## Spatial Descriptions and Transformations Related Exercise and Solution

## Matrix Multiplication

$$
\left[\begin{array}{l}
a b \\
c d
\end{array}\right]\left[\begin{array}{c}
e f \\
g h
\end{array}\right]=\left[\begin{array}{ll}
a e+b g & a f+b h \\
c e+d g & c f+d h
\end{array}\right]
$$

$$
\begin{gathered}
{\left[\begin{array}{cccc}
a x_{1} & a x_{2} & a x_{3} & a x_{4} \\
a y_{1} & a y_{2} & a y_{3} & a y_{4} \\
a z_{1} & a z_{2} & a z_{3} & a z_{4} \\
a w_{1} & a w_{2} & a w_{3} & a w_{4}
\end{array}\right] \times\left[\begin{array}{llll}
b x_{1} & b x_{2} & b x_{3} & b x_{4} \\
b y_{1} & b y_{2} & b y_{3} & b y_{4} \\
b z_{1} & b z_{2} & b z_{3} & b z_{4} \\
b w_{1} & b w_{2} & b w_{3} & b w_{4}
\end{array}\right]} \\
=
\end{gathered}
$$

$$
\left[\begin{array}{rr}
a x_{1} * b x_{1}+a x_{2} * b y_{1}+a x_{3} * b z_{1}+a x_{4} * b w_{1} & a x_{1} * b x_{2}+a x_{2} * b y_{2}+a x_{3} * b z_{2}+a x_{4} * b w_{2} \\
a y_{1} * b x_{1}+a y_{2} * b y_{1}+a y_{3} * b z_{1}+a y_{4}{ }^{*} * b w_{1} & a y_{1} * b x_{2}+a y_{2} * b y_{2}+a y_{3} * b z_{2}+a y_{4} * b w_{2} \\
a z_{1} * b x_{1}+a z_{2} * b y_{1}+a z_{3} * b z_{1}+a z_{4} * b w_{1} & a z_{1} * b x_{2}+a z_{2} * b y_{2}+a z_{3} * b z_{2}+a z_{4} * b w_{2} \\
a w_{1} * b x_{1}+a w_{2} * b y_{1}+a w_{3} * b z_{1}+a w_{4} * b w_{1} & a w_{1} * b x_{2}+a w_{2} * b y_{2}+a w_{3} * b z_{2}+a w_{4} * b w_{2} \\
a x_{1} * b x_{3}+a x_{2} * b y_{3}+a x_{3} * b z_{3}+a x_{4} * b w_{3} & a x_{1} * b x_{4}+a x_{2} * b y_{4}+a x_{3} * b z_{4}+a x_{4} * b w_{4} \\
a y_{1} * b x_{3}+a y_{2} * b y_{3}+a y_{3} * b z_{3}+a y_{4} * b w_{3} & a y_{1} * b x_{4}+a y_{2} * b y_{4}+a y_{3} * b z_{4}+a y_{4} * b w_{4} \\
a z_{1} * b x_{3}+a z_{2} * b y_{3}+a z_{3} * b z_{3}+a z_{4} * b w_{3} & a z_{1} * b x_{4}+a z_{2} * b y_{4}+a z_{3} * b z_{4}+a z_{4} * b w_{4} \\
a w_{1} * b x_{3}+a w_{2} * b y_{3}+a w_{3} * b z_{3}+a w_{4} * b w_{3} & a w_{1} * b x_{4}+a w_{2} * b y_{4}+a w_{3} * b z_{4}+a w_{4} * b w_{4}
\end{array}\right]
$$

