Circuit Theorms

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- Linearity property
- Superposition.
- Source transformation

Linearity property

A **linear circuit** is one whose output is linearly related (or directly proportional) to its input.

- A linear circuit consists of only linear elements, linear dependent sources, and independent sources.
- In general, a circuit is linear if it is both additive and homogeneous

Two properties to be a linear Circuit

1. Homogenity property(Scaling)
2. Additive property

Linearity property

1. Homogenity property(Scaling)

- $V = iR$
- $V = iR$
- $KV = KiR$

The homogeneity property requires that if the input (also called the excitation) is multiplied by a constant, then the output (also called the response) is multiplied by the same constant.

Linearity property

1. Additive property

- $V_1 = i_1 R$
- $V_2 = i_2 R$
- V = (i_1+i_2)R= V_1+V_2

Resistor is a linear element because the voltage-current relationship satisfies both the homogeneity and the additivity properties.

Linearity property

For the circuit in Fig. 4.2, find I_o when $v_s = 12$ V and $v_s = 24$ V.

Example 4.1

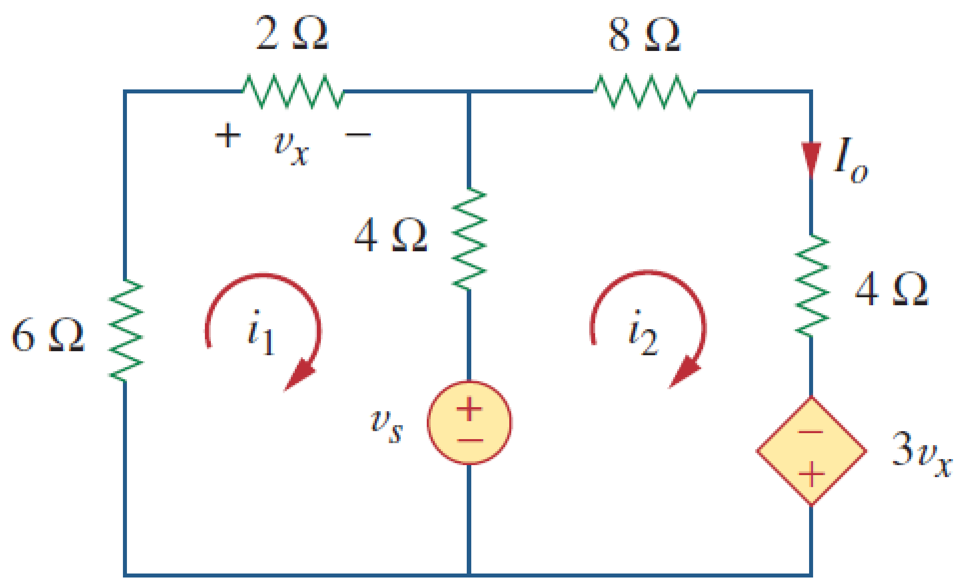


Figure 4.2
For Example 4.1.

- $v_x = 2i_1$
- $I_o = i_2$

Let $v_s = 12V$

Mesh Equation – 1

- $12i_1 - 4i_2 + v_s = 0$
- $12i_1 - 4i_2 + 12 = 0$
- $12i_1 - 4i_2 = -12$

Mesh Equation – 2

- $16i_2 - 4i_1 - v_s - 3v_x = 0$
- $16i_2 - 4i_1 - 12 - 3 \text{ X } 2i_1 = 0$
- $16i_2 - 4i_1 - 12 - 6i_1 = 0$
- $16i_2 - 10i_1 = 12$

Answer

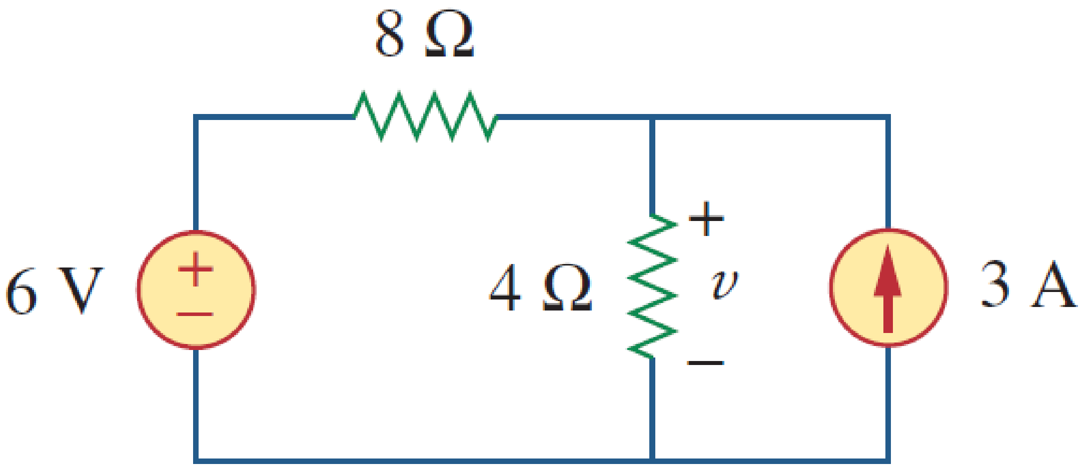
$I_o = i_2 = \mathbf{0.1578A}$

Similiarly for $v_s = 24V$

$I_o = i_2 = \mathbf{0.3157A}$

Superposition Theorm

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.



