16

Composite Steel-Concrete Construction

16.1 HISTORICAL BACKGROUND

Steel framing supporting cast-in-place reinforced concrete slab construction was historically designed on the assumption that the concrete slab acts independently of the steel in resisting loads. No consideration was given to the composite effect of the steel and concrete acting together. This neglect was justified on the basis that the bond between the concrete floor or deck and the top of the steel beam could not be depended upon. However, with the advent of welding, it became practical to provide mechanical shear connectors to resist the horizontal shear which develops during bending.

Steel beams encased in concrete were widely used from the early 1900s until the development of lightweight materials for fire protection in the past 50 years. Some such beams were designed compositely and some were not. In the early 1930s bridge construction began to use composite sections. Not until the early 1960s was it economical to use composite construction for buildings. However, current practice (2008) utilizes composite action in nearly all situations where concrete and steel are in contact, both on bridges and buildings.

Composite construction, as treated in this chapter, consists either of a solid cast-in-place concrete slab placed upon and interconnected to a steel rolled W section or welded I-shaped girder, as shown in Fig. 16.1.1, or most commonly, the concrete slab is cast upon

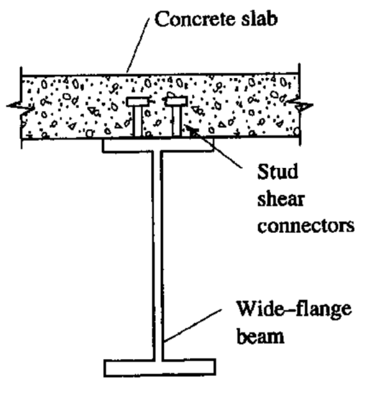
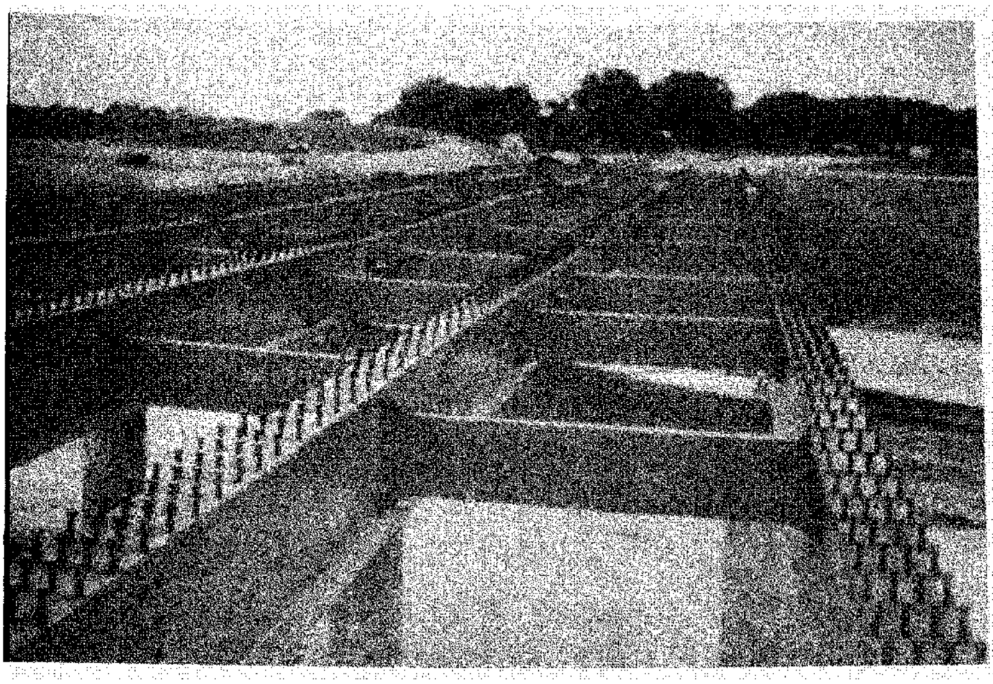


Figure 16.1.1
Conventional composite steel-concrete beam.



Shear stud connectors on flanges of bridge girders to be embedded in the concrete slab in order to make steel section and concrete slab act as a unit (i.e., compositely). (Photo by C. G. Salmon)

cold-formed steel deck (Fig. 16.1.2), which itself is supported on a steel I-shaped section. The corrugations (ribs) may be either parallel to or perpendicular to the supporting beam. When the ribs are parallel to the beam, the behavior is essentially that of a variable thickness slab supported directly on the steel beam. When the ribs are perpendicular to the steel W section, special treatment is required. The many varieties of composite steel–concrete construction are discussed in the State-of-the Art Report [16.1].

The composite beam is one having a wide flange (concrete slab), typically spanning 8 to 15 ft between parallel beams. Ordinary beam theory, where the stress is assumed constant across the width of a beam at a given distance from the neutral axis, *does not apply*. Plate theory indicates the stress decreases the more distant a point is from the stiff part (steel section in this case) of the beam. Similarly to the treatment of T-sections in reinforced concrete, an *equivalent width* is used in place of the actual width, so that ordinary

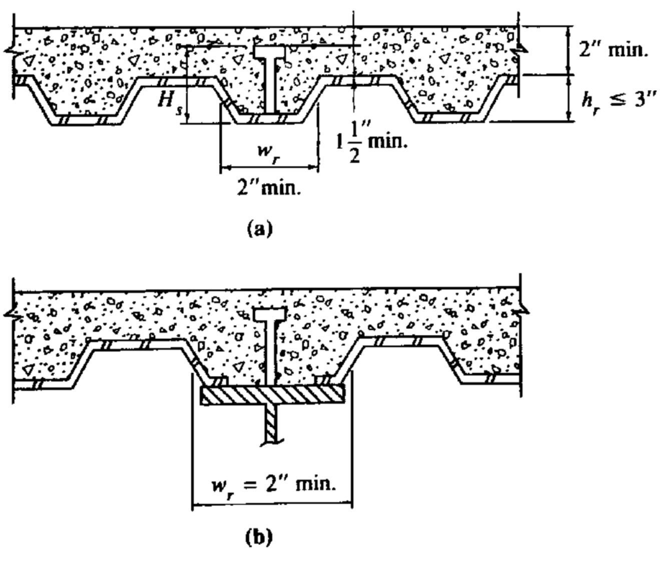


Figure 16.1.2
Composite section using formed steel deck. Steel beam supporting deck and slab may be parallel to ribs of formed deck (as in b.) or perpendicular to the ribs. (AISC-I3.2c)(Adapted from AISC Commentary [1.14])

beam theory can be used. An excellent summary of the factors involved in obtaining an effective width is given by Brendel [16.2] and Heins and Fan [16.3]. Vallenilla and Bjorhovde [16.4] have reviewed the effective width in the context of LRFD and the use of steel deck to support the slab.

Viest [16.5], in his 1960 review of research, notes that the important factor in composite action is that the bond between concrete and steel remain unbroken. As designers began to place slabs on top of supporting steel beams, investigators began to study the behavior of mechanical shear connectors. The shear connectors provided the interaction necessary for the concrete slab and steel beam to act as a unit; i.e., no slip between the concrete and steel beam parallel to the beam. For the earlier encased beams there had been sufficient contact area between concrete and steel so that friction provided the necessary interaction between the two materials.

The State-of-the-Art Report of 1974 [16.1] provides an overall survey of the subject of composite construction, including bibliography. Hansell, Galambos, Ravindra, and Viest [16.6] have provided the background for Load and Resistance Factor Design. Iyengar and Iqbal [16.7] have provided a modern review of composite construction in building design, and Lorenz and Stockwell [16.8] and Lorenz [16.9] have provided treatment of basic design concepts for Load and Resistance Factor Design.

A thorough treatment of steel-concrete composite construction in the context of *Eurocode 4* has been developed by IABSE [16.42].

16.2 COMPOSITE ACTION

Composite action is developed when two load-carrying structural members such as a concrete floor system and the supporting steel beam (Fig. 16.2.1a) are integrally connected and deflect as a single unit as in Fig. 16.2.1b. The extent to which composite action is developed depends on the provisions made to insure a single linear strain from the top of the concrete slab to the bottom of the steel section.

In developing the concept of composite behavior, consider first the noncomposite beam of Fig. 16.2.1a, wherein if friction between the slab and beam is neglected, the beam and slab each carry separately a part of the load. This is further shown in Fig. 16.2.2a. When the slab deforms under vertical load, its lower surface is in tension and elongates; while the upper surface of the beam is in compression and shortens. Thus a discontinuity will occur at the plane of contact. Since friction is neglected, only vertical internal forces act between the slab and beam.

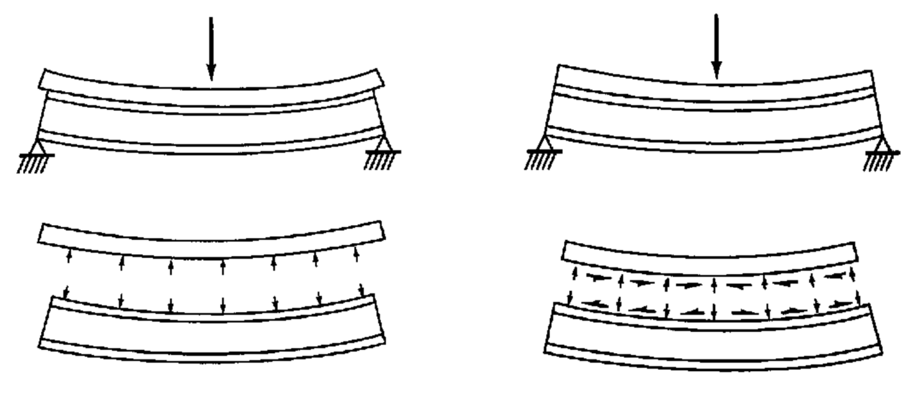


Figure 16.2.1 Comparison of deflected beams with and without composite action.

(a) Deflected noncomposite beam

(b) Deflected composite beam

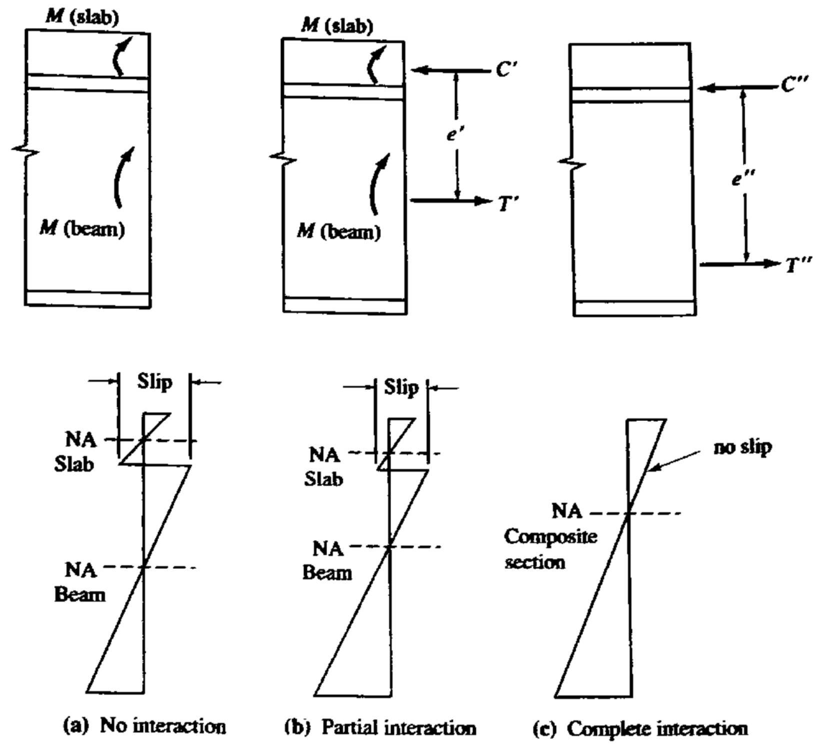


Figure 16.2.2
Strain variation in composite beams.

When a system acts compositely (Fig. 16.2.1b and 16.2.2c) no relative slip occurs between the slab and beam. Horizontal forces (shears) are developed that act on the lower surface of the slab to compress and shorten it, while simultaneously they act on the upper surface of the beam to elongate it.

By an examination of the strain distribution that occurs when there is no interaction between the concrete slab and the steel beam (Fig. 16.2.2a), it is seen that the total resisting moment is equal to

$$\Sigma M = M_{\text{slab}} + M_{\text{beam}} \tag{16.2.1}$$

It is noted that for this case there are two neutral axes; one at the center of gravity of the slab and the other at the center of gravity of the beam. The horizontal slip resulting from the bottom of the slab in tension and the top of the beam in compression is also indicated.

Consider next the case where only partial interaction is present, Fig. 16.2.2b. The neutral axis of the slab is closer to the beam and that of the beam closer to the slab. Due to the partial interaction, the horizontal slip has now decreased. The result of the partial interaction is the partial development of the maximum compressive and tensile forces C' and T', in the concrete slab and steel beam, respectively. The resisting moment of the section would then be increased by the amount $T'e'$ or $C'e'$.

When complete interaction (known as full composite action) between the slab and the beam is developed, no slip occurs and the resulting strain diagram is shown in Fig. 16.2.2c. Under this condition, a single neutral axis occurs which lies below that of the slab and above that of the beam. In addition, the compressive and tensile forces C'' and T'', respectively, are larger than the C' and T' existing with partial interaction. The resisting moment of the fully developed composite section then becomes

$$\Sigma M = T''e'' \quad \text{or} \quad C''e'' \tag{16.2.2}$$

16.3 ADVANTAGES AND DISADVANTAGES

The basic advantages resulting from composite design are

1. Reduction in the weight of steel
2. Shallower steel beams
3. Increased floor stiffness
4. Increased span length for a given number

A weight savings in steel of 20 to 30% is often possible by taking full advantage of a composite system. Such a weight reduction in the supporting steel beams usually permits the use of a shallower as well as a lighter member. This advantage may reduce the height of a multistoried building significantly so as to provide savings in other building materials such as outside walls and stairways. The overall economy of using composite construction when considering total building cost appears to be increasingly favorable [16.10, 16.11].

The stiffness of a composite floor is substantially greater than that of a concrete floor with its supporting beams acting independently. Normally the concrete slab acts as a one-way plate spanning between the supporting beams. In composite design, an additional use is made of the slab by its action in a direction parallel to and in combination with the supporting steel beams. The net effect is to greatly increase the moment of inertia of the floor system in the direction of the steel beams. The increased stiffness considerably reduces the live load deflections and, if shoring is provided during construction, also reduces dead load deflections. Assuming full composite action, the nominal strength of the section greatly exceeds the sum of the strengths of the slab and the beam considered separately, providing high overload capacity.

While there are no major disadvantages, some limitations should be recognized. In continuous construction, the negative moment region will have a different stiffness because the concrete slab in tension is expected to be cracked and not participating. In general, it is considered acceptable to assume the moment of inertia to be constant through both positive and negative moment regions, using the positive moment composite section moment of inertia AISC-I1. Tension in the concrete is neglected.

Long-term deflection caused by concrete creep and shrinkage could be important when the composite section resists a substantial part of the dead load, or when the live load is of long duration. This is discussed in Sec. 16.12.

16.4 EFFECTIVE WIDTH

The concept of *effective width* is useful in design when strength must be determined for an element subject to nonuniform distribution of stress. Referring to Fig. 16.4.1, the concrete slab of a composite section is considered to be infinitely wide. The intensity of extreme fiber stress f_c is a maximum over the steel beam and decreases nonlinearly as the distance from the supporting beam increases.

The effective width b_E of a flange for a composite member may be expressed

$$b_E = b_f + 2b' \tag{16.4.1}$$

where $2b'$ times the maximum stress f_c equals the area under the curves for f_c. Various investigators, including Timoshenko and Goodier [16.12] and von Kármán [16.13], have derived expressions for the effective width of homogeneous beams having wide flanges; and Johnson and Lewis [16.14] have shown such expressions are valid for beams in which the flange and web are of different materials.

The analysis for effective width involves theory of elasticity applied to plates, using an infinitely long continuous beam on equidistant supports, with an infinitely wide flange

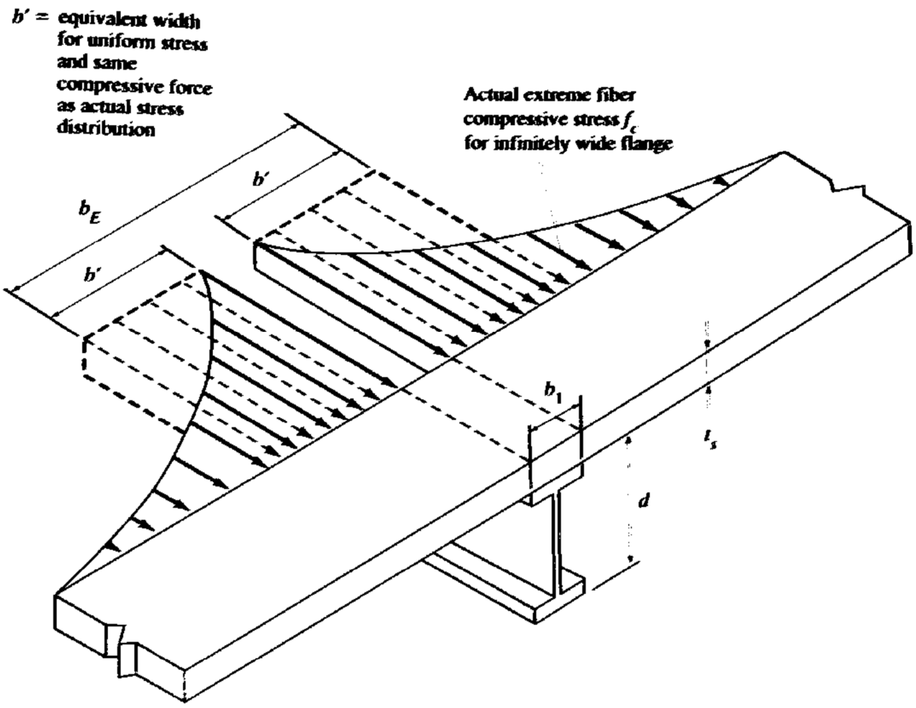


Figure 16.4.1 Actual and equivalent stress distribution over flange width.

having a small thickness compared to the beam depth. The total compression force carried by the equivalent system must be the same as that carried by the real system.

The practical simplifications for design purposes are given by AISC-I3.1a the *same* for service load calculations as for nominal strength calculations when failure is imminent.

1. For an *interior girder*, referring to Fig. 16.4.2,

$$b_E \le \frac{L}{4} \tag{16.4.2a}$$

$$b_E \le b_0 \quad \text{(for equal beam spacing)} \tag{16.4.2b}$$

2. For an *exterior girder*,

$$b_E \le \frac{L}{8} - \begin{pmatrix}\text{distance from beam center}\\ \text{to edge of slab}\end{pmatrix} \tag{16.4.3a}$$

$$b_E \le \frac{1}{2}b_0 + \begin{pmatrix}\text{distance from beam center}\\ \text{to edge of slab}\end{pmatrix} \tag{16.4.3b}$$

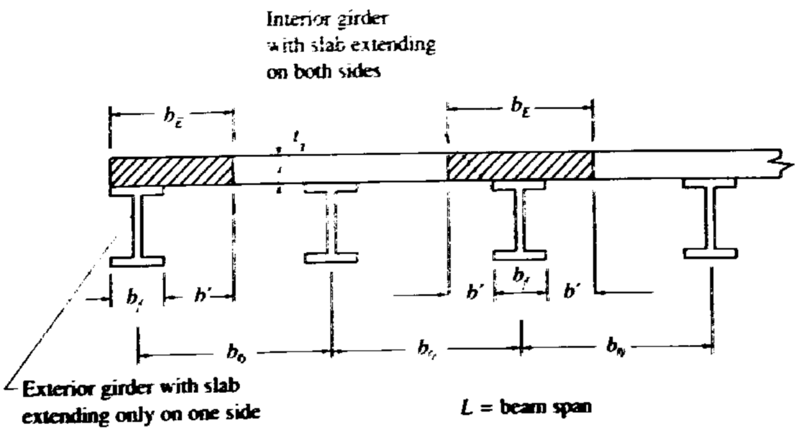


Figure 16.4.2 Dimensions governing effective width b_E on composite steel-concrete beams.

