Correlation Analysis (Part 2)

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2. Partial correlation

- It is used to explore the relationship between two variables while controlling for the effects of one or more additional variables, known as "control variables" or "covariates."
- The purpose of partial correlation is to isolate the unique association between the two main variables of interest, removing the influence of the control variables.

$$r_{12.3} = \frac{r_{12} - r_{13} r_{23}}{\sqrt{1 - r_{13}^2}\sqrt{1 - r_{23}^2}}$$

Partial Correlation

- Useful with three variables
  - predictor variable
  - predicted variable
  - control variable
- is
  - relationship between two variables while controlling for a third variable
  - type of Pearson correlation coefficient
- Used in models which assume a linear relationship
- data is supposed to be interval in nature
- example relationship between height & weight, while controlling for age
- assist in understanding regression
- range in value from -1 to +1
- conducted to understand why two variables are correlated

Data of academic achievement, anxiety and intelligence for 10 subjects

Subject	Academic Achievement	Anxiety	Intelligence
1	15	6	25
2	18	3	29
3	13	8	27
4	14	6	24
5	19	2	30
6	11	3	21
7	17	4	26
8	20	4	31
9	10	5	20
10	16	7	25

A Academic Achievement

B Anxiety

C Intelligence

$r_{AB} = -0.369$ $\quad r_{AC} = 0.918$

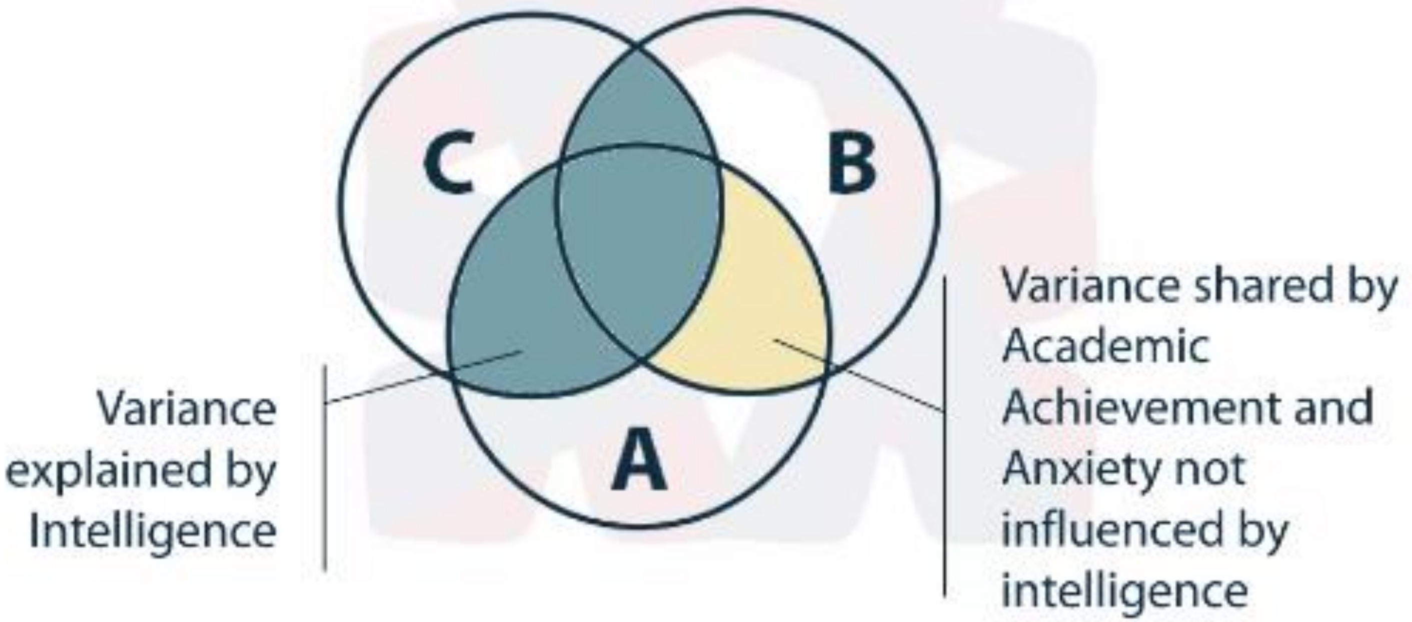
$r_{BC} = -0.245$

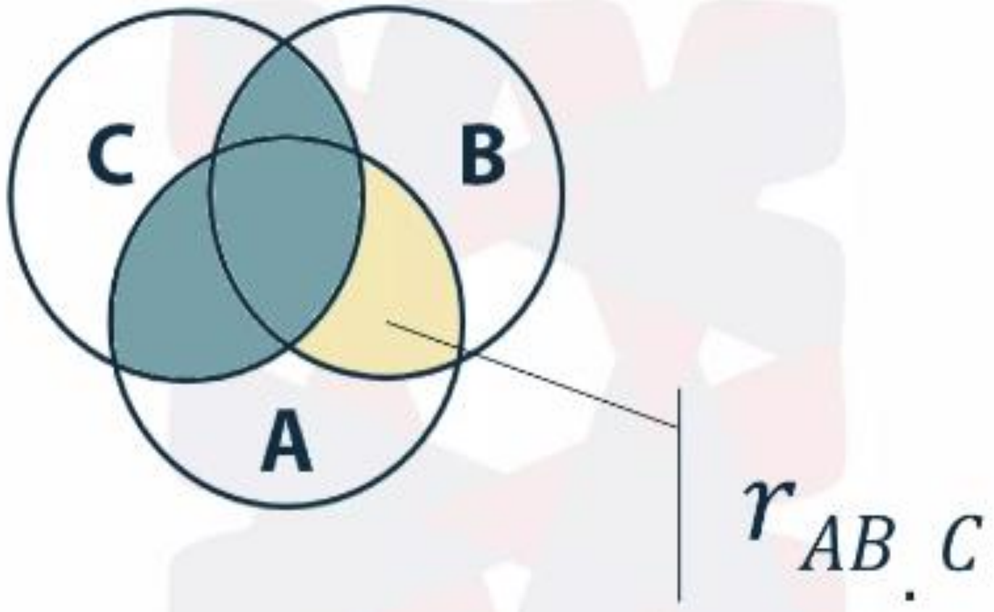
$$r_p = r_{AB.C} = \frac{(-0.369) - (0.918)(-0.245)}{\sqrt{(1 - 0.918^2)(1 - (-0.245)^2)}}$$

