Course Code: CE 455
Course Title: Traffic Engineering and Management

Lecture 13: Traffic Flow Studies

Course Teacher: Saurav Barua (SB)
Assistant Professor, Dept. Of Civil Engineering, DIU
Contact No: 01715334075
Email: saurav.ce@diu.edu.bd

Outline

- Flow, density and headway of traffic stream
- Time space diagram
- Time mean speed, space mean speed
- Fundamental diagram of traffic flow, speed-flow relation, speed-density relation, flow-density diagram
- Greenshield’s macroscopic stream model
- Other macroscopic stream model: Greenberg’s logarithmic stream model, Underwood’s exponential model, pipes generalized model
- Shockwaves
- Calculations for Moving observer methods
- Peak hour factor calculation,
- Determination of PCU, headway methods of PCU calculations
- Miscellaneous: Calculation for Control delay, Poisson vehicle arrival pattern computation, Acceptability of simulation models, License plate method of parking study.

Flow

There are practically two ways of counting the number of vehicles on a road. One is flow or volume, which is defined as the number of vehicles that pass a point on a highway or a given lane or direction of a highway during a specific time interval. The measurement is carried out by counting the number of vehicles, n_t, passing a particular point in one lane in a defined period t. Then the flow q expressed in vehicles/hour is given by

$$q = \frac{n_t}{t} \tag{1}$$

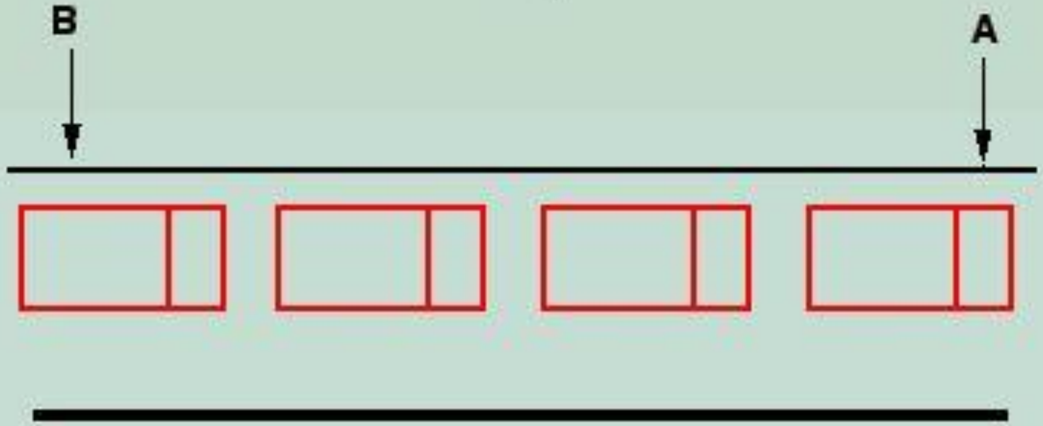
Flow is expressed in planning and design field taking a day as the measurement of time.

Density

Density is defined as the number of vehicles occupying a given length of highway or lane and is generally expressed as vehicles per km. One can photograph a length of road x, count the number of vehicles, n_x, in one lane of the road at that point of time and derive the density k as,

$$k = \frac{n_x}{x} \tag{1}$$

This is illustrated in figure 1. From the figure, the density is the number of vehicles between the point A and B divided by the distance between A and B. Density is also equally important as flow but from a different angle as it is the measure most directly related to traffic demand. Again it measures the proximity of vehicles in the stream which in turn affects the freedom to maneuver and comfortable driving.



Time headway

The microscopic character related to volume is the time headway or simply headway. Time headway is defined as the time difference between any two successive vehicles when they cross a given point. Practically, it involves the measurement of time between the passage of one rear bumper and the next past a given point. If all headways h in time period, t, over which flow has been measured are added then,

$$\sum_{1}^{n_t} h_i = t \tag{1}$$

But the flow is defined as the number of vehicles n_t measured in time interval t, that is,

$$q = \frac{n_t}{t} = \frac{n_t}{\sum_{1}^{n_t} h_i} = \frac{1}{h_{av}} \tag{2}$$

where, h_{av} is the average headway. Thus average headway is the inverse of flow. Time headway is often referred to as simply the headway.