The American Concrete Institute (ACI) Code [16.15] has long used the following effective flange widths for T-sections:

1. For an *interior girder*, referring to Fig. 16.4.2,

$$b_E \leq \frac{L}{4} \tag{16.4.4a}$$

$$b_E \leq b_0 \quad \text{(for equal beam spacing)} \tag{16.4.4b}$$

$$b_E \leq b_f + 16t_s \tag{16.4.4c}$$

2. For an *exterior girder*,

$$b_E \leq b_f + \frac{L}{12} \tag{16.4.5a}$$

$$b_E \leq b_f + 6t_s \tag{16.4.5b}$$

$$b_E \leq b_f + 0.5(\text{clear distance to next beam}) \tag{16.4.5c}$$

These 2008 *ACI Code* effective widths are identical to AISC effective widths used prior to the 1986 LRFD Specification. The present AISC rules are simpler, eliminating the beam flange width b_f and the slab thickness t_s as variables.

16.5 COMPUTATION OF ELASTIC SECTION PROPERTIES

The elastic section properties of a composite section can be computed by the transformed section method. In contrast to reinforced concrete, where the reinforcing bar steel is transformed into an equivalent concrete area, the concrete slab in the composite section is transformed into equivalent steel. As a result, the concrete area is reduced by using a slab width equal to b_E/n, where n is the modulus of elasticity ratio E_s/E_c. E_s is the modulus of elasticity of steel, taken as 29,000 ksi, and E_c in psi is given by the ACI Code [16.15], as follows:

$$E_c = 33(w^{1.5})\sqrt{f'_c}, \text{ psi} \tag{16.5.1*}$$

where w is the density of concrete in pcf and f'_c is in psi. Since the *AISC Specification* uses stress in ksi for all formulas, AISC-I2.1b converts Eq. 16.5.1 approximately to the following for E_c in ksi:

$$E_c = w^{1.5}\sqrt{f'_c}, \text{ ksi} \tag{16.5.2*}$$

Note that $\sqrt{1000}$ is 31.6; thus, Eq. 16.5.2 gives E_c about 4% lower than the ACI Code. For normal-weight concrete, weighing approximately 145 pcf, Eq. 16.5.2 gives E_c in ksi as

$$E_c = 1750\sqrt{f'_c}, \text{ ksi} \tag{16.5.3*}$$

Within the accuracy that the modulus of elasticity of concrete may be predicted, either the *ACI Code* [16.15] value or the suggested value of AISC-I2.1b is acceptable. The modulus of elasticity ratio n is commonly taken to the nearest whole number. Table 16.5.1 indicates practical values usually used in computing elastic section properties.

*For SI units, giving E_c in MPa,

$$E_c = w^{1.5}(0.043)\sqrt{f'_c} \tag{16.5.1}$$

$$E_c = w^{1.5}(0.041)\sqrt{f'_c} \tag{16.5.2}$$

$$E_c = 4600\sqrt{f'_c} \tag{16.5.3}$$

where w is in kg/m^3 and f'_c is in MPa.

TABLE 16.5.1 Practical Values for Modular Ratio n

f'_c (psi)	Modular ratio $n = E_s/E_c$	f'_c (MPa)
3000	9	21
3500	$8\frac{1}{2}$	24
4000	8	28
4500	$7\frac{1}{2}$	31
5000	7	35
6000	$6\frac{1}{2}$	42

Effective Elastic Section Modulus

A complete beam may be considered as a steel member to which has been added a cover plate on the top flange. This "cover plate" being concrete is considered to be effective only when the top flange is in compression. In continuous beams, the concrete slab is usually ignored in regions of negative moment. If the neutral axis falls within the concrete slab, present practice is to consider only that portion of the concrete slab which is in compression.

AISC-I3.2 permits reinforcement parallel to the steel beam and lying within the effective slab width to be included in computing properties of composite sections. These reinforcing bars usually make little difference to the composite section modulus in the positive moment region and their effect is frequently neglected.

EXAMPLE 16.5.1

Compute the elastic section properties of the composite section shown in Fig. 16.5.1 assuming $f'_c = 3000$ psi and $n = 9$. Use the effective flange width according to AISC-I3.1a.

Solution:
First, determine effective width (AISC-I3.1a).

$$b_E = L/4 = 0.25(30)12 = 90 \text{ in.} \quad \textit{controls}$$

$$b_E = b_0 = 8(12) = 96 \text{ in.}$$

The width of equivalent steel is $b_E/n = 10.0$ in. The computation of the moment of inertia I_x about the center of gravity of the W21×62 is shown, as follows:

Element	Transformed Area A (sq in.)	Moment Arm from Centroid y (in.)	Ay (in.^{3})	Ay^2 (in.^{4})	I_0 (in.^{4})
Slab	40.0	+12.495	+500	6245	53
W21×62	18.3	0	0	0	1330
Cover plate	7.0	−10.995	−77	846	1
	65.3		+423	7091	1384

$$I_x = I_0 + Ay^2 = 1384 + 7091 = 8475 \text{ in.}^4$$

$$\bar{y} = \frac{+423}{65.3} = +6.48 \text{ in.}$$

$$I_{tr} = I_x - A\bar{y}^2 = 8475 - 65.3\,(6.48)^2 = 5737 \text{ in.}^4$$

$$y_t = 10.50 - 6.48 + 4.0 = 8.02 \text{ in.}$$

$$y_b = 10.50 + 6.48 + 1.0 = 17.98 \text{ in.}$$

The symbol I_{tr} is used for the *fully composite uncracked transformed section* moment of inertia. The elastic section modulus S_{conc} referred to the top fiber of the concrete slab is

$$S_{conc} = I_{tr}/y_t = 5737/8.02 = 715 \text{ in.}^3$$

The elastic section modulus S_{tr} referred to the extreme fiber at the tension flange of the steel section (in this case the cover plate) is

$$S_{tr} = I_{tr}/y_b = 5737/17.98 = 319 \text{ in.}^3$$

The addition of a cover plate at the tension flange brings the neutral axis down and permits more economical use of the composite section. However, the cost of welding a cover plate to the rolled section usually exceeds any material saving; thus, a cover plate is rarely used. ■

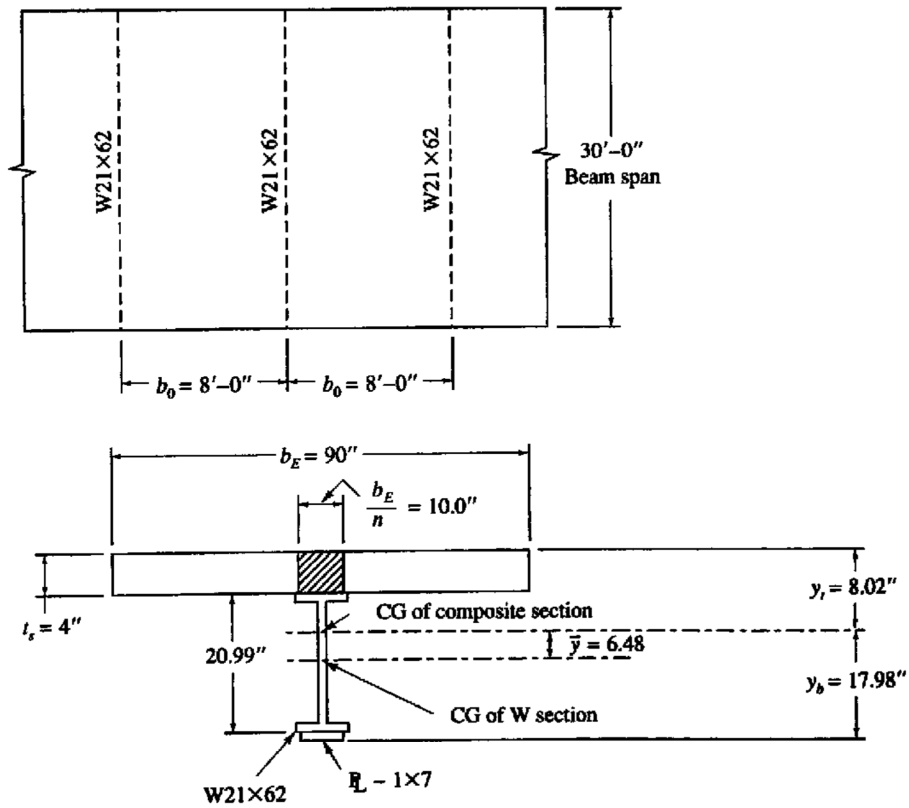


Figure 16.5.1
Composite section for Example 16.5.1.

16.6 SERVICE LOAD STRESSES WITH AND WITHOUT SHORING

The actual stresses that result due to a given loading on a composite member are dependent upon the manner of construction.

The simplest construction occurs when the steel beams are placed first and used to support the concrete slab formwork. In this case the steel beam acting noncompositely (i.e., by itself) supports the weight of the forms, the wet concrete, and its own weight. Once forms are removed and concrete has cured, the section will act compositely to resist all dead and live loads placed after the curing of concrete. Such construction is said to be *without temporary shoring* (i.e., unshored).

Alternatively, to reduce the service load stresses, the steel beams may be supported on temporary shoring; in which case, the steel beam, forms, and wet concrete, are carried by the shores. After curing of the concrete, the shores are removed and the section acts compositely to resist all loads. This system is called *shored* construction.

The following example illustrates the difference in service load stresses under the two systems of construction.

EXAMPLE 16.6.1

For the steel W21×62 with the 1 by 7-in. plate of Fig. 16.5.1, determine the service load stresses considering that (a) construction is without temporary shoring, and (b) construction uses temporary shores. The dead- and live-load moment M_L to be superimposed on the system after the concrete has cured is 560 ft-kips.

Solution:
The composite section properties as computed in Example 16.5.2 are

$$S_{\text{top}} = 715 \text{ in.}^3 \quad \text{(top of concrete)}$$

$$S_{\text{bottom}} = S_{\text{tr}} = 319 \text{ in.}^3 \quad \text{(bottom of steel)}$$

The noncomposite properties for the steel section alone (see Fig. 16.6.1) are computed as follows:

$$\bar{y} = \frac{7.0(10.995)}{7.0 + 18.3} = 3.04 \text{ in.}$$

$$y_b = 10.495 - 3.04 + 1.00 = 8.45 \text{ in.}$$

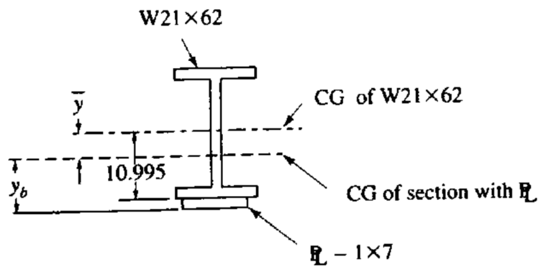


Figure 16.6.1
Steel section for Example 16.6.1.

$$I_s = I_0\,(\text{W21}\times62) + A_p y^2 - A\bar{y}^2$$
$$= 1330 + 7.0(10.995)^2 - 25.3(3.04)^2$$
$$= 1330 + 846 - 234 = 1942 \text{ in.}^4$$
$$S_{st} = \frac{1942}{13.55} = 143 \text{ in.}^3 \quad \text{(top)}$$
$$S_{sb} = \frac{1942}{8.45} = 230 \text{ in.}^3 \quad \text{(bottom)}$$

(a) *Without Temporary Shores.* Weight due to the concrete slab and steel beam,

$$w \text{ (concrete slab)}, (4/12)(8)0.15 = 0.40$$
$$w \text{ (steel beam)} \qquad = \underline{0.06}$$
$$0.46 \text{ kips/ft}$$

$$M_D \text{ (DL on noncomposite)} = \tfrac{1}{8}(0.46)(30)^2 = 51.8 \text{ ft-kips}$$
$$f_{\text{top}} = \frac{M_D}{S_{st}(\text{steel section})} = \frac{51.8(12)}{143} = 4.3 \text{ ksi}$$
$$f_{\text{bottom}} = \frac{M_D}{S_{sb}(\text{steel section})} = \frac{51.8(12)}{230} = 2.7 \text{ ksi}$$

The additional stresses after the concrete has cured are

$$f_{\text{top}} = \frac{M_L}{nS_{\text{top}}(\text{composite})} = \frac{560(12)}{9(715)} = 1.04 \text{ ksi (concrete stress)}$$

where the stress in the concrete is $1/n$ times the stress on equivalent steel (transformed section).

$$f_{\text{bottom}} = \frac{M_L}{S_{\text{tr}}} = \frac{560(12)}{319} = 21.1 \text{ ksi}$$

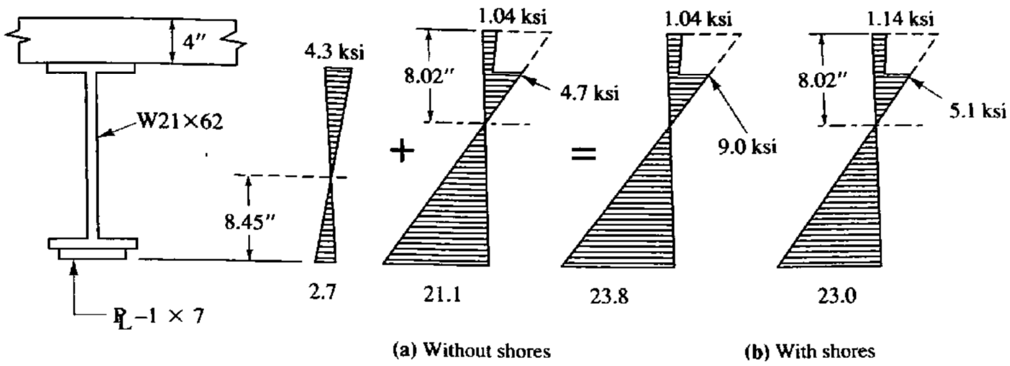
The total maximum tensile stress in the steel is

$$f = f(\text{noncomposite}) + f(\text{composite}) = 2.7 + 21.1 = 23.8 \text{ ksi}$$

(b) *With Temporary Shores.* Under this condition all loads are resisted by the composite section.

$$f_{\text{top}} = \frac{M_D + M_L}{S_{\text{top}}(\text{composite})} = \frac{(560 + 51.8)12}{715(9)} = 1.14 \text{ ksi} \quad \text{on concrete}$$
$$f_{\text{bottom}} = \frac{M_D + M_L}{S_{\text{tr}}} = \frac{(560 + 51.8)12}{319} = 23.0 \text{ ksi}$$

Stress distributions for both with and without shores are given in Fig. 16.6.2. Since the dead load was small in this example, use of shores gave insignificant reduction in service load stress. Where thicker slabs are used, the dead load stresses may become as high as 30%, in which case using or not using shores will make a significant difference. ■



igure 16.6.2 ervice load stresses for xample 16.6.1.

6.7 NOMINAL MOMENT STRENGTH OF FULLY COMPOSITE SECTIONS

The nominal strength M_n of a composite section having its slab in compression (positive moment) depends on the yield stress F_y and section properties (including slenderness $\lambda = h/t_w$ for the web) for the steel beam, the concrete slab strength f'_c, and the strength of shear connectors providing the interface shear transfer between slab and beam.

The nominal strength (commonly called *ultimate strength*) concepts were first applied to design practice as recommended by the ASCE-ACI Joint Committee on Composite Construction [16.16], and further modified by Slutter and Driscoll [16.17]. Ultimate strength was reviewed in the State-of-the-Art Report [16.1], and treated in the context of Load and Resistance Factor Design by Hansell et al. [16.6].

Traditionally, since the Joint Committee Report [16.16] the design of composite beams has been based on nominal moment strength even though Allowable Stress Design was used. Load and Resistance Factor Design is particularly adapted to using composite flexural members since the concepts of strength are easier to understand without trying to convert them into a service load based Allowable Stress Design.

The nominal moment strength M_n when the slab is in compression (positive moment) is divided into two categories according to AISC-I3.2a, depending on web slenderness, as follows:

1. For $h/t_w \le \left[3.76\sqrt{E/F_y} = 640/\sqrt{F_y}\right]$

M_n = nominal moment strength based on plastic stress distribution on the composite section (plastic moment)

$\phi_b = 0.90$

2. For $h/t_w > \left[3.76\sqrt{E/F_y} = 640/\sqrt{F_y}\right]$

M_n = nominal moment strength based on superposition of elastic stresses (shown in Sec. 16.6), considering the effects of shoring, for the limit state of yielding (yielding moment)

$\phi_b = 0.90$

Since the elastic properties and effects of shoring have been treated in Sec. 16.6, this section focuses on strength based on plastic stress distribution. It is noted that all current ASTM A6 W shapes satisfy the limit for Case 1.

The nominal strength M_n based on plastic stress distribution may be divided into two general categories: (1) the plastic neutral axis (PNA) occurs in the slab; and (2) the plastic neutral axis occurs in the steel section. When the PNA occurs in the steel section, the nominal strength M_n calculation will differ depending on whether the PNA is in the flange or the web.

The concrete is assumed to develop only compression forces. Although concrete is able to sustain a limited amount of tension, the tensile strength is negligible at the strains occurring when nominal strength is reached.

Case 1—Plastic Neutral Axis (PNA) in the Slab

Referring to Fig. 16.7.1b and assuming the Whitney rectangular stress distribution* (uniform stress of $0.85f'_c$ acting over a depth a), the compressive force C is

$$C = 0.85 f'_c a b_E \tag{16.7.1}$$

The tensile force T is the yield stress on the beam times its area:

$$T = A_s F_y \tag{16.7.2}$$

Equating the compressive force C to the tensile force T gives

$$a = \frac{A_s F_y}{0.85 f'_c b_E} \tag{16.7.3}$$

According to the ACI-accepted [16.15, Sec. 10.2.7] rectangular stress distribution, the neutral axis distance x, as shown in Fig. 16.7.1d, equals $a/0.85$ for $f'_c \leq 4000$ psi. The nominal moment strength M_n, from Fig. 16.7.1b, becomes

$$M_n = Cd_1 \quad \text{or} \quad Td_1 \tag{16.7.4}$$

When the slab is capable of developing a compressive force at least equal to the full yield strength of the steel beam, the PNA will be in the slab, the common situation for fully composite sections. Expressing the nominal strength in terms of the steel force gives

$$M_n = A_s F_y \left(\frac{d}{2} + t_s - \frac{a}{2} \right) \tag{16.7.5}$$

The usual procedure for computing nominal strength is to assume the depth a for the rectangular stress distribution will not exceed t_s; i.e., use Eq. 16.7.3. If a is verified to not exceed t_s, Eq. 16.7.5 can be used to obtain nominal strength M_n.

In the past, Case 1 has been referred to as "slab adequate"; meaning that the slab is capable of developing in compression the full nominal strength of the steel beam in tension.

