## Spatial Descriptions and Transformations Related Exercise and Solution

### **Matrix Multiplication**

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

$ax_1$ $ay_1$ $az_1$ $aw_1$	$ax_{2}$ $ay_{2}$ $az_{2}$ $aw_{2}$	ax <sub>3</sub> ay <sub>3</sub> az <sub>3</sub> aw <sub>3</sub>	ax <sub>4</sub> ay <sub>4</sub> az <sub>4</sub> aw <sub>4</sub>	x	$bx_1$ $by_1$ $bz_1$ $bw_1$	οx <sub>2</sub> by <sub>2</sub> bz <sub>2</sub> bw <sub>2</sub>	bx <sub>3</sub> by <sub>3</sub> bz <sub>3</sub> bw <sub>3</sub>	bx4 by4 bz4 bw4	
ar	ar	ar	ar		hr	hr	hr	hr	

 $\begin{array}{rll} ax_{1}*bx_{1}+ax_{2}*by_{1}+ax_{3}*bz_{1}+ax_{4}*bw_{1}&ax_{1}*bx_{2}+ax_{2}*by_{2}+ax_{3}*bz_{2}+ax_{4}*bw_{2}\\ ay_{1}*bx_{1}+ay_{2}*by_{1}+ay_{3}*bz_{1}+ay_{4}*bw_{1}&ay_{1}*bx_{2}+ay_{2}*by_{2}+ay_{3}*bz_{2}+ay_{4}*bw_{2}\\ az_{1}*bx_{1}+az_{2}*by_{1}+az_{3}*bz_{1}+az_{4}*bw_{1}&az_{1}*bx_{2}+az_{2}*by_{2}+az_{3}*bz_{2}+az_{4}*bw_{2}\\ aw_{1}*bx_{1}+aw_{2}*by_{1}+aw_{3}*bz_{1}+aw_{4}*bw_{1}&aw_{1}*bx_{2}+aw_{2}*by_{2}+aw_{3}*bz_{2}+aw_{4}*bw_{2}\\ \end{array}$ 

## **Exercise and Solution**

Q1: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along Z axis by anti-clock 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

#### Solution : [T(3,-5,3) \* RotY(60) \* RotZ(45) \* T(2,3,5) ]-1 \* Q[2,5,7] = P

Q2: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along X axis by 60 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 45 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

### Solution: [T(3,-5,3) \* RotY(45) \* RotX(60) \* T(2,3,5) ]-1 \* Q[2,5,7] = P

Q3: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along Z axis by anti-clock 30 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 45 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

### Solution: [ T(3,-5,3) \* RotY(45) \* RotZ(30) \* T(2,3,5) ]-1 \* Q[2,5,7] = P

Q4: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along X axis by 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 30 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution: [ T(3,-5,3) \* RotY(30) \* RotX(45) \* T(2,3,5) ]-1 \* Q[2,5,7] = P

Q1: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along Z axis by anti-clock 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution : [T(3,-5,3) \* RotY(60) \* RotZ(45) \* T(2,3,5) ]-1 \* Q[2,5,7] = P

$RotZ(45) \wedge I(2,3,5) =$	$\cos(45)$	$-\sin(45)$	0	0		[1	0	0	2]		0.707	-0.707	0	0		<b>[</b> 1	0	0	2]	1
	$\sin(45)$	$\cos(45)$	0	0	~	0	1	0	3	_	0.707	0.707	0	0		0	1	0	3	_
	0	0	1	0	Ŷ	0	0	1	5	_	0	0	1	0	^	0	0	1	5	-
	0	0	0	1		0	0	0	1		0	0	0	1		0	0	0	1	

	0.707	-0.707	0	-0.707	
	0.707	0.707	0	3.535	
=	0	0	1	5	
	0	0	0	1	

Q1: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along Z axis by anti-clock 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution : [T(3,-5,3) \* RotY(60) \* RotZ(45) \* T(2,3,5) ]-1 \* Q[2,5,7] = P  $T(3,-5,3) * RotY(60) = \begin{bmatrix} 1 & 0 \end{bmatrix}$ 

