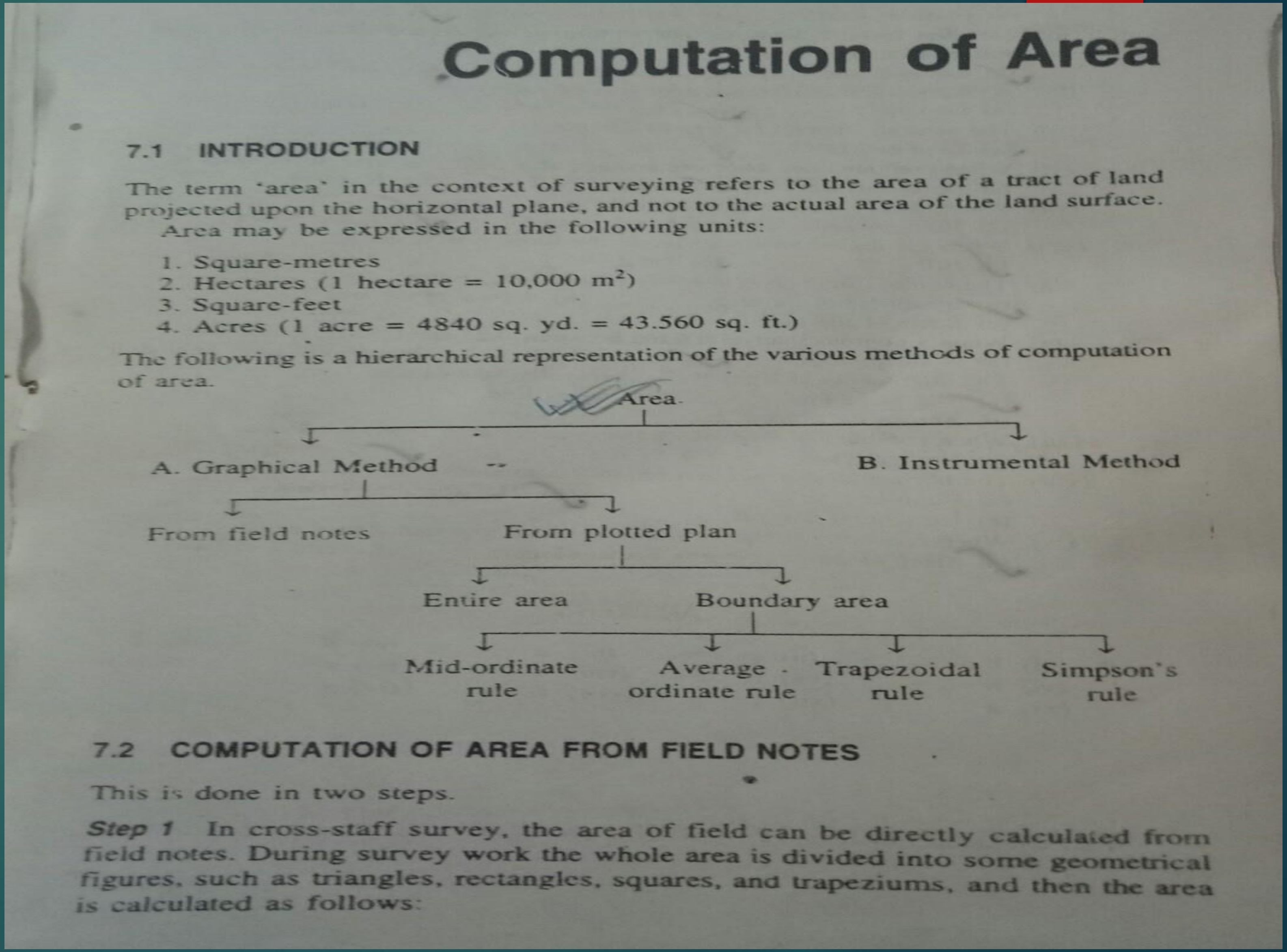
Calculation of Area

Computation of Area

Area is mainly calculated by two methods :

- Graphical Method
- Instrumental Method



Computation of Area

7.1 INTRODUCTION

The term 'area' in the context of surveying refers to the area of a tract of land projected upon the horizontal plane, and not to the actual area of the land surface.

