CE 103: Surveying

Lecture 11: Volume and area calculation (Contd.)

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Outline

- ❑Comparison of trapezoidal and Simpson’s rule
- ❑Area computed from map
- ❑Level section and two level section
- ❑Prismoidal correction

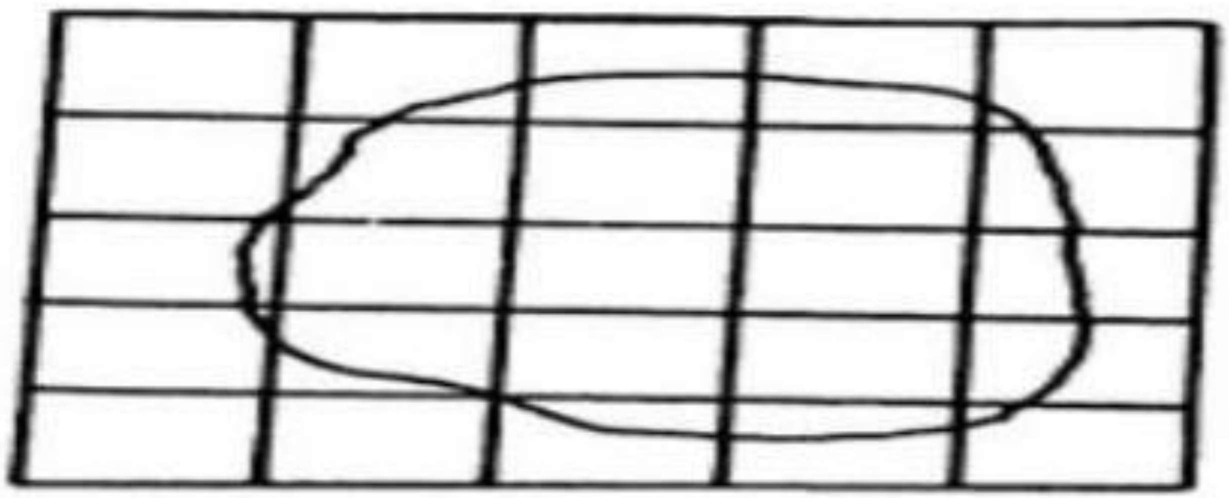
10.1.4 Trapezoidal Rule VS Simpson's Rule

Trapezoidal Rule	Simpson's Rule
The boundary between the ordinates is considered to be straight.	The Boundary between the ordinates is considered to be an arc of a parabola.
There is no limitation. It can be applied for any number of ordinates.	To apply this rule, the number of ordinates must be odd. That is, the number of divisions must be even.
It Gives an approximate Result.	It gives a more accurate result.

10.2 Area Computed from Map measurements

Area can be computed by subdividing the map into some regular geometric figures (squares, trapezoids etc.).

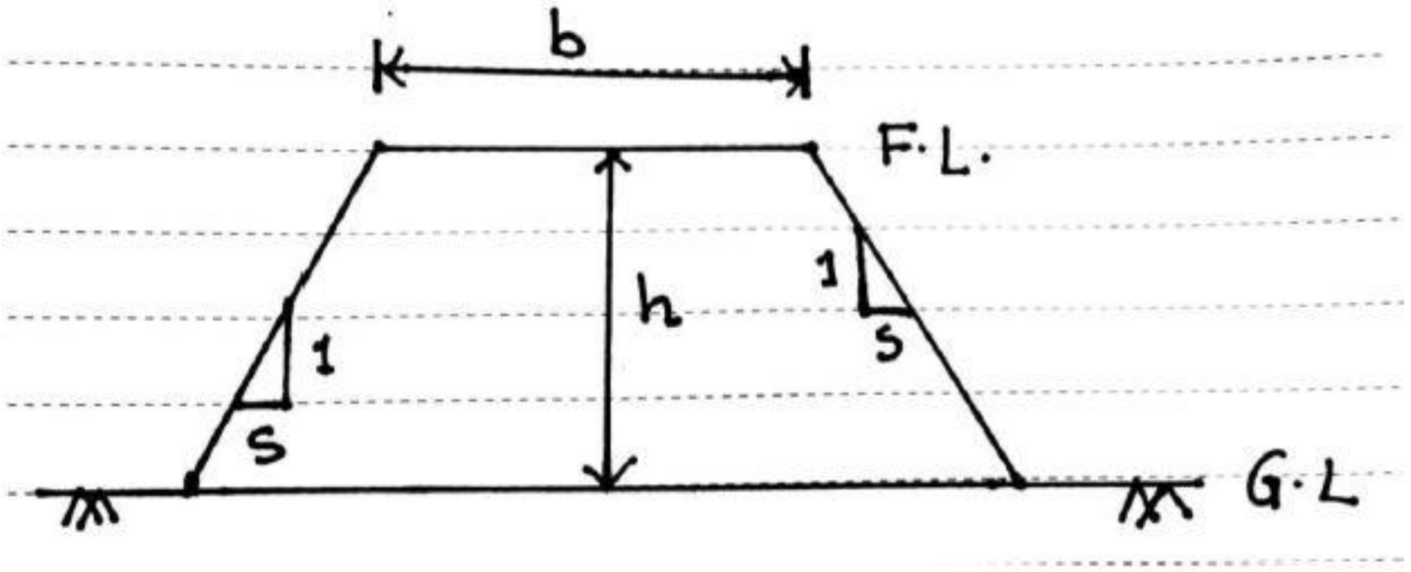
Total Area = Number of squares enclosed in the figure X Area of each square