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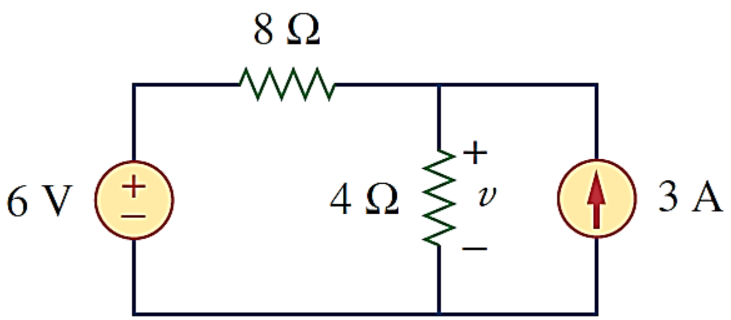
Superposition Theorm

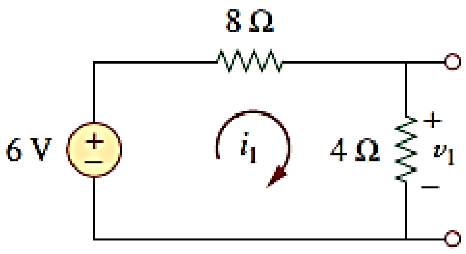
Steps to Apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques before.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Superposition Theorm

Using Superposition theorm determine v.





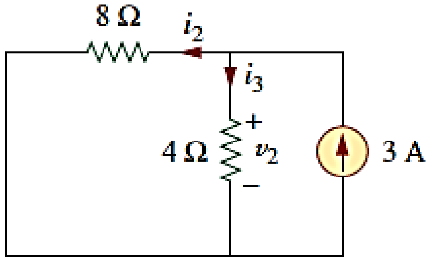
(a)

Thus,

$$v_1 = 4i_1 = 2\text{ V}$$

We may also use voltage division to get v_1 by writing

$$v_1 = \frac{4}{4+8}(6) = 2\text{ V}$$



(b)

Figure 4.7
For Example 4.3: (a) calculating v_1, (b) calculating v_2.

To get v_2, we set the voltage source to zero, as in Fig. 4.7(b). Using current division,

$$i_3 = \frac{8}{4+8}(3) = 2\text{ A}$$

Hence,

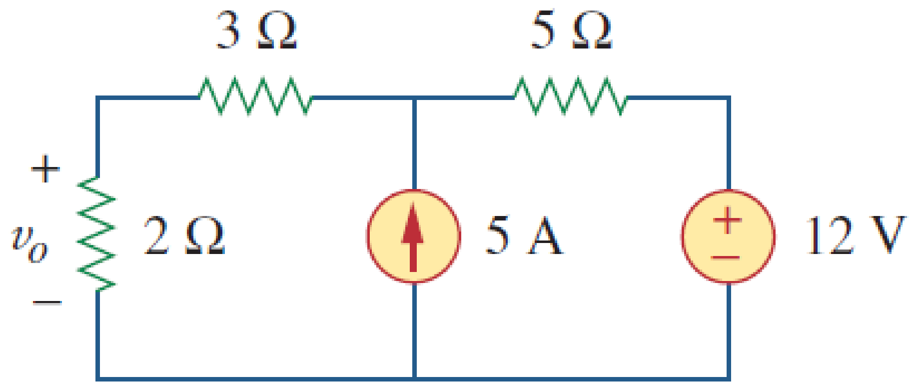
$$v_2 = 4i_3 = 8\text{ V}$$

And we find

$$v = v_1 + v_2 = 2 + 8 = 10\text{ V}$$

Superposition Theorm

Using the superposition theorem, find v_o in the circuit of Fig. 4.8.



Try Yourself

Ans : 7.4 V

$$\frac{5}{10} \times 5 \times 2 + \frac{2}{10} \times 12 = 7.4$$

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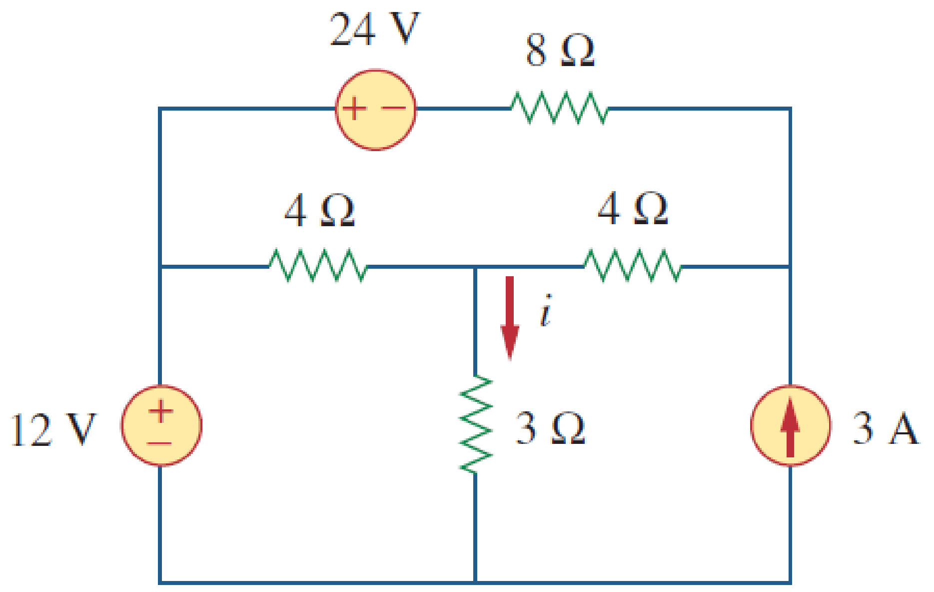
Superposition Theorm

Using Superposition theorm determine i.



