CE 103: Surveying

Lecture 10: Volume and area calculation

Course Instructor: Saurav Barua (SB)

Assistant Professor, Dept. of Civil Engineering, DIU

Email: saurav.ce@diu.edu.bd

Phone: 01715334075

Outline

- ❑Trapezoidal rule
- ❑Simpson’s one third rule
- ❑Irregular intervals problem

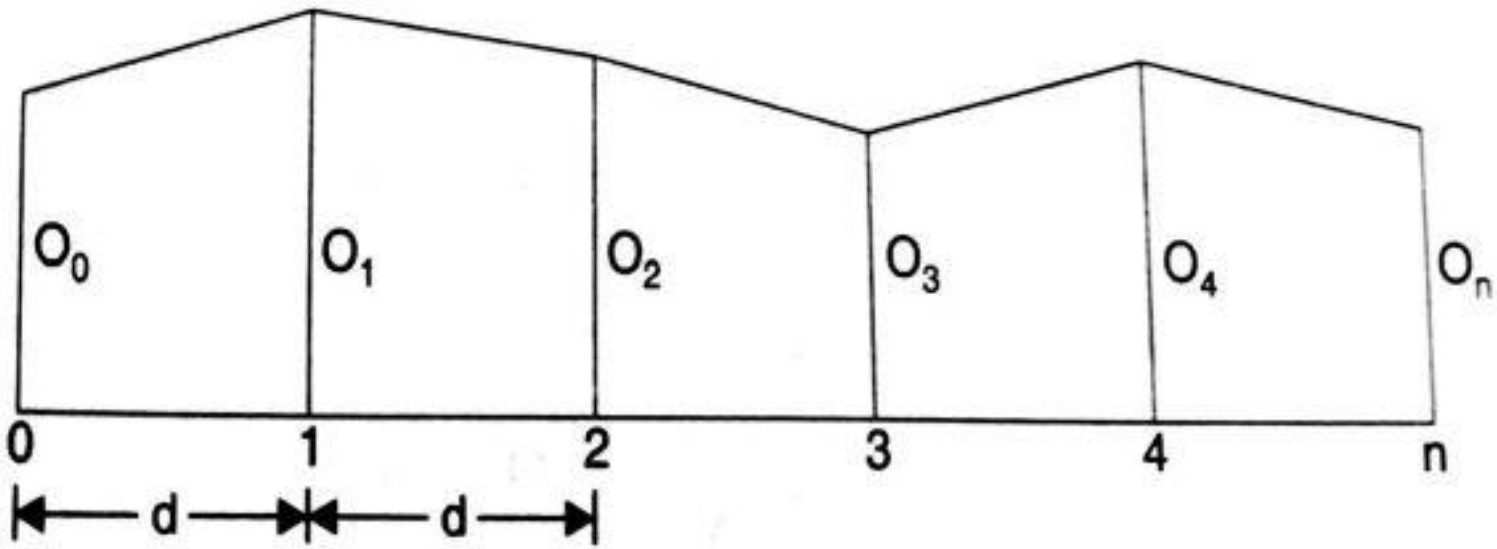
10.1 Method of calculation of area

There are two methods of calculating area:

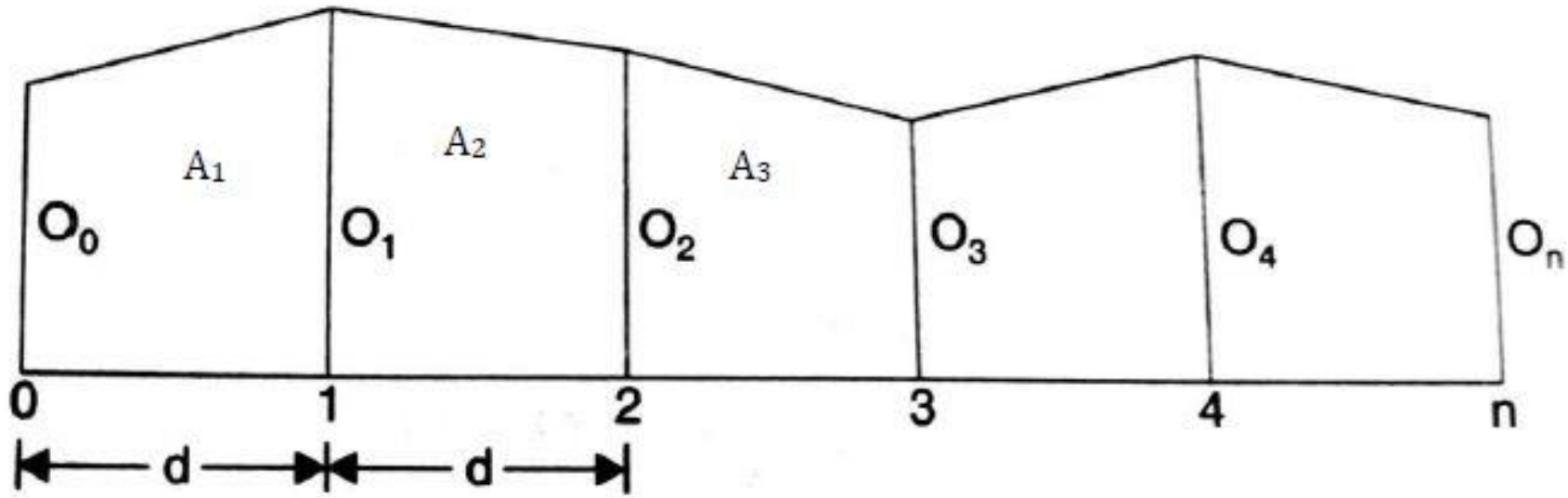
- Trapezoidal Rule.
- Simpson's one third Rule .

10.1.1 Trapezoidal Rule

- Assumption : straight line between points "**a**" and "**b**" .



Area of each segment can be calculated using area formula for trapezoid:



$$A_1 = \frac{O_0 + O_1}{2} \times d$$

$$A_2 = \frac{O_1 + O_2}{2} \times d$$

$$A_3 = \frac{O_2 + O_3}{2} \times d$$

$$A_n = \frac{O_{n-1} + O_n}{2} \times d$$

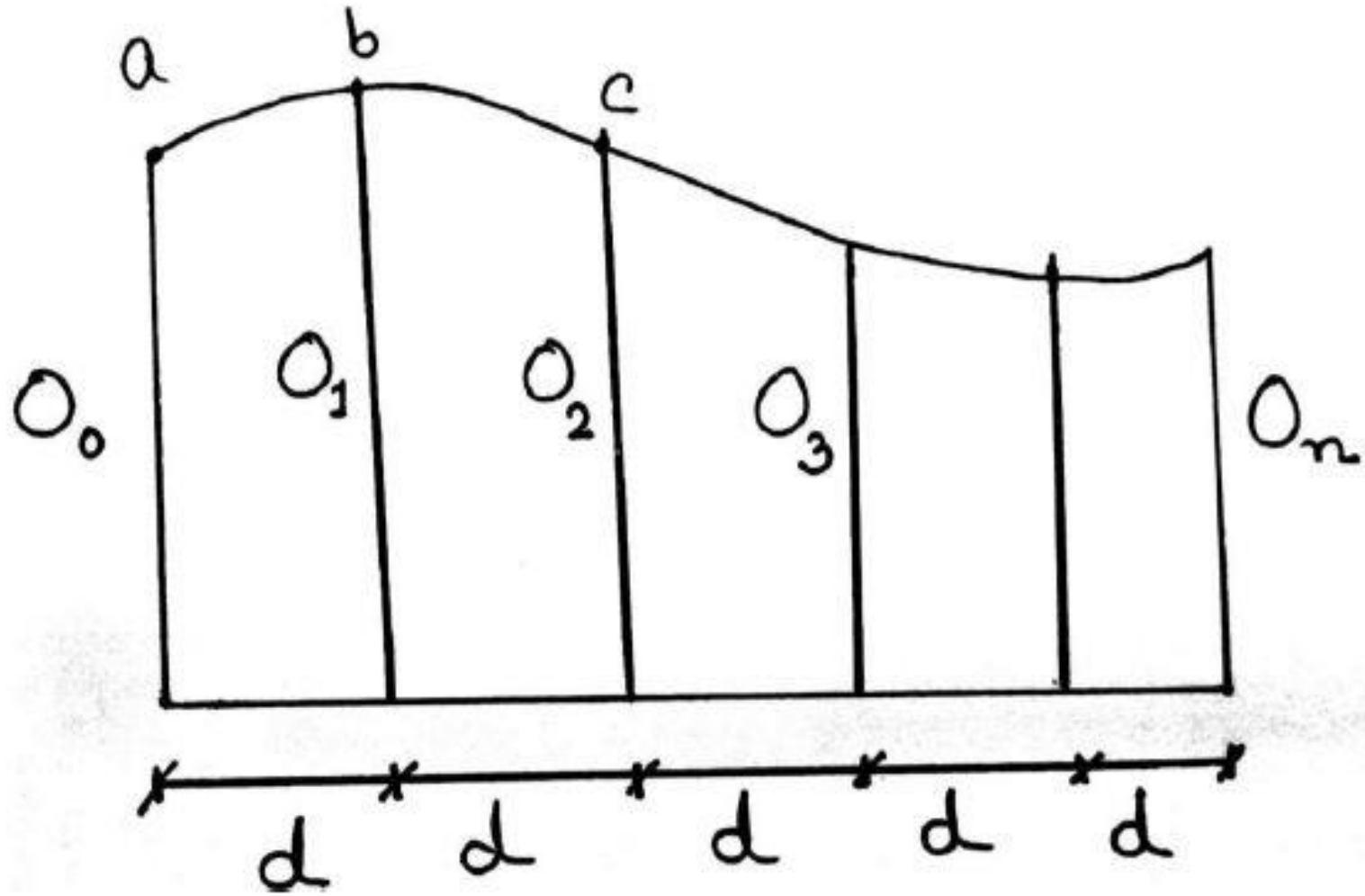
Total Area = $A_1 + A_2 + A_3 + \ldots\ldots\ldots + A_n$