## Exercise and Solution

Q1: A frame Fnoa is located in the position P.After the following transformation the frame position has changed to $Q[2,5,7] T$. A rotation along $Z$ axis by anti-clock 45 degree but before that a translation along all axis by $[2,3,5$ ]. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by $[3,-5,3]$. Find the position P with respect to Q .
Solution : $[\mathrm{T}(3,-5,3)$ * $\operatorname{Rot} \mathbf{Y}(60)$ * $\operatorname{Rot} Z(45)$ * $\mathrm{T}(2,3,5)]-1$ * $\mathrm{Q}[2,5,7]=\mathbf{P}$
Q2: A frame Fnoa is located in the position P.After the following transformation the frame position has changed to $\mathrm{Q}[2,5,7] \mathrm{T}$. A rotation along X axis by 60 degree but before that a translation along all axis by [ $2,3,5]$. After those two, another rotation along Y axis by 45 degree followed by a translation along all axis by $[3,-5,3]$. Find the position $P$ with respect to Q .
Solution: [T(3,-5,3) * $\operatorname{RotY}(45)$ * $\operatorname{RotX}(60)$ * $\mathrm{T}(2,3,5)]-1$ * $\mathrm{Q}[2,5,7]=P$
Q3: A frame Fnoa is located in the position P.After the following transformation the frame position has changed to $\mathrm{Q}[2,5,7]$ T. A rotation along $Z$ axis by anti-clock 30 degree but before that a translation along all axis by $[2,3,5]$. After those two, another rotation along Y axis by 45 degree followed by a translation along all axis by $[3,-5,3]$. Find the position $P$ with respect to $Q$.
Solution: [ T(3,-5,3) * RotY(45) * RotZ(30) *T(2,3,5) ]-1 * $\mathbf{Q}[2,5,7]=\mathbf{P}$
Q4: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to $Q[2,5,7]$ T. A rotation along $X$ axis by 45 degree but before that a translation along all axis by [ $2,3,5]$. After those two, another rotation along Y axis by 30 degree followed by a translation along all axis by $[3,-5,3]$. Find the position P with respect to Q .
Solution: [ $\mathrm{T}(3,-5,3)$ * $\operatorname{Rot} \mathrm{Y}(30)$ * $\operatorname{Rot} \mathrm{X}(45)$ * $\mathrm{T}(2,3,5) \mathrm{J}-1$ * $\mathrm{Q}[2,5,7]=\mathrm{P}$

## Solution

Q1: A frame Fnoa is located in the position P.After the following transformation the frame position has changed to $\mathrm{Q}[2,5,7]$ T. A rotation along $Z$ axis by anti-clock 45 degree but before that a translation along all axis by $[2,3,5]$. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by $[3,-5,3]$. Find the position P with respect to Q .

$$
\begin{aligned}
& \text { Solution : }\left[T(3,-5,3){ }^{*} \operatorname{Rot} Y(60){ }^{*} \operatorname{RotZ}(45){ }^{*} \mathrm{~T}(2,3,5)\right]-1 \text { * } \mathrm{Q}[2,5,7]=\mathbf{P} \\
& \operatorname{RotZ}(45) \text { * } \mathrm{T}(2,3,5)= \\
& {\left[\begin{array}{cccc}
\cos (45) & -\sin (45) & 0 & 0 \\
\sin (45) & \cos (45) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \times\left[\begin{array}{llll}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & 3 \\
0 & 0 & 1 & 5 \\
0 & 0 & 0 & 1
\end{array}\right]=\left[\begin{array}{cccc}
0.707 & -0.707 & 0 & 0 \\
0.707 & 0.707 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \times\left[\begin{array}{llll}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & 3 \\
0 & 0 & 1 & 5 \\
0 & 0 & 0 & 1
\end{array}\right]=}
\end{aligned}
$$