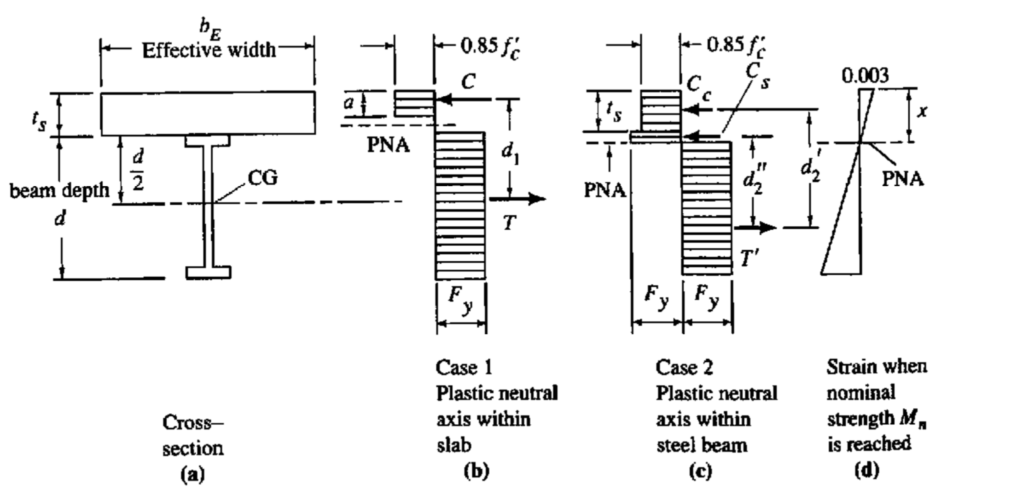


Figure 16.7.1
Plastic stress distribution at nominal moment strength M_n. (PNA = plastic neutral axis)

*For the development of the concept of replacing the true distribution of compressive stress by a rectangular stress distribution, see for example, Chu-Kia Wang, Charles G. Salmon, and José Pinchiera. *Reinforced Concrete Design*, 7th ed. (Wiley, 2006, Chap. 3).

Case 2—Plastic Neutral Axis (PNA) in the Steel Beam

If the depth a of the stress block as determined in Eq. 16.7.3 exceeds the slab thickness, the stress distribution will be as shown in Fig. 16.7.1c. The compressive force C_c in the slab is

$$C_c = 0.85 f'_c b_E t_s \tag{16.7.6}$$

The compressive force in the steel beam resulting from the portion of the beam above the neutral axis is shown in Fig. 16.7.1c as C_s.

The tensile force T' which is now less than $A_s F_y$ must equal the sum of the compressive forces:

$$T' = C_c + C_s \tag{16.7.7}$$

Also,

$$T' = A_s F_y - C_s \tag{16.7.8}$$

Equating Eqs. 16.7.7 and 16.7.8, C_s becomes

$$C_s = \frac{A_s F_y - C_c}{2}$$

or

$$C_s = \frac{A_s F_y - 0.85 f'_c b_E t_s}{2} \tag{16.7.9}$$

Considering the compressive forces C_c and C_s, the nominal moment strength M_n for Case 2 is

$$M_n = C_c d'_2 + C_s d''_2 \tag{16.7.10}$$

where the moment arms d'_2 and d''_2 are as shown in Fig. 16.7.1c.

When the Case 2 situation occurs, the steel beam must be capable of accommodating plastic strain in both tension and compression to achieve the nominal strength condition. The lower the PNA occurs in the steel section the more local buckling may influence the behavior. As indicated earlier in this section, in order to use the plastic stress distribution at all, AISC-I3.2a requires the web $\lambda \leq \lambda_p$.

When the flange of the steel section adjacent to the slab is in compression, there might be concern regarding flange local buckling. The combination of concrete bearing against the compression flange and the shear connectors used to attach the slab and steel beam together eliminates flange local buckling as well as lateral-torsional buckling as controlling limit states. AISC-I3.2 addresses only the issue of web local buckling; it is silent regarding flange local buckling.

EXAMPLE 16.7.1

Determine the nominal moment strength M_n of the composite section shown in Fig. 16.7.2. Use A992 steel, $f'_c = 4000$ psi, and $n = 8$.

Solution:

Assume the plastic neutral axis (PNA) is within the slab; i.e., that $a \leq t_s$ (Case 1),

$$a = \frac{A_s F_y}{0.85 f'_c b_E} = \frac{10.6(50)}{0.85(4)60} = 2.60 \text{ in.} < t_s \qquad \textbf{OK}$$

$$C = 0.85 f'_c a b_E = 0.85(4)(2.6)60 = 530 \text{ kips}$$

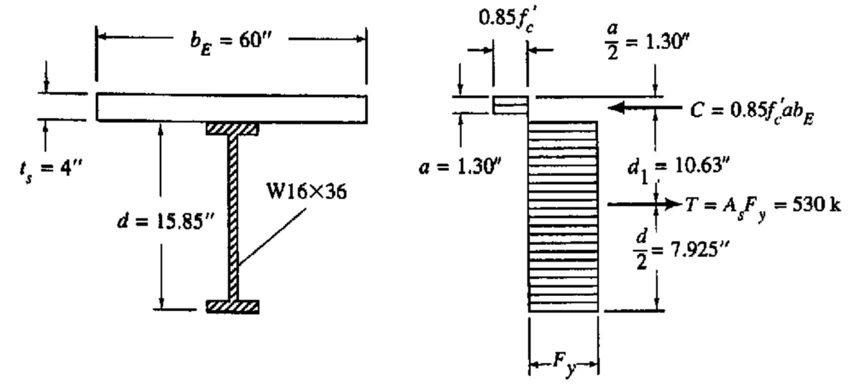


Figure 16.7.2
Example 16.7.1.

$$T = A_sF_y = 10.6(50) = 530 \text{ kips} \quad \text{(checks)}$$

$$\text{Arm } d_1 = \frac{d}{2} + t - \frac{a}{2} = 7.925 + 4.0 - 1.30 = 10.63 \text{ in.}$$

The nominal moment strength M_n is then

$$M_n = Cd_1 = 530(10.63)\tfrac{1}{12} = 470 \text{ ft-kips}$$ ■

EXAMPLE 16.7.2

Determine the nominal moment strength M_n of the composite section shown in Fig. 16.7.3. Use A992 steel, $f'_c = 4000$ psi, and $n = 8$.

Solution:
Referring to Fig. 16.7.3, assume the plastic neutral axis (PNA) is within the flange (i.e., Case 1),

$$a = \frac{A_sF_y}{0.85f'_cb_E} = \frac{47.0(50)}{0.85(4)(72)} = 9.60 \text{ in.} > \left[t_s = 7.0 \text{ in.}\right] \qquad \textbf{NG}$$

Since the concrete slab is only 7 in. thick, the slab cannot develop enough strength to balance the tension force A_sF_y capable of developing in the steel section; thus the PNA will be within the steel section; thus, Case 2 applies. Using Eq. 16.7.6,

$$C_c = 0.85f'_cb_Et_s = 0.85(4)72(7) = 1714 \text{ kips}$$

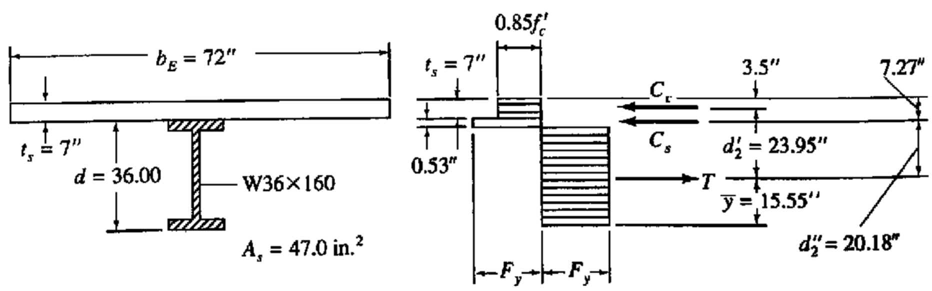


Figure 16.7.3
Example 16.7.2.

Using Eq. 16.7.9.

$$C_s = \frac{A_sF_y - 0.85\,f'_c b_E t_s}{2} = \frac{47.0(50) - 1714}{2} = 318 \text{ kips}$$

Assuming only the flange of the W36×160 ($b_f = 12.00$ in.) is in compression, the portion of the flange d_f to the neutral axis is

$$d_f = \frac{318}{50(12.00)} = 0.53 \text{ in.} < [t_f = 1.020 \text{ in.}]$$

Thus, the PNA is within the flange. The location of the centroid of the tension portion of the steel beam from the bottom is

$$\bar{y} = \frac{47.0(18) - 0.53(12)35.76}{47.0 - 0.53(12)} = 15.22 \text{ in.}$$

Referring to Fig. 16.7.3, the nominal composite moment strength M_n from Eq. 16.7.10 is

$$M_n = C_c d'_2 + C_s d''_2$$
$$= [1714(23.95) + 318(20.18)]/12 = 3960 \text{ ft-kips}$$ ■

The nominal strength M_n has inherently assumed that shear connectors will provide sufficient shear transfer at the slab-to-flange interface to develop however much of the slab compressive strength that is required to balance the tension force developed in the steel beam. Shear connectors are treated in Sec. 16.8.

The nominal strength M_n is *independent* of whether or not the system is shored during construction. Even though service load stresses are different, as illustrated in Sec. 16.6, the nominal strength is the same, shored or unshored.

16.8 SHEAR CONNECTORS

The horizontal shear that develops between the concrete slab and the steel beam during loading must be resisted so that the slip shown in Fig. 16.2.2 will be restrained. A fully composite section will have no slip at the concrete-steel interface. Although some bond may develop between the steel and the concrete, it is not sufficiently predictable to provide the required interface shear strength. Neither can friction between the concrete slab and the steel beam develop such strength.

Instead, mechanical shear connectors are *required* (AISC-I3.2d), except for the totally concrete-encased steel beam. Some mechanical shear connectors are shown in Fig. 16.8.1. The only connectors specifically provided for in the *AISC Specification* are stud shear connectors [AISC-I3.2d(3)] and channel connectors [AISC-I3.2d(4)]. Currently (2008), nearly all shear connectors are headed studs.

Ideally, to obtain a *fully composite section* the shear connectors should be stiff enough to provide the complete interaction (i.e., no slip at the interface) shown in Fig. 16.2.2c. This, however, would require that the connectors be infinitely rigid. Also, by referring to the shear diagram for a uniformly loaded beam as shown in Fig. 16.8.2, it would be inferred, theoretically at least, that more shear connectors are required near the ends of the span where the shear is high, than near midspan where the shear is low.

Consider the shear stress distribution of Fig. 16.8.2b wherein the stress v_1 must be developed by the connection between the slab and beam. Under elastic conditions the shear stress at any point in the cross-section will vary from a maximum at the support to zero at

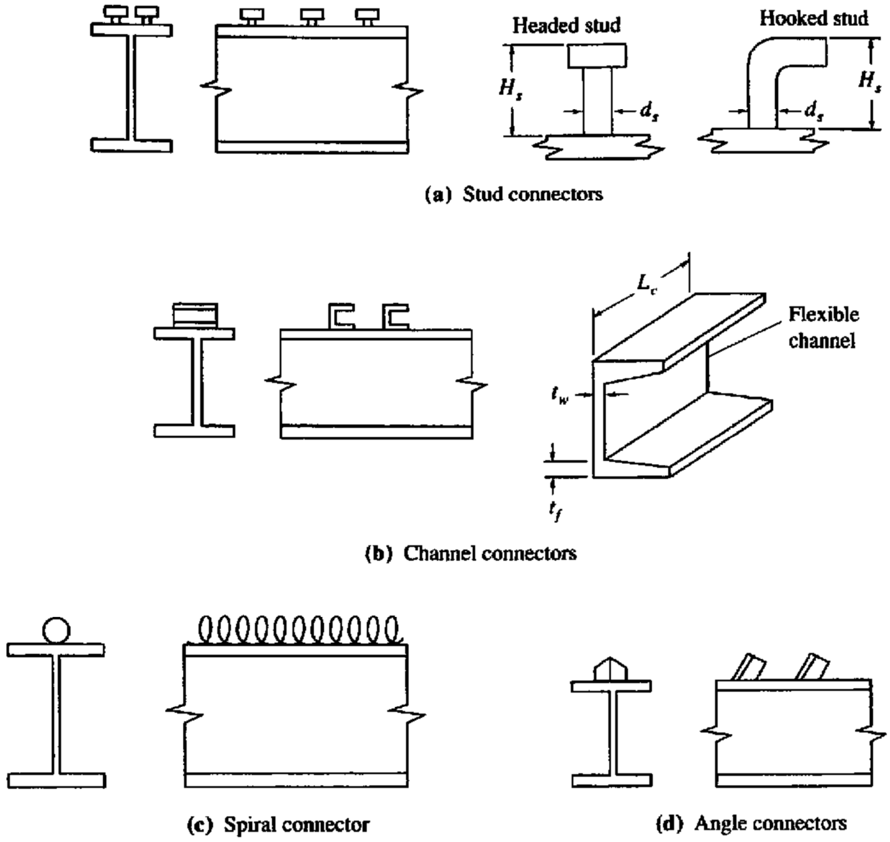


Figure 16.8.1
Shear connectors.

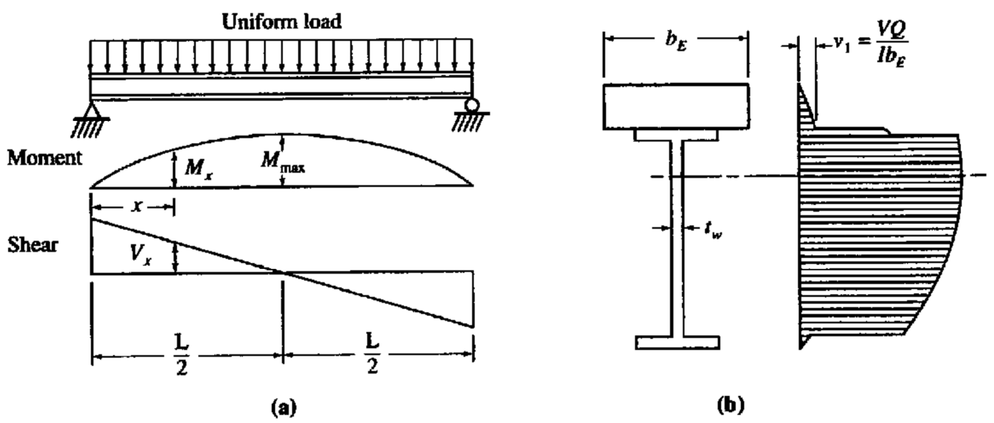


Figure 16.8.2
Shear variation for uniform loading and distribution of shear stress over the depth of a steel-concrete composite section.

midspan. Next, examine the equilibrium of an elemental slice of the beam, as in Fig. 16.8.3. The shear force per unit distance along the span is $dC/dx = v_1 b_E = V(\int y\,dA)/I$. (The $\int y\,dA$ is commonly given the symbol Q in elastic beam theory; this should not be confused with the nominal connector strength Q_n used below.) Thus, if a given connector has an allowable service load capacity q (kips), the maximum spacing p to provide the required strength is

$$p = \frac{q}{V(\int y\,dA)/I} \tag{16.8.1}$$

where $\int y\,dA$ is the statical moment of the transformed compressive concrete area (the slab) taken about the neutral axis of the composite section. Equation 16.8.1 is based on *elastic* beam theory and a fully composite section.

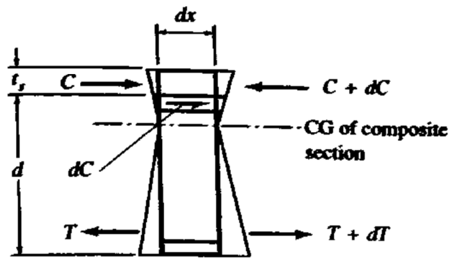


Figure 16.8.3
Force required from shear connectors at service load level.

Until recent years, Eq. 16.8.1 was used to space shear connectors. AASHTO-6.10.10.1.2 [1.3] requires using Eq. 16.8.1 to design for fatigue, a service load limit state related to the *range* of force applied, in this case the range of shear V_r resulting from live load (and impact). AASHTO also requires a strength limit state check.

According to the strength limit state, the shear connectors at nominal moment strength share equally in transmitting the shear at the interface between concrete slab and steel beam. This means, referring to Fig. 16.8.2a, that shear connectors are required to transfer the compressive force developed in the slab at midspan to the steel beam in the distance $L/2$, since no compressive force can exist in the slab at the end of the span where zero moment exists. The nominal shear transfer strength cannot exceed the maximum force the concrete can develop, namely

$$V' = C_{\max} = 0.85f'_c b_E t_s \tag{16.8.2}$$

where b_E is the effective slab width and t_s is the slab thickness. When the maximum force $T_{\max}$ that can develop in the steel is less than $C_{\max}$, the maximum shear transfer strength will be

$$V' = T_{\max} = A_s F_y \tag{16.8.3}$$

where A_s is the cross-sectional area of the steel section.

Thus, when the nominal strength Q_n of one shear connector is known, the total number N of shear connectors required between points of maximum and zero bending moment is

$$N = \frac{C_{\max}}{Q_n} \quad \text{or} \quad \frac{T_{\max}}{Q_n}, \quad \text{whichever is } \textit{smaller} \tag{16.8.4}$$

Thus, the strength is achieved when the total number N of shear connectors is placed between the maximum moment and zero moment locations. Uniform spacing will be the simplest procedure, because the number of connectors rather than the spacings affects the strength.

The determination of the connector capacity analytically is complex, since the shear connector deforms under load and the concrete which surrounds it is also a deformable material. Moreover, the amount of deformation a shear connector undergoes is dependent upon factors such as its own shape and size, its location along the beam, the location of the maximum moment, and the manner in which it is attached to the top flange of the steel beam. In addition, any particular shear connector may yield sufficiently to cause slip between the beam and the slab. In the latter case the adjacent shear connectors pick up the additional shear.

As a result of the complex behavior of shear connectors, their capacities are not based solely on a theoretical analysis. In order to develop a rational approach, a number of research programs, summarized by Viest [16.1, 16.5], were undertaken to develop the strengths of the various types of shear connectors.

Investigators determined that shear connectors will not fail when the average load per connector is below that causing 0.003 in. (0.076 mm) residual slip between concrete and steel. The amount of slip is also a function of the strength of the concrete that surrounds the shear connector. Relating connector capacity to a specified slip may be realistic for bridge design where fatigue strength is important, but it is overly conservative with respect to failure loads. So-called "ultimate" capacities used prior to 1965 [16.17] were based on slip limitation, giving values about one-third of the strengths obtained when failure of a connector is the criterion.

When flexural strength of the composite section is the basis for design, the connectors must be adequate to satisfy equilibrium of the concrete slab between the points of maximum and zero moment, as discussed in the development of Eqs. 16.8.2, 16.8.3, and 16.8.4. Slip is not a criterion for this equilibrium requirement. As stated by Slutter and Driscoll [16.17], "the magnitude of slip will not reduce the ultimate moment provided that (1) the equilibrium condition is satisfied, and (2) the magnitude of slip is no greater than the lowest value of slip at which an individual connector might fail." Studies by Ollgaard, Slutter, and Fisher [16.19] and McGarraugh and Baldwin [16.20] included the effect of lightweight concrete on stud connector capacity.

All research work cited above was based on experimental work that used solid slabs or steel decks from flat steel plates. Most of composite steel floor decks used in buildings today are formed and have a stiffening rib in the middle of each deck flute. Recent research by Rambo-Roddenberry and others [16.47] concluded that shear stud strength equations in past AISC Specifications are unconservative. The stud strength whether the deck was perpendicular or parallel to the beams, is higher than those derived from either pushout or beam tests for studs embedded in modern steel decks. Also, because of the stiffener, studs must be welded off-center in the deck rib. Rambo-Roddenberry et al [16.47] have shown that shear studs behave differently depending upon their position within the deck rib. The "weak" (unfavorable) and "strong" (favorable) positions are shown in Figure 16.8.4.

Two currently accepted expressions for the nominal strength Q_n of shear connectors are as follows:

1. *Headed steel stud connectors welded to flange* (Fig. 16.8.1a). Load and Resistance Factor Design (AISC-I3.2) gives essentially the expression developed at Lehigh [16.19], and subsequently modified at Virginia Tech [16.47]

$$Q_n = 0.5A_{sc}\sqrt{f'_c E_c} \leq R_g R_p A_{sc} F_u \tag{16.8.5}$$

where A_{sc} = cross-sectional area of stud shear connector, sq in.