Area may be expressed in the following units:

1. Square-metres
2. Hectares (1 hectare = 10,000 m^2)
3. Square-feet
4. Acres (1 acre = 4840 sq. yd. = 43.560 sq. ft.)

The following is a hierarchical representation of the various methods of computation of area.

7.2 COMPUTATION OF AREA FROM FIELD NOTES

This is done in two steps.

Step 1 In cross-staff survey, the area of field can be directly calculated from field notes. During survey work the whole area is divided into some geometrical figures, such as triangles, rectangles, squares, and trapeziums, and then the area is calculated as follows:

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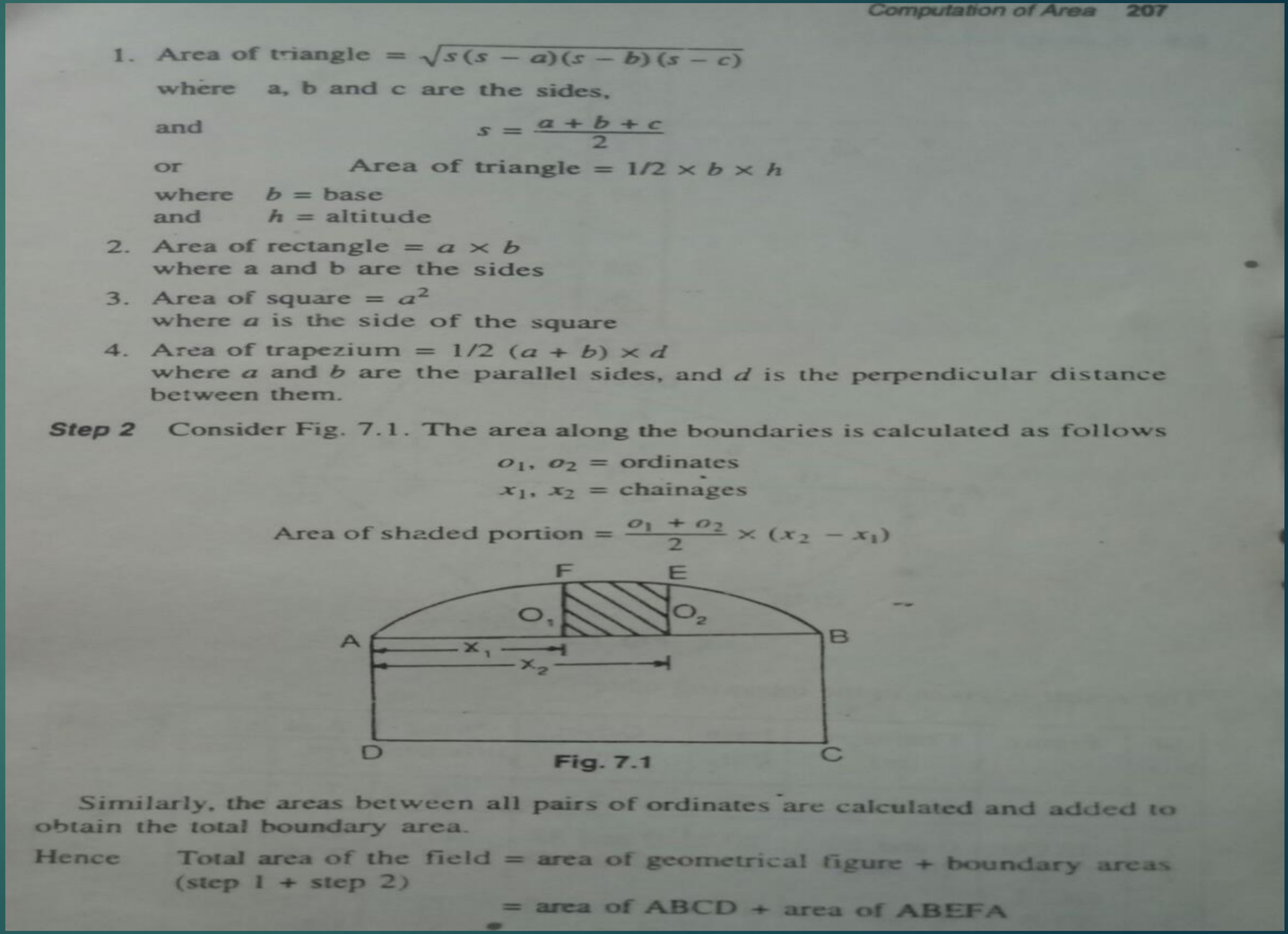
Area of a Triangle = a x b
Where a & b are the sides



1. Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

   where a, b and c are the sides,

   and $s = \frac{a+b+c}{2}$

   or Area of triangle $= 1/2 \times b \times h$

   where b = base

   and h = altitude

2. Area of rectangle $= a \times b$

   where a and b are the sides

3. Area of square $= a^2$

   where a is the side of the square

4. Area of trapezium $= 1/2\ (a + b) \times d$

   where a and b are the parallel sides, and d is the perpendicular distance between them.

Step 2 Consider Fig. 7.1. The area along the boundaries is calculated as follows

o_1, o_2 = ordinates

x_1, x_2 = chainages

$$\text{Area of shaded portion} = \frac{o_1 + o_2}{2} \times (x_2 - x_1)$$

Fig. 7.1

Similarly, the areas between all pairs of ordinates are calculated and added to obtain the total boundary area.

Hence Total area of the field = area of geometrical figure + boundary areas (step 1 + step 2)

= area of ABCD + area of ABEFA

Computation of Area

Considering the entire area:

- By dividing the area into triangles
- By dividing the area into squares
- By drawing parallel lines and converting them into rectangles

7.4 COMPUTATION OF AREA FROM PLOTTED PLAN

The area may be calculated in the two following ways.

Case I—Considering the entire area The entire area is divided into regions of a convenient shape, and calculated as follows:

(a) By dividing the area into triangles The triangles are so drawn as to equalise the irregular boundary line.

Then the bases and altitudes of the triangles are determined according to the scale to which the plan was drawn. After this, the areas of these triangles are calculated (area = 1/2 × base × altitude). The areas are then added to obtain the total area (Fig. 7.6).

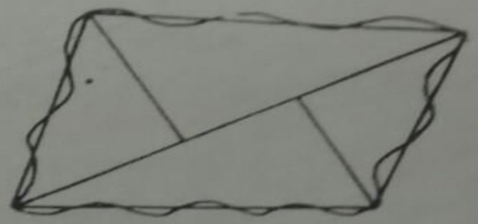


Fig. 7.6

(b) By dividing the area into squares In this method, squares of equal size are ruled out on a piece of tracing paper. Each square represents a unit area, which could be 1 cm^2 or 1 m^2. The tracing paper is placed over the plan and the number of full squares are counted. The total area is then calculated by multiplying the number of squares by the unit area of each square (Fig. 7.7).

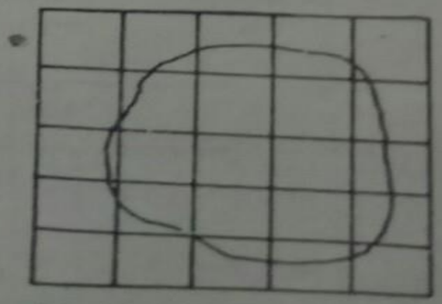


Fig. 7.7

(c) By drawing parallel lines and converting them to rectangles In this method, a series of equidistant parallel lines are drawn on a tracing paper (Fig. 7.8). The constant distance represents a metre or centimetre. The tracing paper is placed

Computation of Area

- The Mid- ordinate rule
- The Average Ordinate rule
- The Trapezoidal rule
- Simpson's rule

over the plan in such a way that the area is enclosed between the two parallel lines at the top and bottom. Thus the area is divided into a number of strips. The curved ends of the strips are replaced by perpendicular lines (by give and take principle) and a number of rectangles are formed. The sum of the lengths of the rectangles is then calculated.

