

prime numbers, **p** and **q**.

n is called the modulus for encryption and decryption.

C = cipher text

M= plain text.

e = public key

d= private key

Important Theorem

- $n = p \times q$
- $\phi(n) = (p - 1) \times (q - 1)$
- $d = (1 + k \cdot \phi(n)) / e$
- $e = (1 + k \cdot \phi(n)) / d$
- $C = m^e \pmod n$
- $m = c^d \pmod n$

Example 1:

In an RSA cryptosystem, a particular A uses two prime numbers, 13 and 17, to generate the public and private keys. If the public of A is 35. Then the private key of A is?.

Explanation:

Step 1: in the first step, select two large prime numbers, **p** and **q**.

$$p = 13$$

$$q = 17$$

Step 2:

$$\phi(n) = (p - 1) \times (q - 1)$$

$$\phi(n) = (13 - 1) \times (17 - 1)$$

$$\phi(n) = 12 \times 16$$

$$\phi(n) = 192$$

Step 3:

$$d = (1 + k \cdot \phi(n)) / e \quad [\text{let } k = 0, 1, 2, 3, \dots]$$

Put **k = 0**

$$d = (1 + 0 \times 192)/35$$

$$d = 1/35$$

Put k = 1

$$d = (1 + 1 \times 192)/35$$

$$d = 193/35$$

Put k = 2

$$d = (1 + 2 \times 192)/35$$

$$d = 385/35$$

$$d = 11$$

The private key is $\langle d, n \rangle = (11, 221)$

Hence, private key i.e. $d = 11$

Example 2:

A RSA cryptosystem uses two prime numbers 3 and 13 to generate the public key= 3 and the private key = 7. What is the value of cipher text for a plain text=5?

Explanation:

Step 1: In the first step, select two large prime numbers, **p** and **q**.

$$p = 3$$

$$q = 13$$

Step 2: Multiply these numbers to find $n = p \times q$, where **n** is called the modulus for encryption and decryption.

First, we calculate

$$n = p \times q$$

$$n = 3 \times 13$$

$$n = 39$$

Step 3:

To find ciphertext from the plain text following formula is used to get ciphertext C.

$$C = m^e \text{ mod } n$$

$$C = 5^3 \text{ mod } 39$$

$$C = 125 \text{ mod } 39$$

$$C = 8$$

Hence, the ciphertext generated from plain text, $C = 8$.

Example 3:

In an RSA cryptosystem, a particular A uses two prime numbers, 3 and 11, to generate the public and private keys. If the private key of A is 7. Then the public key of A is

Explanation:

Step 1: in the first step, select two large prime numbers, p and q .

$$p = 3$$

$$q = 11$$

Step 2:

$$\phi(n) = (p - 1) \times (q - 1)$$

$$\phi(n) = (3 - 1) \times (11 - 1)$$

$$\phi(n) = 2 \times 10$$

$$\phi(n) = 20$$

Step 3:

$$e = (1 + k \cdot \phi(n)) / d \quad [\text{let } k = 0, 1, 2, 3, \dots]$$

Put $k = 0$

$$e = (1 + 0 \times 20) / 7$$

$$e = 1/7$$

Put $k = 1$

$$e = (1 + 1 \times 20) / 7$$

$$e = 21/7$$

$$e = 3$$

The public key is $\langle e, n \rangle = (3, 33)$

Hence, public key i.e. $e = 3$