F_u = specified minimum tensile strength of a stud shear connector, ksi

R_g = 1.0;

(a) for one stud welded in a steel deck rib with the deck oriented perpendicular to the steel shape;

(b) for any number of studs welded in a row directly to the steel shape;

(c) for any number of studs welded in a row through steel deck with the deck oriented parallel to the steel shape and the ratio of the average rib width to rib depth ≥ 1.5

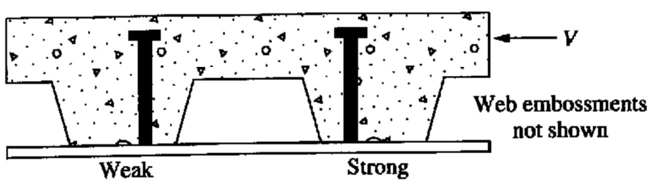


Figure 16.8.4
Weak and strong stud positions [16.47]

R_g = 0.85;

(a) for two studs welded in a steel deck rib with the deck oriented perpendicular to the steel shape;

(b) for one stud welded through steel deck with the deck oriented parallel to the steel shape and the ratio of the average rib width to rib depth <1.5

R_g = 0.7 for three or more studs welded in a steel deck rib with the deck oriented perpendicular to the steel shape

R_p = 1.0 for studs welded directly to the steel shape (in other words, not through steel deck or sheet) and having a haunch detail with not more than 50 percent of the top flange covered by deck or sheet steel closures

R_p = 0.75;

(a) for studs welded in a composite slab with the deck oriented perpendicular to the beam and $e_{mid\text{-}ht} \geq 2$ in.;

(b) for studs welded through steel deck, or steel sheet used as girder filler material, and embedded in a composite slab with the deck oriented parallel to the beam

R_p = 0.6 for studs welded in a composite slab with deck oriented perpendicular to the beam and $e_{mid\text{-}ht} < 2$ in.

$e_{mid\text{-}ht}$ = distance from the edge of stud shank to the steel deck web, measured at mid-height of the deck rib, and in the load bearing direction of the stud (in other words, in the direction of maximum moment for a simply supported beam), in.

w_c = weight of concrete per unit volume (90 pcf $\leq w_c \leq$ 155 pcf)

E_c = modulus of elasticity of concrete, ksi

= $(w^{1.5})\sqrt{f'_c}$, according to AISC-I2.1b, using f'_c in ksi. For normal-weight concrete having density w = 145 pcf, $E_c = 1746\sqrt{f'_c}$. Note that the ACI Code [16.15] gives slightly different values using $E_c = w^{1.5}33\sqrt{f'_c}$, with f'_c in psi instead of ksi.

2. *Channel connectors* (Fig. 16.8.1b). AISC-I3.2D(4) gives for the nominal connector strength Q_n,

$$Q_n = 0.3(t_f - 0.5t_w)L_c\sqrt{f'_cE_c} \tag{16.8.6}$$

where Q_n = nominal strength of one channel, kips

t_f = channel flange thickness (Fig. 16.8.1), in.

t_w = channel web thickness, in.

L_c = length of channel, in.

f'_c = 28-day compressive strength of concrete, ksi

E_c = modulus of elasticity of concrete (defined following Eq. 16.8.5), ksi

Connector Design—AISC LRFD Method

The nominal strength Q_n of the connectors is directly used in the AISC Design Methods. AISC-I3.2d requires "... the entire horizontal shear at the interface between the steel beam and the concrete slab shall be assumed to be transferred by shear connectors." For fully composite sections, the nominal horizontal shear strength V_{nh} to be provided by connectors is the smaller of Eqs. 16.8.2 and 16.8.3.

The section may also be designed as *partially composite*, where the forces utilized of the internal couple are less than either the nominal compression strength available from the concrete, or the nominal tension strength available from the steel section. In partially composite sections, the strength ΣQ_n of the shear connectors determines the magnitude of the forces of the internal couple and nominal moment strength M_n, and correspondingly the required nominal horizontal shear strength V_{nh}. Lorenz and Stockwell [16.8] have discussed stresses in partial composite beams. Bradford and Gilbert [16.44] have provided recent work on partial interaction under sustained loads.

For the positive moment situations (i.e., compression in the concrete slab), the shear strength V_{nh} required is, therefore, the *smallest* of the following:

1. V_{nh} required $= 0.85\, f'_c b_E t_s$ [16.8.2]
2. V_{nh} required $= A_s F_y$ [16.8.3]
3. V_{nh} required $= \Sigma Q_n$ provided

When 3. applies, the number of connectors controls the nominal strength M_n of the section.

As stated earlier, and as specifically stated in AISC-I3.2d(6), the strength V_{nh} must be provided "... each side of the point of maximum moment ..." to the points of zero moment. Further, AISC-I3.2d(6) states that the connectors *shall* be distributed uniformly between the point of maximum moment and the point of zero moment. This is "unless otherwise specified", whatever that may mean. As long as adequate strength is provided, the spacing of the connectors is not important.

The nominal strengths Q_n for stud and channel connectors from AISC-I3.2d(3) and (4) are given by Eqs. 16.8.5 and 16.8.6; values for common stud diameters and some channels are given in Table 16.8.1.

When a formed steel deck is used (see Fig. 16.1.2) with shear studs embedded in the supported concrete slab, reduction factors must be applied to Q_n in accordance with AISC-I3.2d(3).

In the case of continuous beams (also see Sec. 16.13), the longitudinal reinforcing bar steel within the effective width of the concrete slab is permitted (AISC-I3.2d(2)) to be assumed to act compositely with the steel beam in the areas of negative moment. The total nominal horizontal strength V_{nh} needed from shear connectors between the interior support and each adjacent point of inflection (zero moment) equals the tension force available from the reinforcement (since the tension in the concrete is neglected),

$$T_{\text{slab}} = A_r F_{yr} \tag{16.8.7}$$

where A_r = total area of adequately developed longitudinal reinforcing steel within the effective width b_E of the concrete slab

F_{yr} = minimum specified yield stress of the reinforcing steel

EXAMPLE 16.8.1

Determine the number of $\frac{3}{4}$-in.-diam × 3-in. shear stud connectors required to develop the fully composite section of Fig. 16.8.5. Assume the applied loading is uniform and the beam is simply supported. Use A992 steel, f'_c = 4000 psi, n = 8, and Load and Resistance Factor Design.

Solution:

Using Eqs. 16.8.2 and 16.8.3,

$$V_{nh} = C_{\max} = 0.85 f'_c b_E t_s = 0.85(4.0)(72)7 = 1714 \text{ kips}$$

or

$$V_{nh} = T_{\max} = A_s F_y = 47.0(50) = 2350 \text{ kips}$$

TABLE 16.8.1 Nominal Strength Q_n (kips) for Stud and Channel Shear Connectors Used with No Decking ($R_g = R_p = 1.0$) and Normal-Weight Concrete†

Connector	Concrete strength f'_c (ksi) 3.0	3.5	4.0
1/2″ diam × 2″ headed stud	9.4	10.5	11.6
5/8″ diam × 2-1/2″ headed stud	14.6	16.4	18.1
3/4″ diam × 3″ headed stud	21.0	23.6	26.1
7/8″ diam × 3-1/2″ headed stud	28.6	32.1	35.5
Channel C3×4.1	$10.2L_c$*	$11.5L_c$	$12.7L_c$
Channel C4×5.4	$11.1L_c$	$12.4L_c$	$13.8L_c$
Channel C5×6.7	$11.9L_c$	$13.3L_c$	$14.7L_c$

†AISC Formula (I3-3), Eq. 16.8.5, used for studs and AISC Formula (I3-4), Eq. 16.8.6, used for channels. Studs, A108Type 2, F_u^b = 60 ksi.

*L_c = Length of channel, in.

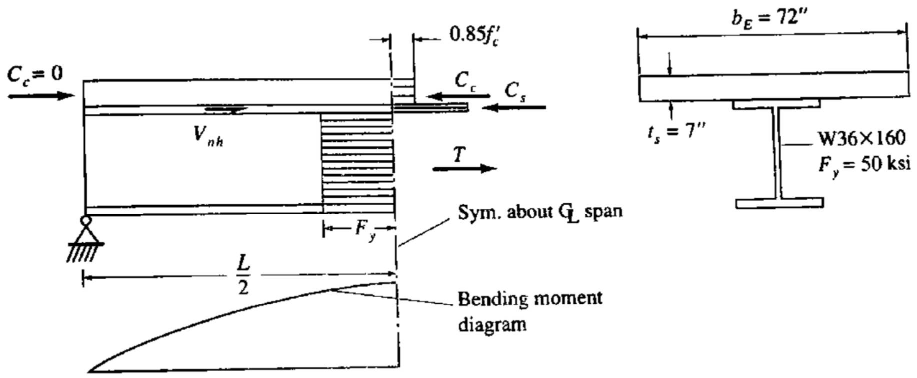


Figure 16.8.5 Example 16.8.1.

As found from the analysis in Example 16.7.2, the neutral axis is located within the steel section; thus, $C_{max} < T_{max}$. The force in the concrete to be carried by shear connectors is 1714 kips.

The nominal strength Q_n per connector, from Eq. 16.8.5 or Table 16.8.1, is 26.1 kips. The number N of shear connectors required for each half span is

$$N = \frac{1714}{26.1} = 66$$

Use 66—$\frac{3}{4}$-in.-diam × 3-in. studs per half span.

Connector Design—Elastic Concept for Fatigue Strength

The 1992 AASHTO Specification [1.3] requirements for fatigue are based largely on the work of Slutter and Fisher [16.21]. For fatigue, the *range* of service load shear rather than strength under overload is the major concern. Fatigue strength may be expressed

$$\log N = A + BS_r \tag{16.8.8}$$

where S_r is the range of service load horizontal shear; N is the number of cycles to failure; and A and B are empirical constants. The equation used for design is shown in Fig. 16.8.6.

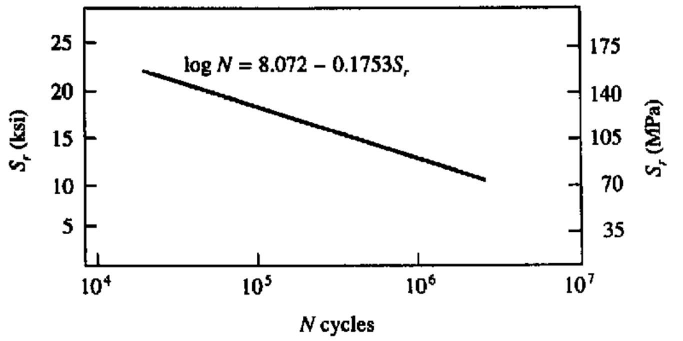


Figure 16.8.6
Fatigue strength of stud shear connectors. (From Ref. 16.21)

Since the magnitude of shear force transmitted by individual connectors when service loads act agrees well with prediction by elastic theory, the horizontal shear is calculated by the elastic relation VQ/I. Fatigue is critical under repeated application of service load; thus it is reasonable to determine variation in shear using elastic theory. The spacing of the connectors will vary along the span in accordance with V.

For *cyclical load,* Eq. 16.8.1 gives

$$V_{sr} = \frac{(V_{\max} - V_{\min})Q}{I} = \frac{\text{Allowable range } \Sigma Z_r}{p} \tag{16.8.9}$$

where p is the connector spacing. AASHTO-6.10.10.1.2 [1.3] gives Eq. 16.8.9 as

$$p \leq \frac{\Sigma Z_r}{V_{sr}} \tag{16.8.10}$$

where V_{sr} = horizontal fatigue shear range per unit length, kip/in.

Z_r = allowable range of load per connector, lb/stud connector

$= \alpha d^2 \geq \dfrac{5.5d^2}{2}$ (AASHTO 6.10.10.2)

Σ = indicates the sum of Z_r for connectors at the section is to be used.

d = stud diameter, in.

$\alpha = 34.5 - 4.28 \log N$

EXAMPLE 16.8.2

Redesign the shear conncetors for the beam of Example 16.8.1 (Fig 16.8.5) using the service load stress fatigue requirement of AASHTO with $\frac{3}{4}$-in.-diam × 3-in. stud connectors. Design for 500,000 cycles of loading of live load. Whether or not the beam is shored, only the live load is the cylical load. Use uniform live load of 3.5 kips/ft, a spacing of 6 ft for beams, a beam span of 45 ft, $F_y = 50$ ksi, and $f'_c = 4$ ksi.

Solution:

(a) Loads and shears. For the fatigue requirement in AASHTO-6.10.10.1.2 only the *range* of service live load is needed. At the support with full span loaded,

$$V = \tfrac{1}{2}wL = 0.5(3.5)45 = 78.8 \text{ kips}$$

Using partial span loading of live load,

$$\text{Max } V\left(\text{at } \tfrac{1}{4} \text{ point}\right) = 3.5(45)(0.75)(0.375) = 44.3 \text{ kips}$$

$$\text{Max } V(\text{at midspan}) = \tfrac{1}{8}wL = \tfrac{1}{8}(3.5)45 = 19.7 \text{ kips}$$

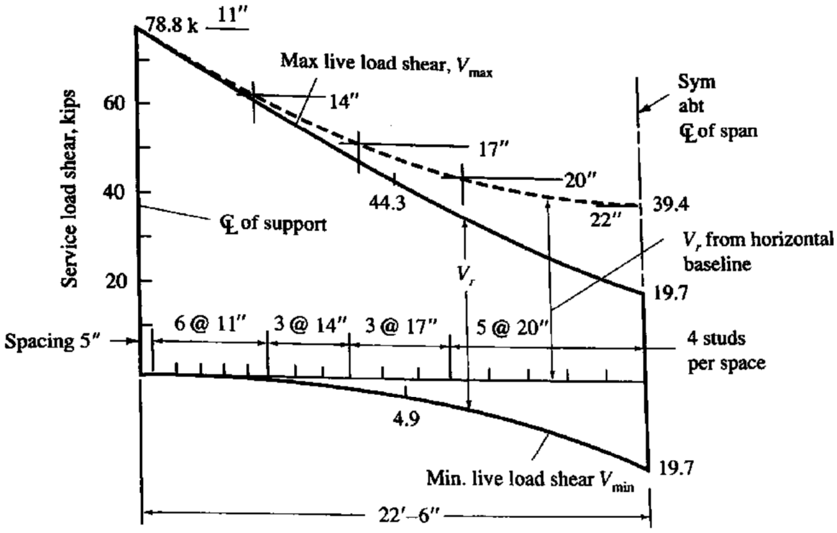


Figure 16.8.7
Shear range diagram and stud spacing according to elastic fatigue theory used by AASHTO-Example 16.8.2.

The envelope showing the *range* of live load shear is given in Fig. 16.8.7. Inclusion of dead load shear would change both V_{max} and V_{min} by the same amount at any section along the beam; however, $(V_{max} - V_{min})$, that is, the range V_r would not be affected.

(b) Compute elastic composite section properties ($n = 8$) (see Fig. 16.8.5).

$$\text{Effective slab width } b_E = b_0 = 72 \text{ in.}$$

Element	Effective area, A (sq in.)	Arm from CG of steel beam, y (in.)	Ay (sq in.)	Ay^2 (in.^{3})	I_0 (in.^{4})
Slab, 72(7)/8	63.0	21.5	1355	29,120	257
W36×160	47.0	—	—	—	9760
	110		1355	29,120	10,017

$$I_x = Ay^2 + I_0 = 29{,}120 + 10{,}017 = 39{,}100 \text{ in.}^4$$

$$\bar{y} = \frac{1355}{110} = 12.32 \text{ in.}$$

$$I_{tr} = 39{,}100 - 110(12.32)^2 = 22{,}400 \text{ in.}^4$$

$$y_t = 18.0 + 7 - 12.32 = 12.68 \text{ in.}$$

$$y_b = 18.0 + 12.32 = 30.32 \text{ in.}$$

$$S_t = \frac{22{,}400}{12.68} = 1767 \text{ in.}^3 \quad \text{(concrete at top)}$$

$$S_b = S_{tr} = \frac{22{,}400}{30.32} = 739 \text{ in.}^3 \quad \text{(steel at bottom)}$$

Determine the static moment of the effective concrete area about the centroid of the composite section,

$$Q = 63.0(y_t - 3.5) = 63.0(9.18) = 578 \text{ in.}^3$$

(c) Determine the allowable load for $\frac{3}{4}$-in.-diam × 3-in. stud connectors. AASHTO-6.10.10.1.2 gives an allowable service load range Z_r per connector based on fatigue for 500,000 cycles of loading as

$$\alpha = 34.5 - 4.28 \log N$$
$$= 34.5 - 4.28 \log(500{,}000) = 10.1$$

$$Z_r = \alpha d^2 = 10.1(0.75)^2 = 5.68 \text{ kips} > \left[\frac{5.5d^2}{2} = 1.55 \text{ kips}\right] \qquad \textbf{OK}$$

The AASHTO allowable values are based on a slip limitation.

(d) Determine spacing of connectors. Use 4 studs across the beam flange width at each location:

$$\Sigma Z_r \text{ for 4 studs} = 4(5.68) = 22.7 \text{ kips}$$

Using Eq. 16.8.10,

$$p = \frac{\Sigma Z_r}{V_{sr}} = \frac{\Sigma Z_r}{(V_{max} - V_{min})Q/I}$$

where $I/Q = 22{,}400/578 = 38.8$ in.

$$p = \frac{22.7(38.8)}{(V_{max} - V_{min})} = \frac{881}{(V_{max} - V_{min})(\text{kips})}$$

The values are computed in the table below and the spacing is determined graphically on the shear diagram of Fig. 16.8.7.

p (in.)	V_r (kips)	p (in.)	V_r (kips)
11	79	20	45
14	61	22	39
17	51		

$V_r = V_{max} - V_{min}$

The fatigue service load criterion requires 8% more connectors (68 vs 61 per half span) than the procedure based on strength. ■

16.9 COMPOSITE FLEXURAL MEMBERS CONTAINING FORMED STEEL DECK

Composite flexural members may be made using formed steel deck, as shown in Fig. 16.1.2. The formed metal deck may be placed perpendicular to or parallel with the supporting beam. Furthermore, the beam may actually be an open web joist. Typically, the deck plate varies in thickness from 22 ga. (0.0336 in., 0.853 mm) to 12 ga. (0.1084 in., 2.75 mm). The deck rib height typically is $1\frac{1}{2}$, 2, and 3 in. for spans of, say, 8, 10, and 15 ft. As shown in Fig. 16.1.2, the thickness of the concrete slab above the top of the ribs must be at least 2 in. AISC-I3.2c and the embedment of the stud connectors into the concrete above the top of the ribs must be at least $1\frac{1}{2}$ in.

When the steel deck ribs are perpendicular to the steel beam, the stud strength Q_n may have to be reduced from that given by Eq. 16.8.5 by a reduction factor as explained earlier. Easterling, Gibbings, and Murray [16.43] provide a study of strength of shear studs in steel deck on composite beams.

Full treatment of formed steel deck supported slab composite beams is outside the scope of this chapter. The reader is referred to Grant, Fisher, and Slutter [16.23], and particularly with regard to LRFD design, to Vinnakota, Foley, and Vinnakota [16.24]. Composite open-web joists have been treated by Tide and Galambos [16.25] and Rongoe [16.26]. Two-way acting composite slabs with steel deck have been treated by Porter [16.29], and design *Specifications* and *Commentary* [16.27, 16.28] are available from ASCE. The special considerations regarding the design of "stubgirders" are treated by Buckner, Deville, and McKee [16.30].

16.10 DESIGN PROCEDURE—AISC LRFD AND ASD METHODS

The design of composite beams involves providing sufficient plastic strength ϕM_p of the composite section to equal the factored moment. Using rolled W shapes, local buckling ordinarily is not a controlling limit state although h/t_w should be checked when the PNA is in the web, and because the compression flange is attached to the concrete slab lateral-torsional buckling is precluded as a controlling limit state. Thus, it is required that

$$\phi_b M_p \geq M_u \qquad \text{(LRFD)} \tag{16.10.1a}$$

$$M_p/\Omega_b \geq M_a \qquad \text{(ASD)} \tag{16.10.1b}$$

where $\phi_b = 0.90$ and $\Omega_b = 1.67$ for a composite beam.