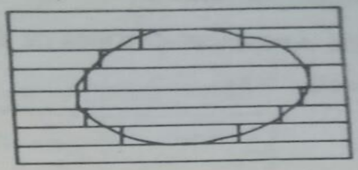


Fig. 7.8

Then,

Required area = Σ length of rectangles × constant distance

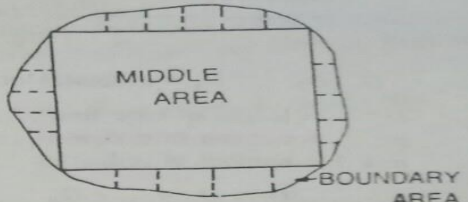


Fig. 7.9

Case II In this method, a large square or rectangle is formed within the area in the plan. Then ordinates are drawn at regular intervals from the side of the square to the curved boundary. The middle area is calculated in the usual way. The boundary area is calculated according to one of the following rules:

1. The mid-ordinate rule
2. The average ordinate rule
3. The trapezoidal rule
4. Simpson's rule

The various rules are explained in the following sections.

7.5 THE MID-ORDINATE RULE

Consider Fig. 7.10

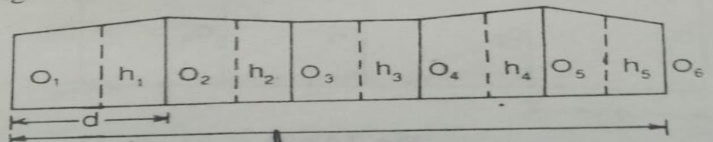


Fig. 7.10

Let $O_1, O_2, O_3, \ldots, O_n$ = ordinates at equal intervals

l = length of base line

d = common distance between ordinates

$h_1, h_2, \ldots h_n$ = mid-ordinates

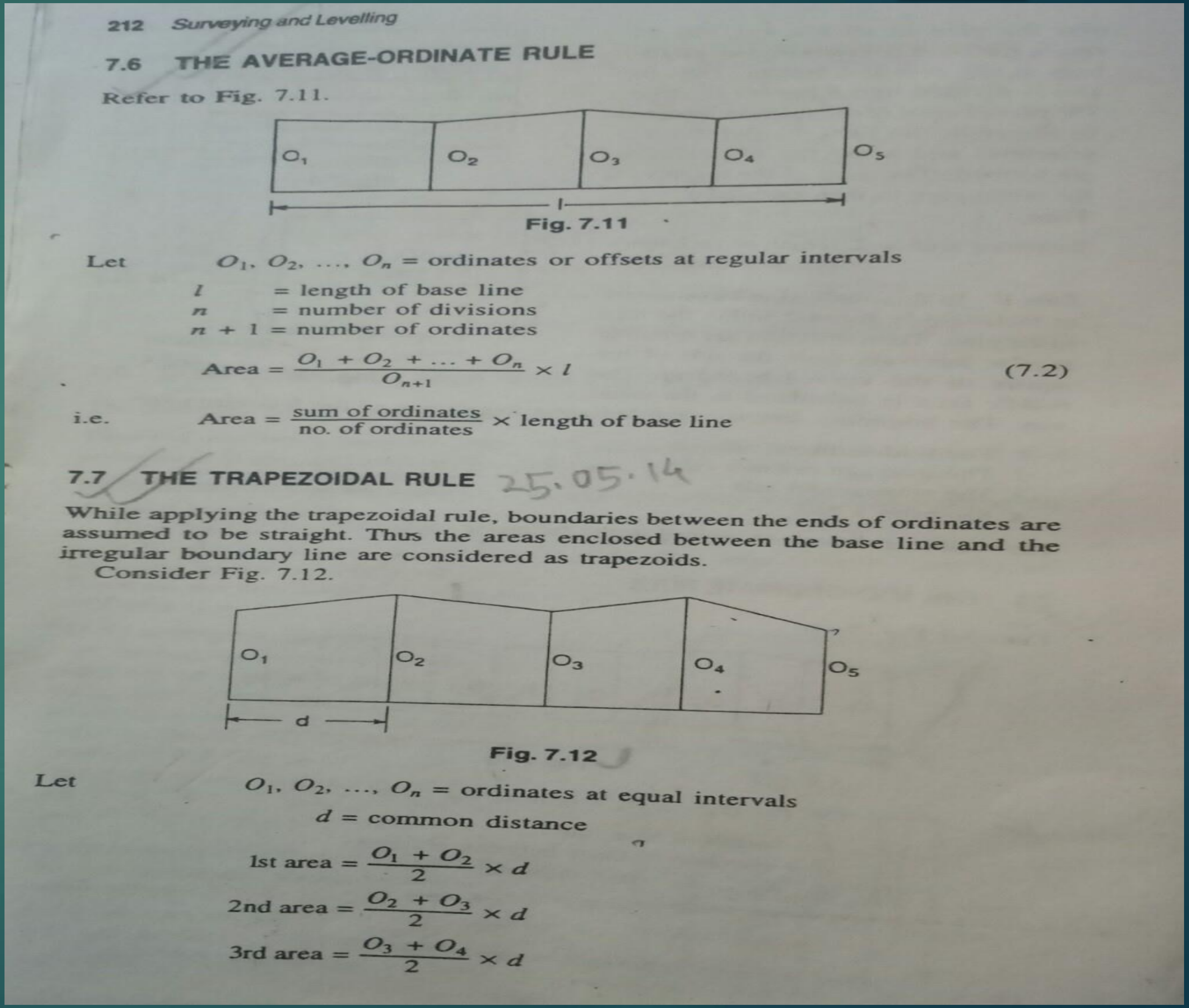
$$\text{Area of plot} = h_1 \times d + h_2 \times d + \ldots + h_n \times d = d(h_1 + h_2 + \ldots + h_n) \quad (7.1)$$

