

Rules of Inferences Discrete Mathematics — CSE 131

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Outline	Motivation Definitions Rules of Inference Fallacies Using Rules of Inference to Build Arguments Rules of Inference and Quantifiers





Rules of Inference

- Motivation
- Operation Definitions
- Rules of Inference
- Fallacies
- Using Rules of Inference to Build Arguments
- Rules of Inference and Quantifiers

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Example: Existence of Superman

If Superman were able and willing to prevent evil, then he would so. If Superman were unable to prevent evil, then he would be impotent; if he were unwilling to prevent evil, then he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.

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Is this argument valid ?

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Definitions		

By an **argument**, we mean a sequence of statements that ends with a **conclusion**.

The **conclusion** is the last statement of the argument.

The **premises** are the statements of the argument preceding the conclusion.

By a **valid argument**, we mean that the conclusion must follow from the truth of the premises.

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Rule of Interence

Some tautologies are **rules of inference**. The general form of a rule of inference is

$$(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \rightarrow c$$

where

p_i are the **premises**

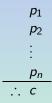
and

c is the conclusion.

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A rule of inference is written as



where the symbol \therefore denotes "**therefore**". Using this notation, the hypotheses are written in a column, followed by a horizontal bar, followed by a line that begins with the therefore symbol and ends with the conclusion.

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modus ponens

The rule of inference

$$p
ightarrow q$$

 p
 $\therefore q$

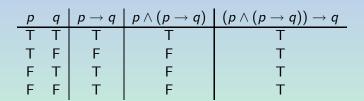
is denoted the **law of detachment** or *modus ponens* (Latin for *mode that affirms*). If a conditional statement and the hypothesis of the conditional statement are both true, therefore the conclusion must also be true.

The basis of the modus ponens is the tautology

$$((p \rightarrow q) \land p) \rightarrow q.$$

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modus ponens



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Example of modus ponens

If it rains, then it is cloudy. It rains. Therefore, it is cloudy.

r is the proposition "it rains."
c is the proposition "it is cloudy."

 $r \to c$ r r $\therefore c$

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The rule of inference

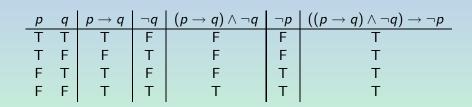
$$p
ightarrow q$$
 $\neg q$
. $\neg p$

is denoted the *modus tollens* (Latin for *mode that denies*). This rule of inference is based on the contrapositive. The basis of the *modus ponens* is the tautology

$$((p
ightarrow q) \land \neg q)
ightarrow \neg p.$$

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modus tollens



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Example of *modus tollens*

If it rains, then it is cloudy. It is not cloudy. Therefore, it is not the case that it rains.

r is the proposition "it rains."c is the proposition "it is cloudy."

 $r \to c$ $\neg c$ \overrightarrow{r}

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The rule of inference

 $\frac{p}{\therefore p \lor q}$

is the rule of addition.

This rule comes from the tautology

 $p
ightarrow (p \lor q).$

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The Simplification

The rule of inference

$$p \wedge q$$

 $\therefore p$

is the rule of **simplification**.

This rule comes from the tautology

 $(p \wedge q) \rightarrow p.$

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The Hypothetical Syllogism

The rule of inference

$$p \to q$$

$$q \to r$$

$$\therefore p \to r$$

is the rule of **hypothetical syllogism** (syllogism means "argument made of three propositions where the last one, the conclusion, is necessarily true if the two firsts, the hypotheses, are true").

This rule comes from the tautology

$$((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r).$$

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The Disjunctive Syllogism

The rule of inference

$$\begin{array}{c} p \lor q \\ \neg p \\ \hline \ddots q \end{array}$$

is the rule of disjunctive syllogism.

This rule comes from the tautology

$$((p \lor q) \land \neg p) \rightarrow q.$$

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The rule of inference

 $\begin{array}{c} p \\ q \\ \hline \vdots p \land q \end{array}$

is the rule of **conjunction**.

This rule comes from the tautology

 $((p) \land (q)) \rightarrow (p \land q).$

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The rule of inference

 $p \lor q$ $\neg p \lor r$ $\therefore q \lor r$

is the rule of **resolution**.

This rule comes from the tautology

$$((p \lor q) \land (\neg p \lor r)) \to (q \lor r).$$

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Fallacies are incorrect arguments.

Fallacies resemble rules of inference but are based on <u>contingencies</u> rather than tautologies.

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The Fallacy of Affirming the Conclusion

The wrong "rule of inference"

$$p \rightarrow q$$
 q
 p

is denoted the fallacy of affirming the conclusion.

The basis of this fallacy is the contingency

$$(q \land (p
ightarrow q))
ightarrow p$$

that is a misuse of the modus ponens and is not a tautology.