In general, the design should be started by assuming the plastic neutral axis (PNA) is within the slab (Case 1—Fig. 16.7.1b). Thus, using Eq. 16.7.5, the required area A_s for the steel section is

$$\text{Required } A_s = \frac{M_u}{\phi_b F_y\left(\dfrac{d}{2} + t_s - \dfrac{a}{2}\right)} \qquad \text{(LRFD)} \tag{16.10.2a}$$

$$\text{Required } A_s = \frac{M_u}{\dfrac{F_y}{\Omega_b}\left(\dfrac{d}{2} + t_s - \dfrac{a}{2}\right)} \qquad \text{(ASD)} \tag{16.10.2b}$$

Typically $a/2$ can be estimated as 1 in. for preliminary design.

In addition to the strength requirement under full dead and live load, AISC-I3.1c requires that when temporary shores are not used during construction, the steel section alone must have adequate strength "to support all loads applied prior to the concrete attaining 75% of its specified strength f'_c." For this condition, local buckling of the beam elements and lateral-torsional buckling must be considered.

16.11 AISC EXAMPLES—SIMPLY SUPPORTED BEAMS

EXAMPLE 16.11.1

Design an interior composite beam for the floor whose plan is shown in Fig. 16.11.1 assuming the beam is to be constructed without temporary shoring. Use 50 ksi, $f'_c = 4$ ksi ($n = 8$), a 4-in. slab, and the AISC LRFD Method.

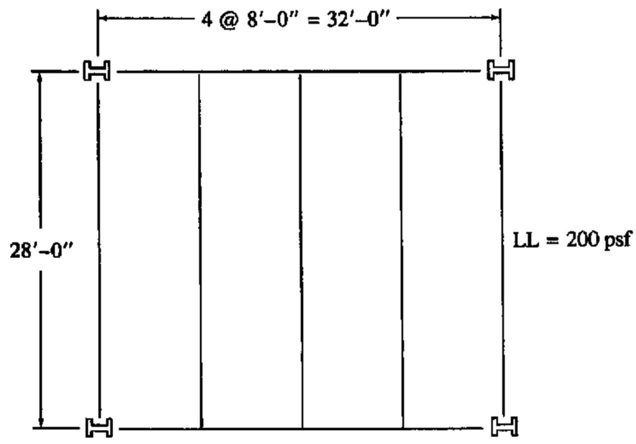


Figure 16.11.1
Beam framing plan for Examples 16.11.1 and 16.12.1.

Solution:

(a) Compute factored and service loads.

Loads carried on steel beam:

$$\begin{aligned} \text{concrete slab, } \tfrac{4}{12}(0.15)8 &= 0.40 \text{ kip/ft} \\ \text{beam weight (estimated)} &= \underline{0.04} \text{ kip/ft} \\ \text{service dead load} &= 0.44 \text{ kip/ft} \\ \text{factored dead load} &= 0.44(12) = 0.53 \text{ kip/ft} \end{aligned}$$

Load carried by composite action:

$$\begin{aligned} \text{service live load, } 0.2(8) &= 1.6 \text{ kips/ft} \\ \text{factored live load} &= 1.6(1.6) = 2.56 \text{ kips/ft} \end{aligned}$$

(b) Compute service load and factored load moments.

$$\begin{aligned} M_D &= \tfrac{1}{8}(0.44)(28)^2 = 43 \text{ ft-kips} \quad \text{(service load)} \\ M_L &= \tfrac{1}{8}(1.60)(28)^2 = 157 \text{ ft-kips} \quad \text{(service load)} \\ M_u &= \tfrac{1}{8}(0.53 + 2.56)(28)^2 = 303 \text{ ft-kips} \quad \text{(factored)} \end{aligned}$$

(c) Select the section. Use Eq. 16.10.2 assuming the PNA (plastic neutral axis) is within the slab. Estimate $a \approx 1.0$ for preliminary selection.

$$\text{Required } A_s = \frac{M_u}{\phi_b F_y\left(\dfrac{d}{2} + t_s - \dfrac{a}{2}\right)} \qquad [16.10.2a]$$

From Eq. 16.10.2, the design strength $\phi_b M_n$ provided can be computed as A_s times the denominator. For a given value of $(t_s - a/2)$, $\phi_b M_n$ can be tabulated for a steel W section for any given yield stress; such tabulated information is given in the *AISC Manual.* Thus, for the 4-in. slab and estimated a of 1 in.,

$$t_s - \frac{a}{2} = 4 - 0.50 = 3.5 \text{ in.}$$

$$\text{Required } A_s = \frac{303(12)}{0.90(50)(7 + 3.5)} = 7.7 \text{ sq in.} \quad (\text{for W14})$$

$$\text{Required } A_s = \frac{303(12)}{0.90(50)(8 + 3.5)} = 7.0 \text{ sq in.} \quad (\text{for W16})$$

Using *AISC Manual*, Table 3-19 "Composite W Shapes-Available Strength in Flexure" entering with $Y2 = t_s - a/2 = 3.5$ in. and required $\phi M_n = 303$ ft-kips, find

W16×26	$\phi_b M_n = 327$ ft-kips	$A_s = 7.68$ sq in.
W14×26	$\phi_b M_n = 302$ ft-kips	$A_s = 7.69$ sq in.

The tabulated values selected are for the PNA within the slab (that is, $Y1$ = distance from PNA to top of steel beam = 0 in.). When these tables are available, their use will be faster and more accurate than putting estimated d into Eqs. 16.10.2.

(d) Compute the plastic neutral axis location and check strength.

Try W16×26: Properties of the steel section alone are:

$$A_s = 7.68 \text{ sq in.} \qquad I_x = 301 \text{ in.}^4 \qquad b_f = 5.50 \text{ in.}$$

$$d = 15.70 \text{ in.}$$

Determine effective width of slab:

$$b_E = \tfrac{1}{4} \text{ of span} = 0.25(28)12 = 84 \text{ in.} \quad \textit{controls}$$

$$\text{or} \quad b_E = \text{beam spacing} = 8(12) = 96 \text{ in.}$$

The compressive force in the concrete, assuming $a < t_s$, and the tension force in the steel section, are

$$C = 0.85 f'_c b_E a = 0.85(4)84a = 286a$$

$$T = A_s F_y = 7.68(50) = 384 \text{ kips}$$

Statics requires

$$C = T$$

$$a = 1.34 \text{ in.} < t_s \qquad \textbf{OK as assumed}$$

The nominal moment strength M_n is

$$M_n = T\left(\frac{d}{2} + t_s - \frac{a}{2}\right)$$

$$M_n = 384\left(\frac{15.7}{2} + 4.0 - \frac{1.34}{2}\right)\frac{1}{12} = 358 \text{ ft-kips}$$

$$\phi_b M_n = 0.90(358) = 322 \text{ ft-kips} > (M_u = 301 \text{ ft-kips}) \qquad \textbf{OK}$$

Note that M_u has been revised to include the correct beam weight.

(e) Check the strength of the steel section to support construction loads (AISC-I3.1c). This check is required when shores are not used. Assume adequate lateral support is provided during construction such that $L_b \leq L_p$ and the section is compact for local buckling; therefore $\phi_b M_n = \phi_b M_p$, and $\phi_b = 0.90$ for the steel section acting noncompositely. There are no AISC-prescribed construction loads. It is prudent to consider that some of the wet concrete load should be treated as live load, say 50% of it (accomplished by using an average overload factor of 1.4). Further, other construction live load on the order of 20 to 25 psf should be included (20 psf used here).

$$\begin{aligned} \text{Slab} &= 0.40(1.4) \;\;= 0.56 \;\;\text{kip/ft} \\ \text{Construction} &= 0.02(8)1.6 = 0.26 \;\;\text{kip/ft} \\ \text{Steel section} &= 0.026(1.2) = \underline{0.03} \;\;\text{kip/ft} \\ w_u &= 0.85 \;\;\text{kip/ft} \\ M_u &= \tfrac{1}{8}(0.85)(28)^2 = 83 \text{ ft-kips} \\ \phi_b M_p \text{ for W16}\times 26 &= 166 \text{ ft-kips} > 83 \text{ ft-kips} \qquad \textbf{OK} \end{aligned}$$

(f) Design shear connectors. The compressive force in the slab must be carried by shear connectors,

$$C = 0.85 f'_c b_E a = 0.85(4)84a = 286a = 286(1.34) = 385 \text{ kips}$$

Since $a < t_s$, V_{nh} will be based on the 385 kips, which does equal $T_{max} = A_s F_y$. Using $\frac{3}{4}$-in.-diam × 3-in. headed studs, $Q_n = 26.1$ kips/stud from Table 16.8.1. The number N of connectors required to carry 385 kips is

$$N = \frac{V_{nh}}{Q_n} = \frac{385}{26.1} = 14.8, \quad \text{say 15}$$

which is the number of connectors required for the region between maximum moment and the support (zero moment location). Thus, 30 studs are needed for the entire span. Using a uniform spacing with two studs at each location, the spacing p required would be

$$p = \frac{L}{N} = \frac{28(12)}{15} = 22 \text{ in.}$$

$$\text{Maximum } p = 8t_s = 8(4) = 32 \text{ in.} \quad \text{(AISC-I3.2)}$$

$$\text{Minimum } p = 6(\text{diam}) = 6(0.75) = 4.5 \text{ in.} \quad \text{(AISC-I3.2)}$$

Use W16×26 section of A992 steel, along with 30—$\frac{3}{4}$-in.-diam × 3-in. headed stud connectors over the entire span, spaced at 22 in. The connectors are to be placed in pairs starting at the support. ■

EXAMPLE 16.11.2

Design an interior composite beam to span 30 ft with a beam spacing of 8 ft, using the minimum number of $\frac{3}{4}$-in.-diam × 3-in. stud shear connectors. The slab is 5 in. thick. The beam is to be constructed without shores. The beam must support a ceiling of 7 psf, partitions and other dead load of 25 psf, and live load of 150 psf. Use A572 Grade 50 steel and $f'_c = 3$ ksi ($n = 9$) concrete. Use the AISC LRFD Design Method.

Solution:

(a) Compute factored loads and bending moments. The dead load and moment that must be carried by the steel beam alone during construction are

$$\begin{aligned} \text{5-in. slab, } \tfrac{5}{12}(8)0.15 &= 0.50 \text{ kips/ft} \\ \text{Steel beam (assumed)} &= \underline{0.03} \\ &\ 0.53 \text{ kips/ft} \end{aligned}$$

$$M_D = \tfrac{1}{8}(0.53)(30)^2 = 60 \text{ ft-kips}$$
$$M_{u1} = 1.2(60) = 72 \text{ ft-kips}$$

The partition and ceiling dead loads, and the live load that must be carried by the composite section are

$$\begin{aligned}
&\text{Live load } 0.15(8) &&= 1.2 \text{ kips/ft}\\
&\text{Partitions } 0.025(8) &&= 0.2\\
&\text{Ceiling } 0.007(8) &&= \underline{0.06}\\
& && \quad 1.46 \text{ kips/ft}
\end{aligned}$$

$$M_L = \tfrac{1}{8}(1.46)(30)^2 = 164 \text{ ft-kips}$$
$$w_{u2} = 1.2(0.25) + 1.6(1.2) = 2.22 \text{ kips/ft}$$
$$M_{u2} = \tfrac{1}{8}(2.22)(30)^2 = 250 \text{ ft-kips}$$
$$M_u = M_{u1} + M_{u2} = 72 + 250 = 322 \text{ ft-kips}$$

(b) Select the section. One could use Eq. 16.10.2 assuming the PNA (plastic neutral axis) is within the slab and solve for required A_s as illustrated in Example 16.11.1 (part c). Alternatively, it will be simpler to use AISC Table 3-19, "Composite W Shapes-Available Strength in Flexure." Equation 16.10.2 is tabulated for the steel W shapes for various values of $Y2$. Estimate $a \approx 1.0$ for preliminary selection as in Example 16.12.1. For the 5-in. slab,

$$Y2 = t_s - \frac{a}{2} = 5 - 0.50 = 4.5 \text{ in.}$$
$$\text{Required } \phi M_n = 322 \text{ ft-kips}$$

Find:

$$\text{W}16\times26 \qquad \phi M_n = 356 \text{ ft-kips}$$
$$\text{W}14\times26 \qquad \phi M_n = 330 \text{ ft-kips}$$

The tabulated values selected are for the PNA within the slab (that is, $Y1$ = distance from PNA to top of steel beam = 0 in.).

(c) Investigate the W16×26 further. For fully composite action, compute the plastic neutral axis location and check strength.

Try W16×26: Properties of the steel section alone are:

$$A_s = 7.68 \text{ sq in.} \qquad I_x = 301 \text{ in.}^4 \qquad b_f = 5.500 \text{ in.}$$
$$d = 15.69 \text{ in.}$$

Determine effective width of slab:

$$b_E = \tfrac{1}{4} \text{ of span} = 0.25(30)12 = 90 \text{ in.} \quad \textit{controls}$$
$$\text{or} \quad b_E = \text{beam spacing} = 8(12) = 96 \text{ in.}$$

The compressive force in the concrete, assuming $a < t_s$, and the tension force in the steel section are

$$C = 0.85 f'_c b_E a = 0.85(3)90a = 229.5a$$
$$T = A_s F_y = 7.68(50) = 384 \text{ kips}$$

Statics requires

$$C = T$$

$$a = 1.67 \text{ in.} < t_s \qquad \textbf{OK as assumed}$$

The nominal moment strength M_n is

$$M_n = T\left(\frac{d}{2} + t_s - \frac{a}{2}\right)$$

$$M_n = 384\left(\frac{15.69}{2} + 5.0 - \frac{1.67}{2}\right)\frac{1}{12} = 384 \text{ ft-kips}$$

$$[\phi_b M_n = 0.90(384) = 346 \text{ ft-kips}] > [M_u = 322 \text{ ft-kips}] \qquad \textbf{OK}$$

The W16×26 section is adequate as a *fully composite section.* However, when a minimum number of shear connectors is desired and only partial composite action is used, the steel section usually must be heavier. Try W16×31 section.

(d) Minimum number of shear connectors required. The maximum spacing p along the span is

$$\text{Maximum } p = 8t_s = 8(5) = 40 \text{ in.} \quad \text{(AISC-I3.2)}$$

$$N = \frac{L}{p} = \frac{30(12)}{40} = 9 \text{ spaces}$$

The connectors would be in pairs which would mean 20 connectors for the 30-ft span, with 5 pairs (10 connectors) located between midspan and the end of the beam. When $\frac{3}{4}$-in.-diam studs are used, 10 connectors provide nominal strength ΣQ_n,

$$\Sigma Q_n = 10(21.0) = 210 \text{ kips}$$

Since the force in the slab based on connector strength is less than the maximum steel force,

$$T_{\text{max}} = A_s F_y = 9.12(50) = 456 \text{ kips}$$

the plastic neutral axis (PNA) is within the steel section.

(e) Locate plastic neutral axis (PNA) and compute nominal strength. Check if PNA occurs within the flange,

$$\Sigma Q_n = 210 \text{ kips}$$

$$\text{Max force in flange} = t_f b_f F_y = 0.440(5.525)50 = 121.6 \text{ kips}$$

$$T_{\text{max}} - 121.6 = 334.5 \text{ kips} > \Sigma Q_n$$

Thus, PNA is in the web. For equilibrium of internal forces, referring to Fig. 16.11.2, compute the compression force in the web,

$$\Sigma Q_n + C_f + C_w = T_{\text{max}} - C_f - C_w$$

$$210 + 121.6 + C_w = 334.5 - C_w$$

$$2C_w = 2.9 \text{ kips}$$

$$\begin{array}{l}\text{Depth to PNA from}\\ \text{inside of flange}\end{array} = \frac{C_w}{F_y t_w} = \frac{1.45}{50(0.275)} = 0.11 \text{ in.}$$

$$\text{PNA from top of slab} = t_s + t_f + 0.11$$

$$= 5 + 0.44 + 0.11 = 5.55 \text{ in.}$$

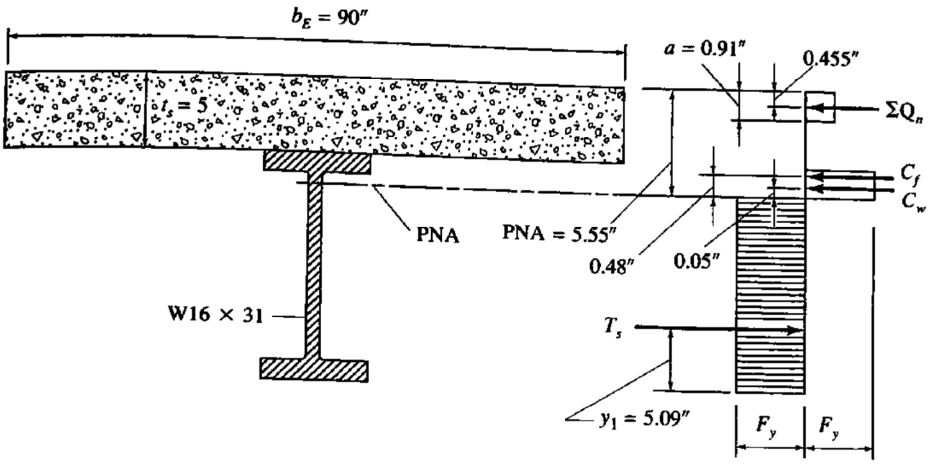


Figure 16.11.2 Example 16.11.2, showing stress distribution to obtain plastic neutral axis.

Locate the centroid y_1 of the portion of the steel section in tension measured from the bottom of the steel section,

	Area, A	Arm, y	Ay
W section	9.12	7.94	72.41
Flange	−2.43	15.66	−38.07
Web	−0.03	15.39	−0.45
	6.66 sq in.		33.90 in.^{3}

$$y_1 = \frac{33.90}{6.66} = 5.09 \text{ in.}$$

(f) Compute the nominal moment strength M_n. Since the ΣQ_n representing the strength of the shear connectors used is less than the force in the concrete when there is fully composite action, force ΣQ_n is taken equivalent to $C_c = 0.85 f'_c b_E a$, the concrete force represented by the rectangular stress distribution in the concrete. That means

$$a = \frac{\Sigma Q_n}{0.85 f'_c b_E} = \frac{210}{0.85(3)90} = 0.91 \text{ in.}$$

Referring to Fig. 16.11.2, taking internal moments about the point of action of T_s gives

$$\begin{aligned}
\Sigma Q_n\text{:} \quad M_{n1} &= \Sigma Q_n(d - 5.09 + t_s - a/2)\\
&= 210(15.88 - 5.09 + 5 - 0.91/2)\tfrac{1}{12}\\
&= 210(15.33)\tfrac{1}{12} = 268.3 \text{ ft-kips}\\
C_f\text{:} \quad M_{n2} &= C_f(d - 5.09 - t_f/2)\\
&= 121.6(15.88 - 5.09 - 0.440/2)\tfrac{1}{12}\\
&= 121.6(10.57)\tfrac{1}{12} = 107.1 \text{ ft-kips}\\
C_w\text{:} \quad M_{n3} &= C_w(d - 5.09 - t_f - 0.11/2)\tfrac{1}{12}\\
&= 1.45(15.88 - 5.09 - 0.440 - 0.055)\tfrac{1}{12}\\
&= 1.45(10.30)\tfrac{1}{12} = 1.2 \text{ ft-kips}\\
M_n &= M_{n1} = M_{n2} + M_{n3}\\
&= 268.3 + 107.1 + 1.2 = 376.6 \text{ ft-kips}\\
\phi_b M_n &= 0.90(376.6) = 340 \text{ ft-kips}
\end{aligned}$$

After correcting the dead load for the W16×31 section, the factored moment M_u becomes 321 ft-kips. Thus, $\phi_b M_n > M_u$ and the design is acceptable.