$[0.707 \times 1+-0.707 \times 0+0 \times 0+0 \times 00.707 \times 0+-0.707 \times 1+0 \times 0+0 \times 00.707 \times 0+-0.707 \times 0+0 \times 1+0 \times 00.707 \times 2+-0.707 \times 3+0 \times 5+0 \times 17$ $0.707 \times 1+0.707 \times 0+0 \times 0+0 \times 0 \quad 0.707 \times 0+0.707 \times 1+0 \times 0+0 \times 0 \quad 0.707 \times 0+0.707 \times 0+0 \times 1+0 \times 0 \quad 0.707 \times 2+0.707 \times 3+0 \times 5+0 \times 1$ $0 \times 1+0 \times 0+1 \times 0+0 \times 0 \quad 0 \times 0+0 \times 1+1 \times 0+0 \times 0 \quad 0 \times 0+0 \times 0+1 \times 1+0 \times 0 \quad 0 \times 2+0 \times 3+1 \times 5+0 \times 1$
$0 \times 1+0 \times 0+0 \times 0+1 \times 0 \quad 0 \times 0+0 \times 1+0 \times 0+1 \times 0$
$0 \times 0+0 \times 0+0 \times 1+1 \times 0$
$0 \times 2+0 \times 3+0 \times 5+1 \times 1$

$$
=\left[\begin{array}{cccc}
0.707 & -0.707 & 0 & -0.707 \\
0.707 & 0.707 & 0 & 3.535 \\
0 & 0 & 1 & 5 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

## Solution

Q1: A frame Fnoa is located in the position P.After the following transformation the frame position has changed to $\mathrm{Q}[2,5,7] \mathrm{T}$. A rotation along $Z$ axis by anti-clock 45 degree but before that a translation along all axis by $[2,3,5]$. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by $[3,-5,3]$. Find the position P with respect to Q .

## Solution : $\left[\mathbf{T}(3,-5,3)\right.$ * $\left.\operatorname{Rot} Y(60){ }^{*} \operatorname{RotZ}(45){ }^{*} \mathrm{~T}(2,3,5)\right]-1$ * $\mathbf{Q}[2,5,7]=\mathbf{P}$

$\mathrm{T}(3,-5,3)$ * $\operatorname{Rot}(60)=$

$$
\left[\begin{array}{cccc}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & -5 \\
0 & 0 & 1 & 3 \\
0 & 0 & 0 & 1
\end{array}\right] \times\left[\begin{array}{cccc}
\cos (60) & 0 & \sin (60) & 0 \\
0 & 1 & 0 & 0 \\
-\sin (60) & 0 & \cos (60) & 0 \\
0 & 0 & 0 & 1
\end{array}\right]=\left[\begin{array}{cccc}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & -5 \\
0 & 0 & 1 & 3 \\
0 & 0 & 0 & 1
\end{array}\right] \times\left[\begin{array}{cccc}
0.5 & 0 & 0.866 & 0 \\
0 & 1 & 0 & 0 \\
-0.866 & 0 & 0.5 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]=
$$

$$
\left[\begin{array}{cccc}
1 \times 0.5+0 \times 0+0 \times-0.866+3 \times 0 & 1 \times 0+0 \times 1+0 \times 0+3 \times 0 & 1 \times 0.866+0 \times 0+0 \times 0.5+3 \times 0 & 1 \times 0+0 \times 0+0 \times 0+3 \times 1 \\
0 \times 0.5+1 \times 0+0 \times-0.866+-5 \times 0 & 0 \times 0+1 \times 1+0 \times 0+-5 \times 0 & 0 \times 0.866+1 \times 0+0 \times 0.5+-5 \times 0 & 0 \times 0+1 \times 0+0 \times 0+-5 \times 1 \\
0 \times 0.5+0 \times 0+1 \times-0.866+3 \times 0 & 0 \times 0+0 \times 1+1 \times 0+3 \times 0 & 0 \times 0.866+0 \times 0+1 \times 0.5+3 \times 0 & 0 \times 0+0 \times 0+1 \times 0+3 \times 1 \\
0 \times 0.5+0 \times 0+0 \times-0.866+1 \times 0 & 0 \times 0+0 \times 1+0 \times 0+1 \times 0 & 0 \times 0.866+0 \times 0+0 \times 0.5+1 \times 0 & 0 \times 0+0 \times 0+0 \times 0+1 \times 1
\end{array}\right]
$$

$$
=\left[\begin{array}{cccc}
0.5 & 0 & 0.866 & 3 \\
0 & 1 & 0 & -5 \\
-0.866 & 0 & 0.5 & 3 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