10.3 Cross Sectional Area

Level Section:

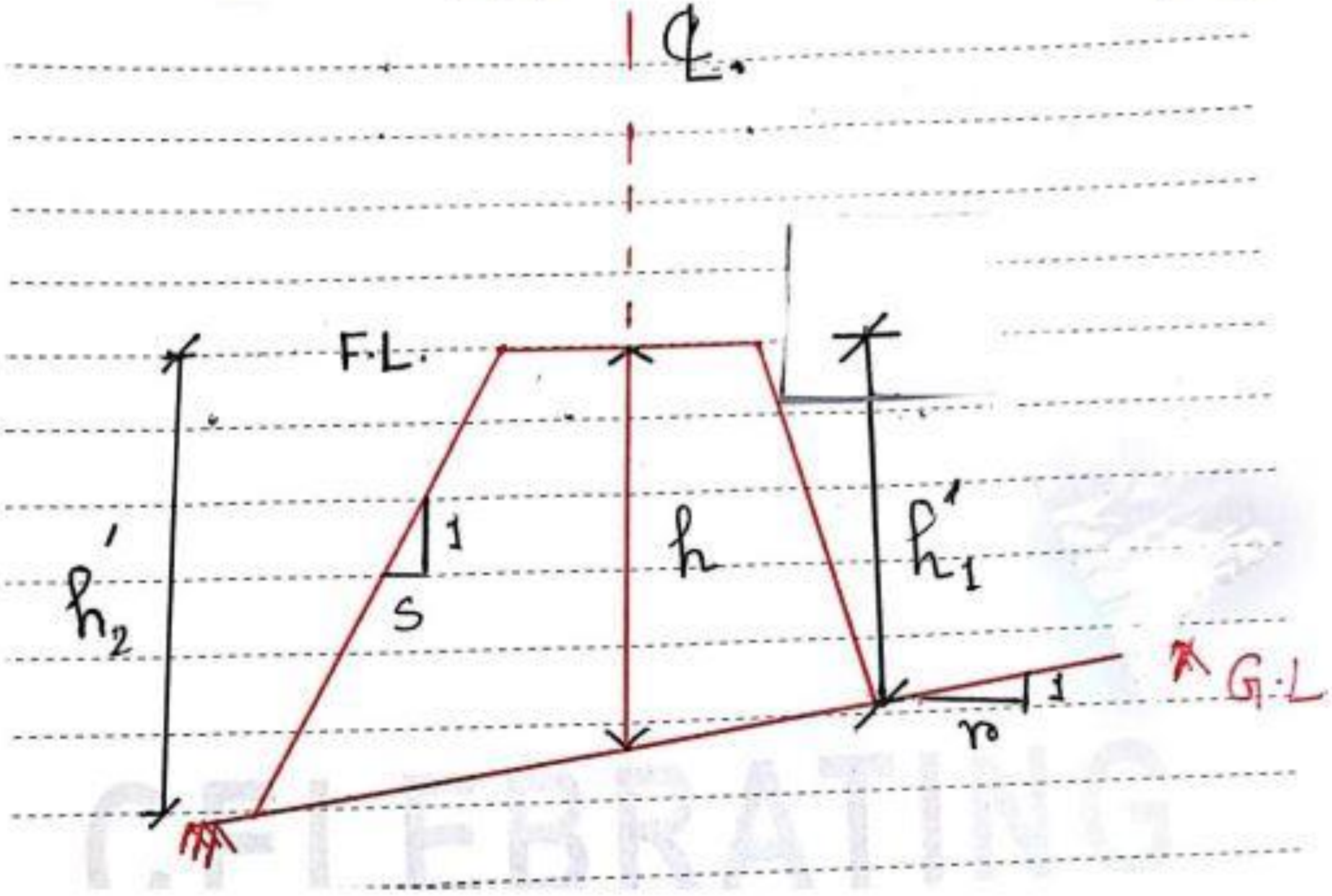
- F.L. and G.L. are both horizontal.
- Area = h (b + sh)