The designer should compare the economics of the W16×26 using connectors to develop a fully composite section with W16×31 using the minimum 20 connectors needed for this span length. To obtain a fully composite section the force to be carried by shear connectors would have been $T_{max} = A_s F_y = 384$ kips for the W16×26 section. The number of $\frac{3}{4}$-in.-diam studs needed would be

$$N = \frac{384}{21.0} = 18.3, \quad \text{say 20 for half the span}$$

Thus, the 40 connectors required for fully composite action can be reduced to 20 using partial composite action with the next heavier section.

(g) Check the strength of the W16×31 steel section to support construction loads (AISC-I3.1c). Refer to discussion in Example 16.14.1, part (e). Assume construction live load consists of 50% of the wet concrete (accomplished by using an average overload factor of 1.4), plus 20 psf for other construction loads.

$$\begin{aligned}
&\text{Slab} &&= 0.50(1.4) &&= 0.70 \quad \text{kip/ft}\\
&\text{Construction} &&= 0.02(8)1.6 &&= 0.26 \quad \text{kip/ft}\\
&\text{Steel section} &&= 0.031(1.2) &&= \underline{0.04} \quad \text{kip/ft}\\
& && &w_u &= 1.00 \quad \text{kip/ft}
\end{aligned}$$

$$M_u = \tfrac{1}{8}(1.00)(30)^2 = 113 \text{ ft-kips}$$

$$\phi_b M_p \text{ for W16×31} = 203 \text{ ft-kips} > 113 \text{ ft-kips} \qquad \textbf{OK}$$

<u>*Use* W16×31 section ($F_y = 50$ ksi), with 20—$\frac{3}{4}$-in.-diam connectors over the entire span, spaced at 40 in.</u> ■

16.12 ASD EXAMPLE—SIMPLY SUPPORTED BEAM

EXAMPLE 16.12.1

Redesign the composite beam of Example 16.11.1 (see Fig. 16.11.1) using the AISC ASD Method. The materials are $F_y = 50$ ksi, $f'_c = 4$ ksi ($n = 8$), and a 4-in. slab.

Solution:

(a) Service load bending moments. From Example 16.11.1,

$$M_D = 43 \text{ ft-kips}$$
$$M_L = 157 \text{ ft-kips}$$

(b) Select steel section. Use Eq. 16.11.2b, assuming the PNA is within the slab. Estimate $a = 1.0$ in for preliminary selection.

The required allowable strength $M_a = 43 + 157 = 200$ ft-kips

$$\text{Required } A_s = \frac{M_a}{\dfrac{F_y}{\Omega_b}\left(\dfrac{d}{2} + t_n - \dfrac{a}{2}\right)} \qquad [16.11.2b]$$

$$\text{Required } A_s = \frac{200(12)}{\frac{50}{1.67}(7 + 3.5)} = 7.63 \text{ sq in.} \quad (\text{for W14})$$

$$\text{Required } A_s = \frac{200(12)}{\frac{50}{1.67}(8 + 3.5)} = 6.97 \text{ sq in.} \quad (\text{for W16})$$

Using AISC-Table 3-19, "Composite W Shapes-Available Strength in Flexure" with $Y2 = t_n - a/2 = 3.5$ in. and required $M_n/\Omega = 200$ ft-kips, find

W16×26	$M_n/\Omega = 217$ ft-kips	$A_s = 7.68$ sq in.
W14×26	$M_n/\Omega = 201$ ft-kips	$A_s = 7.69$ sq in.

This is identical to the sections determined by the LRFD Method.

(c) Compute the plastic neutral axis and check allowable strength.

From Example 16.11.1,

$$M_n = 358 \text{ ft-kips}$$

$$M_n/\Omega = 358/1.67 = 214 \text{ ft-kips} > (M_a = 200 \text{ ft-kips}) \qquad \textbf{OK}$$

(d) Check the allowable strength of the steel section to support construction loads. Loads values are obtained from Example 16.11.1.

Slab	$= 0.40$ kips/ft
Construction	$= 0.16$ kips/ft
Steel Section	$= \underline{0.026 \text{ kips/ft}}$
w_a	$= 0.586$ kips/ft

$$M_a = \tfrac{1}{8}(0.586)(28)^2 = 57.4 \text{ ft-kips}$$

$$M_p/\Omega_b \text{ for W16×26} = 110 \text{ ft-kips} > 57.4 \text{ ft-kips} \qquad \textbf{OK}$$

(e) Designing the shear connectors is identical to the LRFD Method.

Use W16×26 of A992 steel along with 30–3/4-in.-diameter × 3-in. headed stud connectors over the entire span, spaced at 22 in. The connectors are to be placed in pairs starting at support. ■

16.13 DEFLECTIONS

The deflection of a composite beam will depend on whether it is shored or unshored during construction. Creep and shrinkage of the concrete in the slab also affect the result. Calculation of deflection requires obtaining the elastic cracked transformed section moment of inertia I_{tr} for the composite beam, and if unshored, also the elastic moment of inertia of the steel section alone.

When the steel beam is shored from below during the hardening of the concrete slab, the composite section will carry both the dead and live load. However, if the steel beam is *not* shored, the steel beam alone must carry the dead load.

When the construction is *without* shoring, the total deflection will be the sum of the dead load deflection of the steel beam alone and the live load deflection of the composite section.

When shoring provides the support during the hardening of the concrete slab, the composite section resists the entire load. Account should be taken to reflect the fact that concrete is subject to creep under long time load and that shrinkage will occur. This inelastic behavior may be approximated by multiplying the modulus of elasticity ratio n by a time-dependent factor such as two; thus reducing the effective width b_E/n. The result is a reduced moment of inertia I_{tr} to be used for computing the sustained load (dead load) deflection. The live load deflection would be computed using the elastic cracked transformed section moment of inertia.

Because the concrete slab in building construction is normally not too thick (say $t_s \leq 6$ in.) creep deflection is often not considered. The *AISC Specification* [1.15] gives no indication of any concern with creep of a concrete slab in composite construction. However, as discussed in Sec. 7.6, AISC-L3 states "Deflection in structural members and structural systems under appropriate service load combinations shall not impair the serviceability of the structure."

The ACI-ASCE Joint Committee [16.16] recommends using $E_c/2$ as the sustained concrete modulus of elasticity instead of E_c when computing sustained load creep deflection. AASHTO-6.10.1.1b [1.3] uses $E_c/3$ instead of E_c. Such arbitrary procedures can at best give an estimate of creep effects, probably no better than ±30%. The steel section, exhibiting no creep, and representing the principal carrying element, ensures that creep problems will usually be minimal.

More accurate procedures for computing deflections to account for creep and shrinkage on composite steel-concrete beams are available in a paper by Roll [16.31], and particularly in *Deformation of Concrete Structures* by Branson [16.32]. Lamport and Porter [16.45] have treated deflection prediction for concrete slabs reinforced with steel decking.

EXAMPLE 16.13.1

Compute the service dead and live load deflections for the composite beam consisting of W16×26 with 4-in. slab designed in Example 16.12.1 (see Fig. 16.13.1).

Solution:

Regardless of whether the selection of the steel section has been done by Load and Resistance Factor Design or by Allowable Strength Design, the deflections must be computed for *service* loads acting on the elastic section.

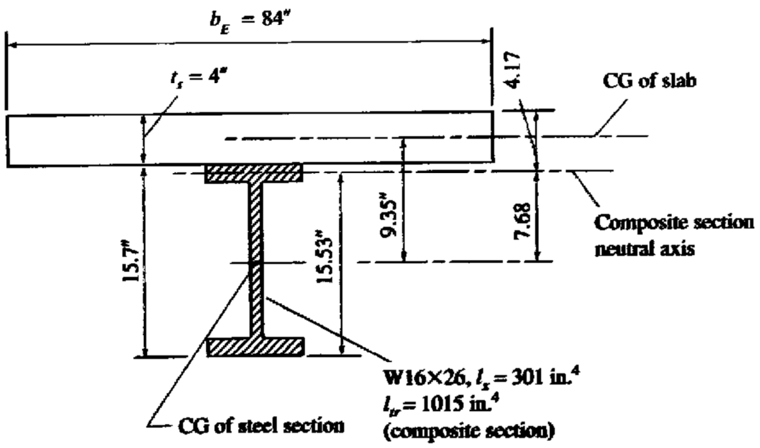


Figure 16.13.1
Beam cross-section for Example 16.13.1.

(a) Compute the dead load deflection. From Example 16.11.1, part (a), the service dead load is 0.44 kip/ft, and all must be carried by the steel beam alone when the beam is unshored.

$$\Delta_{DL} = \frac{5wL^4}{384E_sI_s} = \frac{5(0.44)(28)^4(12)^3}{384(29{,}000)448} = 0.47 \text{ in.,} \quad \text{say } \tfrac{1}{2} \text{ in.}$$

The beam can be cambered or the slab can be thickened toward midspan so that this amount of deflection is compensated for during construction.

(b) Compute the live load deflection. From Example 16.11.1, part (a), the service live load is 1.6 kips/ft. This load must be carried by the composite section; thus, the elastic composite moment of inertia is required. Compute composite elastic section properties. Referring to Fig. 16.13.1 determine effective width b_E (ASCE-I3.1).

$$b_E = \tfrac{1}{4} \text{ of span} = 0.25(28)12 = 84 \text{ in.} \quad \textit{controls}$$

or

$$b_E = \text{beam spacing} = 8(12) = 96 \text{ in.}$$

The width of equivalent steel is $b_E/n = 84/8 = 10.5$ in. The moment of inertia and elastic section modulus values are computed as follows:

Element	Transformed area A (sq in.)	Moment arm from centroid y (in.)	Ay (in.^{3})	Ay^2 (in.^{4})	I_0 (in.^{4})
Slab	42.0	9.85	413.7	4075	78.8
W16×36	7.68	0	0	0	301
	49.7		413.7	4075	379.8

$$I_x = I_0 + Ay^2 = 380 + 4075 = 4455 \text{ in}^4$$

$$\bar{y} = \frac{413.3}{49.7} = 8.32 \text{ in. above centroid of W16×26}$$

$$I_{tr} = I_x - A\bar{y}^2 = 4455 - 49.7(8.32)^2 = 1015 \text{ in.}^4$$

Often some of the dead load, such as partition loads and other items placed after the concrete slab has cured, acts on the composite section.

$$\Delta_{LL} = \frac{5wL^4}{384E_bI_b} = \frac{5(1.6)(28)^4(12)^3}{384(29{,}000)1015} = 0.75 \text{ in.}$$

As discussed in Sec. 7.6, it has been traditional to consider that live load deflection exceeding $L/360$ may cause cracking of plaster. On the other hand, the *ACI Code* [16.15] restricts the live load plus creep and shrinkage deflection to a maximum of $L/480$. Thus, in the absence of any specific AISC limitation, a limit of approximately $L/400$ will likely give satisfactory serviceability for the floor system. In this case,

$$\Delta_{\text{limit}} = \frac{L}{400} = \frac{28(12)}{400} = 0.84 \text{ in.} > \Delta_{LL} \qquad \textbf{OK}$$

One may conclude that deflection will not cause concern. Note that $L/400$ is *not* an AISC limit. It is the designer's responsibility to establish any limit.

In the solution above, the full thickness of the slab was assumed to be contributing to the elastic stiffness of the composite beam. However, there is some uncertainty about the thickness of the slab that is actually contributing to the stiffness of the composite beam. The *AISC Manual* adopts a more conservative approach by assuming that the thickness of the concrete slab that is contributing to the strength is also defining the stiffness of the beam. So, regardless of whether the beam is fully composite or partially composite, the elastic stiffness of the composite beam is determined using the depth of the rectangular stress block a. The moment of inertia computed using a for the concrete thickness is defined as the lower bound moment of inertia I_{Lb}, because the actual moment of inertia would be always larger. I_{Lb} can be determined from AISC-Table 3-24 as follows:

$$\sum Q_n = 26.1(15) = 392 \text{ kips}$$

$$Y2 = t_s - \frac{a}{2} = 4 - \frac{1.34}{2} = 3.33 \text{ in.}$$

Enter AISC-Table-3.20 with these values.

By interpolation: $I_{Lb} = 781 \text{ in.}^4$

$$\Delta_{LL} = \frac{5wL^4}{384E_sI_{Lb}} = \frac{5(1.6)(28)^4(12)^3}{384(29{,}000)781} = 0.98 \text{ in.} > [\Delta_{\text{limit}} = 0.84 \text{ in.}] \qquad \textbf{NG}$$

The authors believe the "exact" deflection probably falls near the $L/400$ limit. ■

16.14 CONTINUOUS BEAMS

Traditionally on continuous beams the positive moment region has been designed as a composite section and the negative moment region where the concrete slab is in tension as a noncomposite section. However, some composite action has been known to exist in the negative moment region. Continuous composite beams have been studied by Barnard and Johnson [16.33], Johnson, Van Dalen, and Kemp [16.34], Daniels and Fisher [16.35], Hamada and Longworth [16.36, 16.37], and Kubo and Galambos [16.38]. Kubo and Galambos extended the treatment to plate girders (that is, beams having h/t_w exceeding $970/\sqrt{F_y}$).

According to AISC-I3.2b the negative moment strength is determined for the steel section alone. AISC allows for calculating the negative moment strength using the composite section, which accounts for the reinforcement contribution. However to analyze the beam as a composite section, the following conditions must be met:

1. The steel beam is compact and adequately braced.
2. Shear connectors are provided over the support region.
3. The slab reinforcement is within the effective width b_E and properly developed.

When the steel reinforcing bars in the concrete slab are utilized to contribute to composite action, the force developed by such bars must be transferred by shear connectors. The nominal strength developed would be

$$T_n(\text{for } -M \text{ region}) = A_{sr}F_{yr} \qquad (16.16.1)$$

$$C_n(\text{for } +M \text{ region}) = A'_sF_{yr} \qquad (16.16.2)$$

where A_{sr} = total area of longitudinal reinforcing steel at the interior support located within the effective flange width b_E

A'_s = total area of longitudinal compression steel acting with the concrete slab at the location of maximum positive moment and lying within the effective width b_E

F_{yr} = specified minimum yield stress of the longitudinal reinforcing steel

Thus, the nominal strength V_{nh} for which shear connectors must be provided in the *negative moment zone* is

$$V_{nh} = A_{sr}F_{yr} \tag{16.16.3}$$

In the *positive moment zone*, when compression steel is included in the computation of composite section properties (plastic neutral axis), the nominal strength V_{nh2} from the compression steel is

$$V_{nh2} = A'_sF_{yr} \tag{16.16.4}$$

The total horizontal shear force between the point of zero moment and the point of maximum moment is the smallest of $(0.85f'_cA_c + V_{nh2})$, A_sF_y, and ΣQ_n. AISC has no specific mention of the compression reinforcement in the positive moment zone; thus, inclusion of V_{nh2} is optional.

As discussed in Sec. 16.7, the usual limit state for composite sections in the positive moment zones is crushing of the concrete at the top of the slab. This assumes no shear connector failure, no longitudinal splitting because of inadequate reinforcing bar development, and no shear failure in the slab. In the negative moment region, the usual limit state is flange local buckling [16.37].

Regarding the lateral-torsional buckling limit state, the usual provisions for noncomposite steel sections apply to the negative moment regions of continuous composite beams. The limits on λ from AISC-B4 for the flange and web local buckling limit states must be applied in the negative moment zone.

EXAMPLE 16.14.1

Compute the plastic neutral axis (PNA) location and the nominal strength M_n for the section of Fig. 16.14.1 subject to negative bending moment. The W12×26 steel section is of A992 steel and the reinforcement in the slab has F_{yr} = 60 ksi.

Solution:

(a) Determine the plastic neutral axis location. The concrete slab will be in tension; therefore, none of the concrete is assumed to be effective. The reinforcing bars contribute

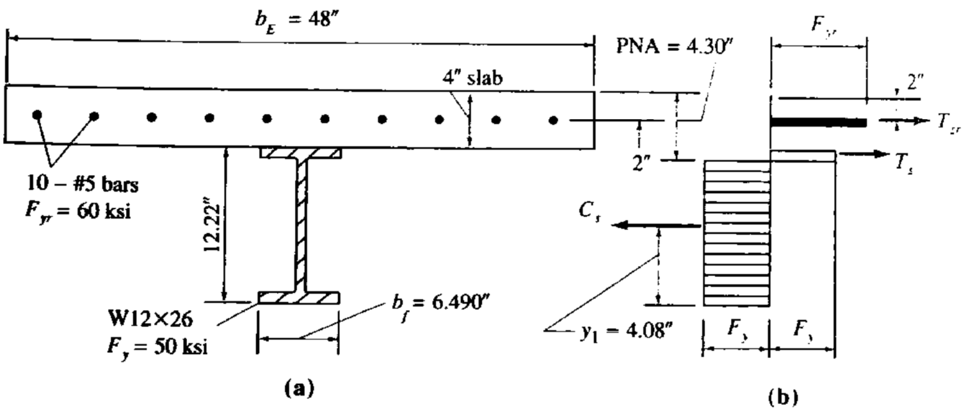


Figure 16.14.1 Composite section for negative bending of Example 16.14.1, including plastic stress distribution according to AISC-13.2b.

the nominal tension strength T_{sr},

$$T_{sr} = A_{sr}F_{yr} = 10(0.31)60 = 186 \text{ kips}$$

The maximum nominal compression force in the W12 section is

$$C_{\max} = A_s F_y = 7.65(50) = 382.5 \text{ kips}$$

Since $C_{\max}$ exceeds T_{sr}, the PNA is within the steel W12 section. In which case the force equilibrium requirement may be expressed,

$$\begin{aligned} T_{sr} + T_s &= C_{\max} - T_s \\ 2T_s &= C_{\max} - T_{sr} = 382.5 - 186 = 196.5 \\ T_s &= 98.3 \text{ kips} \end{aligned}$$

Assuming that the PNA is within the flange of the W12,

$$\text{From top of flange to PNA} = \frac{T_s}{F_y b_f} = \frac{98.3}{50(6.49)} = 0.30 \text{ in.}$$

The assumption that PNA is within the flange is confirmed since $0.30 < (t_f = 0.38 \text{ in.})$. Thus, the distance PNA from top of slab is

$$\text{PNA} = t_s + 0.30 = 4.0 + 0.30 = 4.30 \text{ in.}$$

(b) Compute the nominal moment strength M_n. Locate the center of gravity y_1 of the compression force C_s in the steel section, measured from the bottom of the steel section,

	Area, A	Arm, y	Ay
W12 section	7.65	6.11	46.74
Flange	−1.95	12.07	− 23.5
	5.7		23.24

$$y_1 = \frac{23.24}{5.70} = 4.08 \text{ in.}$$

Referring to Fig. 16.14.1, taking internal moments about the point of action of C_s gives

$$\begin{aligned} T_{sr}: \quad M_{n1} &= T_{sr}(d - 4.08 + t_s - 2.00) \\ &= 186(12.22 - 4.08 + 4 - 2.00)\tfrac{1}{12} \\ &= 186(10.14)\tfrac{1}{12} = 157.2 \text{ ft-kips} \\ T_s: \quad M_{n2} &= T_s(d - 4.08 - 0.30/2) \\ &= 98.3(12.22 - 4.08 - 0.30/2)\tfrac{1}{12} \\ &= 98.3(7.99)\tfrac{1}{12} = 65.5 \text{ ft-kips} \\ M_n &= M_{n1} + M_{n2} \\ &= 157.2 + 65.5 = 223 \text{ ft-kips} \\ \phi_b M_n &= 0.90(223) = 201 \text{ ft-kips} \end{aligned}$$

Note that for composite action in the negative moment region, shear connectors must be used throughout the entire region. The required ΣQ_n equals the force T_{sr} in the reinforcement.

When partial composite action is used, ΣQ_n will be less than T_{sr}. In such a case, the PNA location and the nominal moment strength are computed using ΣQ_n instead of T_{sr}. ■

16.15 COMPOSITE COLUMNS

A composite column can be defined as "a steel column fabricated from rolled or built-up steel shapes and encased in structural concrete or fabricated from steel pipe or tubing and filled with structural concrete." An example of the former is shown in Fig. 16.15.1, where a steel W section is encased in concrete; the concrete must contain longitudinal reinforcing bars and these must be surrounded by lateral ties in the manner of a reinforced concrete column.

