately, available shear strengths can be selected directly from Table I.12-1 located at the end of this example.

Available Tensile Strength

The nominal tensile strength of a steel headed stud anchor is determined using AISC *Specification* Equation I8-4 provided the edge and spacing limitations of AISC *Specification* Section I8.3b are met as follows:

(1) Minimum distance from centerline of anchor to free edge: $1.5h = 1.5(6.00 \text{ in.}) = 9.00 \text{ in.}$

There are no free edges, therefore this limitation does not apply.

(2) Minimum distance between centerlines of adjacent anchors: $3h = 3(6.00 \text{ in.}) = 18.0 \text{ in.}$

$18.0 \text{ in.} < 24 \text{ in.}$ **o.k.**

Equation I8-4 may therefore be used as follows:

$$Q_{nt} = F_u A_{sa} \qquad (Spec.\ \text{Eq. I8-4})$$

$$Q_{nt} = (65 \text{ ksi})(0.442 \text{ in.}^2)$$
$$= 28.7 \text{ kips}$$

LRFD	ASD
$\phi_t = 0.75$ $\phi_t Q_{nt} = 0.75(28.7 \text{ kips})$ $= 21.5 \text{ kips}$	$\Omega_t = 2.00$ $\dfrac{Q_{nt}}{\Omega_t} = \dfrac{28.7 \text{ kips}}{2.00}$ $= 14.4 \text{ kips}$

Alternately, available tension strengths can be selected directly from Table I.12-1 located at the end of this example.

Interaction of Shear and Tension

The detailing limits on edge distances and spacing imposed by AISC *Specification* Section I8.3c for shear and tension interaction are the same as those previously reviewed separately for tension and shear alone. Tension and shear interaction is checked using *Specification* Equation I8-5 which can be written in terms of LRFD and ASD design as follows:

LRFD	ASD
$\left[\left(\dfrac{Q_{ut}}{\phi_t Q_{nt}}\right)^{5/3} + \left(\dfrac{Q_{uv}}{\phi_v Q_{nv}}\right)^{5/3}\right] \le 1.0$ $\left[\left(\dfrac{15.6 \text{ kips}}{21.5 \text{ kips}}\right)^{5/3} + \left(\dfrac{10.4 \text{ kips}}{18.7 \text{ kips}}\right)^{5/3}\right] = 0.96$ $0.96 < 1.0$ **o.k.**	$\left[\left(\dfrac{Q_{at}}{Q_{nt}/\Omega_t}\right)^{5/3} + \left(\dfrac{Q_{av}}{Q_{nv}/\Omega_v}\right)^{5/3}\right] \le 1.0$ $\left[\left(\dfrac{10.5 \text{ kips}}{14.4 \text{ kips}}\right)^{5/3} + \left(\dfrac{7.00 \text{ kips}}{12.4 \text{ kips}}\right)^{5/3}\right] = 0.98$ $0.98 < 1.0$ **o.k.**

Thus, a ¾-in.-diameter × 6³⁄₁₆-in.-long headed stud anchor is adequate for the applied loads.

Limits of Application

The application of the steel anchors in composite component provisions have strict limitations as summarized in the User Note provided at the beginning of AISC *Specification* Section I8.3. These provisions do not apply to typical composite beam designs nor do they apply to hybrid construction where the steel and concrete do not resist loads together via composite action such as in embed plates. This design example is intended solely to illustrate the calculations associated with an isolated anchor that is part of an applicable composite system.

Available Strength Table

Table I.12-1 provides available shear and tension strengths for standard Type B steel headed stud anchors conforming to the requirements of AWS D1.1 for use in composite components.

Table I.12-1. Steel Headed Stud Anchor Available Strengths					
Anchor Shank Diameter	A_{sa}	Q_{nv} / Ω_v	$\phi_v Q_{nv}$	Q_{nt} / Ω_t	$\phi_t Q_{nt}$
		kips	kips	kips	kips
in.	in.²	ASD	LRFD	ASD	LRFD
½	0.196	5.52	8.30	6.38	9.57
⅝	0.307	8.63	13.0	9.97	15.0
¾	0.442	12.4	18.7	14.4	21.5
⅞	0.601	16.9	25.4	N/A[a]	N/A[a]
1	0.785	22.1	33.2	25.5	38.3
ASD	LRFD	[a] ⅞-in.-diameter anchors conforming to AWS D1.1 Figure 7.1 do not meet the minimum head-to-shank diameter ratio of 1.6 as required for tensile resistance per AISC *Specification* Section I8.3.			
$\Omega_v = 2.31$	$\phi_v = 0.65$				
$\Omega_t = 2.00$	$\phi_t = 0.75$				

CHAPTER I DESIGN EXAMPLE REFERENCES

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