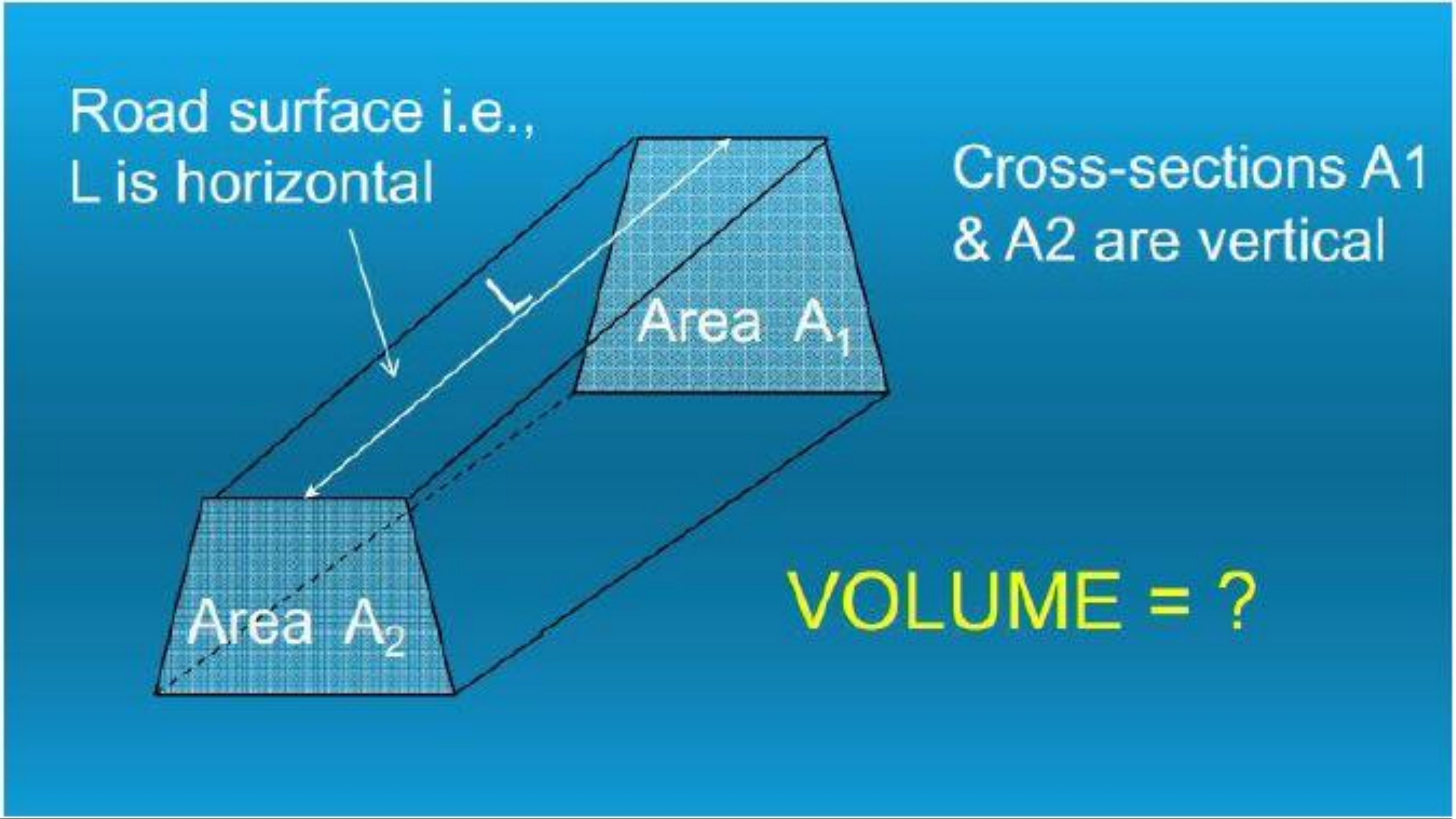
Two Level Section

- G.L. is inclined.
- Area = $\frac{r^2\,(bh)+s\,(0.5b)^2+r^2sh^2}{r^2-s^2}$

where, $h_1' = \frac{rh - 0.5b}{r+s}$ $h_2' = \frac{rh + 0.5b}{r-s}$



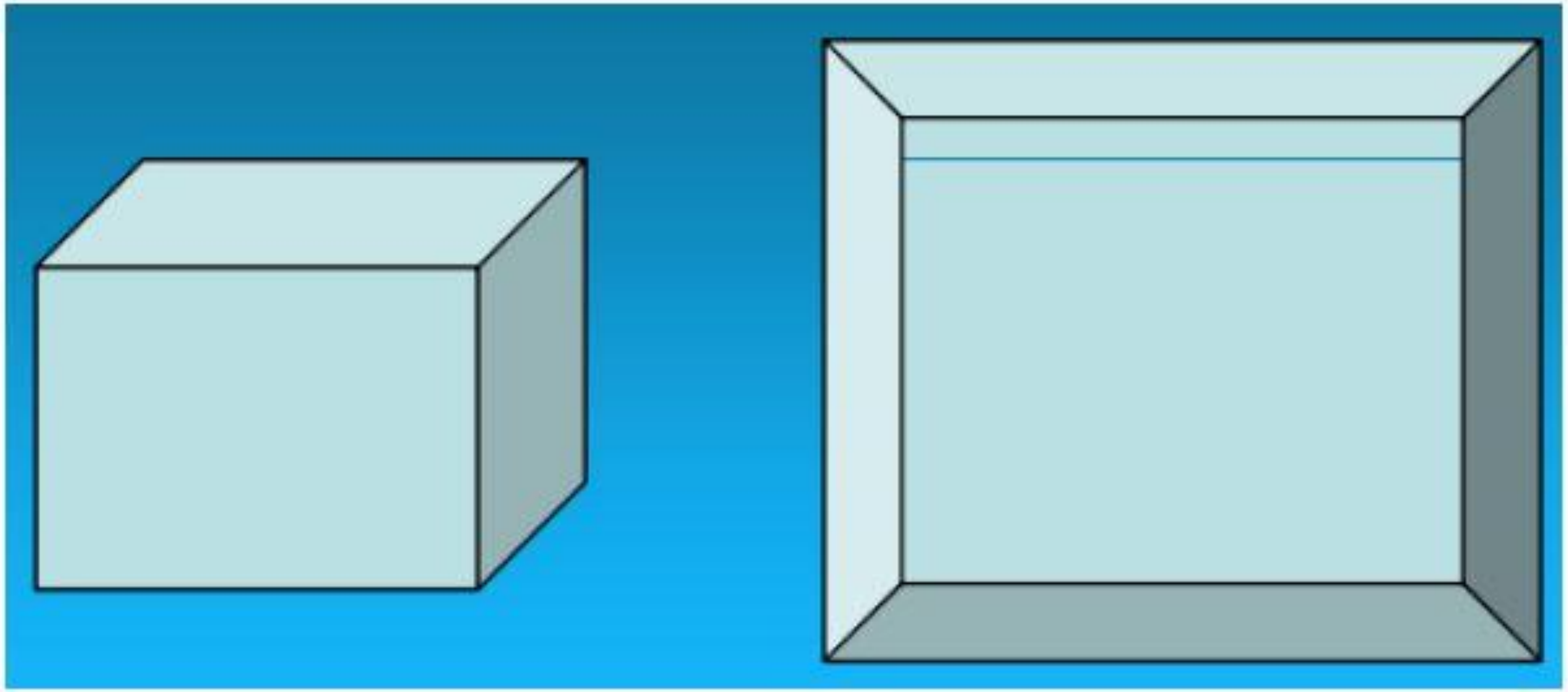
10.4 Volume Calculation



- Apply Prismoidal Rule
- Apply Trapezoidal Rule and apply correction.

Prismoid:

A solid, bounded by two parallel plane ends having the form of polygons, joined by longitudinal faces which are plane surfaces.



Prismoidal Rule

Volume of earth between two successive cross-sections A_1 & A_2 at distance L apart, is considered as prismoid.

$$V_p = \frac{L}{6} (A_1 + 4A_m + A_2)$$

Trapezoidal Rule

where A_m is cross-sectional area midway between A_1 & A_2.

Volume of earth between two successive cross-sections A_1 & A_2 at distance L apart is calculated as:

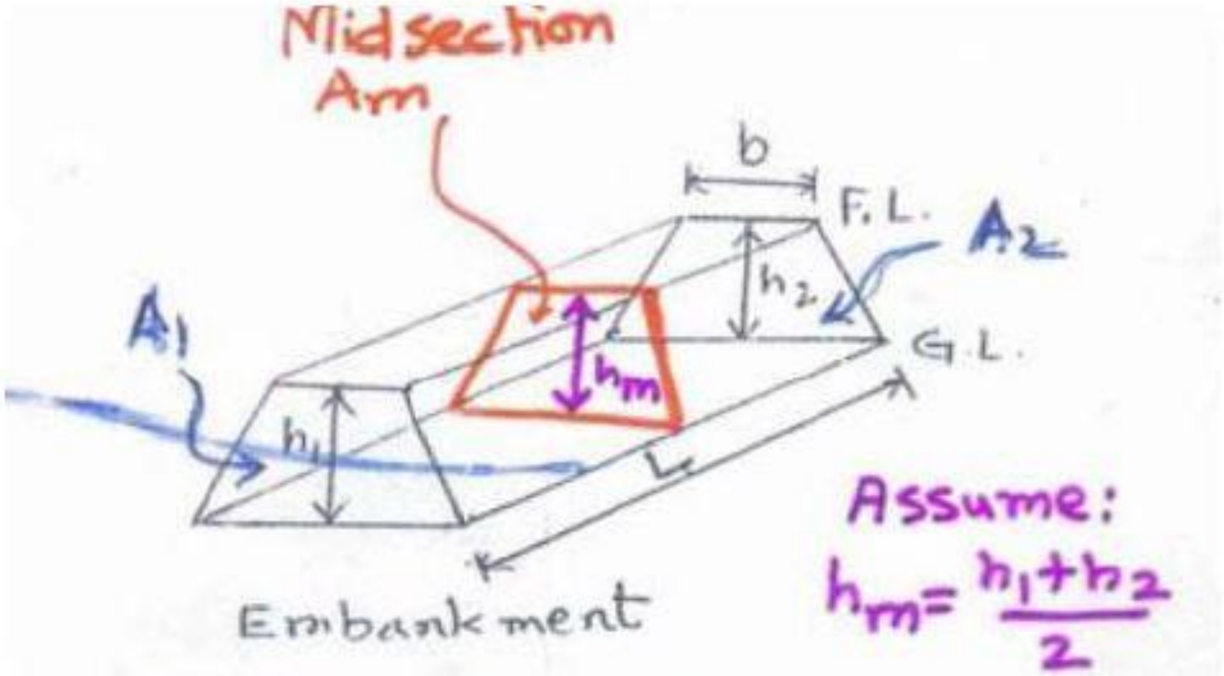
$$V_T = \frac{L}{2}\,(A_1 + A_2)$$

This equation agrees with the prismoidal volume if $A_m = 1/2\ (A_1 + A_2)$.

This formula is applicable for prisms but not applicable for pyramids as for pyramids $A_m \neq 1/2\ (A_1 + A_2)$.

Prismoidal Correction

Prismoidal correction , $C_p = V_T - V_p$



$$A_1 = (b + sh_1)h_1$$

$$A_2 = (b + sh_2)h_2$$

$$A_m = (b + sh_m)h_m$$

$$= b\left(\frac{h_1 + h_2}{2}\right) + s\left(\frac{h_1 + h_2}{2}\right)^2$$

$$V_p = \frac{L}{6}\left[A_1 + 4A_m + A_2\right]$$

$$= \frac{L}{6}\left[(b+sh_1)h_1 + 4(b+sh_m)h_m + (b+sh_2)h_2\right]$$

$$= \frac{L}{6}\left[bh_1 + sh_1^2 + 4\left(b + s\frac{h_1+h_2}{2}\right)\left(\frac{h_1+h_2}{2}\right) + bh_2 + sh_2^2\right]$$

$$= \frac{L}{6}\left[bh_1 + sh_1^2 + 2b(h_1+h_2) + s(h_1+h_2)^2 + bh_2 + sh_2^2\right]$$

$$= \frac{L}{6}\left[b(h_1+h_2) + 2b(h_1+h_2) + s(2h_1^2 + 2h_2^2 + 2h_1h_2)\right]$$

$$= \frac{L}{6}\left[3b(h_1+h_2) + s(2h_1^2 + 2h_2^2 + 2h_1h_2)\right]$$

$$= L\left[b\left(\frac{h_1+h_2}{2}\right) + \frac{s}{3}(h_1^2 + h_2^2 + h_1h_2)\right]$$

$$V_T = \frac{L}{2}\left[A_1 + A_2\right]$$

$$= \frac{L}{2}\left[h_1\,(b + sh_1) + h_2\,(b + sh_2)\right]$$

$$= \frac{L}{2}\left[b\,(h_1 + h_2) + s\,(h_1^2 + h_2^2)\right]$$

$$= L\left[b\left(\frac{h_1 + h_2}{2}\right) + \frac{s}{2}\left(h_1^2 + h_2^2\right)\right]$$

$$C_P = V_T - V_P$$

$$= L\left[\frac{S}{2}\left(h_1^2 + h_2^2\right) - \frac{S}{3}\left(h_1^2 + h_1 h_2 + h_2^2\right)\right]$$