## Solution

Q1: A frame Fnoa is located in the position P.After the following transformation the frame position has changed to $\mathrm{Q}[2,5,7]$ T. A rotation along $Z$ axis by anti-clock 45 degree but before that a translation along all axis by $[2,3,5]$. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by $[3,-5,3]$. Find the position P with respect to Q .

$$
\begin{aligned}
& \text { Solution : }[T(3,-5,3) \text { * } \operatorname{Rot} Y(60) \text { * } \operatorname{RotZ}(45) \text { * } T(2,3,5)]-1 \text { * } \mathbf{Q}[2,5,7]=P \\
& \mathrm{~T}(3,-5,3) \text { * } \operatorname{Rot} Y(60) \text { * } \operatorname{Rot}(45) \text { * } \mathrm{T}(2,3,5)= \\
& 0.5 \times 0.707+0 \times 0.707+0.866 \times 0+3 \times 0 \quad 0.5 \times-0.707+0 \times 0.707+0.866 \times 0+3 \times 0 \\
& 0 \times 0.707+1 \times 0.707+0 \times 0+-5 \times 0 \quad 0 \times-0.707+1 \times 0.707+0 \times 0+-5 \times 0 \\
& -0.866 \times 0.707+0 \times 0.707+0.5 \times 0+3 \times 0 \quad-0.866 \times-0.707+0 \times 0.707+0.5 \times 0+3 \times 0 \\
& 0 \times 0.707+0 \times 0.707+0 \times 0+1 \times 0 \\
& 0 \times-0.707+0 \times 0.707+0 \times 0+1 \times 0
\end{aligned}
$$

$$
\begin{aligned}
& 0.5 \times 0+0 \times 0+0.866 \times 1+3 \times 0 \quad 0.5 \times-0.707+0 \times 3.535+0.866 \times 5+3 \times 1 \\
& 0 \times 0+1 \times 0+0 \times 1+-5 \times 0 \quad 0 \times-0.707+1 \times 3.535+0 \times 5+-5 \times 1 \\
& -0.866 \times 0+0 \times 0+0.5 \times 1+3 \times 0 \quad-0.866 \times-0.707+0 \times 3.535+0.5 \times 5+3 \times 1 \\
& 0 \times 0+0 \times 0+0 \times 1+1 \times 0 \quad 0 \times-0.707+0 \times 3.535+0 \times 5+1 \times 1 \\
& =\left[\begin{array}{cccc}
0.3535 & -0.3535 & 0.866 & 6.9765 \\
0.707 & 0.707 & 0 & -1.465 \\
-0.612262 & 0.612262 & 0.5 & 6.112262 \\
0 & 0 & 0 & 1
\end{array}\right]
\end{aligned}
$$

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## Solution

Q1: A frame Fnoa is located in the position P.After the following transformation the frame position has changed to $\mathrm{Q}[2,5,7] \mathrm{T}$. A rotation along Z axis by anti-clock 45 degree but before that a translation along all axis by $[2,3,5]$. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by $[3,-5,3]$. Find the position P with respect to Q .

$\left[\begin{array}{cccc}0.35 & 0.707 & -0.61 & -(6.98 \times 0.35+-1.465 \times(-0.35)+6.11 \times 0.866) \\ -0.355 & 0.707 & 0.61 & -(6.98 \times(-0.35)+-1.465 \times 0.707+6.11 \times 0.61) \\ 0.866 & 0 & 0.5 & -(6.98 \times 0.866+-1.465 \times 0+6.11 \times 0.5) \\ 0 & 0 & 0 & 1\end{array}\right]=\left[\begin{array}{cccc}0.35 & 0.707 & -0.61 & -8.24401 \\ -0.35 & 0.707 & 0.61 & -0.248345 \\ 0.866 & 0 & 0.5 & -9.09968 \\ 0 & 0 & 0 & 1\end{array}\right]$

## Solution

Q1: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to $Q[2,5,7]$. A rotation along $Z$ axis by anti-clock 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by $[3,-5,3]$. Find the position P with respect to Q .