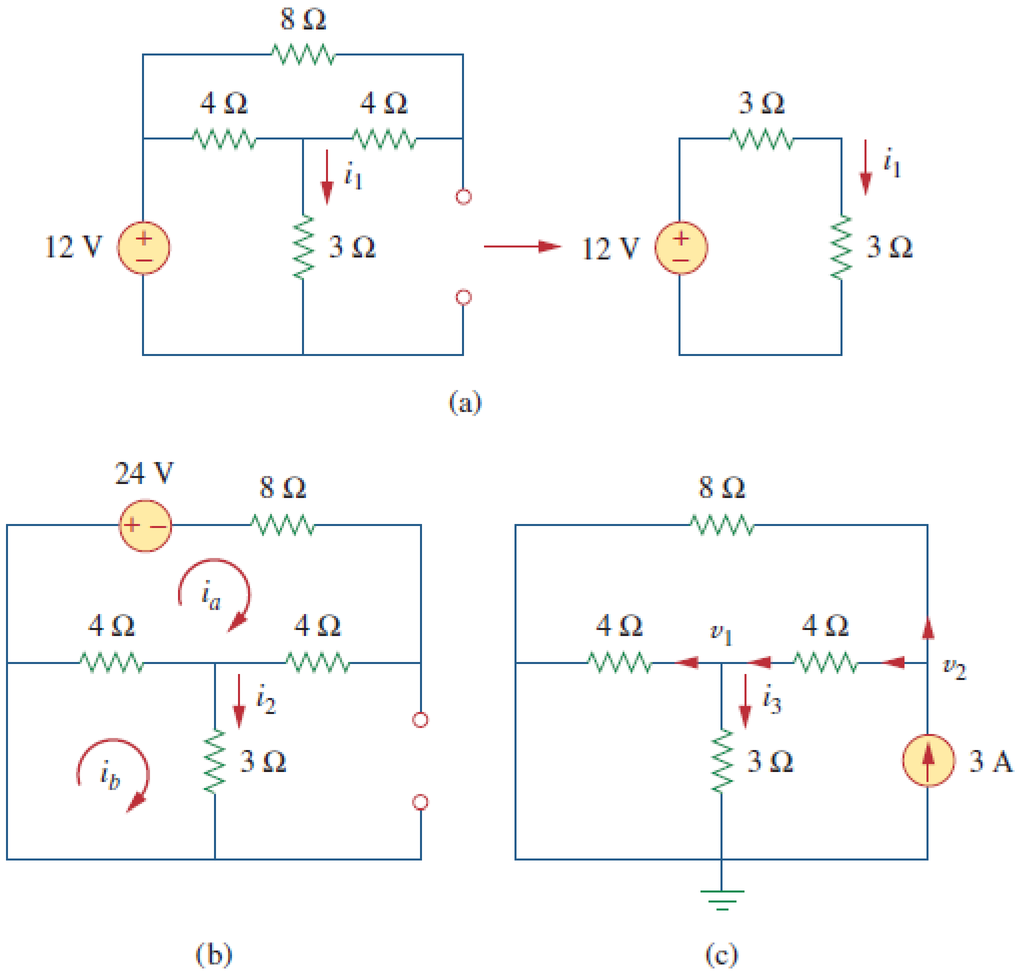
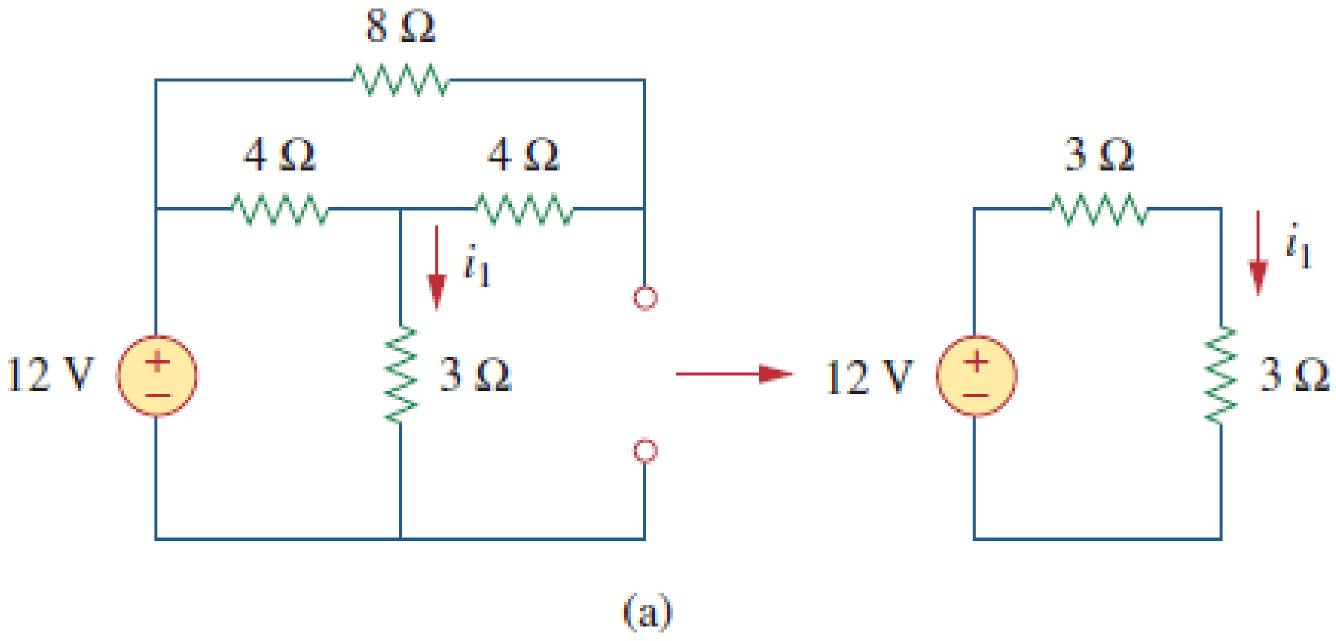


