Calculation of Volume

Computation of Volume

Volume is mainly calculated by two methods :

- Trapezoidal rule
- Prismoidal rule

Computation of Volume

8.1 INTRODUCTION

For computation of the volume of earth work, the sectional areas of the cross-section which are taken transverse to the longitudinal section during profile levelling are first calculated. Again, the cross-sections may be of different types, namely: (i) level (ii) two-level, (iii) three-level, (iv) side-hill two-level, and (v) multi-level.

The methods of calculating the areas of such sections are discussed in Sec. 8.2.

After calculation of cross-sectional areas, the volume of earth work is calculated by: (i) the trapezoidal (or average end area) rule, and (ii) the prismoidal rule.

Notes: 1. The prismoidal rule gives the correct volume directly.

2. The trapezoidal rule does not give the correct volume. Prismoidal correction should be applied for this purpose. This correction is always subtractive.

3. Cutting is denoted by a positive sign and filling by a negative sign.

8.2 FORMULAE FOR CALCULATION OF CROSS-SECTIONAL AREA

A. Level Section

When the ground is level along the transverse direction:

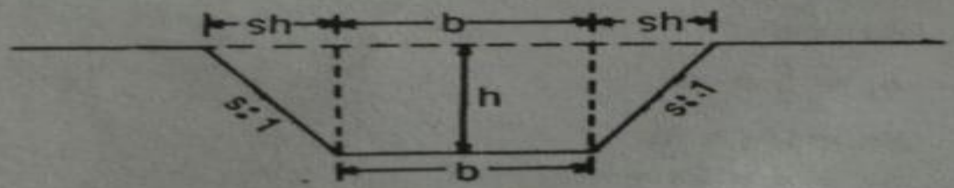


Fig. 8.1

$$\text{Cross-sectional area} = \frac{b + b + 2sh}{2} \times h$$
$$= (b + sh)h \qquad (1)$$

Computation of Volume

Problem -01

Example Calculate the sectional area of an embankment 10 m wide, with a side slope of 2 : 1. The ground is level in a transverse direction to the centre line. The central height of the embankment is 2.5 m.

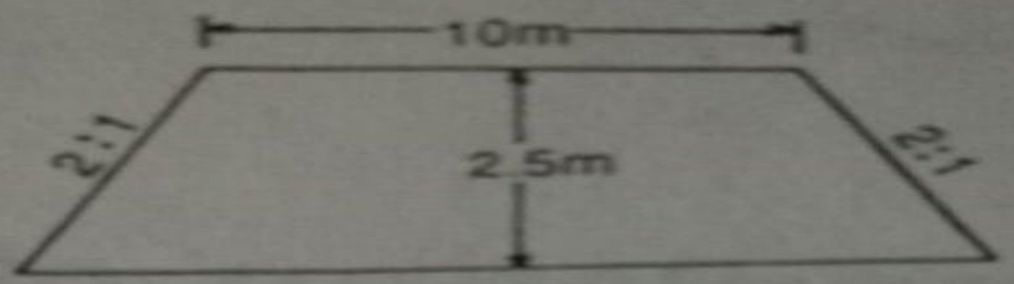


Fig. 8.2

Solution

Here

$b = 10$ m

$S = 2$

$h = 2.5$ m

From Eq. 1,

$$\text{Cross-sectional area} = (b + sh)h$$
$$= (10 + 2 \times 2.5) \times 2.5 = 37.5 \text{ m}^2$$

Computation of Volume

Formula for calculation of volume:

- ❑ Trapezoidal rule
- ❑ Prismoidal rule

8.3 FORMULA FOR CALCULATION OF VOLUME

D = common distance between sections

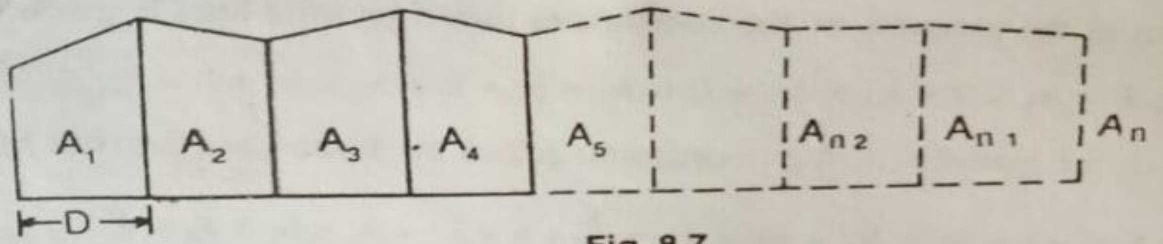


Fig. 8.7

A. Trapezoidal Rule (Average End Area Rule)

Volume (cutting or filling), $V = \frac{D}{2}\{A_1 + A_n + 2(A_2 + A_3 + \ldots + A_{n-1})\}$

i.e. volume = $\frac{\text{common distance}}{2}$ {area of 1st section + area of last section + 2 (sum of area of other sections)}