 $\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos(60) & 0 & \sin(60) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(60) & 0 & \cos(60) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.5 & 0 & 0.866 & 0 \\ 0 & 1 & 0 & 0 \\ -0.866 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$  $0 \times 0.5 + 1 \times 0 + 0 \times -0.866 + -5 \times 0 \quad 0 \times 0 + 1 \times 1 + 0 \times 0 + -5 \times 0 \quad 0 \times 0.866 + 1 \times 0 + 0 \times 0.5 + -5 \times 0 \quad 0 \times 0 + 1 \times 0 + 0 \times 0 + -5 \times 1$  $\begin{bmatrix} 0 \times 0.5 + 0 \times 0 + 0 \times -0.866 + 1 \times 0 & 0 \times 0 + 0 \times 1 + 0 \times 0 + 1 \times 0 & 0 \times 0.866 + 0 \times 0 + 0 \times 0.5 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix}$ 

	0.5	0	0.866	3 ]
	0	1	0	-5
-	-0.866	0	0.5	3
	0	0	0	1

Q1: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along Z axis by anti-clock 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution : [T(3,-5,3) \* RotY(60) \* RotZ(45) \* T(2,3,5) ]-1 \* Q[2,5,7] = P

 $\mathbf{T(3,-5,3)} * \operatorname{Rot}\mathbf{Y(60)} * \operatorname{Rot}\mathbf{Z(45)} * \mathbf{T(2,3,5)} = \begin{bmatrix} 0.5 & 0 & 0.866 & 3 \\ 0 & 1 & 0 & -5 \\ -0.866 & 0 & 0.5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.707 & -0.707 & 0 & -0.707 \\ 0.707 & 0.707 & 0 & 3.535 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$ 

 $\begin{array}{lll} 0.5 \times 0.707 + 0 \times 0.707 + 0.866 \times 0 + 3 \times 0 & 0.5 \times -0.707 + 0 \times 0.707 + 0.866 \times 0 + 3 \times 0 \\ 0 \times 0.707 + 1 \times 0.707 + 0 \times 0 + -5 \times 0 & 0 \times -0.707 + 1 \times 0.707 + 0 \times 0 + -5 \times 0 \\ -0.866 \times 0.707 + 0 \times 0.707 + 0.5 \times 0 + 3 \times 0 & -0.866 \times -0.707 + 0 \times 0.707 + 0.5 \times 0 + 3 \times 0 \\ 0 \times 0.707 + 0 \times 0.707 + 0 \times 0 + 1 \times 0 & 0 \times -0.707 + 0 \times 0.707 + 0 \times 0 + 1 \times 0 \\ \end{array}$ 

0.3535	-0.3535	0.866	6.9765
0.707	0.707	0	-1.465
-0.612262	0.612262	0.5	6.112262
0	0	0	1
	$\begin{bmatrix} 0.3535 \\ 0.707 \\ -0.612262 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.3535 & -0.3535 \\ 0.707 & 0.707 \\ -0.612262 & 0.612262 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.3535 & -0.3535 & 0.866 \\ 0.707 & 0.707 & 0 \\ -0.612262 & 0.612262 & 0.5 \\ 0 & 0 & 0 \end{bmatrix}$

Q1: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along Z axis by anti-clock 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution : [T(3,-5,3) \* RotY(60) \* RotZ(45) \* T(2,3,5) ]-1 \* Q[2,5,7] = P

[T(3,-5,3) \* RotY(60) \* RotZ(45) \* T(2,3,5) ] -1 =

0.3535	-0.3535	0.866	6.9765	-1
0.707	0.707	0	-1.465	
-0.612262	0.612262	0.5	6.112262	
0	0	0	1	