$$r_p = r_{AB.C} = \frac{-0.1441}{0.499}$$

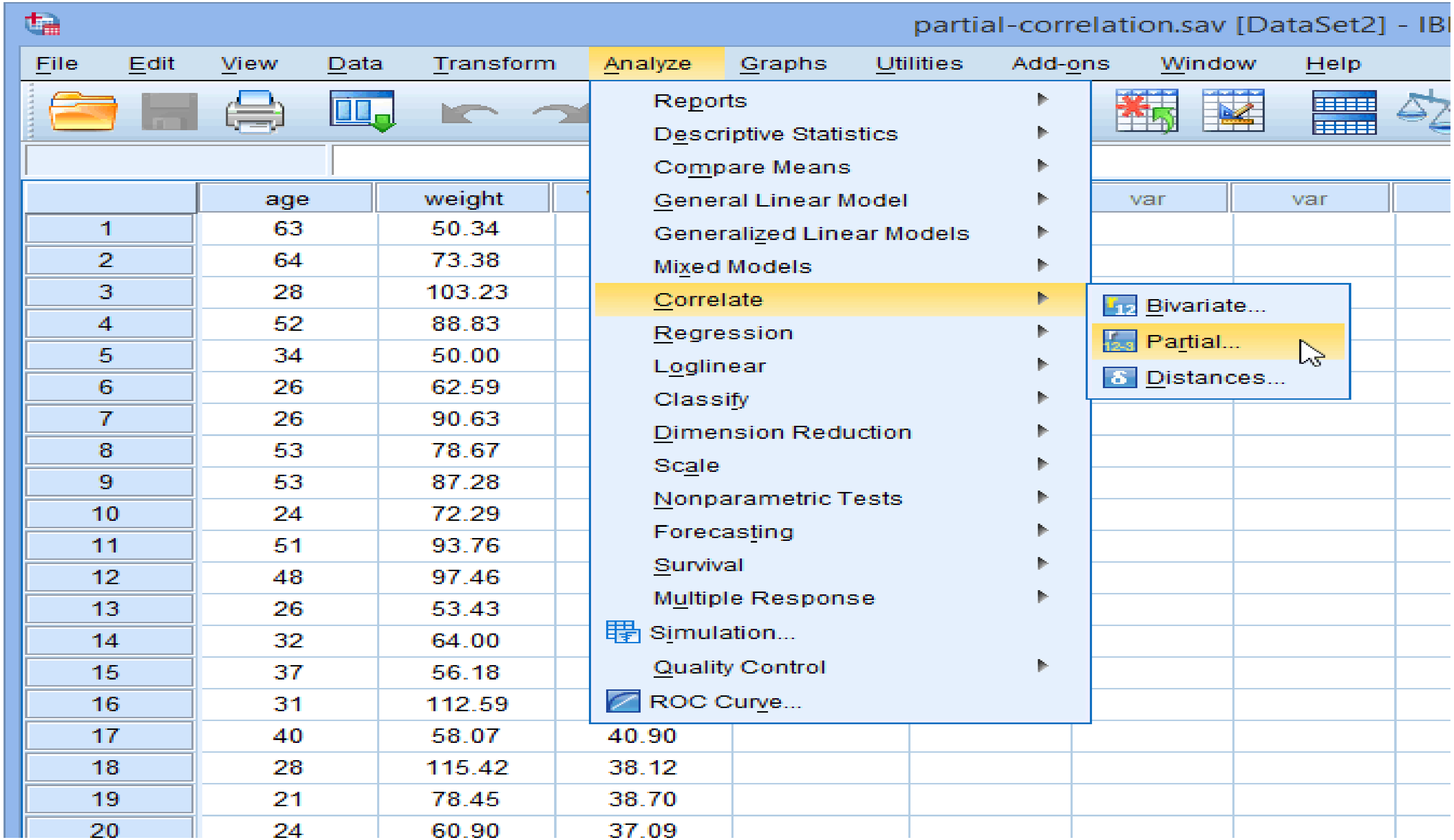
$$r_p = r_{AB.C} = -0.375$$

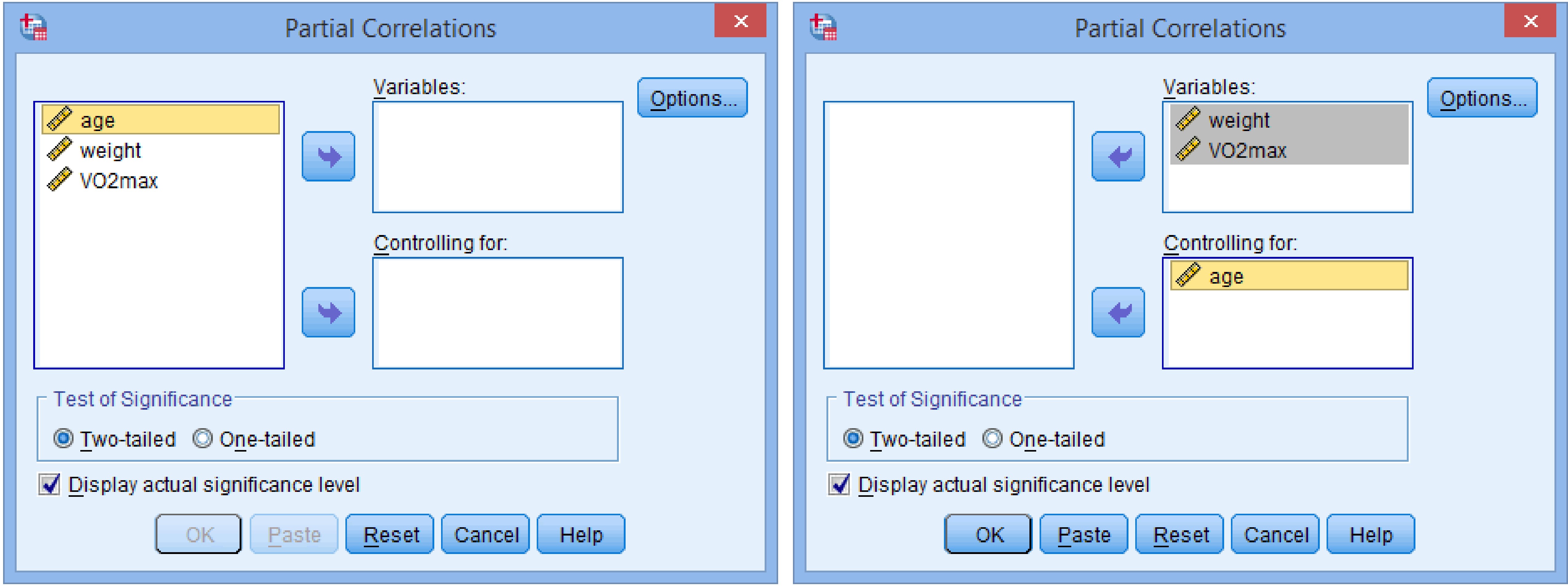
Graphical Explanation of partial correlation between Academic Achievement and Anxiety controlled for Intelligence





Calculate Partial correlation in SPSS




Partial Correlations
age
weight
VO2max
Variables:
Controlling for:
Options...
Test of Significance
Two-tailed
One-tailed
Display actual significance level
OK
Paste
Reset
Cancel
Help
Partial Correlations
Variables:
weight
VO2max
Controlling for:
age
Options...
Test of Significance
Two-tailed
One-tailed
Display actual significance level
OK
Paste
Reset
Cancel
Help

The **Correlations** table is split into two main parts:

(a) the Pearson product-moment correlation coefficients for all your variables – that is, your dependent variable, independent variable, and one or more control variables – as highlighted by the blue rectangle; and

(b) the results from the partial correlation where the Pearson product-moment correlation coefficient between the dependent and independent variable has been adjusted to take into account the control variable(s), as highlighted by the red rectangle.