i.e. Area = common distance × sum of mid-ordinates

Computation of Area

Average Ordinate Rule:

- Area = (Sum of Ordinates / No of Ordinates) x Length of Base Line



7.6 THE AVERAGE-ORDINATE RULE

Refer to Fig. 7.11.

Fig. 7.11

Let $O_1, O_2, \ldots, O_n$ = ordinates or offsets at regular intervals

l = length of base line

n = number of divisions

$n + 1$ = number of ordinates

$$\text{Area} = \frac{O_1 + O_2 + \ldots + O_n}{O_{n+1}} \times l \qquad (7.2)$$

i.e.

$$\text{Area} = \frac{\text{sum of ordinates}}{\text{no. of ordinates}} \times \text{length of base line}$$

7.7 THE TRAPEZOIDAL RULE 25.05.14

While applying the trapezoidal rule, boundaries between the ends of ordinates are assumed to be straight. Thus the areas enclosed between the base line and the irregular boundary line are considered as trapezoids.

Consider Fig. 7.12.

Fig. 7.12

Let $O_1, O_2, \ldots, O_n$ = ordinates at equal intervals

d = common distance

$$\text{1st area} = \frac{O_1 + O_2}{2} \times d$$

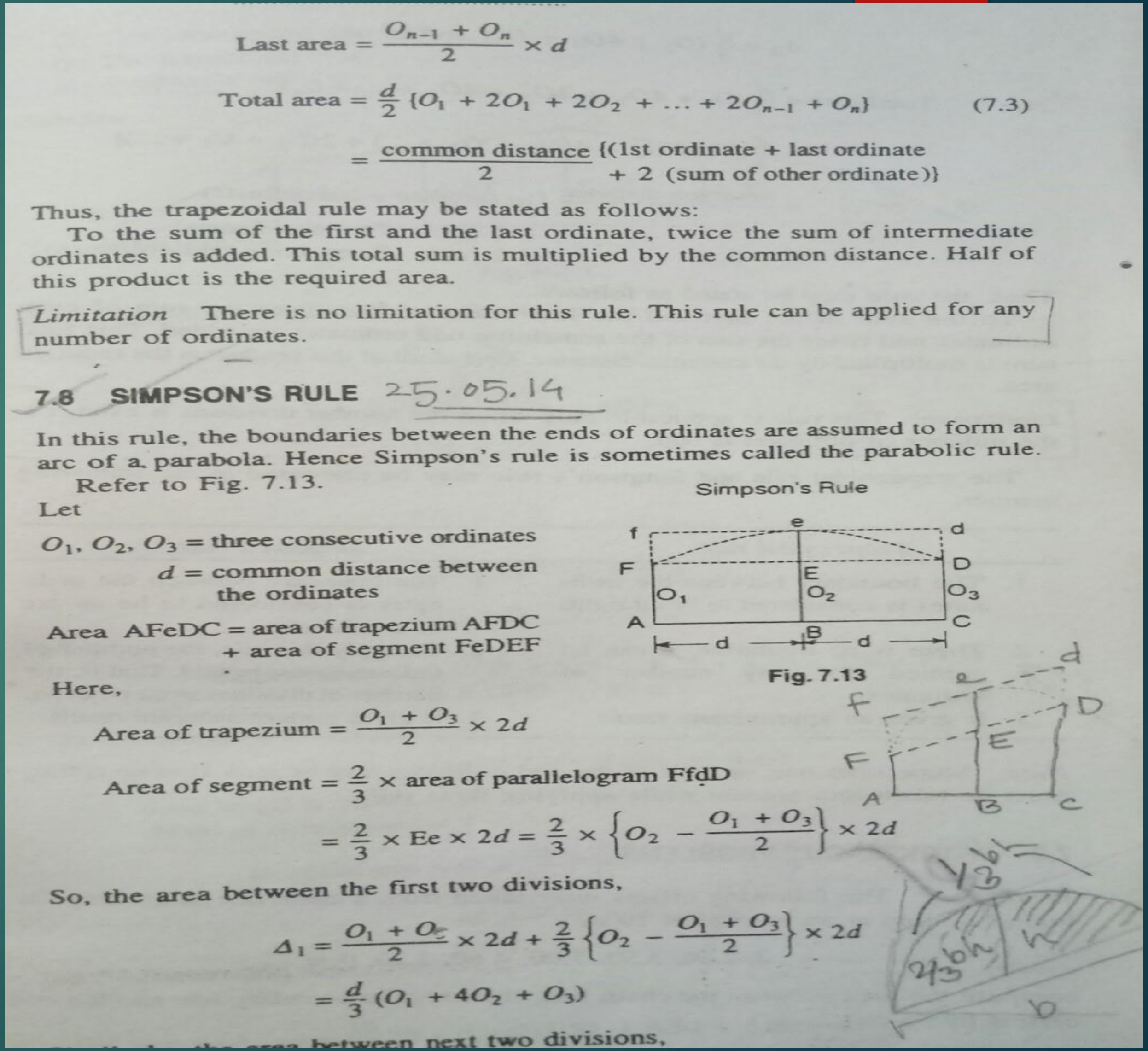
$$\text{2nd area} = \frac{O_2 + O_3}{2} \times d$$

$$\text{3rd area} = \frac{O_3 + O_4}{2} \times d$$

Computation of Area

Limitation of Trapezoidal rule:

- ✓ NO limitation
- ✓ Can be applied for any number of ordinates



$$\text{Last area} = \frac{O_{n-1} + O_n}{2} \times d$$

$$\text{Total area} = \frac{d}{2}\{O_1 + 2O_1 + 2O_2 + \ldots + 2O_{n-1} + O_n\} \qquad (7.3)$$

$$= \frac{\text{common distance}}{2}\{(\text{1st ordinate} + \text{last ordinate} + 2\ (\text{sum of other ordinate})\}$$

Thus, the trapezoidal rule may be stated as follows:

To the sum of the first and the last ordinate, twice the sum of intermediate ordinates is added. This total sum is multiplied by the common distance. Half of this product is the required area.

Limitation There is no limitation for this rule. This rule can be applied for any number of ordinates.

7.8 SIMPSON'S RULE 25.05.14

In this rule, the boundaries between the ends of ordinates are assumed to form an arc of a parabola. Hence Simpson's rule is sometimes called the parabolic rule.