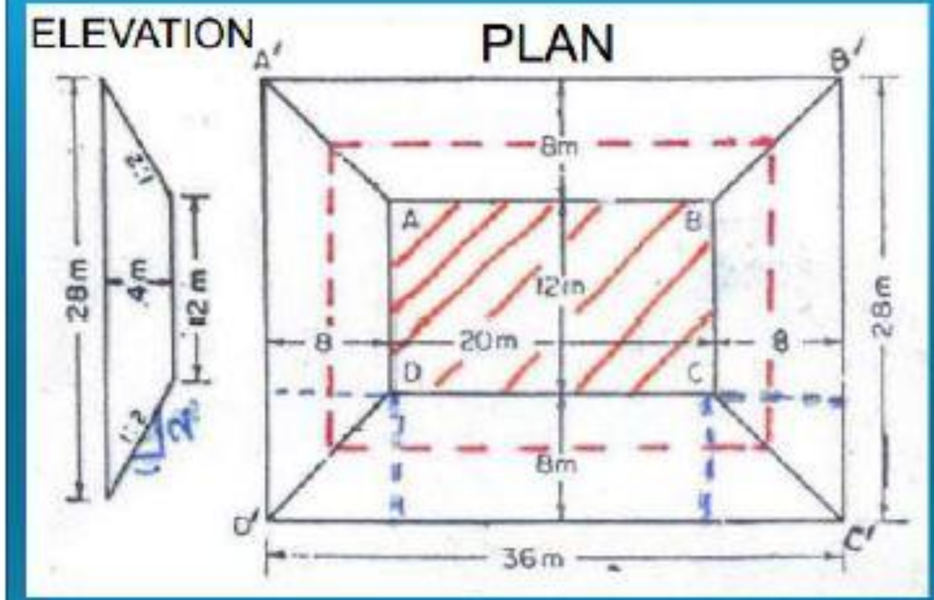
= [illegible]

$$= \frac{Ls}{6}\left[3\left(h_1^2 + h_2^2\right) - 2\left(h_1^2 + h_1 h_2 + h_2^2\right)\right]$$

$$= \frac{Ls}{6}\left[h_1^2 - 2h_1 h_2 + h_2^2\right]$$

$$= \frac{Ls}{6}\left[h_1 - h_2\right]^2$$

Problem 1: Determine Volume of Pond by Prismoidal & Trapezoidal Rule



Top and bottom planes of pond are horizontal & parallel, its volume may be considered as prismoid.

A_{bottom} = 20 x 12 = 240 m^2
A_{top} = 28 x 36 = 1008 m^2
V_T = 4/2 [240+1008]= 2496 m^3

A_{mid} = 28 x 20 = 560 m^2
V_P = 4/6 [240+4x560+1008]
= 2325 m^3

Note: $A_{mid} \neq (A_{bottom} + A_{top})/2$
$V_T > V_P$

Mathematical Problem

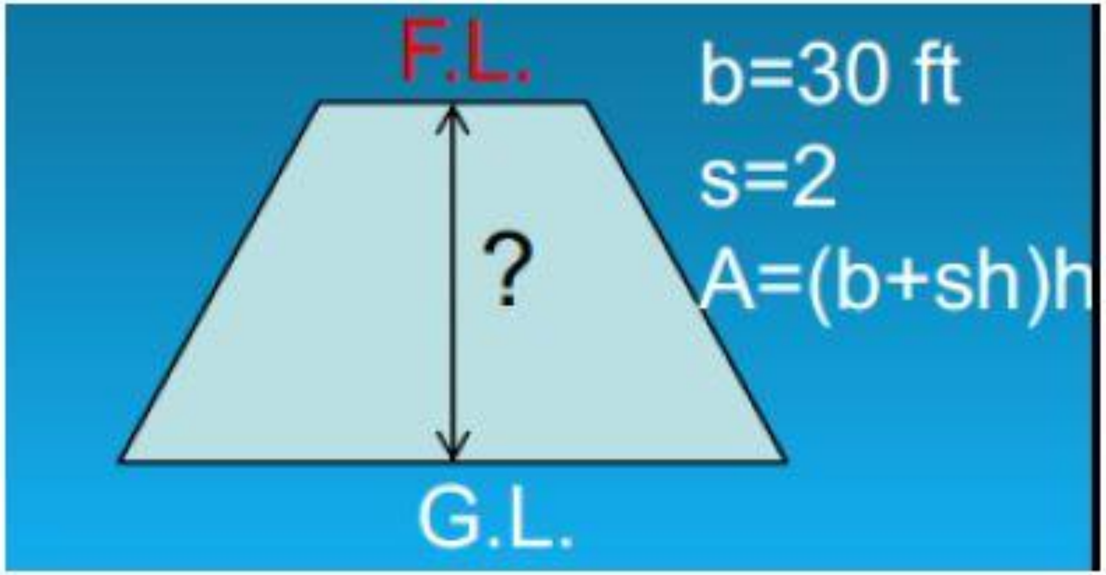
A railway embankment is 300ft long is 30ft wide at the formation level and has a side slope 2 to 1.

