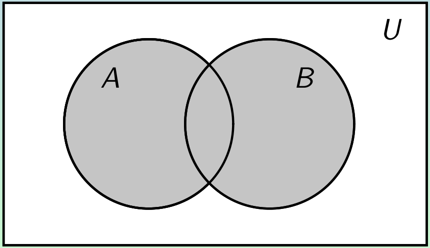
Set Operations
Discrete Mathematics

Definition

Let A and B be sets. The **union** of the sets A and B, denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.

$$A \cup B = \{x \mid (x \in A) \vee (x \in B)\}.$$

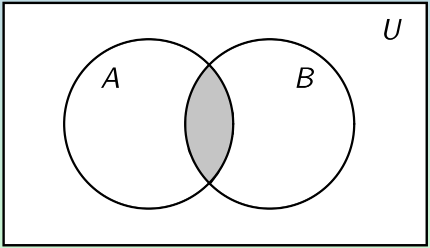


Definition: Intersection of Sets

Definition

Let A and B be sets. The **intersection** of the sets A and B, denoted by $A \cap B$, is the set containing those elements in both A and B.

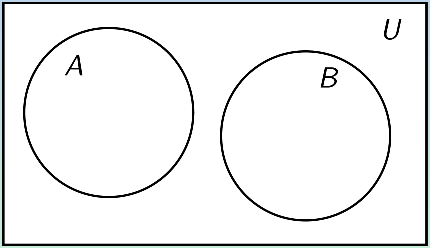
$$A \cap B = \{x \mid (x \in A) \wedge (x \in B)\}.$$



Definition: Disjoint Sets

Definition

Two sets are called **disjoint** if their intersection is the empty set.



Principle of Inclusion-Exclusion

The number of elements in the union of two sets is equal to the number of elements in the first set plus the number of elements in the second one, minus the number of elements in the intersection of the two sets because they were counted twice.

Theorem

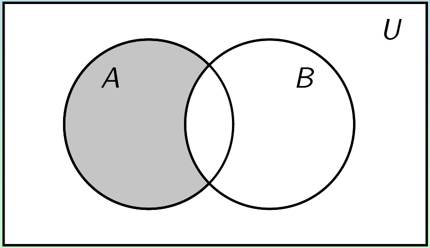
$$|A \cup B| = |A| + |B| - |A \cap B|$$

Definition: Difference of Sets

Definition

Let A and B be sets. The **difference** of A and B, denoted by $A - B$, is the set containing those elements that are in A but not in B. The difference of A and B is also called the **complement of** B **with respect to** A.

$$A - B = \{x \mid (x \in A) \wedge (x \notin B)\}.$$

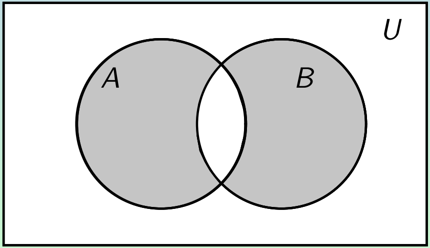


Definition: Symmetric Difference of Sets

Definition

Let A and B be sets. The **symmetric difference** of A and B, denoted by $A \oplus B$, is the set containing those elements in either A or B, but not in both A and B.

$$A \oplus B = \{x \mid (x \in A) \oplus (x \in B)\}.$$

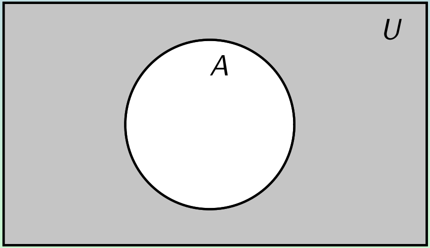


Definition: Complement of Sets

Definition

Let U be the universal set. The **complement** of the set A, denoted by $\overline{A}$ or A^c, is the set containing those elements that are in U but not in A. In other words, the complement of the set A is the complement of A with respect to U, i.e. $U - A$.

$$\overline{A} = \{x \mid x \notin A\}.$$



Set Identities

Identity	Name
$A \cup \emptyset = A$ $A \cap U = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$\overline{(\overline{A})} = A$	Complementation law

Set Identities (continued)

Identity	Name
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

Membership Table

We consider each combination of sets that an element can belong to and verify that elements in the same combinations of sets belong to both the sets in the identity. To indicate that an element is in a set, a 1 is used; to indicate that an element is not in a set, a 0 is used.

Example: De Morgan's law: $\overline{A \cup B} = \overline{A} \cap \overline{B}$

A	B	$A \cup B$	$\overline{A \cup B}$	$\overline{A}$	$\overline{B}$	$\overline{A} \cap \overline{B}$
1	1	1	0	0	0	0
1	0	1	0	0	1	0
0	1	1	0	1	0	0
0	0	0	1	1	1	1

Definition: Generalized Union of Sets

Definition

The **union** of a collection of sets is the set that contains those elements that are members of at least one set in the collection.

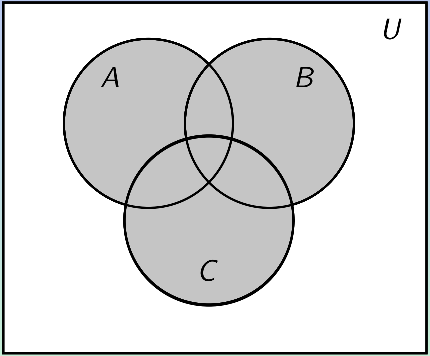
We use the notation

$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$$

to denote the union of the sets $A_1, A_2, \ldots, A_n$.

Example of a Generalized Union of Sets

This Venn diagram shows the union of the sets A, B and C.



Definition: Generalized Intersection of Sets

Definition

The **intersection** of a collection of sets is the set that contains those elements that are members of all the sets in the collection.

We use the notation

$$A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$$

to denote the intersection of the sets $A_1, A_2, \ldots, A_n$.

Example of a Generalized Intersection of Sets

This Venn diagram shows the intersection of the sets A, B and C.