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Fallacy of Affirming the Conclusion



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Example of the Fallacy of Affirming the Conclusion

- If it rains, then it is cloudy. It is cloudy. Therefore, it rains (wrong).
- r is the proposition "it rains."c is the proposition "it is cloudy."

$$r \to c$$

$$c$$

$$\therefore r \text{ (wrong)}$$

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The Fallacy of Denying the Hypothesis

The wrong "rule of inference"

$$p \rightarrow q$$
 $\neg p$
 $\overline{\neg q}$

is denoted the fallacy of denying the hypothesis.

The basis of this fallacy is the contingency

$$(\neg p \land (p
ightarrow q))
ightarrow \neg q$$

that is a misuse of the modus tollens and is not a tautology.

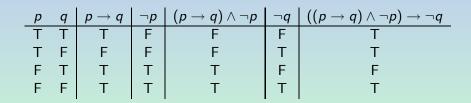
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Fallacy of Denying the Hypothesis



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Example of the Fallacy of Denying the Hypothesis

If it rains, then it is cloudy. It is not the case that it rains. Therefore, it is not cloudy (wrong).

r is the proposition "it rains."c is the proposition "it is cloudy."

$$r \to c$$

$$\neg r$$

$$\therefore \neg c \text{ (wrong)}$$

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Example: Existence of Superman

If Superman were able and willing to prevent evil, then he would so. If Superman were unable to prevent evil, then he would be impotent; if he were unwilling to prevent evil, then he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.

- w is "Superman is willing to prevent evil"
- a is "Superman is able to prevent evil"
- *i* is "Superman is impotent"
- *m* is "Superman is malevolent"
- p is "Superman prevents evil"
- x is "Superman exists"

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Example: Existence of Superman

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- h1. $(a \wedge w) \rightarrow p$
- h2. $\neg a \rightarrow i$
- $h3. \neg w \rightarrow m$
- *h*4. ¬*p*
- *h*5. $x \rightarrow \neg i$
- *h*6. $x \rightarrow \neg m$

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Example: Existence of Superman

Argument:

1.	eg i ightarrow a	contrapositive of h_2 .
2.	$x \rightarrow a$	h_5 and step 1 with hyp. syll.
3.	$\neg m \rightarrow w$	contrapositive of h_3 .
4.	$x \to w$	h_6 ans step 3 with hyp. syll.
5.	$x ightarrow (a \wedge w)$	Step 2 and 4 with conjunction.
6.	x ightarrow p	Step 5 and h_1 with hyp. syll.
7.	$\neg x$	Step 6 and h_4 with modus tollens.

Q.E.D.

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Rules of Inference and Quantifiers

There are four rules of inference for quantifiers:

- Universal instantiation (UI),
- Universal generalization (UG),
- Existential instantiation (EI),
- Existential generalization (EG).

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Universal Instantiation

 $\frac{\forall x P(x)}{\therefore P(c)}$

If a propositional function is true for all element x of the universe of discourse, then it is true for a particular element c of the universe of discourse.

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Universal Instantiation and modus ponens

The universal instantiation and the *modus ponens* are used together to form the **universal modus ponens**. Example: *All humans have two legs. John Smith is a human. Therefore, John Smith has two legs.*

- H(x) is "x is a human."
- L(x) is "x has two legs."
- j is John Smith, a element of the universe of discourse.

1.	$\forall x (H(x) \to L(x))$	Premise.
2.	$H(j) \rightarrow L(j)$	Universal instantiation from 1.
3.	H(j)	Premise.
·.	L(j)	Modus ponens from 2. et 3.

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Universal Generalization

 $\begin{array}{c} P(c) & \text{for an arbitrary } c \\ \therefore \quad \forall x P(x) \end{array}$

We must first define the universe of discourse. Then, we must show that P(c) is true for an arbitrary, and not a specific, element c of the universe of discourse. We have no control over c and we can not make any other assumptions about c other than it comes from the domain of discourse. The error of adding unwarranted assumptions about the arbitrary element c is common and is an incorrect reasoning.

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Existential Instantiation

$$\exists x P(x)$$

$$\therefore P(c) \text{ for some element } c$$

The existential instantiation is the rule that allow us to conclude that there is an element c in the universe of discourse for which P(c) is true if we know that $\exists x P(x)$ is true. We can not select an arbitrary value of c here, but rather it must be a c for which P(c)is true.
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Existential Generalization

$$\begin{array}{c} P(c) \quad \text{for some element } c \\ \therefore \quad \exists x \, P(x) \end{array}$$

If we know one element c in the universe of discourse for which P(c) is true, therefore we know that $\exists x P(x)$ is true.