Solution : [T(3,-5,3) * RotY(60) * RotZ(45) * $\mathrm{T}(2,3,5)]-1$ * $\mathrm{Q}[2,5,7]=\mathrm{P}$
$\left[\begin{array}{cccc}0.35 & 0.707 & -0.61 & -8.25 \\ -0.35 & 0.707 & 0.61 & -0.25 \\ 0.866 & 0 & 0.5 & -0.91 \\ 0 & 0 & 0 & 1\end{array}\right] \times\left[\begin{array}{l}2 \\ 5 \\ 7 \\ 1\end{array}\right]=\left[\begin{array}{c}0.35 \times 2+0.707 \times 5+-0.61 \times 7+-8.25 \times 1 \\ -0.35 \times 2+0.707 \times 5+0.61 \times 7+-0.25 \times 1 \\ 0.866 \times 2+0 \times 5+0.5 \times 7+-0.91 \times 1 \\ 0 \times 2+0 \times 5+0 \times 7+1 \times 1\end{array}\right]=\left[\begin{array}{c}-8.285 \\ 6.855 \\ 4.322 \\ 1\end{array}\right]$

Answer: P [ - 8.3 , $6.86,4.3] \mathrm{T}$

## Solution

Q5: A frame Fnoa is located in the position $\mathrm{P}[1,1,1]$ T. After the following transformation the frame position has changed to $Q$. A rotation along $Z$ axis by anti-clock 45 degree followed by a translation along all axis by [2,3,5], after those two, another rotation along Y axis by 45 degree followed by a translation along all axis by [3,3,3]. Find the position $Q$ with respect to $P$.

Solution: $\mathrm{T}(3,3,3)$ * $\operatorname{Rot} \mathbf{Y}(45)$ * $\mathrm{T}(2,3,5)$ * $\operatorname{Rot}(45)$ * $\mathrm{P}[1,1,1]=\mathbf{Q}$ $\mathbf{T}(\mathbf{2 , 3 , 5})$ * $\operatorname{Rot} \mathbf{Z ( 4 5 )}=\left[\begin{array}{llll}1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1\end{array}\right] \times\left[\begin{array}{cccc}\cos (45) & -\sin (45) & 0 & 0 \\ \sin (45) & \cos (45) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]=\left[\begin{array}{llll}1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1\end{array}\right] \times\left[\begin{array}{ccc}0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0 \\ 0 & 0 & 0\end{array}\right]=$ $1 \times 0.707+0 \times 0.707+0 \times 0+2 \times 0 \quad 1 \times-0.707+0 \times 0.707+0 \times 0+2 \times 0 \quad 1 \times 0+0 \times 0+0 \times 1+2 \times 0 \quad 1 \times 0+0 \times 0+0 \times 0+2 \times 1$ $0 \times 0.707+1 \times 0.707+0 \times 0+3 \times 0 \quad 0 \times-0.707+1 \times 0.707+0 \times 0+3 \times 0 \quad 0 \times 0+1 \times 0+0 \times 1+3 \times 0 \quad 0 \times 0+1 \times 0+0 \times 0+3 \times 1$ $0 \times 0.707+0 \times 0.707+1 \times 0+5 \times 0 \quad 0 \times-0.707+0 \times 0.707+1 \times 0+5 \times 0 \quad 0 \times 0+0 \times 0+1 \times 1+5 \times 0 \quad 0 \times 0+0 \times 0+1 \times 0+5 \times 1$ $[0 \times 0.707+0 \times 0.707+0 \times 0+1 \times 0 \quad 0 \times-0.707+0 \times 0.707+0 \times 0+1 \times 0 \quad 0 \times 0+0 \times 0+0 \times 1+1 \times 0 \quad 0 \times 0+0 \times 0+0 \times 0+1 \times 1]$

$$
=\left[\begin{array}{cccc}
0.707 & -0.707 & 0 & 2 \\
0.707 & 0.707 & 0 & 3 \\
0 & 0 & 1 & 5 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