0.35	0.707	-0.61	$-(6.98{ imes}0.35{ imes}-1.465{ imes}(-0.35){ imes}0.11{ imes}0.866)$		0.35	0.707	-0.61	-8.24701
-0.35	0.707	0.61	$-\left(6.98{ imes}\left(-0.35 ight)+-1.465{ imes}0.707+6.11{ imes}0.61 ight)$	_	-0.35	0.707	0.61	-0.248345
0.866	0	0.5	$-\left(6.98{ imes}0.866+{-}1.465{ imes}0+6.11{ imes}0.5 ight)$	-	0.866	0	0.5	-9.09968
0	0	0	1		0	0	0	1

Q1: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along Z axis by anti-clock 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution : [T(3,-5,3) \* RotY(60) \* RotZ(45) \* T(2,3,5) ]-1 \* Q[2,5,7] = P

$$\begin{bmatrix} 0.35 & 0.707 & -0.61 & -8.25 \\ -0.35 & 0.707 & 0.61 & -0.25 \\ 0.866 & 0 & 0.5 & -0.91 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 5 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.35 \times 2 + 0.707 \times 5 + -0.61 \times 7 + -8.25 \times 1 \\ -0.35 \times 2 + 0.707 \times 5 + 0.61 \times 7 + -0.25 \times 1 \\ 0.866 \times 2 + 0 \times 5 + 0.5 \times 7 + -0.91 \times 1 \\ 0 \times 2 + 0 \times 5 + 0 \times 7 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} -8.285 \\ 6.855 \\ 4.322 \\ 1 \end{bmatrix}$$

Answer: P [ -8.3 , 6.86 , 4.3 ]T

Q5: A frame Fnoa is located in the position P[1,1,1]T. After the following transformation the frame position has changed to Q. A rotation along Z axis by anti-clock 45 degree followed by a translation along all axis by [2,3,5], after those two, another rotation along Y axis by 45 degree followed by a translation along all axis by [3,3,3]. Find the position Q with respect to P.

Solution: T(3,3,3) \* RotY(45) \* T(2,3,5) \* RotZ(45) \* P[1,1,1] = Q

 $\mathbf{T(2,3,5)} * \mathbf{RotZ(45)} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} \cos(45) & -\sin(45) & 0 & 0 \\ \sin(45) & \cos(45) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.707 & -0.707 & 0 & 0 \\ 0.707 & 0.707 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \times 0.707 + 0 \times 0.707 + 1 \times 0.707 + 0 \times 0.707 +$ 

	0.707	-0.707	0	2]	
	0.707	0.707	0	3	
=	0	0	1	5	
	0	0	0	1	

Q5: A frame Fnoa is located in the position P[1,1,1]T. After the following transformation the frame position has changed to Q. A rotation along Z axis by anti-clock 45 degree followed by a translation along all axis by [2,3,5], after those two, another rotation along Y axis by 45 degree followed by a translation along all axis by [3,3,3]. Find the position Q with respect to P.

Solution: T(3,3,3) \* RotY(45) \* T(2,3,5) \* RotZ(45) \* P[1,1,1] = Q

 $\mathbf{T(3,3,3)}^{*} \operatorname{Rot} \mathbf{Y(45)} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos(45) & 0 & \sin(45) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(45) & 0 & \cos(45) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.707 & 0 & 0.707 & 0 \\ 0 & 1 & 0 & 0 \\ -0.707 & 0 & 0.707 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \times 0.707 + 0 \times 0 + 0 \times 1 + 0 \times 0 + 0$ 

 $0 \times 0.707 + 0 \times 0 + 0 \times -0.707 + 1 \times 0 \quad 0 \times 0 + 0 \times 1 + 0 \times 0 + 1 \times 0 \quad 0 \times 0.707 + 0 \times 0 + 0 \times 0.707 + 1 \times 0 \quad 0 \times 0 + 0 \times 0 + 0 \times 0 + 1 \times 1$ 