Refer to Fig. 7.13.

Let

O_1, O_2, O_3 = three consecutive ordinates

d = common distance between the ordinates

Area AFeDC = area of trapezium AFDC + area of segment FeDEF

Simpson's Rule

Fig. 7.13

Here,

$$\text{Area of trapezium} = \frac{O_1 + O_3}{2} \times 2d$$

$$\text{Area of segment} = \frac{2}{3} \times \text{area of parallelogram FfdD}$$

$$= \frac{2}{3} \times \text{Ee} \times 2d = \frac{2}{3} \times \left\{O_2 - \frac{O_1 + O_3}{2}\right\} \times 2d$$

So, the area between the first two divisions,

$$\Delta_1 = \frac{O_1 + O_3}{2} \times 2d + \frac{2}{3}\left\{O_2 - \frac{O_1 + O_3}{2}\right\} \times 2d$$

$$= \frac{d}{3}(O_1 + 4O_2 + O_3)$$

... the area between next two divisions,

Computation of Area

- **Simpson's rule** = Common distance / 3 (1st ordinate + Last ordinate + 4 (sum of even ordinates) + (sum of remaining odd ordinates)

$$\Delta_2 = \frac{d}{3}(O_3 + 4O_4 + O_5) \text{ and so on.}$$

$$\therefore \quad \text{Total area} = \frac{d}{3}(O_1 + 4O_2 + 2O_3 + 4O_4 + \ldots + O_n)$$

$$= \frac{d}{3}\{O_1 + O_n + 4(O_2 + O_4 + \ldots) + 2(O_3 + O_5 + \ldots)\}$$

$$= \frac{\text{common distance}}{3}\{\text{Ist ordinate} + \text{last ordinate}) + 4\,(\text{sum of even ordinates}) + 2\,(\text{sum of remaining odd ordinates})\}$$

Thus, the rule may be stated as follows.

To the sum of the first and the last ordinate, four times the sum of even ordinates and twice the sum of the remaining odd ordinates are added. This total sum is multiplied by the common distance. One-third of this product is the required area.

Limitation This rule is applicable only when the number divisions is even, i.e. the number of ordinates is odd.

The trapezoidal rule and Simpson's rule may be compared in the following manner:

Trapezoidal rule	*Simpson's rule*
1. The boundary between the ordinates is considered to be straight.	1. The boundary between the ordinates is considered to be an arc of a parabola.
2. There is no limitation. It can be applied for any number of ordinates.	2. To apply this rule, the number of ordinates must be odd. That is, the number of divisions must be even.
3. It gives an approximate result	3. It gives a more accurate result

Note Sometimes one, or both, of the end ordinates may be zero. However, they must be taken into account while applying these rules.

Computation of Area

Problem -01

7.9 WORKED-OUT PROBLEMS

Problem 1 The following offsets were taken from a chain line to an irregular boundary line at an interval of 10 m:

0, 2.50, 3.50, 5.00, 4.60, 3.20, 0 m

compute the area between the chain line, the irregular boundary line and the end offsets by:

(a) The mid-ordinate rule

Computation of Area

Problem -01

(b) The average-ordinate rule
(c) The trapezoidal rule
(d) Simpson's rule

Solution

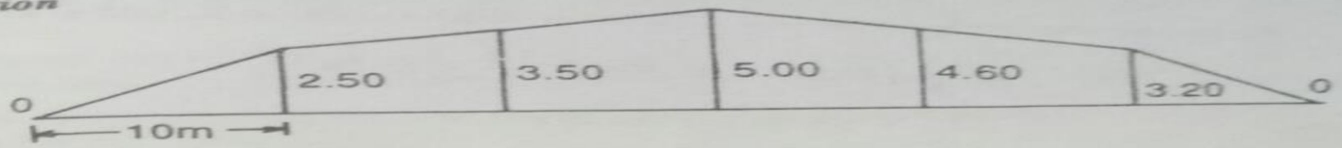


Fig. P.7.1

(a) *By mid-ordinate rule*: The mid-ordinates are

$$h_1 = \frac{0 + 2.50}{2} = 1.25 \text{ m}$$

$$h_2 = \frac{2.50 + 3.50}{2} = 3.00 \text{ m}$$

$$h_3 = \frac{3.50 + 5.00}{2} = 4.25 \text{ m}$$

$$h_4 = \frac{5.00 + 4.60}{2} = 4.80 \text{ m}$$

$$h_5 = \frac{4.60 + 3.20}{2} = 3.90 \text{ m}$$

$$h_6 = \frac{3.20 + 0}{2} = 1.60 \text{ m}$$

$$\text{Required area} = 10\,(1.25 + 3.00 + 4.25 + 4.80 + 3.90 + 1.60)$$
$$= 10 \times 18.80 = 188 \text{ m}^2$$

(b) *By average-ordinate rule*:

Here $d = 10$ m and $n = 6$ (no. of divs)

Base length = 10 × 6 = 60 m
Number of ordinates = 7

$$\text{Required area} = 60 \times \left\{\frac{0 + 2.50 + 3.50 + 5.00 + 4.60 + 3.20 + 0}{7}\right\}$$
$$= 60 \times \frac{18.80}{7} = 161.14 \text{ m}^2$$

(c) *By trapezoidal rule*:

Here $d = 10$

$$\text{Required area} = \frac{10}{2}\{0 + 0 + 2(2.50 + 3.50 + 5.00 + 4.60 + 3.20)\}$$
$$= 5 \times 37.60 = 188 \text{ m}^2$$

Computation of Area

Problem -02

(d) *By Simpson's rule:*

$d = 10$

$$\text{Required area} = \frac{10}{3}\{0 + 0 + 4(2.50 + 5.00 + 3.20) + 2(3.50 + 4.60)\}$$

$$= \frac{10}{3}\{42.80 + 16.20\} = \frac{10}{3} \times 59.00$$

$$= \frac{10}{3} \times 59.00 = 196.66\ \text{m}^2$$

Solved in class

Problem 2 The following offsets were taken at 15 m intervals from a survey line to an irregular boundary line:

3.50, 4.30, 6.75, 5.25, 7.50, 8.80, 7.90, 6.40, 4.40, 3.25 m

Calculate the area enclosed between the survey line, the irregular boundary line, and the first and last offsets, by:

(a) The trapezoidal rule

(b) Simpson's rule

Solution (Fig. P-7.2)

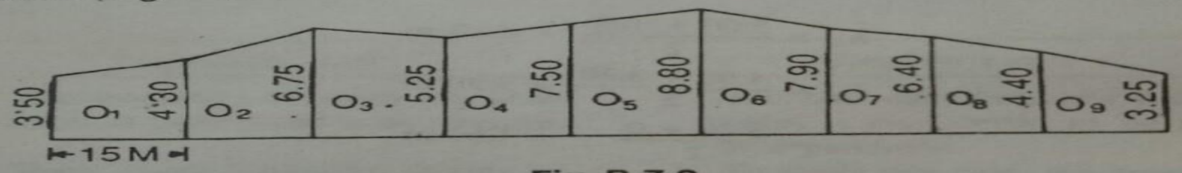


Fig. P-7.2

(a) *By trapezoidal rule:*

$$\text{Required area} = \frac{15}{2}\{3.50 + 3.25 + 2(4.30 + 6.75 + 5.25 + 7.50 + 8.80 + 7.90 + 6.40 + 4.40)\}$$

$$= \frac{15}{2}\{6.75 + 102.60\} = 820.125\ \text{m}^2$$

(b) *Simpson's rule:* If this rule is to be applied, the number of ordinates must be odd. But here the number of ordinate is even (ten).

So, Simpson's rule is applied from O_1 to O_9 and the area between O_9 and O_{10} is found out by the trapezoidal rule.

$$A_1 = \frac{15}{3}\{3.50 + 4.40 + 4(4.30 + 5.25 + 8.80 + 6.40) + 2(6.75 + 7.50 + 7.90)\}$$

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THANK YOU