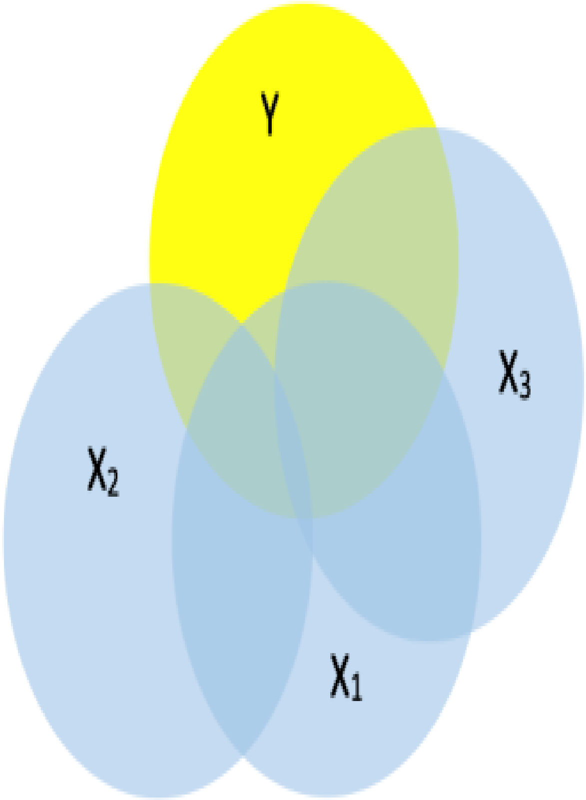
Correlations

Control Variables			Weight	VO2max	Age
-none-[a]	Weight	Correlation	1.000	-.307	-.004
		Significance (2-tailed)	.	.002	.972
		df	0	98	98
	VO2max	Correlation	-.307	1.000	-.191
		Significance (2-tailed)	.002	.	.057
		df	98	0	98
	Age	Correlation	-.004	-.191	1.000
		Significance (2-tailed)	.972	.057	.
		df	98	98	0
Age	Weight	Correlation	1.000	-.314	
		Significance (2-tailed)	.	.002	
		df	0	97	
	VO2max	Correlation	-.314	1.000	
		Significance (2-tailed)	.002	.	
		df	97	0	

a. Cells contain zero-order (Pearson) correlations.

3. Multiple Correlation:

- It examines the relationship between one dependent variable and two or more independent variables, an extension of simple correlation.
- In multiple correlation, the goal is to understand how a single dependent variable is related to a combination of several independent variables.
- Multiple correlation is often used in regression analysis, where the dependent variable is predicted based on the values of multiple independent variables.
- The multiple correlation coefficient is represented by "R" and is the square root of the coefficient of determination (R-squared) in regression analysis. R-squared represents the proportion of the variance in the dependent variable that is explained by the independent variables. It ranges from 0 to 1.



Multiple Correlation Coefficient

The formula for R is

$$R = \sqrt{\frac{r_{yx_1}^2 + r_{yx_2}^2 - 2r_{yx_1} \cdot r_{yx_2} \cdot r_{x_1x_2}}{1 - r_{x_1x_2}^2}}$$

where

r_{yx_1} = correlation coefficient for y and x_1

r_{yx_2} = correlation coefficient for y and x_2

$r_{x_1x_2}$ = correlation coefficient for x_1 and x_2

Problem 3

If the simple correlation coefficients have the values $r_{12} = 0.6$, $r_{13} = 0.65$, $r_{23} = 0.8$, find the multiple correlation coefficient $R_{1.23}$

Solution:

We have

$$R_{1.23} = \frac{\sqrt{r^2_{12} + r^2_{13} - 2\, r_{12}\, r_{13}\, r_{23}}}{\sqrt{1 - r^2_{23}}}$$

$$= \frac{\sqrt{(0.6)^2 + (0.65)^2 - 2x0.6x0.65x0.8}}{\sqrt{1 - (0.8)^2}}$$

Data of academic achievement, anxiety and intelligence for 10 subjects

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A= Academic achievement

B= Anxiety

C= Intelligence

r_{AB} = -0.369

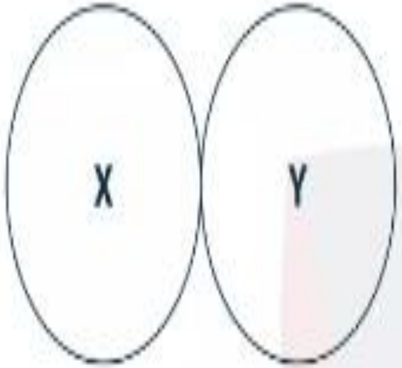
r_{AC} = 0.918

r_{BC} = -0.245

$R_{A.BC}$ = 0.929

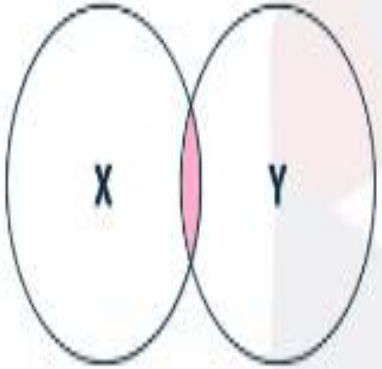
Interpretation of multiple correlation coefficient

- What is the meaning of $R_{A.BC} = 0.929$?
- This means that the multiple correlation between academic achievement and the linear combination of intelligence and anxiety is 0.929 or 0.93.
- What is R^2 ?
- The R^2 is the percentage of variance in academic achievement explained by the linear combination of intelligence and anxiety.
- In this sample R=0.929 or 0.93; so, the value of R^2 is about 0.865. It means that the linear combination of intelligence and anxiety explain 86.5 percent variance in the academic achievement.