$$= \frac{O_0 + O_1}{2} d + \frac{O_1 + O_2}{2} d + \ldots\ldots + \frac{O_{n-1} + O_n}{2} d$$

$$= \left(\frac{O_0 + O_n}{2} + O_1 + O_2 + \ldots\ldots + O_{n-1} \right) d$$

10.1.2 Simpson's Rule

- Assumption : Parabola between points "**a**" , "**b**" and "**c**" .

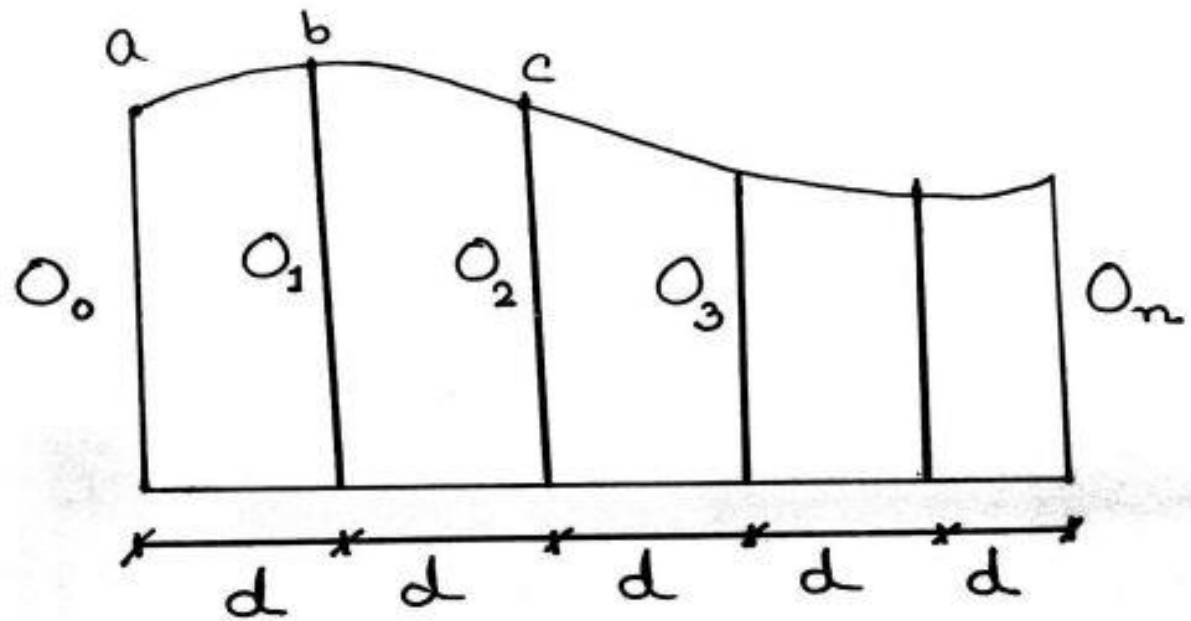


Total Area

$$= \frac{2d}{6}(O_0 + 4O_1 + O_2) + \frac{2d}{6}(O_2 + 4O_3 + O_4) + \ldots\ldots\ldots\ldots$$