Distance (m)	0	100	200	300
Ground Level (ft)	18	13	12	13
Formation Level (ft)	15	15	15	15

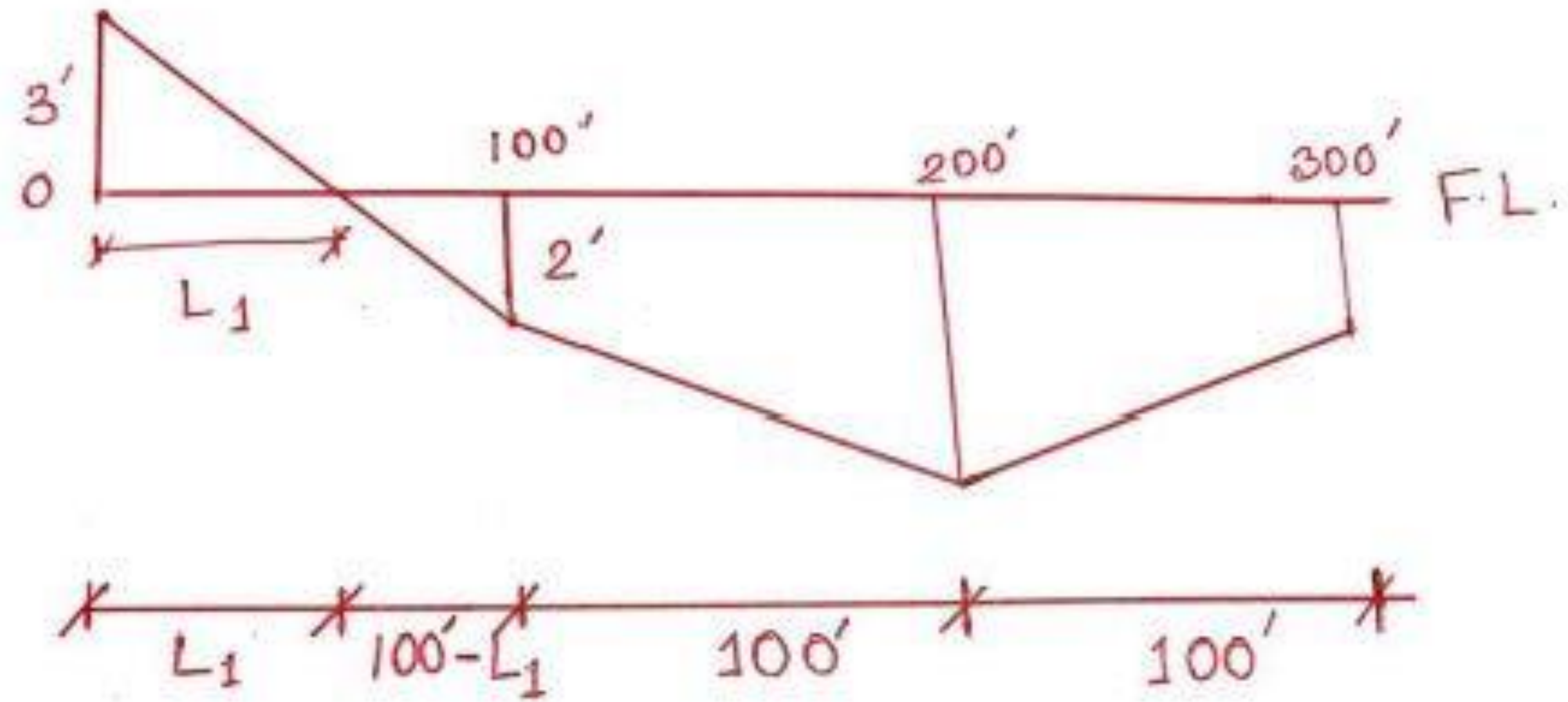
Calculate the volume of earthwork.

Solution:

Here, b = 30ft , s = 2.



Distance (ft)	Ground Level (ft)	Formation Level (ft)	Depth ,h (ft)
0	18	15	3
L_1	15	15	0
100	13	15	2
200	12	15	3
300	13	15	2


3'
0
100'
200'
300'
F.L.
2'
L1
L1
100'-L1
100'
100'

From similar triangle,

$$\frac{3'}{L_1} = \frac{2'}{100 - L_1}$$

$$\Rightarrow 300 - 3L_1 = 2L_1$$

$$\Rightarrow 5L_1 = 300$$

$$\therefore L_1 = 60'$$

Distance (ft)	Ground Level (ft)	Formation Level (ft)	Depth ,h (ft)	Area, A = h(b+sh)	Remarks
0	18	15	3	108	Cut
$L_1 = 60$	15	15	0	0	-
100	13	15	2	68	Fill
200	12	15	3	108	Fill
300	13	15	2	68	Fill