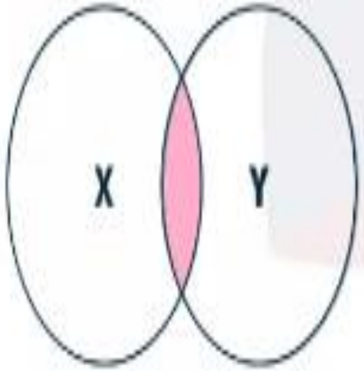
$R = 0$ $R^2 = 0$

$Variance = 0\%$



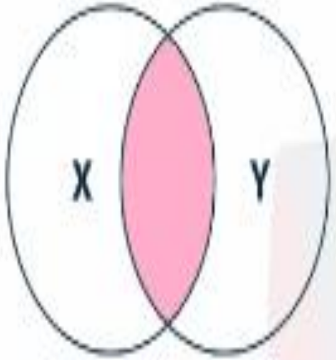
$R = \pm 0.2$ $R^2 = 0.04$

$Variance = 4\%$



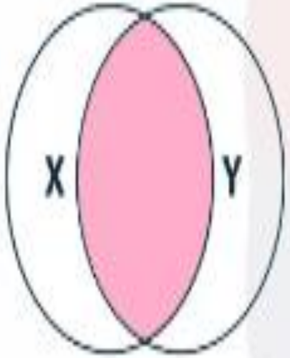
$R = \pm 0.4$ $R^2 = 0.16$

$Variance = 16\%$



$R = \pm 0.6$ $R^2 = 0.36$

$Variance = 36\%$



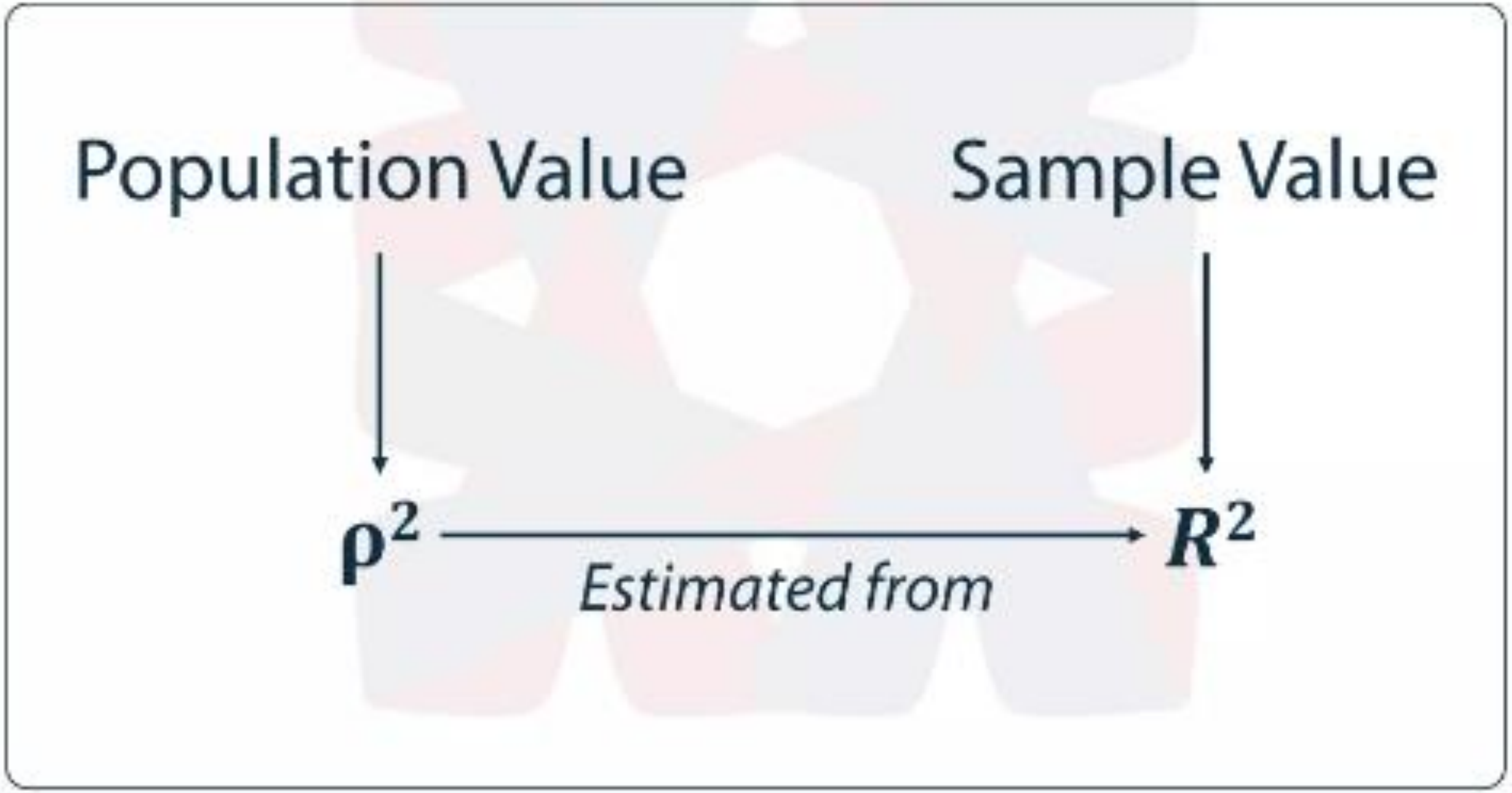
$R = \pm 0.8$ $R^2 = 0.64$

$Variance = 64\%$

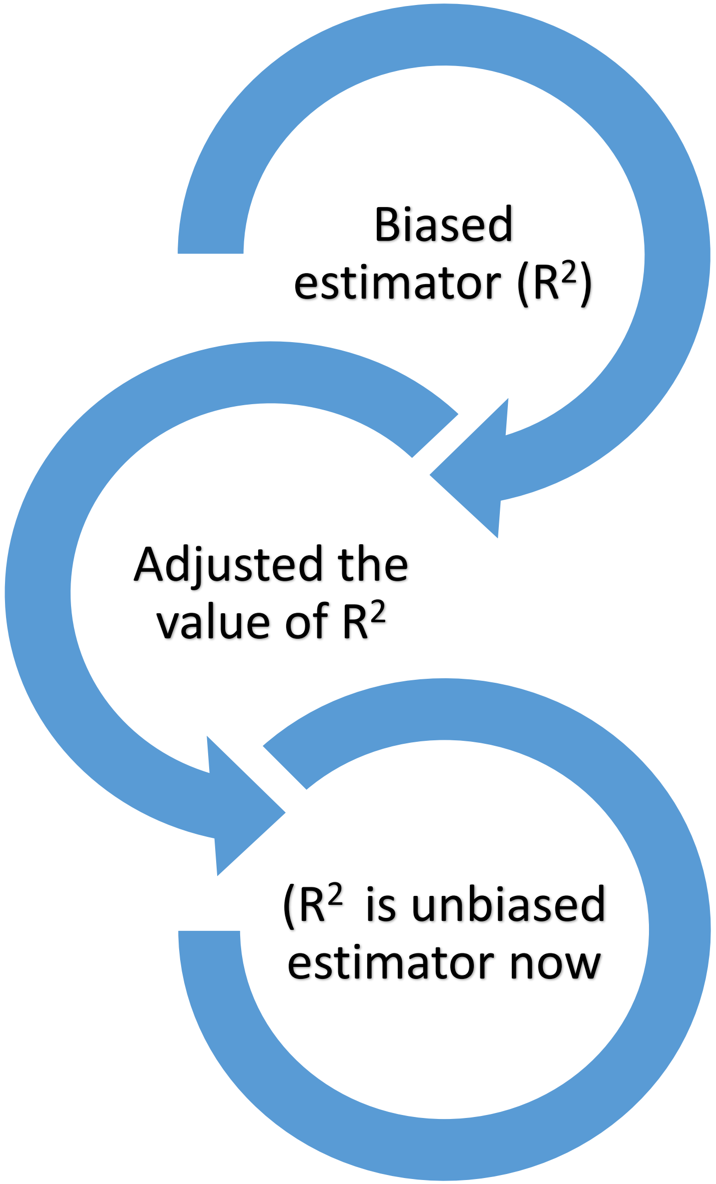
$R = \pm 1$ $R^2 = 1$

$Variance = 100\%$

Multiple Correlation Coefficient



Reduced biasness



A biased estimator is one that deviates from the true population value.
An unbiased estimator is one that does not deviate from the true population parameter.
Formula: R*2 = 1 −(1−R*2)(n−1)/(n−k−1)

Where,
n = number of participants / sample size
k = number of predicted variables
R*2 = is the adjusted value of R