$$= \frac{d}{3}[O_0 + 4(O_1 + O_3 + O_5 + \ldots) + 2(O_2 + O_4 + O_6 + \ldots) + O_n]$$

$$= \frac{d}{3}[O_0 + O_n + 4(O_1 + O_3 + O_5 + \ldots\ldots) + 2(O_2 + O_4 + O_6 + \ldots)]$$



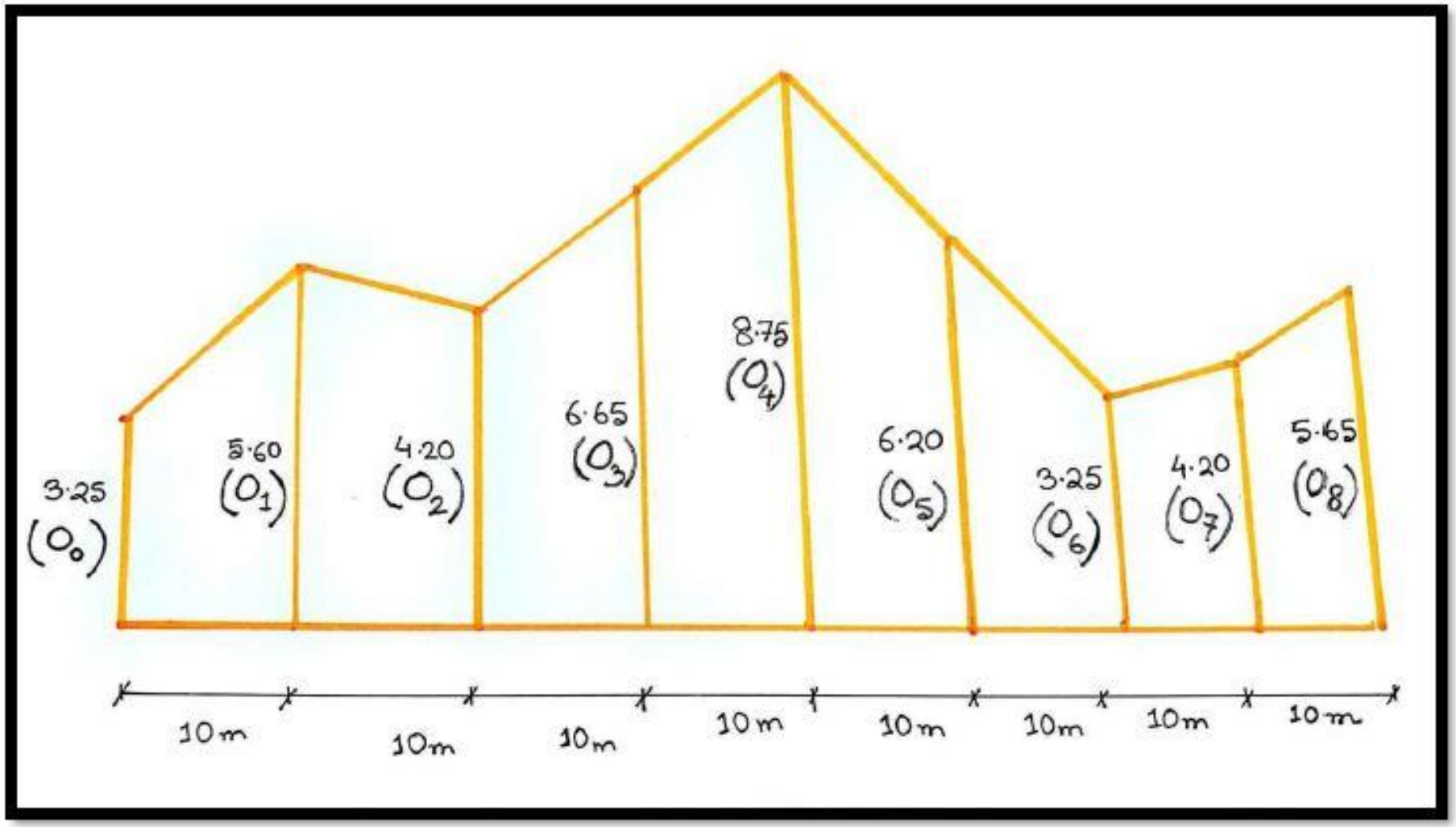
Mathematical Problem

The following perpendicular offsets were taken at 10 metres intervals from a survey line to an irregular boundary line :

3.25, 5.60, 4.20 , 6.65 , 8.75, 6.20, 3.25, 4.20, 5.65.

Calculate the area enclosed by the survey line, the irregular boundary line and the first and last offsets by Trapezoidal and Simpson's rule.

Solution:


3·25
(O0)
5·60
(O1)
4·20
(O2)
6·65
(O3)
8·75
(O4)
6·20
(O5)
3·25
(O6)
4·20
(O7)
5·65
(O8)
10m
10m
10m
10m
10m
10m
10m
10m

i) Trapezoidal Rule:-

$d = 10\,m$

$$\text{Total Area} = 10m \left[\frac{O_0 + O_8}{2} + O_1 + O_2 + O_3 + O_4 + O_5 + O_6 + O_7 \right]$$

$$= 10m \left[\frac{3.25m + 5.65m}{2} + 5.60m + 4.20m + 6.65m + 8.75m + 6.20m + 3.25m + 4.20m \right]$$

$$= 10m \times 43.3m$$

$$= 433\ m^2$$

$$\text{Total Area} = \frac{10m}{3}\left[O_0 + O_8 + 4(O_1 + O_3 + O_5 + O_7) + 2(O_2 + O_4 + O_6) \right]$$

$$= \frac{10m}{3}\left[3.25m + 5.65m + 4(5.60m + 6.65m + 6.20m + 4.20m) + 2\times(4.20_m + 8.75_m + 3.25_m) \right]$$