## Solution

Q5: A frame Fnoa is located in the position $\mathrm{P}[1,1,1]$ T. After the following transformation the frame position has changed to $Q$. A rotation along $Z$ axis by anti-clock 45 degree followed by a translation along all axis by [2,3,5], after those two, another rotation along Y axis by 45 degree followed by a translation along all axis by [3,3,3]. Find the position $Q$ with respect to $P$.

Solution: $\mathrm{T}(3,3,3)$ * $\operatorname{Rot} Y(45)$ * $\mathrm{T}(2,3,5)$ * $\operatorname{Rot}(45)$ * $\mathrm{P}[1,1,1]=\mathbf{Q}$
$\mathbf{T}(\mathbf{3}, \mathbf{3}, \mathbf{3}) *$ Rot $\mathbf{Y}\left(\mathbf{4 5} \mathbf{)}=\left[\begin{array}{llll}1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1\end{array}\right] \times\left[\begin{array}{cccc}\cos (45) & 0 & \sin (45) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin (45) & 0 & \cos (45) & 0 \\ 0 & 0 & 0 & 1\end{array}\right]=\left[\begin{array}{llll}1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1\end{array}\right] \times\left[\begin{array}{cccc}0.707 & 0 & 0.707 & 0 \\ 0 & 1 & 0 & 0 \\ -0.707 & 0 & 0.707 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]=\right.$
$1 \times 0.707+0 \times 0+0 \times-0.707+3 \times 0 \quad 1 \times 0+0 \times 1+0 \times 0+3 \times 0 \quad 1 \times 0.707+0 \times 0+0 \times 0.707+3 \times 0 \quad 1 \times 0+0 \times 0+0 \times 0+3 \times 1$ $0 \times 0.707+1 \times 0+0 \times-0.707+3 \times 0 \quad 0 \times 0+1 \times 1+0 \times 0+3 \times 0 \quad 0 \times 0.707+1 \times 0+0 \times 0.707+3 \times 0 \quad 0 \times 0+1 \times 0+0 \times 0+3 \times 1$ $0 \times 0.707+0 \times 0+1 \times-0.707+3 \times 0 \quad 0 \times 0+0 \times 1+1 \times 0+3 \times 0 \quad 0 \times 0.707+0 \times 0+1 \times 0.707+3 \times 0 \quad 0 \times 0+0 \times 0+1 \times 0+3 \times 1$ $[0 \times 0.707+0 \times 0+0 \times-0.707+1 \times 0 \quad 0 \times 0+0 \times 1+0 \times 0+1 \times 0 \quad 0 \times 0.707+0 \times 0+0 \times 0.707+1 \times 00 \times 0+0 \times 0+0 \times 0+1 \times 1]$

$$
=\left[\begin{array}{cccc}
0.707 & 0 & 0.707 & 3 \\
0 & 1 & 0 & 3 \\
-0.707 & 0 & 0.707 & 3 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

## Solution

## Solution: $\mathrm{T}(3,3,3)$ * $\operatorname{Rot} \mathrm{Y}(45)$ * $\mathrm{T}(2,3,5)$ * $\operatorname{RotZ(45)~*~} \mathrm{P}[1,1,1]=\mathbf{Q}$