The steel section must comprise at least 1% of the total cross-sectional area, otherwise the column must be designed as an ordinary reinforced concrete column. Research by Furlong [16.39, 16.40] and others was reviewed by Task Group 20 of the Structural Stability Research Council, chaired by Furlong [16.41]. This SSRC Task Group Report forms the basis for design of composite columns under AISC-12.

Limitations

In order to qualify as a composite column, the limitations of AISC-I1.2 and I2 must be satisfied:

1.
$$A_s \geq 0.01A_g \tag{16.15.1}$$

2. For a concrete encasement:

(a) Longitudinal reinforcing bars must be used; load carrying bars must be continuous at framed levels (wherever a beam or slab frames to the column); other longitudinal bars used only to restrain concrete may be interrupted at framed levels.

(b) Lateral ties must be used; spacing of ties may not exceed the smallest of 16 longitudinal bar diameter, 48 tie bar diameter, or 0.5 the least dimension of the composite section.

(c) Area of lateral ties must be at least 0.009 sq in./in. of bar spacing.

(d) The minimum required area of steel for continuous longitudinal reinforcement shall be $0.004A_g$.

(e) Clear cover of at least 15 in. is required.

3. Concrete strength f'_c:

(a) Normal-weight concrete: 3 ksi $\leq f'_c \leq$ 10 ksi

(b) Structural lightweight concrete: 3 ksi $\leq f'_c \leq$ 6 ksi

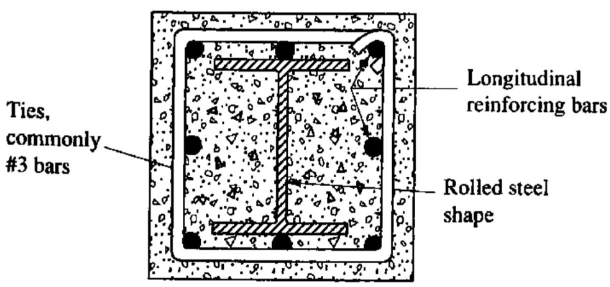


Figure 16.15.1
Composite column section; rolled steel shape encased in concrete.

4. Maximum yield stress of steel used in strength computations is 75 ksi for either structural steel or reinforcing bars.
5. Minimum wall thickness t for concrete-filled pipe or tubing:
   (a) For each face width b in HSS rectangular sections:

$$\frac{b}{t} \leq 2.26\sqrt{\frac{E}{F_y}} \tag{16.15.2}$$

   (b) For outside diameter D in circular sections:

$$\frac{D}{t} \leq 0.15\sqrt{\frac{E}{F_y}} \tag{16.15.3}$$

Nominal Strength

To account for slenderness effects, the AISC equations for composite columns are based on a modified form of the equations for steel columns in AISC-E. The yield strength becomes a modified strength P_e, and the elastic stiffness of the column is defined by an effective elastic stiffness EI_{eff} defined in what follows.

The resistance and safety factors adopted for composite columns are rather conservative in order to account for the uncertainty of composite columns and the use of ultimate strength of two different materials in defining the capacity. The factors are as follows:

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

The nominal compressive strength shall be determined according to AISC-I2.lb as follows

1. When $P_e \geq 0.44P_o$

$$P_n = P_o\left[0.658^{\left(\frac{P_o}{P_e}\right)}\right] \tag{16.15.4}$$

2. When $P_e < 0.44P_o$

$$P_n = 0.877P_e \tag{16.15.5}$$

where $P_e = \pi^2(EI_{eff})/(KL)^2$ (16.15.6)

For filled composite columns, AISC-12.2b defines:

$$P_e = A_sF_y + A_{sr}F_{yt} + C_2A_cf'. \tag{16.15.7}$$

where $C_2 = 0.85$ for irregular sections and 0.95 for circular sections

$$EI_{eff} = E_sI_s + E_sI_{sr} + C_3E_cI_c \tag{16.15.8}$$

$$C_3 = 0.6 + 2\left(\frac{A_s}{A_c + A_r}\right) \leq 0.9 \tag{16.15.9}$$

For encased composite columns. AISC-I2.lb defines:

$$P_e = A_sF_y + A_{sr}F_{yt} + 0.85A_cf'_c \tag{16.15.10}$$

$$EI_{eff} = E_sI_s + 0.5E_sI_{sr} + C_1E_cI_c \tag{16.15.11}$$

$$C_1 = 0.1 + 2\left(\frac{A_s}{A_c + A_s}\right) \leq 0.3 \tag{16.15.12}$$

where A_c = area of concrete
A_r = area of longitudinal reinforcing bars
A_s = gross area of steel shape, pipe, or tube
E_c = modulus of elasticity of concrete, ksi
= $(w^{1.5})\sqrt{f'_c}$, where w is the density of concrete in pcf (i.e., 145 pcf for normal-weight concrete) and f'_c is in ksi
E_s = modulus of elasticity of steel
F_y = specified minimum yield stress of steel shape, pipe, or tube
F_{yr} = specified minimum yield stress of longitudinal reinforcing bars
f'_c = specified 28-day compressive strength of concrete
I_c = moment of inertia of concrete section
I_s = moment of inertia of steel shape
I_{sr} = moment of inertia of reinforcing bars

The *AISC Manual* contains tables for concrete filled HSS sections giving axial strengths ϕP_u and P_n/Ω. Note that $\phi_n = 0.75$ for composite columns.

Composite beam-column design has been treated by Uang, Wattar, and Leet [16.46]. AISC-I4 defines the method of treating composite beam-columns. An interaction curve similar to reinforced concrete needs to be developed, while accounting for the stability requirement of the column. The nominal strength of the section is to be determined using plastic stress distribution or strain compatibility. AISC-I2 is to be used to determine the nominal axial strength of the cross-section, using P_u as determined in AISC-I4.

SELECTED REFERENCES

16.1. Ivan M. Viest. Chairman, "Composite Steel-Concrete Construction," Report of the Subcommittee on the State-of-the-Art Survey of the Task Committee on Composite Construction of the Committee on Metals of the Structural Division, *Journal of the Structural Division*, ASCE, **100**, ST5 (May 1974), 1085–1139.

16.2. Gottfried Brendel. "Strength of the Compression Slab of T-Beams Subject to Simple Bending," *ACI Journal, Proceedings*, **61**, January 1964, 57–76.

16.3. Conrad P. Heins and Horn Ming Fan. "Effective Composite Beam Width at Ultimate Load," *Journal of the Structural Division*, ASCE, **102**, ST11 (November 1978), 2163–2179.

16.4. Cesar R. Vallenilla and Reidar Bjorhovde. "Effective Width Criteria for Composite Beams," *Engineering Journal*, AISC, **22**, 4 (Fourth Quarter 1985), 169–175.

16.5. Ivan M. Viest. "Review of Research on Composite Steel-Concrete Construction," *Journal of the Structural Division*, ASCE, **86**, ST6 (June 1960), 1–21; Also *Transactions*, ASCE, **126** (1961), Part II, 1101–1123.

16.6. William C. Hansell, Theodore V. Galambos, Mayasandra K. Ravindra, and Ivan M. Viest. "Composite Beam Criteria in LRFD," *Journal of the Structural Division*, ASCE, **104**, ST9 (September 1978), 1409–1426; Disc. **106**, ST2 (February 1980), 571–572.

16.7. Srinivasa H. Iyengar and Mohammad Iqbal. "Composite Construction," *Building Structural Design Handbook*, Chapter 23. New York: John Wiley & Sons, 1987.

16.8. Robert F. Lorenz and Frank W. Stockwell, Jr. "Concrete Slab Stresses in Partial Composite Beams and Girders," *Engineering Journal*, AISC, **21**, 3 (Third Quarter 1984), 185–188.

16.9. Robert F. Lorenz. "Understanding Composite Beam Design Methods Using LRFD," *Engineering Journal*, AISC, **25**, 1 (Second Quarter 1988), 35–38.

16.10. Robert F. Lorenz. "Some Economic Considerations for Composite Floor Beams," *Engineering Journal*, AISC, **20**, 2 (Second Quarter 1983), 78–81.

16.11. Mark C. Zahn. "The Economies of LRFD in Composite Floor Beams," *Engineering Journal*, AISC, **24**, 2 (Second Quarter 1987), 87–92.

16.12. S. Timoshenko and J. Goodier. *Theory of Elasticity*. New York: McGraw-Hill Book Company, Inc., 1959, Chap. 6.

16.13. Theodore von Kármán. "Die Mittragende Breitte," *August-Föppel-Festschrift*, 1924. (See also Collected Works of Theodore von Kármán, Volume II, p. 176.)

16.14. John E. Johnson and Albert D. M. Lewis. "Structural Behavior in a Gypsum Roof-Deck System," *Journal of the Structural Division*, ASCE, **92**, ST2 (April 1966), 283–296.

16.15. ACI Committee 318 (ACI 318-08). *Building Code Requirements for Reinforced Concrete*. Farmington Hills, MI: American Concrete Institute, 2008.

16.16. Joint ASCE-ACI Committee on Composite Construction. "Tentative Recommendations for the Design and Construction of Composite Beams and Girders for Buildings," "*Journal of the Structural Division*," ASCE, **86**, ST12 (December 1960), 73–92.

16.17. Roger G. Slutter and George C. Driscoll. "Flexural Strength of Steel-Concrete Composite Beams," *Journal of the Structural Division*, ASCE, **91**, ST2 (April 1965), 71–99.

16.18. Peter Ansourian and Jack William Roderick. "Analysis of Composite Beams," *Journal of the Structural Division*, ASCE, **104**, ST10 (October 1978), 1631–1645.

16.19. Jorgen G. Ollgaard, Roger G. Slutter, and John W. Fisher. "Shear Strength of Stud Connectors in Lightweight and Normal-Weight Concrete," *Engineering Journal*, AISC, **8**, 2 (April 1971), 55–64.

16.20. Jay B. McGarraugh and J. W. Baldwin, Jr. "Lightweight Concrete-on-Steel Composite Beams," *Engineering Journal*, AISC, **8**, 3 (July 1971), 90–98.

16.21. Roger G. Slutter and John W. Fisher. "Fatigue Strength of Shear Connectors," *Highway Research Record No. 147*, Highway Research Board, 1966, pp. 65–88.

16.22. Charles G. Schilling. "Bending Behavior of Composite Hybrid Beams," *Journal of the Structural Division*, ASCE, **94**, ST8 (August 1968), 1945–1964.

16.23. John A. Grant, Jr., John W. Fisher, and Roger G. Slutter. "Composite Beams with Formed Steel Deck," *Engineering Journal*, AISC, **14**, 2 (First Quarter 1977), 24–43.

16.24. Sriramulu Vinnakota, Christopher M. Foley, and Murthy R. Vinnakota. "Design of Partially or Fully Composite Beams, with Ribbed Metal Deck, Using LRFD Specifications," *Engineering Journal*, AISC, **25**, 2 (Second Quarter 1988), 60–78.

16.25. R. H. R. Tide and T. V. Galambos. "Composite Open-Web Steel Joists," *Engineering Journal*, AISC, **7**, 1 (January 1970), 27–36.

16.26. James Rongoe. "A Composite Girder System for Joist Supported Slabs," *Engineering Journal*, AISC, **21**, 3 (Second Quarter 1984), 155–160.

16.27. ASCE. *Specifications for the Design and Construction of Composite Slabs*. New York: Technical Council on Codes and Standards, American Society of Civil Engineers, 1984.

16.28. ASCE. *Commentary on Specifications for the Design and Construction of Composite Slabs*. New York: Technical Council on Codes and Standards, American Society of Civil Engineers, 1984.

16.29. Max L. Porter. "Analysis of Two-Way Acting Composite Slabs," *Journal of Structural Engineering*, ASCE, **111**, 1 (January 1985), 1–17.

16.30. C. Dale Buckner, Danny J. Deville, and Dean C. McKee. "Shear Strength of Slabs in Stub Girders," *Journal of the Structural Division*, ASCE, **107**, ST2 (February 1981), 273–280.

16.31. Frederic Roll. "Effects of Differential Shrinkage and Creep on a Composite Steel-Concrete Structure," *Designing for Effects of Creep, Shrinkage, Temperature in Concrete Structures* (SP-27). Detroit, MI: American Concrete Institute, 1971 (pp. 187–214).

16.32. Dan E. Branson. *Deformation of Concrete Structures*. New York: McGraw-Hill Book Company, Inc., 1977.

16.33. P. R. Barnard and R. P. Johnson. "Plastic Behavior of Continuous Composite Beams," *Proceedings*, Institution of Civil Engineers, October 1965.

16.34. R. P. Johnson, K. Van Dalen, and A. R. Kemp, "Ultimate Strength of Continuous Composite Beams," *Proceedings of the Conference on Structural Steelwork*, British Constructional Steelwork Association, November 1967.

16.35. J. H. Daniels and J. W. Fisher. "Static Behavior of Continuous Composite Beams," *Fritz Engineering Laboratory Report* No. 324.2, Lehigh University, Bethlehem, PA, March 1967.

16.36. Sumio Hamada and Jack Longworth. "Buckling of Composite Beams in Negative Bending," *Journal of the Structural Division*, ASCE, **100**, ST11 (November 1974), 2205–2222.

16.37. Sumio Hamada and Jack Longworth. "Ultimate Strength of Continuous Composite Beams," *Journal of the Structural Division*, ASCE, **102**, ST7 (July 1976), 1463–1478.

16.38. Masahiro Kubo and Theodore V. Galambos. "Plastic Collapse Load of Continuous Composite Plate Girders," *Engineering Journal*, AISC, **25**, 4 (Fourth Quarter 1988), 145–155.

16.39. Richard W. Furlong. "Strength of Steel Encased Concrete Beam Columns," *Journal of the Structural Division*, ASCE, **94**, ST1 (January 1968), 267–281.

16.40. Richard W. Furlong. "AISC Column Design Logic Makes Sense for Composite Columns, Too," *Engineering Journal*, AISC, **13**, 1 (First Quarter 1976), 1–7.

16.41. Task Group 20, Structural Stability Research Council. "A Specification for the Design of Steel-Concrete Composite Columns," *Engineering Journal*, AISC, **16**, 4 (Fourth Quarter 1979), 101–115.

16.42. IABSE. *Composite Steel-Concrete Construction and Eurocode 4*, Short Course Notes, International Association for Bridge and Structural Engineering, Brussels, 1990, 191 pp.

16.43. W. Samuel Easterling, David R. Gibbings and Thomas M. Murray. "Strength of Shear Studs in Steel Deck on Composite Beams and Joists," *Engineering Journal*, AISC, **30**, 2 (2nd Quarter 1993), 44–55.

16.44. Mark Andrew Bradford and R. Ian Gilbert. "Composite Beams with Partial Interaction Under Sustained Loads," *Journal of Structural Engineering*, ASCE, **118**, 7 (July 1992), 1871–1883.

16.45. William B. Lamport and Max L. Porter. "Deflection Predictions for Concrete Slabs Reinforced with Steel Decking," *ACI Structural Journal*, **87**, September-October 1990, 546–570.

16.46. Chia-Ming Uang, Samer W. Wattar, and Kenneth M. Leet. "Proposed Revision of the Equivalent Axial Load Method for LRFD Steel and Composite Beam-Column Design," *Engineering Journal*, AISC, **27**, 4 (4th Quarter 1990), 150–157.

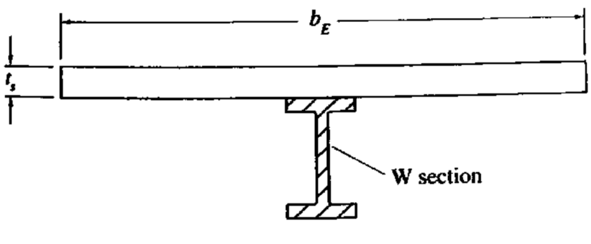
16.47. M. D. Rambo-Roddenberry, W. S. Easterling, and T. M. Murray. "Behavior and Strength of Welded Stud Shear Connectors-Data Report," Report No. CE/VPI-02/04, Virginia Polytechnic Institute and State University, Blackspurg, VA, 2002.

PROBLEMS

All problems are to be done according to the AISC LRFD Method or the ASD Method, as indicated by the instructor. All given loads are service loads unless otherwise indicated.

16.1. For the case (or cases) assigned by the instructor, compute the location of the transformed composite section neutral axis and moment of inertia I_{tr}. Refer to the accompanying figure.

Case	Steel section	F_y (ksi)	Slab t_s (in.)	b_E (in.)	f'_c (ksi)	
2	W14×30	60	4	72	3	$(n = 9)$
4	W18×60	50	4	84	4	$(n = 8)$
5	W24×55	50	4.5	90	4	$(n = 8)$
6	W18×50	50	5	72	4	$(n = 8)$
7	W18×50	50	4	72	3	$(n = 9)$
8	W24×76	50	4.5	72	4	$(n = 8)$
9	W24×94	50	4.5	72	4	$(n = 8)$
10	W21×62	50	5	96	4	$(n = 8)$
11	W21×147	50	4.5	96	4	$(n = 8)$



Problems 16.1, 16.2, and 16.3

16.2. For the case (or cases) listed for Prob. 16.1 and assigned by the instructor, compute the location of plastic neutral axis (PNA) measured from the top of the slab, as well as the nominal strength M_n. Assume the sections are *fully composite*. Refer to the accompanying figure.

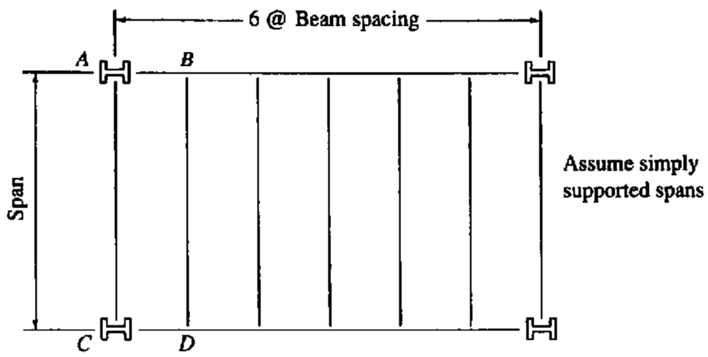
16.3. For the case (or cases) listed for Prob. 16.1 and assigned by the instructor, select an economical size of headed stud shear connector from Table 16.8.1, determine the total number needed to develop a *fully composite* section for the beam, and specify the spacing. Assume the simply supported beam span equals $4b_E$.

16.4. For the case (or cases) assigned by the instructor, select a W section to design a *fully composite* section for span *BD* of the accompanying figure. Assume for simplicity that the slab is the only dead load to be considered. Also, select an economical size of headed stud shear connector from Table 16.8.1, determine the total number needed for the beam, and specify the spacing, to develop a fully composite beam. No shoring is to be used; therefore, assume that during construction wet concrete of 75 psf is live load and that an additional construction live load of 25 psf may act. The final composite beam may not have live load deflection exceeding $L/360$.

Case	Live Load (psf)	F_y (ksi)	Slab t_s (in.)	Span (ft)	Beam Spacing (ft)	f'_c (ksi)	
2	100	60	4	36	8	3	$(n = 9)$
4	80	50	4	36	7	4	$(n = 8)$
5	80	50	4	40	7	4	$(n = 8)$
6	80	50	4	40	7	4	$(n = 8)$
8	125	50	4.5	40	8	3	$(n = 9)$
9	125	50	4.5	42	8	4	$(n = 8)$
10	125	50	4.5	45	8	4	$(n = 8)$
11	125	50	5	45	9	4	$(n = 8)$
12	125	50	5.5	48	9	4	$(n = 8)$

16.5. For the case (or cases) given in Prob. 16.4 assigned by the instructor, select a W section to design a *partially composite* section for span *BD* of the accompanying figure, using the minimum number of $\frac{3}{4}$-in.-diam headed stud shear connectors. Specify number of studs and the spacing. In addition to the slab dead load, using a ceiling load of 7 psf and partition load of 25 psf. No shoring is to be used; therefore, assume that during construction wet concrete of 75 psf is live load and that an additional construction live load of 25 psf may act. The dead load deflection before composite action is effective may not exceed $\frac{7}{8}$ in. and composite beam deflection resulting from superimposed dead load (i.e., ceiling and partitions) and live load may not exceed $L/360$.