Figure 4.13
For Example 4.5.

Superposition Theorm

In this case, we have three sources. Let

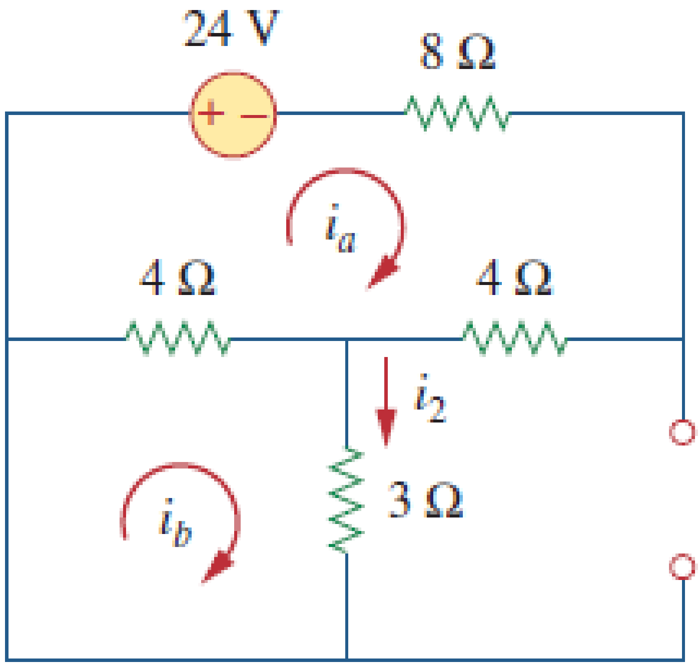
$$i = i_1 + i_2 + i_3$$



$$i_1 = \frac{12}{6} = 2\text{ A}$$

$$R_{eq} = (8 + 4)||4 = 3\Omega$$

Superposition Theorm



(b)

To get i_2, consider the circuit in Fig. 4.13(b). Applying mesh analysis gives

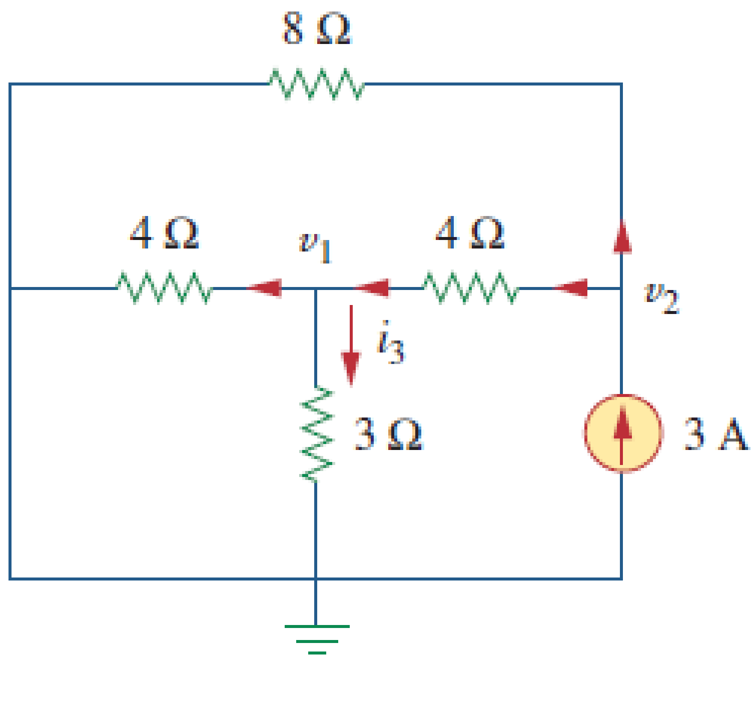
$$16i_a - 4i_b + 24 = 0 \quad \Rightarrow \quad 4i_a - i_b = -6 \qquad \textbf{(4.5.1)}$$

$$7i_b - 4i_a = 0 \quad \Rightarrow \quad i_a = \frac{7}{4}i_b \qquad \textbf{(4.5.2)}$$

Substituting Eq. (4.5.2) into Eq. (4.5.1) gives

$$i_2 = i_b = -1$$

Superposition Theorm



(c)