$\mathbf{T}(\mathbf{3}, \mathbf{3}, \mathbf{3}) * \mathbf{R o t} \mathbf{Y}\left(\mathbf{4 5 )}\right.$ * $\mathbf{T}(\mathbf{2}, \mathbf{3}, \mathbf{5}) * \mathbf{R o t} \mathbf{Z}(\mathbf{4 5})=\left[\begin{array}{cccc}0.707 & 0 & 0.707 & 3 \\ 0 & 1 & 0 & 3 \\ -0.707 & 0 & 0.707 & 3 \\ 0 & 0 & 0 & 1\end{array}\right] \times\left[\begin{array}{cccc}0.707 & -0.707 & 0 & 2 \\ 0.707 & 0.707 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1\end{array}\right]=$

$$
\left[\begin{array}{cc}
0.707 \times 0.707+0 \times 0.707+0.707 \times 0+3 \times 0 & -0.707 \times-0.707+0 \times 0.707+0.707 \times 0+3 \times 0 \\
0 \times 0.707+1 \times 0.707+0 \times 0+3 \times 0 & 0 \times-0.707+1 \times 0.707+0 \times 0+3 \times 0 \\
-0.707 \times 0.707+0 \times 0.707+0.707 \times 0+3 \times 0 & -0.707 \times-0.707+0 \times 0.707+0.707 \times 0+3 \times 0 \\
0 \times 0.707+0 \times 0.707+0 \times 0+1 \times 0 & 0 \times-0.707+0 \times 0.707+0 \times 0+1 \times 0
\end{array}\right.
$$

$$
\begin{aligned}
& 0.707 \times 0+0 \times 0+0.707 \times 1+3 \times 0
\end{aligned} \begin{array}{cc}
0.707 \times 2+0 \times 3+0.707 \times 5+3 \times 1 \\
0 \times 0+1 \times 0+0 \times 1+3 \times 0 & 0 \times 2+1 \times 3+0 \times 5+3 \times 1 \\
-0.707 \times 0+0 \times 0+0.707 \times 1+3 \times 0 & -0.707 \times 2+0 \times 3+0.707 \times 5+3 \times 1 \\
0 \times 0+0 \times 0+0 \times 1+1 \times 0 & 0 \times 2+0 \times 3+0 \times 5+1 \times 1 \\
=\left[\begin{array}{cccc}
0.499849 & -0.499849 & 0.707 & 7.949 \\
0.707 & 0.707 & 0 & 6 \\
-0.499849 & 0.499849 & 0.707 & 5.121 \\
0 & 0 & 0 & 1
\end{array}\right]
\end{array}
$$

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## Solution

Q5: A frame Fnoa is located in the position $\mathrm{P}[1,1,1]$ T. After the following transformation the frame position has changed to $Q$. A rotation along $Z$ axis by anti-clock 45 degree followed by a translation along all axis by [2,3,5], after those two, another rotation along Y axis by 45 degree followed by a translation along all axis by [3,3,3]. Find the position $Q$ with respect to $P$.

Solution: $\mathrm{T}(3,3,3)$ * $\operatorname{Rot} Y(45)$ * $\mathrm{T}(2,3,5)$ * $\operatorname{Rot} Z(45)$ * $\mathrm{P}[1,1,1]=\mathbf{Q}$
$\left[\begin{array}{cccc}0.5 & -0.5 & 0.707 & 7.95 \\ 0.707 & 0.707 & 0 & 6 \\ -0.5 & 0.5 & 0.707 & 5.12 \\ 0 & 0 & 0 & 1\end{array}\right] \times\left[\begin{array}{l}1 \\ 1 \\ 1 \\ 1\end{array}\right]=\left[\begin{array}{c}0.5 \times 1+-0.5 \times 1+0.707 \times 1+7.95 \times 1 \\ 0.707 \times 1+0.707 \times 1+0 \times 1+6 \times 1 \\ -0.5 \times 1+0.5 \times 1+0.707 \times 1+5.12 \times 1 \\ 0 \times 1+0 \times 1+0 \times 1+1 \times 1\end{array}\right]=\left[\begin{array}{c}8.657 \\ 7.414 \\ 5.827 \\ 1\end{array}\right]$

## Keep practicing more and more exercise