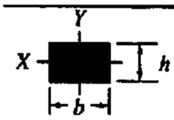
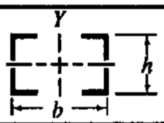
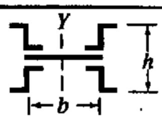
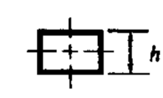
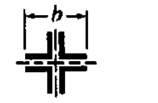
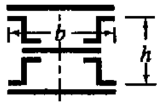
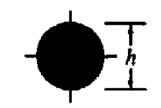
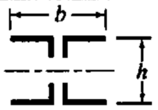
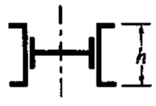
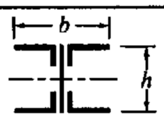
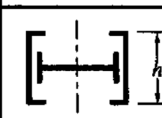
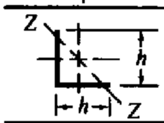
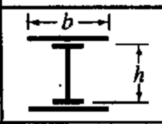
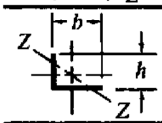
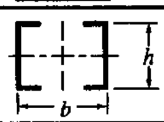
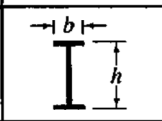
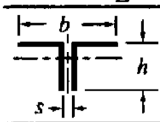
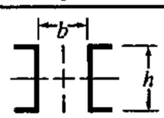
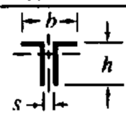
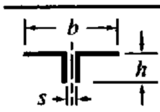
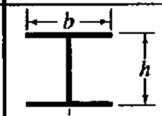
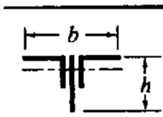
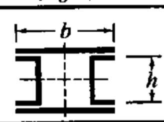
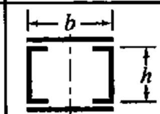
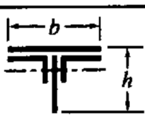
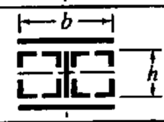
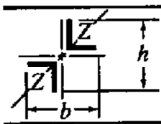
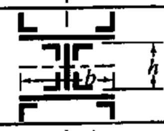
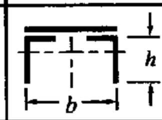
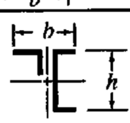
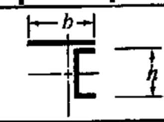
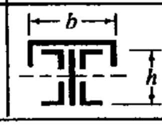
16.6. Design a composite encased W shape column to resist a factored axial compression load P_u of 900 kips. The effective length $KL = 12$ ft, $F_y = 50$ ksi for structural steel, and $f'_c = 4.5$ ksi for concrete.



Problems 16.4 and 16.5
Framing plan.

Appendix

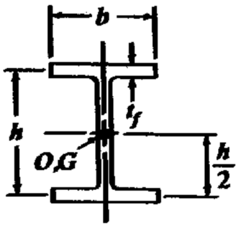
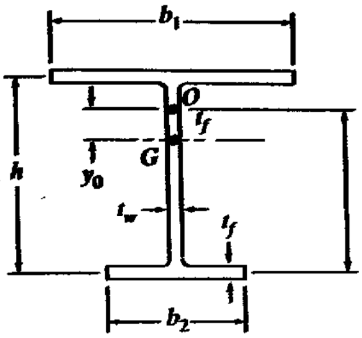
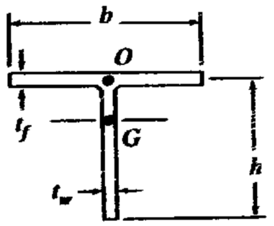
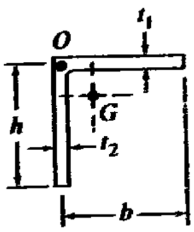
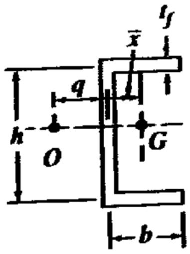
TABLE A1 Approximate Radius of Gyration

 $r_x = 0.29h$ $r_y = 0.29b$	 $r_x = 0.42h$ $r_y = 0.42b$	 $r_x = 0.31h$ $r_y = 0.48b$
 $r_x = 0.40h$ h = mean h	 r_y = same as for 2 L	 $r_x = 0.37h$ $r_y = 0.28b$
 $r_x = 0.25h$	 $r_x = 0.42h$ r_y = same as for 2 L	 $r_x = 0.31h$
 $r = \sqrt{\frac{H^2 + h^2}{16}}$ $r = 0.35H_m$	 $r_x = 0.39h$ $r_y = 0.21b$	 $r_x = 0.31h$
 $r_x = 0.31h$ $r_y = 0.31h$ $r_z = 0.197h$	 $r_x = 0.45h$ $r_y = 0.235b$	 $r_x = 0.40h$ $r_y = 0.21b$
 $r_x = 0.29h$ $r_y = 0.32b$ $r_z = 0.18\,\frac{h + b}{2}$	 $r_x = 0.36h$ $r_y = 0.45b$	 $r_x = 0.38h$ $r_y = 0.22b$
 $r_x = 0.31h$ $r_y = 0.215b$ $= b(0.21 + 0.02s)$	 $r_x = 0.36h$ $r_y = 0.60b$	 $r_x = 0.39h$
 $r_x = 0.32h$ $r_y = 0.21b$ $= b(0.19 + 0.02s)$	 $r_x = 0.36h$ $r_y = 0.53b$	 $r_x = 0.35h$
 $r_x = 0.29h$ $r_y = 0.24b$ $= b(0.23 + 0.02s)$	 $r_x = 0.39h$ $r_y = 0.55b$	 $r_x = 0.435h$ $r_y = 0.25b$
 $r_x = 0.30h$ $r_y = 0.17b$	 $r_x = 0.42h$ $r_y = 0.32b$	 $r_x = 0.42h$
 $r_x = 0.25h$ $r_y = 0.21b$	 $r_x = 0.44h$ $r_y = 0.28b$	 $r_x = 0.42h$
 $r_x = 0.21h$ $r_y = 0.21b$ $r_z = 0.19h$	 $r_x = 0.50h$ $r_y = 0.28b$	 $r_x = 0.285h$ $r_y = 0.37b$
 $r_x = 0.38h$ $r_y = 0.19b$	 $r_x = 0.39h$ $r_y = 0.21b$	 $r_x = 0.42h$ $r_y = 0.23b$

* J.A.L. Waddel. *Bridge Engineering*, Vol. 1. New York: John Wiley & Sons, Inc., 1916, p. 504. Used by permission.

TABLE A2 Torsional Properties

O = shear center J = torsion constant, C_w = warping constant
G = centroid I_p = polar moment of inertia about shear center

	$J = \frac{1}{3}(2bt_f^3 + ht_w^3)$ $C_w = \frac{I_f h^2}{2} = \frac{t_f b^3 h^2}{24} = \frac{h^2 I_y}{4}$ $I_p = I_x + I_y$
 $e = h\frac{b_1^3}{b_1^3 + b_2^3}$	$J = \frac{1}{3}(b_1 t_f^3 + b_2 t_f^3 + ht_w^3)$ $C_w = \frac{t_f h^2}{12}\left(\frac{b_1^3 b_2^3}{b_1^3 + b_2^3}\right)$ $I_p = I_y + I_x + Ay_0^2$
	$J = \frac{1}{3}(bt_f^3 + ht_w^3)$ $C_w = \frac{1}{36}\left(\frac{b^3 t_f^3}{4} + h^3 t_w^3\right)$ ≈ zero for small t
	$J = \frac{1}{3}(bt_1^3 + ht_2^3)$ $C_w = \frac{1}{36}(b^3 t_1^3 + h^3 t_2^3)$ ≈ zero for small t
	$J = \frac{1}{3}(2bt_f^3 + ht_w^3)$ $C_w = \frac{t_f b^3 h^2}{12}\left(\frac{3bt_f + 2ht_w}{6bt_f + ht_w}\right) = \frac{h^2}{4}(I_y + A\bar{x}^2 - q\bar{x}A)$ $q = \frac{th^2 b^2}{4I_x}$

Index

C

D

Q

R

S

U

V

N_i = notional lateral load, Sec. 12.11

p = pitch (spacing) of bolts; connector spacing (Chap. 16); $1/\phi_c P_n$, Eq.12.12.2

P_{bf} = design strength of column web to resist a concentrated factored load

P_{br} = required brace strength

P_{cr} = critical buckling load; compression force at buckling

P_e, P_{ex}, P_{ey} = Euler load $= \pi^2 E A_g/(KL/r)^2$ for axis of bending (using two subscripts for biaxial bending)

P_{e1}, P_{e2} = Euler load $= \pi^2 E A_g/(KL/r)^2$ for axis of bending, for use with magnification factors B_1 and B_2 according to the second subscript

P_n = nominal strength of an axially loaded compression member, $F_{cr}A_g$; nominal strength of weld configuration (Fig. 5.17.2)

P_u = factored axial load (Sec. 1.9); factored reaction or load

P_y = yield load, $F_y A_g$ (Chap. 12)

Q = form factor, $Q_a Q_s$ (Sec. 6.18); first moment of area (i.e., statical moment $\int y\, dA$) about the neutral axis from extreme fiber to section at which elastic shear stress is computed, (see Sec. 7.7)

Q_a = shape factor for stiffened compression element (Sec. 6.18)

Q_f = moment of area of one-half flange about y-axis (Sec. 8.5)

Q_s = shape factor for unstiffened compression element (Sec. 6.18)

r = radius of gyration, $\sqrt{I/A_g}$; radial distance from centroid to point of stress (Sec. 5.18)

r_i = distance from instantaneous center to a weld element (Fig. 5.17.2)

r_0 = distance from instantaneous center to vertical weld line

r_{max} = distance to weld element farthest from instantaneous center

r_t = radius of gyration of a section comprising the compression flange plus one-third of the compression web area, taken about an axis in the plane of the web; used in ASD, Eq. 9.7.14

r_{ts} = effective radius of gyration used in determining L_r (Chap. 9)

r_x, r_y, r_z = radius of gyration about x-, y-, or z-axes, respectively

R_a = required strength (ASD)

R_e = moment strength reduction factor for hybrid girder (Secs. 11.7 and 16.9)

R_g = coefficient to account for group effect (Chap. 16)

R_i = resistance of a bolt at any deformation D (Chap. 4); strength of a fillet weld segment per unit length (Sec. 5.17)

$R_{i,ult}$ = ultimate shear load on an element, Eq. 5.17.3

R_m = cross-section monosymmetry parameter (Chap. 9)

R_n = nominal strength of one fastener in tension, shear, or bearing; nominal reaction strength (Sec. 7.8)

R_{nt} = nominal strength of bolt in tension

R_{nv} = nominal strength of bolt in shear

R_{nw} = nominal strength of weld per inch of length

R_p = position effect factor for shear studs (Chap. 16)

R_{pg} = reduction factor for "bend-buckling" of the web, Eq. 11.4.3

R_s = direct shear component of bolt resistance

R_u = factored load per bolt; factored load per unit length of weld; factored reaction (Sec. 7.8)

R_{ult} = ultimate shear resistance in a bolt, $\tau_u A_b$

R_{us} = factored direct shear on bolt subject to eccentric load

R_{ut} = factored tension load on bolt

R_{uv} = factored direct shear component on bolt

R_{ux}, R_{uy} = factored shear on bolt, in x- or y-direction, respectively

R_v = direct shear component of bolt resistance; shear component of eccentric force on fillet welds; direct shear component of weld resistance/per unit length

R_x, R_y = x-or y-direction component of bolt resistance; x- or y-component of torsional moment force on fillet welds

s = stagger of bolt holes measured in the line of force (Chap. 3); distance from free edge along a thin wall section (Chap 8); band width for tension-field force T (Sec. 11.9)

S, S_x, S_y = elastic section modulus, $I/\bar{y}$ (Table 5.18.1), with respect to x- or y-axes $(I_x/c_y$ or $I_y/c_x)$, according to subscript

S_s	= elastic section modulus of steel section alone, referred to its tension flange
S_{tr}	= elastic section modulus of composite section, I_{tr}/y_b
S_{xc}, S_{xt}	= section modulus S_x referred to the compression flange, S_{xc}, or the compression flange, S_{xt}
t	= thickness; thickness of material against which bolt bears
t_e	= effective throat dimension of a weld (Sec. 5.12)
t_f, t_{fb}, t_{fc}	= flange thickness; for beam, t_{fb}; for column, t_{fc}
t_s	= thickness of stiffener; slab thickness
t_w, t_{wb}, t_{wc}	= web thickness; for beam, t_{wb}; for column, t_{wc}
T	= tensile force; service load tensile force; torsional moment or torsional service load moment (Chap. 8); base metal thickness (Table 5.11.1)
T_0	= required tension strength (ASD)
T_b	= initial force in bolt resulting from installation
	= nominal strength of a tension member
T_u	= factored tension load; factored torsional moment (required tension strength, required torsion strength) (Chap. 8)
u	= displacement in the x-direction
u_f	= lateral deflection of flange
U	= reduction factor to account for shear lag (Sec. 3.9)
U_{bs}	= stress reduction factor for nonuniform stress for block shear rupture
v	= shear stress; displacement in the y-direction
v_s	= St. Venant torsion shear stress (Chap. 8)
v_w	= warping torsion shear stress (Chap. 8)
V	= shear; service load shear force on a bolt
V_f	= warping torsion shear force in flange
V_n	= nominal shear strength
V'_n	= nominal shear strength in the presence of bending moment
V_{nh}	= nominal horizontal shear strength across interface between slab and steel section in a composite beam
V_r	= range of service load shear force, Eq. 16.8.9
V_u	= factored shear force
V_x, V_y	= shear in the x- and y-directions, respectively
w	= uniform loading; service uniformly distributed load on beam; displacement in z-direction (Fig. 6.14.2); width of stiffener plate (Chap. 11); density of concrete, Eq. 16.5.1
w_D, w_L	= service uniform dead and live load, respectively
w_n	= w_u/ϕ_b = required nominal uniform load causing collapse mechanism (Chap. 10)
w_u	= factored uniform load
w_{uh}	= factored uniform horizontal load
W	= total service load on a span; concentrated load on beam; width of stiffener Chap. 11); seat width (Chap. 13)
W_n	= W_u/ϕ_b = required nominal concentrated load causing collapse (Chap. 10)
W_u	= factored concentrated load
x_0, y_0	= shear center distances from centroid measured along the x- and y-axes, respectively
y	= deflection at a location z along axis of member
$\bar{y}$	= center of gravity (CG) of composite section measured from CG of gravity of steel W section
y_0	= $(V'_n/V_n)h$
y_1	= total deflection (including second-order deflection) of beam-column
y_b	= distance to bottom of steel section from CG of composite section
y_c, y_t	= distances from CG of the section to the compression and tension extreme fibers, respectively
Z, Z_x, Z_y	= plastic modulus, $\int y\,dA$, with respect to the axes indicated by subscript
α	= constant $GJ/(2EC_w)$, Eq. 9.4.7; , ratio of web yield stress to flange yield stress, F_{yw}/F_{yf}, (Sec. 11.7); P_u/P_e or $\sum P_u/\sum P_e$ (Chap. 12)
β	= flexure analogy modification factor (Chap. 8); A_w/A_f, ratio of web cross-sectional area to cross-sectional area of the *compression* flange (Sec.11.7)
β_{br}	= required brace stiffness (Chaps. 9 and 15)
β_s	= E_t/E, Eq. 6.9.2 (Table 6.9.1)
β_w	= selected ratio h/t_w for design (Sec. 11.14)

γ	= general term for overload factor; strain angle; angle between the plane of bending and the xz plane (Sec. 7.10)
γ_i	= overload factors (ASCE 7)
δ	= deflection; virtual displacement; sidesway buckling deflection
	= first-order deflection of beam-column
ε	= strain, in./in. or mm/mm
ε_{st}	= strain at onset of strain hardening
$\varepsilon_x, \varepsilon_y$	= strain in the x and y-directions (Sec. 6.14)
ε_y	= strain at first yield, F_y/E_s (Fig. 6.6.1)
Δ	= deflection; shear deformation on a bolt (Chap. 4); maximum deformation on a fillet weld (Chap. 5); sway deflection (Fig. 6.9.3); lateral deflection of a frame, i.e., drift; deformation on framing angles(Sec. 13.2)
Δ_0	= maximum deformation on fillet weld when $\theta = 0°$, 0.11 in.
Δ_1	= deformation on any weld segment (Eq. 5.17.5)
Δ_{1u}	= first-order sway deflection (Sec. 12.11)
Δ_{2u}	= total sway deflection, including second-order effect (Sec. 12.11)
Δ_h	= first-order interstory drift due to lateral force
Δ_{max}	= maximum deflection; maximum shear deformation in a bolt $= 0.34$ in.
Δ_u	= deformation of weld element at ultimate stress (Chap. 5)
λ	= slenderness ratios for plate elements (see AISC-B4.1); torsion parameter, $1/a = \sqrt{GJ/EC_w}$ (Chap. 8)
λ_c	= slenderness parameter; for columns, Eqs. 6.7.2 and 6.7.3; for plate compression elements, Eq. 6.15.1
λ_p	= maximum slenderness ratio for compact element
λ_r	= maximum slenderness ratio for noncompact element
μ	= Poisson's ratio (0.3 for steel); coefficient of friction
ξ	= shape factor, Z/S
ψ	= factor in C_m (Eq. 12.3.8)
ϕ	= resistance factor; strength reduction factor; angle of twist (Chapters 8 and 9); stability parameter $L\sqrt{P/EI}$ (Chap. 14)
ϕ_b	= resistance factor for flexural member, 0.90; for composite section, 0.85
ϕ_c	= resistance factor for compression member, 0.85
ϕ_{cr}	= value of stability parameter when buckling occurs (Chap. 14)
ϕ_t	= resistance factor for tension limit state, (Chap. 3); resistance factor for bolt strength in tension, 0.75
ϕ_v	= resistance factor for shear on beam web, 0.90; resistance factor for bolt strength in shear, 0.75
Ω_b	= safety factor (ASD) for bending
Ω_c	= safety factor (ASD) for compression
Ω_t	= safety factor (ASD) for tension
Ω_v	= safety factor (ASD) for shear
τ	= shear stress (theoretical)
τ_b	= stiffness reduction factor, used in direct analysis method (Chaps. 12 and 15)
τ_{cr}	= buckling stress in shear; $0.6F_{yw}$ or $F_y/\sqrt{3}$ (See Sec. 11.8)
τ_u	= ultimate (fracture) shear strength
τ_{xy}	= shear stress in the xy plane (Sec. 6.14)
τ_y	= shear yield stress
θ	= angle of loading of weld segment measured from the weld longitudinal axis (Sec. 5.17); rotation of beam section (curvature); rate of twist, df/dz (Chap. 8); end slopes on beam (Sec. 13.1)
θ_p	= rotation angle at M_p (see Fig. 7.3.4)
θ_{st}	= rotation angle at onset of strain hardening (Fig. 9.3.2)
θ_u	= rotation angle at plastic hinge M_p (see Figs. 7.3.4 and 10.2.1)
θ_y	= rotation angle of beam section when extreme fiber reaches F_y
σ	= general term for compressive or tensile stress due to bending
σ_x, σ_y	= stress in the x- and y-directions (Sec. 6.14)
σ_y	= tension–compression yield stress
σ_z	= flexural stress (theoretical) in z-direction