To get i_3, consider the circuit in Fig. 4.13(c). Using nodal analysis gives

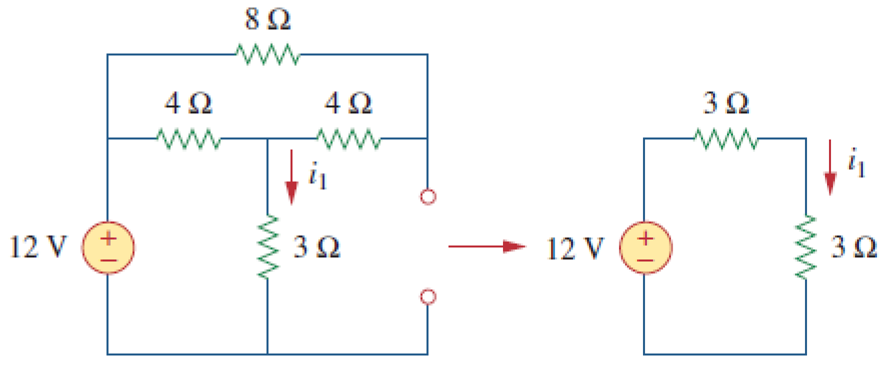
$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Rightarrow \quad 24 = 3v_2 - 2v_1 \qquad (4.5.3)$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Rightarrow \quad v_2 = \frac{10}{3}v_1 \qquad (4.5.4)$$

Substituting Eq. (4.5.4) into Eq. (4.5.3) leads to $v_1 = 3$ and

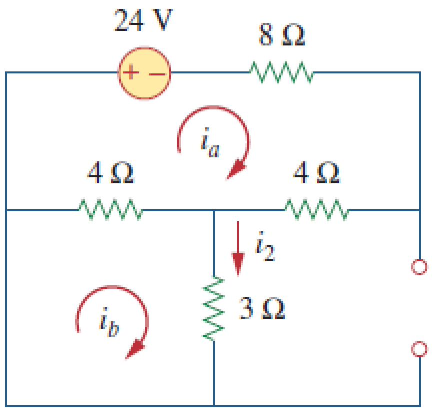
$$i_3 = \frac{v_1}{3} = 1 \text{ A}$$

Superposition Theorm

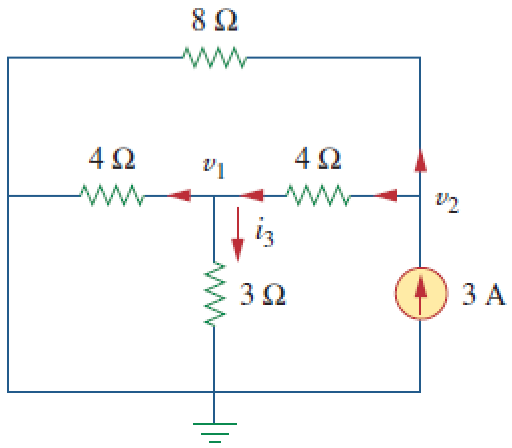


(a)

$$i_1 = 2\ A$$
$$i_2 = -1\ A$$
$$i_3 = 1\ A$$



(b)



(c)

Thus,

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2\ \text{A}$$

Source Transformation

A source transformation is the process of replacing a voltage source *vs* in series with a resistor *R* by a current source *is* in parallel with a resistor *R*, or vice versa.

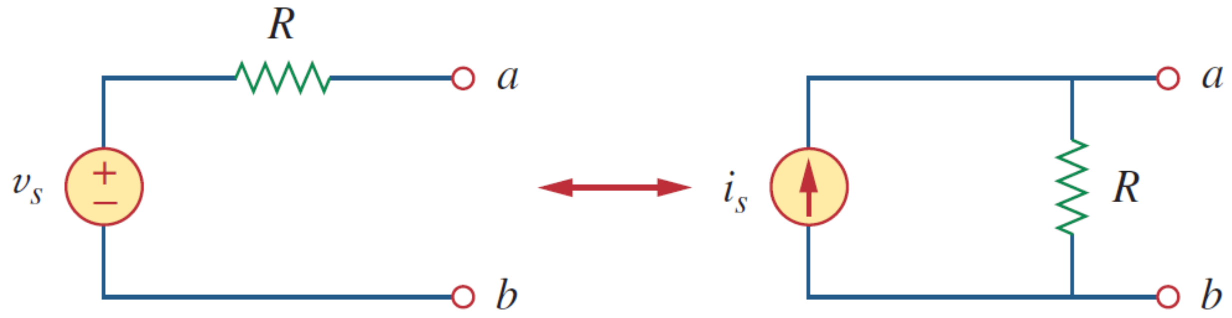


Figure 4.15
Transformation of independent sources

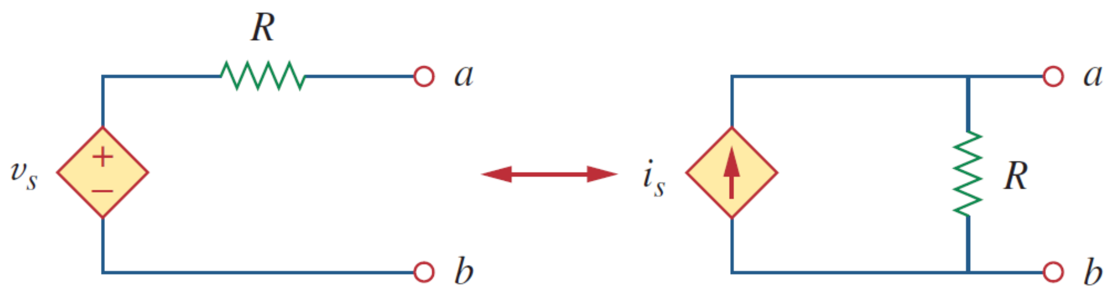


Figure 4.16
Transformation of dependent sources.

$$v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R}$$

Source Transformation

Use source transformation to find v_o in the circuit of Fig. 4.17.