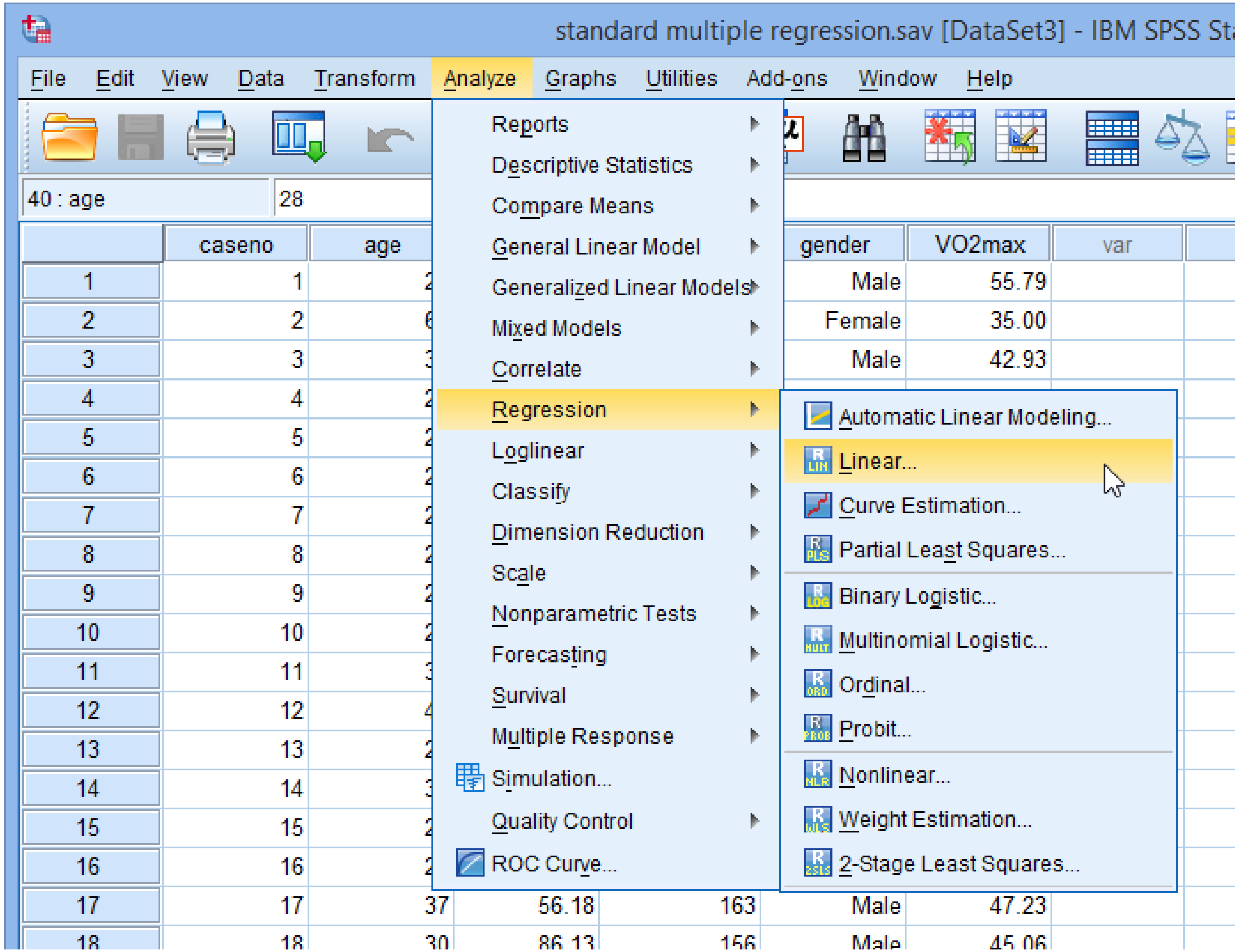
$$\widetilde{R}^2 = 1 - \frac{(1 - R^2)(n - 1)}{n - k - 1}$$

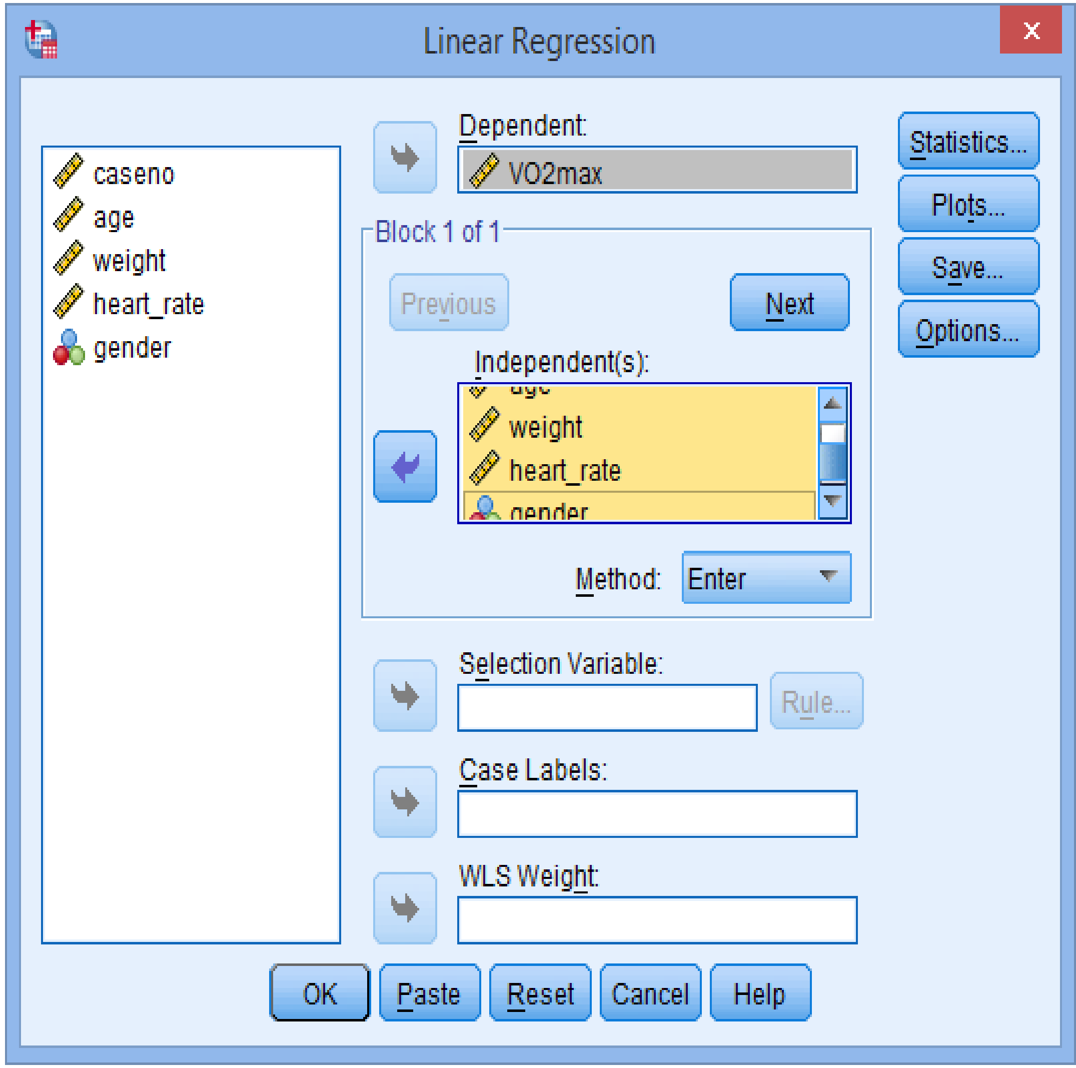
$$= 1 - \frac{(1 - 0.865)(10 - 1)}{10 - 2 - 1}$$