Volume of Cut

$$V_{T(cut)} = \frac{60}{2}\ (108 + 0) = 3240\ ft^3$$

$$C_{P(cut)} = \frac{60\times2}{6}\ (3 - 0)^2 = 180\ ft^3$$

$$V_{P(cut)} = V_T - C_P = 3240\text{-}108 = 3060\ ft^3$$

Volume of Fill

$$V_{T(Fill)} = \frac{40}{2}\ (0 + 68) + \frac{100}{2}(68 + 108) + \frac{100}{2}\ (108 + 68) = 18960\ ft^2$$

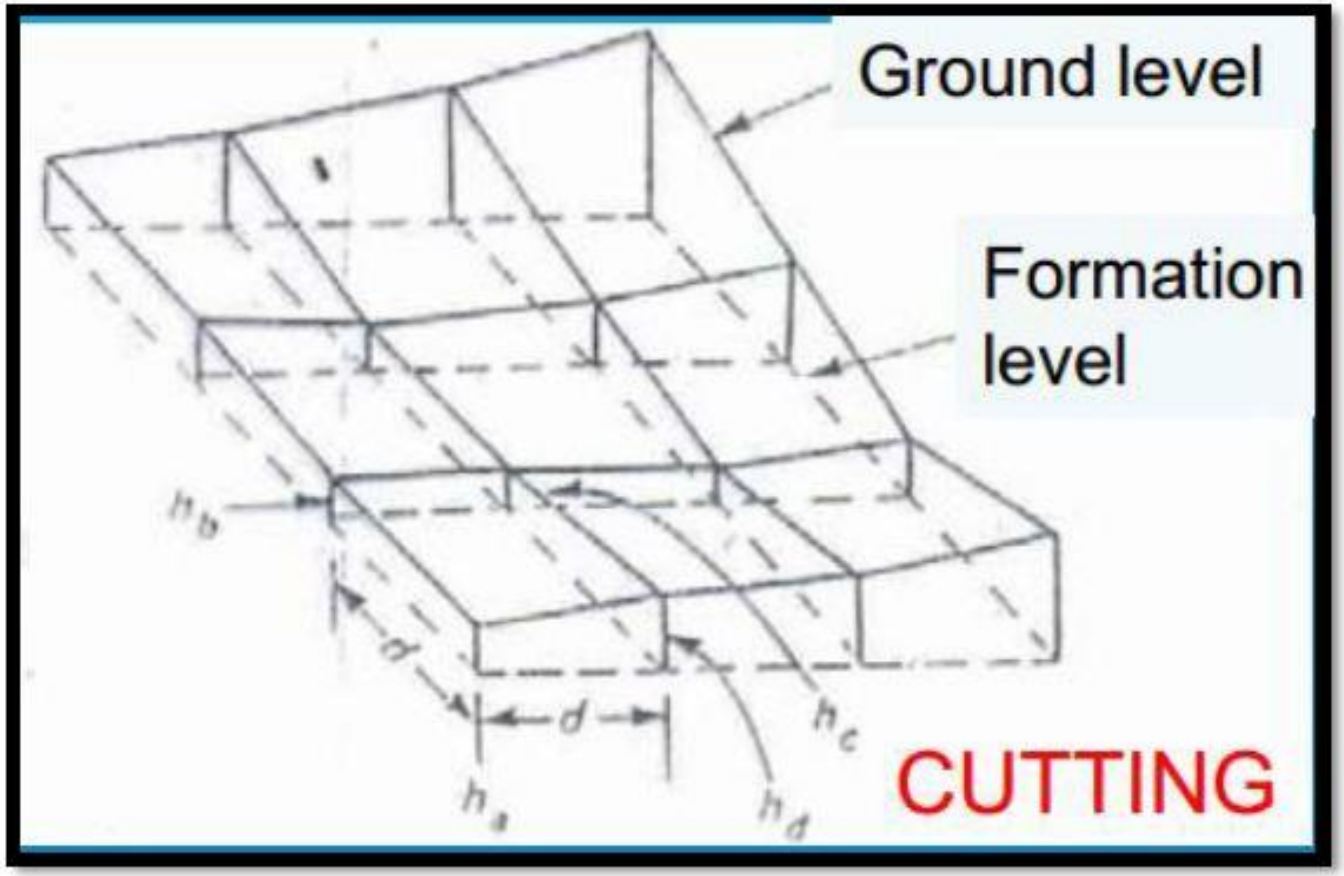
$$C_{P(Fill)} = \frac{40\times2}{6}\ (2 - 0)^2 + \frac{100\times2}{6}\ (2 - 3)^2 + \frac{100\times2}{6}\ (3 - 2)^2$$

$$= 53.33 + 33.33 + 33.33\ ft^2$$

$$= 119.99 ft^2\ \approx 120\ ft^2$$

$V_{P(Fill)} = V_T - C_P = 18960\text{-}120 = 18840\ ft^2$

Volume from spot levels



Volume of square prism = Plan Area X Average depth at four corners.