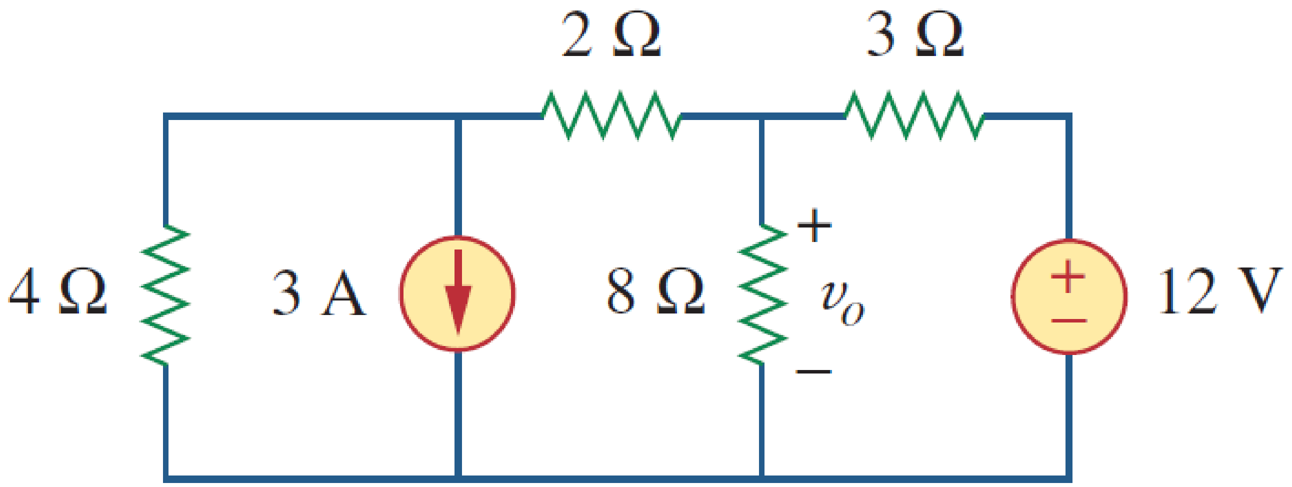


Figure 4.17
For Example 4.6.

Source Transformation

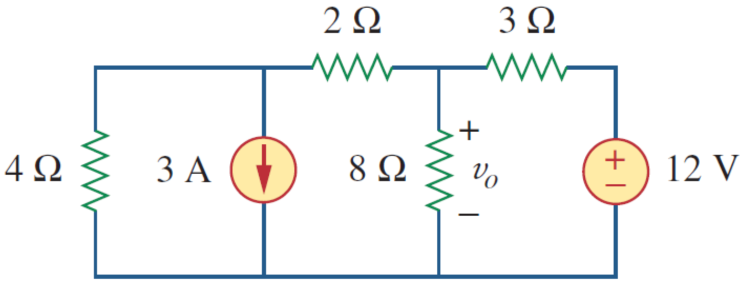
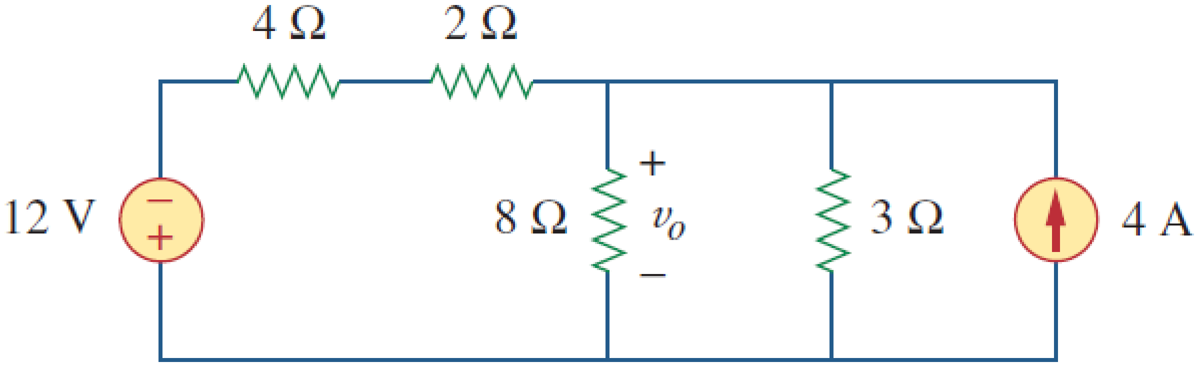
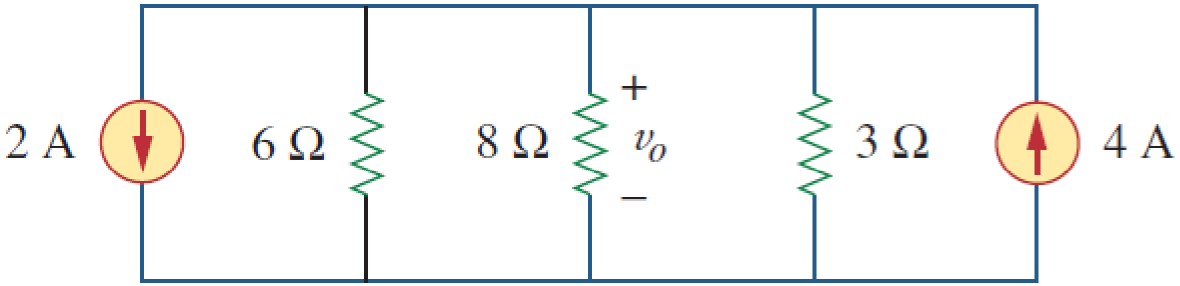