$$= 1 - \frac{1.217}{7}$$

$$= 0.826$$

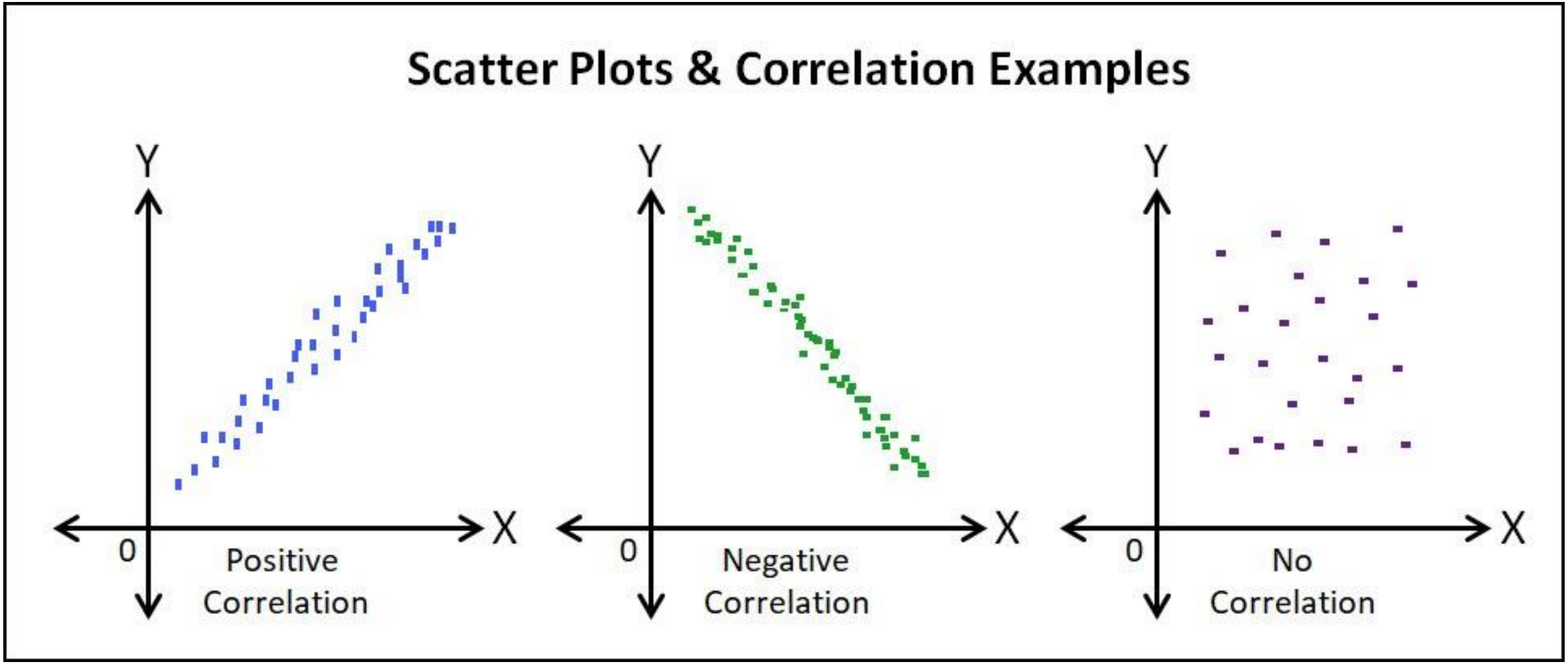
Multiple Correlation Coefficient by SPSS





Scatter Plot or Scatter Diagram

- The scatter plot and correlation complement each other in understanding the relationship between two variables.
- It is a type of graphical representation used to visualize the relationship between two continuous variables.
- Each data point is plotted as a dot on the graph, with one variable represented on the x-axis and the other variable on the y-axis.
- When the points in the scatter plot are tightly clustered along a straight line, the correlation coefficient tends to be close to +1 or -1, indicating a strong linear relationship.
- When the points in the scatter plot are scattered randomly with no apparent pattern, the correlation coefficient tends to be close to 0, indicating a weak or no linear relationship.


Scatter Plots & Correlation Examples
Y
X
0
Positive Correlation
Y
X
0
Negative Correlation
Y
X
0
No Correlation