$$= \frac{10m}{3} * 131.9m = 439.67\ m^2$$

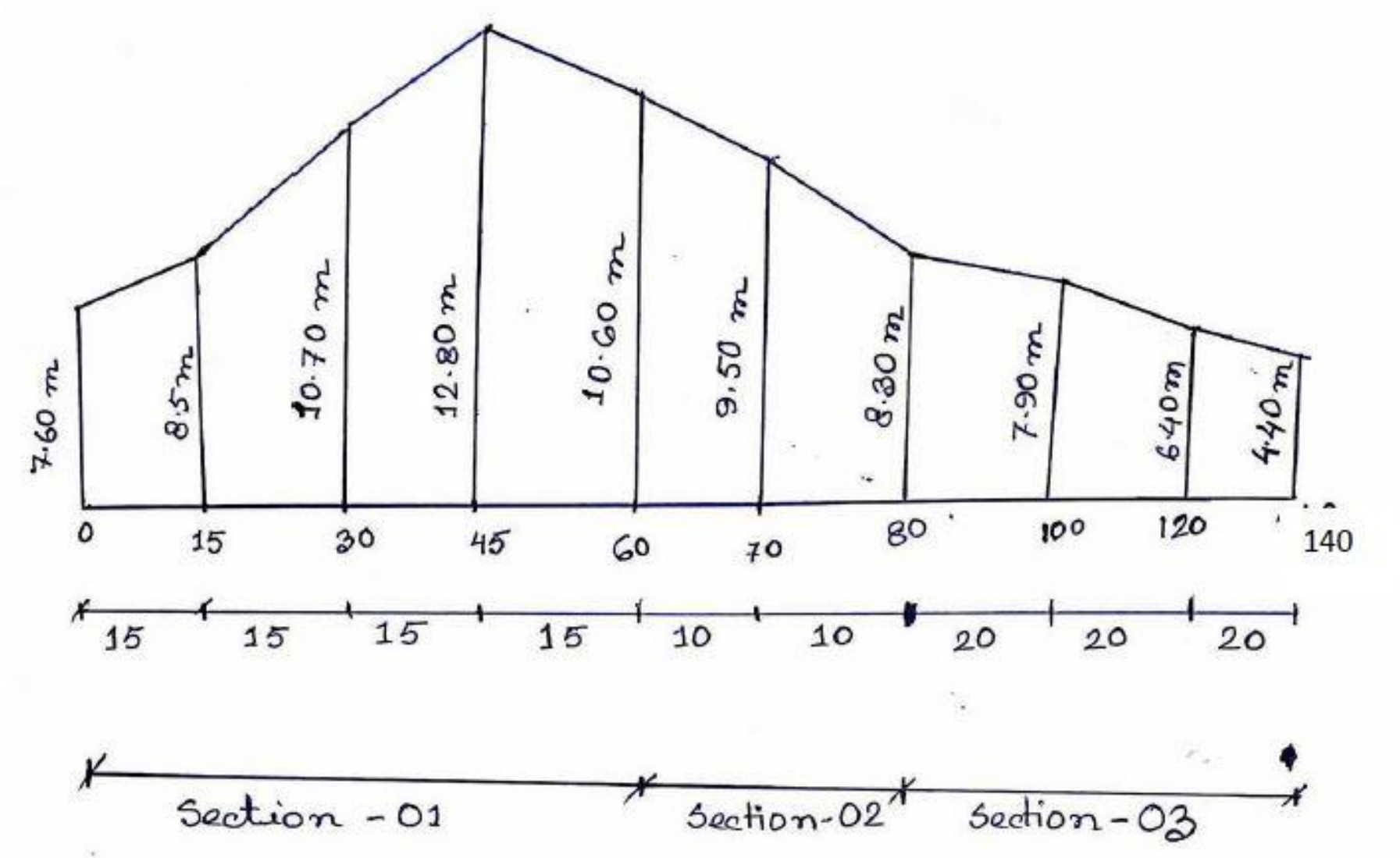
10.1.3 Irregular Intervals

The following offsets were taken from a chain line to an irregular boundary line:

Chainage (m)	0	15	30	45	60	70	80	100	120	140
Offsets (m)	7.60	8.5	10.7	12.8	10.6	9.5	8.3	7.9	6.4	4.4

Calculate the area between the survey line , irregular boundary line and end offsets by Trapezoidal rule and Simpson's rule .

Solution:


7.60 m
8.5 m
10.70 m
12.80 m
10.60 m
9.50 m
8.30 m
7.90 m
6.40 m
4.40 m
0
15
30
45
60
70
80
100
120
140
15
15
15
15
10
10
20
20
20
Section - 01
Section-02
Section-03

Divide the area into three segments with equal intervals.

Section 01 : From chainage 0 to 60 m.

Section 02 : From chainage 60 m to 80 m.

Section 03: From chainage 80 m to 140 m.

Using Trapezoidal Rule:

$$\text{Area of section 01, } A_1 = 15 \times \left(\frac{7.6+10.6}{2} + 8.5 + 10.7 + 12.8\right) = 616.5\, m^2$$

$$\text{Area of section 02, } A_2 = 10 \times \left(\frac{10.6+8.3}{2} + 9.5\right) = 189.5\, m^2$$

$$\text{Area of section 03, } A_3 = 20 \times \left(\frac{8.3+4.4}{2} + 7.9 + 6.4\right) = 413 m^2$$

$$\text{Total Area , } A = A_1 + A_2 + A_3 = 616.5 + 189.5 + 413 = 1219\, m^2$$

Using Simpson's Rule

As the first and second section have odd number of ordinates and therefore, Simpson's rule is directly applicable. The third section has 4 ordinates (even number); the rule is applicable for the first three ordinates only:

Area of section 01, $A_1 = \frac{15}{3} \times \{(7.6 + 10.6) + 4 \times (8.5 + 12.8) + 2 \times 10.7\} = 624\, m^2$

Area of section 02, $A_2 = \frac{10}{3} \times \{(10.6 + 8.3) + 4 \times (9.5)\} = 189.7\, m^2$

Area of section 03, $A_3 = \frac{20}{3} \times \{(8.3 + 6.4) + 4 \times (7.9)\} + \frac{20}{2}(6.4 + 4{,}4) = 416.6\ m^2$

Total Area , $A = A_1 + A_2 + A_3 = 624 + 189.7 + 416.6 = 1230.3\, m^2$