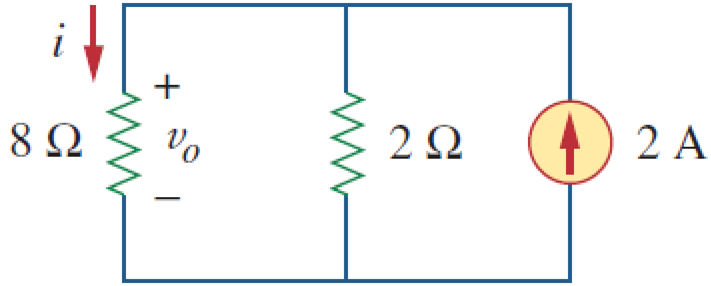
Figure 4.17
For Example 4.6.



(a)

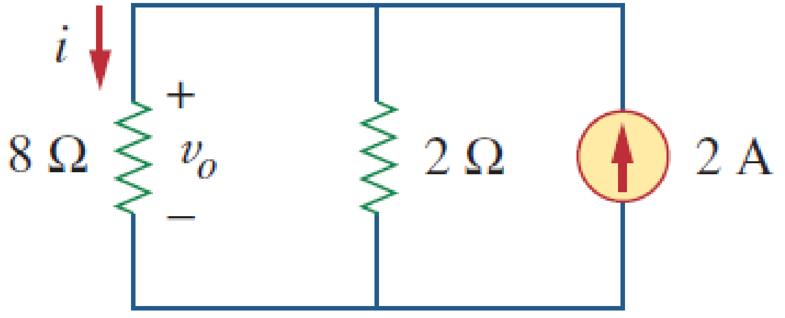


(b)



(c)

Source Transformation



(c)

We use current division in Fig. 4.18(c) to get

$$i = \frac{2}{2 + 8}(2) = 0.4 \text{ A}$$

and

$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

Alternatively, since the 8-Ω and 2-Ω resistors in Fig. 4.18(c) are in parallel, they have the same voltage v_o across them. Hence,

$$v_o = (8 \parallel 2)(2 \text{ A}) = \frac{8 \times 2}{10}(2) = 3.2 \text{ V}$$

Source Transformation

Find v_x in Fig. 4.20 using source transformation.