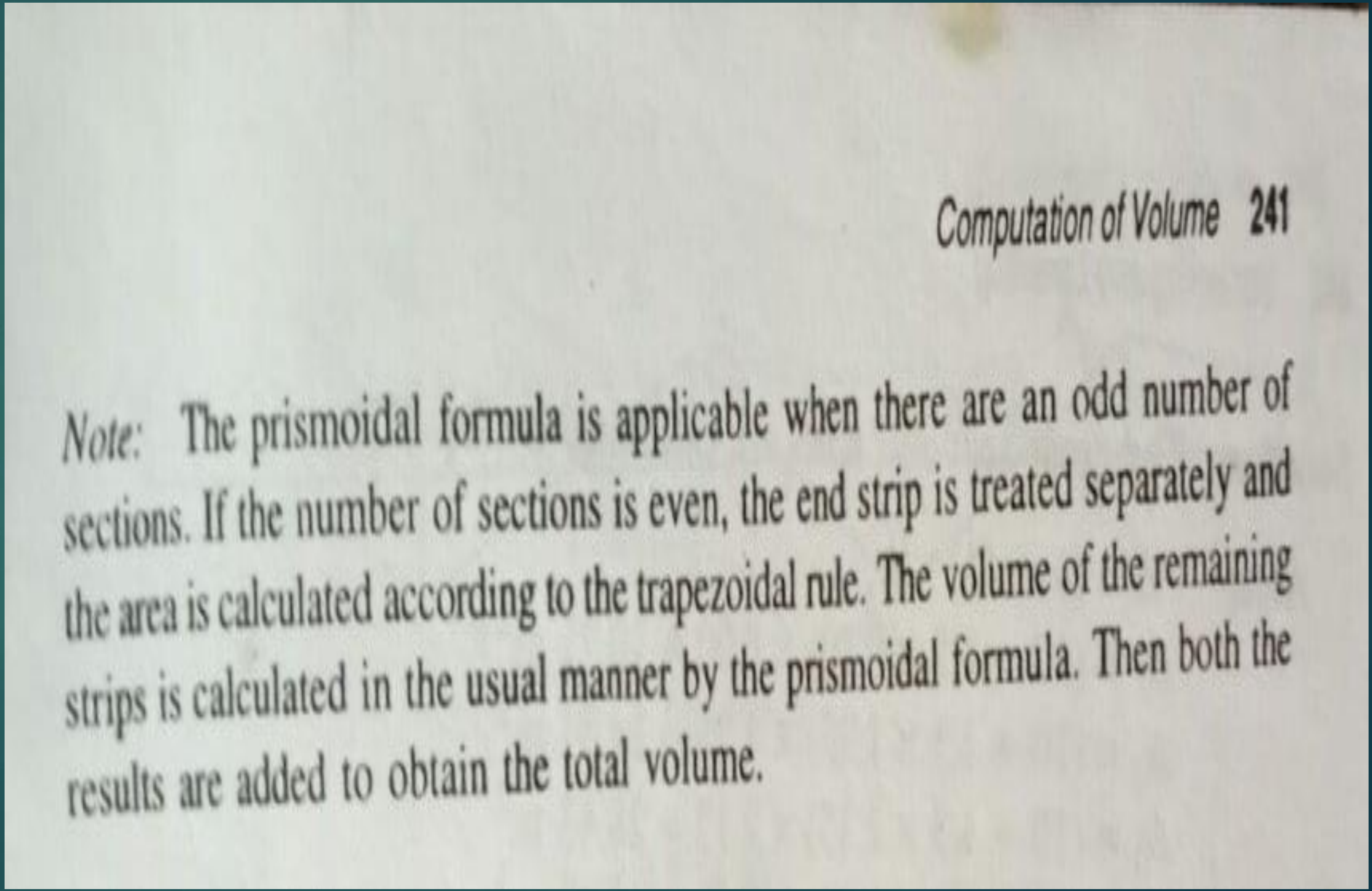
B. Prismoidal Formula

Volume (cutting or filling), $V = \frac{D}{3}\{A_1 + A_n + 4(A_2 + A_4 + A_{n-1}) + 2(A_3 + A_5 + \ldots + A_{n-2})\}$

i.e., $V = \frac{\text{common distance}}{3}$ {Area of 1st section + area of last section + 4 (sum of areas of even sections) + 2 (sum of areas of odd sections)}

Computation of Volume

- **Limitation**



Note: The prismoidal formula is applicable when there are an odd number of sections. If the number of sections is even, the end strip is treated separately and the area is calculated according to the trapezoidal rule. The volume of the remaining strips is calculated in the usual manner by the prismoidal formula. Then both the results are added to obtain the total volume.

Computation of Volume

Problem -02

8.5 WORKED-OUT PROBLEMS

Problem 1 An embankment of width 10 m and side slopes $1\frac{1}{2} : 1$ is required to be made on a ground which is level in a direction transverse to the centre line. The central heights at 40 m intervals are as follows:

0.90, 1.25, 2.15, 2.50, 1.85, 1.35, and 0.85

Calculate the volume of earth work according to (i) the trapezoidal formula and (ii) the prismoidal formula.

Solution The cross-sectional areas are calculated by Eq. (1):

Area, $\Delta = (b + Sh) \times h$

$$\Delta_1 = (10 + 1.5 \times 0.90) \times 0.90 = 10.22 \text{ m}^2$$
$$\Delta_2 = (10 + 1.5 \times 1.25) \times 1.25 = 14.84 \text{ m}^2$$
$$\Delta_3 = (10 + 1.5 \times 2.15) \times 2.15 = 28.43 \text{ m}^2$$
$$\Delta_4 = (10 + 1.5 \times 2.50) \times 2.50 = 34.38 \text{ m}^2$$
$$\Delta_5 = (10 + 1.5 \times 1.85) \times 1.85 = 23.63 \text{ m}^2$$
$$\Delta_6 = (10 + 1.5 \times 1.35) \times 1.35 = 16.23 \text{ m}^2$$
$$\Delta_7 = (10 + 1.5 \times 0.85) \times 0.85 = 9.58 \text{ m}^2$$

(a) Volume according to trapezoidal formula:

$$V = \frac{40}{2}\{10.22 + 9.58 + 2\,(14.84 + 28.43 + 34.38 + 23.63 + 16.23)\}$$
$$= 20\,\{19.80 + 235.02\} = 5{,}096.4 \text{ m}^3$$

(b) Volume calculated in prismoidal formula:

$$V = \frac{40}{3}\{10.22 + 9.58 + 4\,(14.84 + 34.38 + 16.23) + 2\,(28.43 + 23.63)\}$$
$$= \frac{40}{3}\,(19.80 + 261.80 + 104.12) = 5{,}142.9 \text{ m}^3$$

Computation of Volume

Problem -03

Problem 3 The ground level along the centre line of a road is given below:

Chainage (m) —	0	50	100	150	200	250	300
GL (m) —	117.50	116.25	115.95	116.65	117.20	117.85	115.75

It is proposed that the formation level of RL 115.00 should be kept constant of starting from the chainage 'zero'. The formation width of the road is 8 m and the side slope 1 : 1. The ground is level transverse to the centre line.