	0.707	0	0.707	3]
	0	1	0	3
=	-0.707	0	0.707	3
	0	0	0	1

 $-0.707 \times 0.707 + 0 \times 0.707 + 0.707 \times 0 +$ 

Solution: T(3,3,3) \* RotY(45) \* T(2,3,5) \* RotZ(45) \* P[1,1,1] = Q

$$\mathbf{T(3,3,3)} * \operatorname{Rot}\mathbf{Y(45)} * \mathbf{T(2,3,5)} * \operatorname{Rot}\mathbf{Z(45)} = \begin{bmatrix} 0.707 & 0 & 0.707 & 3 \\ 0 & 1 & 0 & 3 \\ -0.707 & 0 & 0.707 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.707 & -0.707 & 0 & 2 \\ 0.707 & 0.707 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 0.707 \times 0.707 + 0 \times 0.707 + 0.707 \times 0 + 3 \times 0 & 0.707 \times -0.707 + 0 \times 0.707 + 0.707 \times 0 + 3 \times 0 \\ 0 \times 0.707 + 1 \times 0.707 + 0 \times 0 + 3 \times 0 & 0 \times -0.707 + 1 \times 0.707 + 0 \times 0 + 3 \times 0 \\ -0.707 \times 0.707 + 0 \times 0.707 + 0.707 \times 0 + 3 \times 0 & -0.707 \times -0.707 + 0 \times 0.707 + 0.707 \times 0 + 3 \times 0 \\ 0 \times 0.707 + 0 \times 0.707 + 0 \times 0 + 1 \times 0 & 0 \times -0.707 + 0 \times 0.707 + 0 \times 0 + 1 \times 0 \\ \end{bmatrix}$$

 $0.707 \times 0 + 0 \times 0 + 0.707 \times 1 + 3 \times 0$   $0.707 \times 2 + 0 \times 3 + 0.707 \times 5 + 3 \times 1$  $0 \times 0 + 1 \times 0 + 0 \times 1 + 3 \times 0 \qquad \qquad 0 \times 2 + 1 \times 3 + 0 \times 5 + 3 \times 1$  $-0.707 \times 0 + 0 \times 0 + 0.707 \times 1 + 3 \times 0 -0.707 \times 2 + 0 \times 3 + 0.707 \times 5 + 3 \times 1$  $0 \times \mathbf{0} + \mathbf{0} \times \mathbf{0} + \mathbf{0} \times \mathbf{1} + \mathbf{1} \times \mathbf{0} \qquad \qquad \mathbf{0} \times \mathbf{2} + \mathbf{0} \times \mathbf{3} + \mathbf{0} \times \mathbf{5} + \mathbf{1} \times \mathbf{1}$ 

	0.499849	-0.499849	0.707	7.949
	0.707	0.707	0	6
-	-0.499849	0.499849	0.707	5.121
	0	0	0	1

Q5: A frame Fnoa is located in the position P[1,1,1]T. After the following transformation the frame position has changed to Q. A rotation along Z axis by anti-clock 45 degree followed by a translation along all axis by [2,3,5], after those two, another rotation along Y axis by 45 degree followed by a translation along all axis by [3,3,3]. Find the position Q with respect to P.

Solution: T(3,3,3) \* RotY(45) \* T(2,3,5) \* RotZ(45) \* P[1,1,1] = Q

$$\begin{bmatrix} 0.5 & -0.5 & 0.707 & 7.95 \\ 0.707 & 0.707 & 0 & 6 \\ -0.5 & 0.5 & 0.707 & 5.12 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \times 1 + -0.5 \times 1 + 0.707 \times 1 + 7.95 \times 1 \\ 0.707 \times 1 + 0.707 \times 1 + 0 \times 1 + 6 \times 1 \\ -0.5 \times 1 + 0.5 \times 1 + 0.707 \times 1 + 5.12 \times 1 \\ 0 \times 1 + 0 \times 1 + 0 \times 1 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} 8.657 \\ 7.414 \\ 5.827 \\ 1 \end{bmatrix}$$

Answer: Q [ 8.657 , 7.414 , 5.827 ]T

### Keep practicing more and more exercise