Example 4.7

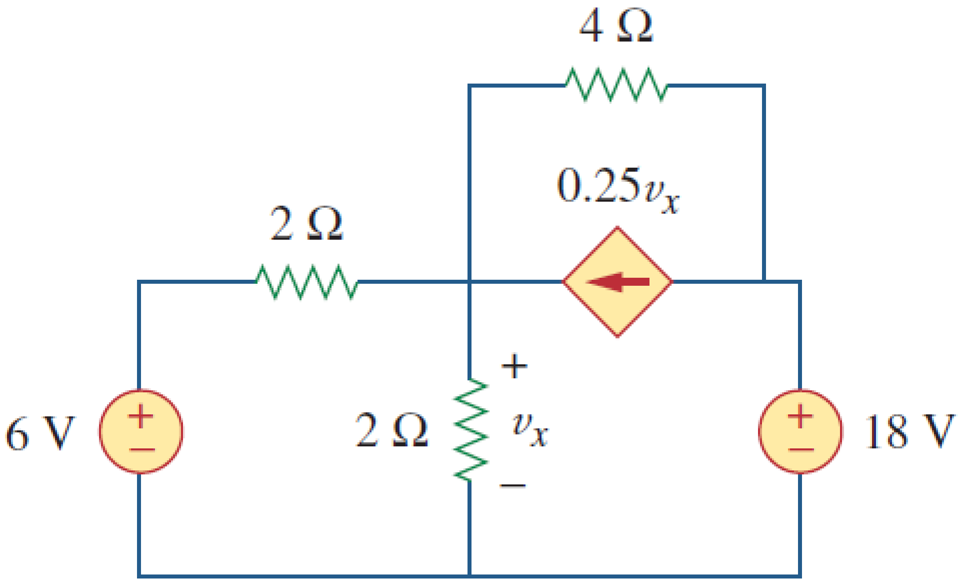
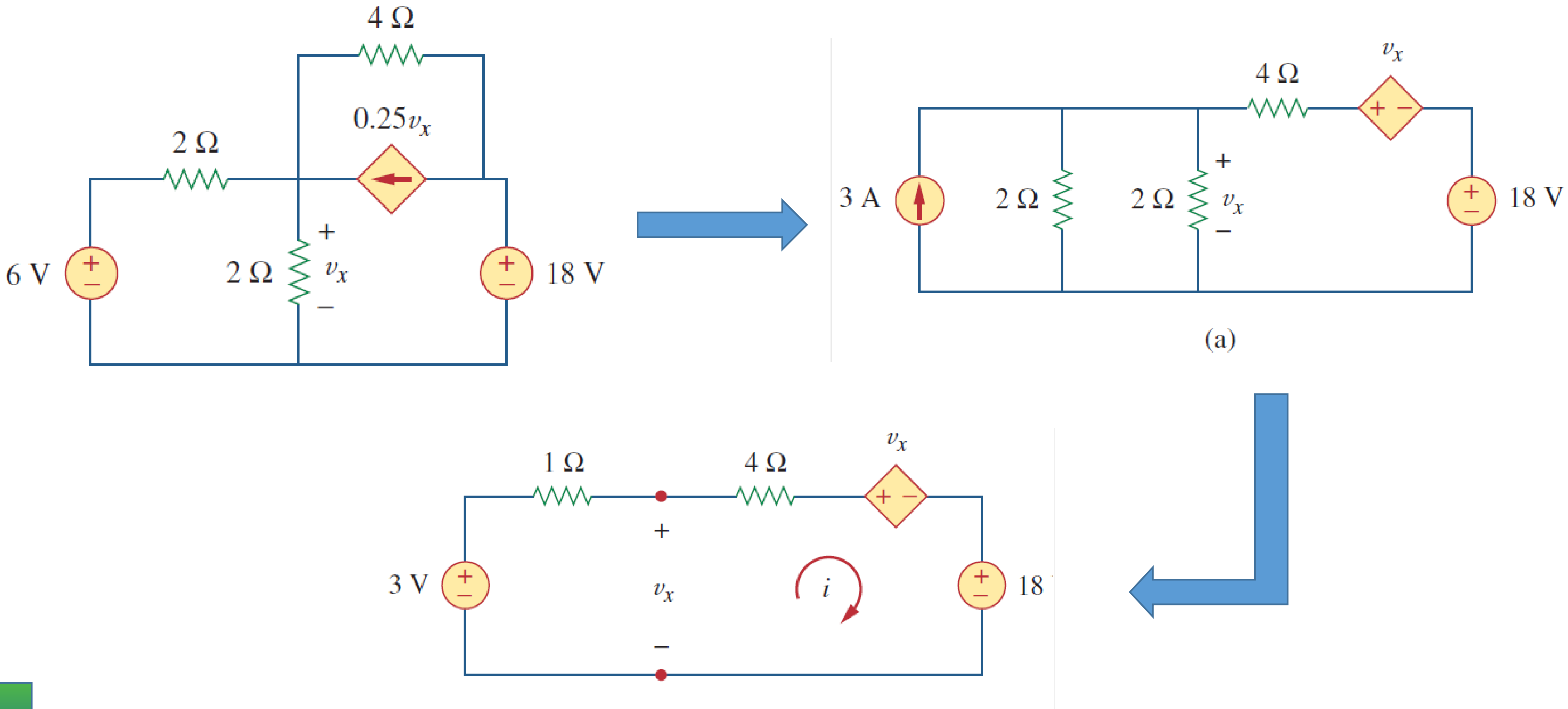


Figure 4.20
For Example 4.7.

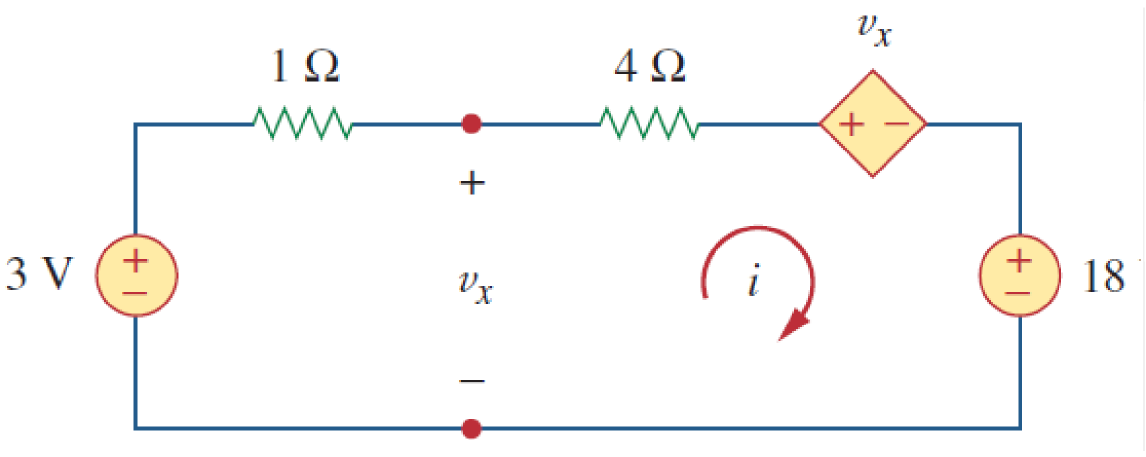
Source Transformation



(a)

(b)

Source Transformation



(b)

$$-3 + 5i + v_x + 18 = 0 \qquad \textbf{(4.7.1)}$$

$$-3 + 1i + v_x = 0 \quad \Rightarrow \quad v_x = 3 - i \qquad \textbf{(4.7.2)}$$

Substituting this into Eq. (4.7.1), we obtain

$$15 + 5i + 3 - i = 0 \quad \Rightarrow \quad i = -4.5 \text{ A}$$

Thus, $v_x = 3 - i = 7.5$ V.

Thank You