Distance headway

Another related parameter is the distance headway. It is defined as the distance between corresponding points of two successive vehicles at any given time. It involves the measurement from a photograph, the distance from rear bumper of lead vehicle to rear bumper of following vehicle at a point of time. If all the space headways in distance x over which the density has been measured are added,

$$\sum_{1}^{n_x} s_i = x \tag{1}$$

But the density (k) is the number of vehicles n_x at a distance of x, that is

$$k = \frac{n_x}{x} = \frac{n_x}{\sum_1^{n_x} s_i} = \frac{1}{s_{av}} \tag{2}$$

Where, s_{av} is average distance headway. The average distance headway is the inverse of density and is sometimes called as spacing.

Time-space diagram

Time space diagram is a convenient tool in understanding the movement of vehicles. It shows the trajectory of vehicles in the form of a two dimensional plot. Time space diagram can be plotted for a single vehicle as well as multiple vehicles. They are discussed below.

Single vehicle

Taking one vehicle at a time, analysis can be carried out on the position of the vehicle with respect to time. This analysis will generate a graph which gives the relation of its position on a road stretch relative to time. This plot thus will be between distance x and time t and x will be a functions the position of the vehicle for every t along the road stretch. This graphical representation of $x(t)$ in a (t, x) plane is a curve which is called as a trajectory.

The trajectory provide an intuitive, clear, and complete summary of vehicular motion in one dimension.

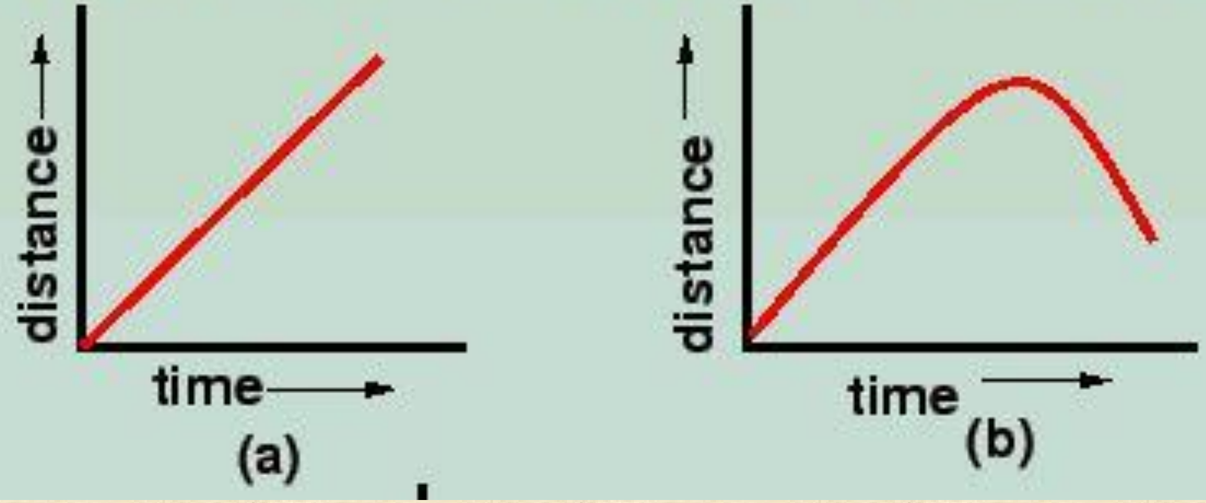


Figure 1: Time space diagram for a single vehicle

In figure 1(a), the the distance x goes on increasing with respect to the origin as time progresses. The vehicle is moving at a smooth condition along the road way. In figure 1(b), the vehicle at first moves with a smooth pace after reaching a position reverses its direction of movement. In figure 1(c), the vehicle in between becomes stationary and maintains the same position.

From the figure, steeply increasing section of $x(t)$ denote a rapidly advancing vehicle and horizontal portions of $x(t)$ denote a stopped vehicle while shallow sections show a slow-moving vehicle. A straight line denotes constant speed motion and curving sections denote accelerated motion; and if the curve is concave downwards it denotes acceleration. But a curve which is convex upwards denotes deceleration.