Solution

Chainage	GL	FL	Cutting
0	117.50	115.00	2.50
50	116.25	115.00	1.25
100	115.95	115.00	0.95
150	116.65	115.00	1.65
200	117.20	115.00	2.20
250	117.85	115.00	2.85
300	115.75	115.00	0.75

Area is calculated according to the equation,

$$A = (b + sh)h$$

where

$$b = 8 \text{ m}$$
$$s = 1$$

and

$$A_1 = (8 + 1 \times 2.50) \times 2.50 = 26.25 \text{ m}^2$$
$$A_2 = (8 + 1 \times 1.25) \times 1.25 = 11.56 \text{ m}^2$$

Computation of Volume

Problem -04

Problem 4 The areas enclosed by the contours in a lake are as follows:

Contour (m)	270	275	280	285	290
Area (m^2)	2,050	8,400	16,300	24,600	31,500

Calculate the volume of water between the contours 270 m and 290 m by: (i) the trapezoidal formula, and (ii) the prismoidal formula.

Solution (a) Volume according to trapezoidal formula

$$= \frac{5}{2}\{2,050 + 31,500 + 2\,(8,400 + 16,300 + 24,600)\}$$

$$= 330,375 \text{ m}^3$$

(b) Volume by prismoidal formula:

$$= \frac{5}{3}\{2,050 + 31,500 + 4\,(8,400 + 24,600) + 2\,(16,300)\}$$

$$= 330,250 \text{ m}^3$$

Computation of Volume

Problem -05

Problem 5 An excavation is to be made for a reservoir 40 m long and 30 m wide at the bottom. The side slope of the excavation has to be 2 : 1. Calculate the volume of earth work if the depth of excavation is 5 m. Assume level ground at the site.

Solution

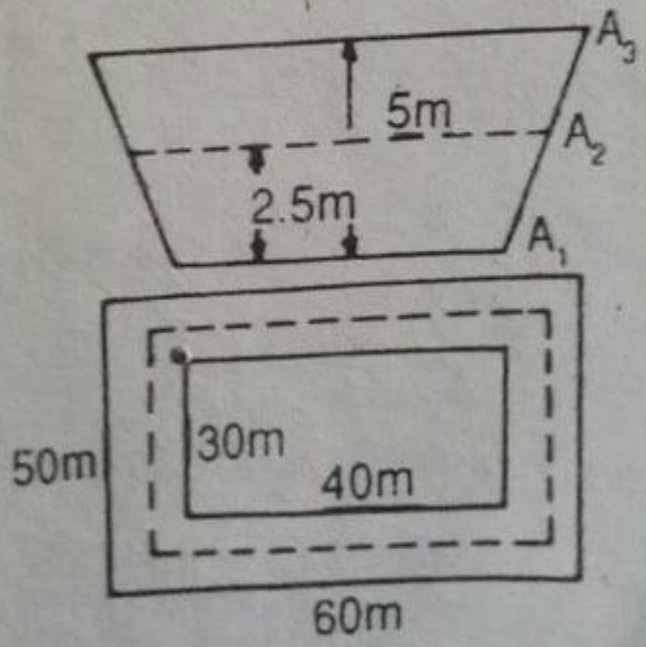


Fig. P.8.2

Computation of Volume

Bottom section: $L = 40$ m $\quad B = 30$ m

$\therefore$ Area $A_1 = 40 \times 30 = 1{,}200$ m^2

Mid-section: $L = b + 2sh = 40 + 2 \times 2 \times 2.5 = 50$ m

$B = 30 + 2 \times 2 \times 2.5 = 40$ m

$\therefore$ Area $A_2 = 50 \times 40 = 2{,}000$ m^2

Top section: $L = 40 + 2 \times 5 = 60$ m

$B = 30 \times 2 \times 2 \times 5 = 50$ m

$\therefore$ Area $A_3 = 60 \times 50 = 3{,}000$ m

Volume according to prismoidal formula $= \frac{2.5}{3}\{1{,}200 + 3{,}000 + 4\,(2{,}000)\}$

$= 10{,}166.66$ m^3

Thank You