Multiple Vehicles

Time-space diagram can also be used to determine the fundamental parameters of traffic flow like speed, density and volume. It can also be used to find the derived characteristics like space headway and time headway. Figure 1 shows the time-space diagram for a set of vehicles traveling at constant speed. Density, by definition is the number of vehicles per unit length. From the figure, an observer looking into the stream can count 4 vehicles passing the stretch of road between x_1 and x_2 at time t. Hence, the density is given as

$$k = \frac{4\ vehicles}{x_2 - x_1} \qquad (1)$$

We can also find volume from this time-space diagram. As per the definition, volume is the number of vehicles counted for a particular interval of time. From the figure 1 we can see that 6 vehicles are present between the time t_1 and t_2. Therefore, the volume q is given as

$$q = \frac{3\ vehicles}{t_2 - t_1} \qquad (2)$$

Again the averages taken at a specific location (i.e., time ranging over an interval) are called time means and those taken at an instant over a space interval are termed as space means.

Another related definition which can be given based on the time-space diagram is the headway. Space headway is defined as the distance between corresponding points of two successive vehicles at any given time. Thus, the vertical gap between any two consecutive lines represents space headway. The reciprocal of density otherwise gives the space headway between vehicles at that time.

Similarly, time headway is defined as the time difference between any two successive vehicles when they cross a given point. Thus, the horizontal gap between the vehicles represented by the lines gives the time headway. The reciprocal of flow gives the average time headway between vehicles at that point.

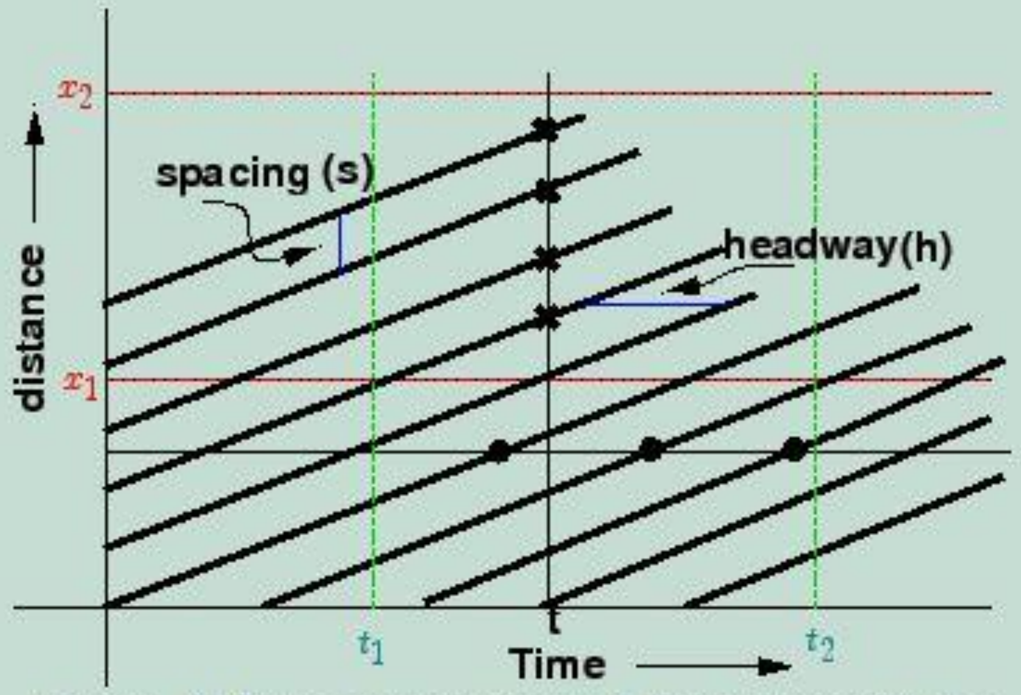


Figure 1: Time space diagram for many vehicles

Time mean speed (v_t)

As noted earlier, time mean speed is the average of all vehicles passing a point over a duration of time. It is the simple average of spot speed. Time mean speed v_t is given by,

$$v_t = \frac{1}{n}\sum_{i=1}^{n} v_i, \tag{1}$$

where v_i is the spot speed of i^{th} vehicle, and n is the number of observations. In many speed studies, speeds are represented in the form of frequency table. Then the time mean speed is given by,

$$v_t = \frac{\sum_{i=1}^{n} q_i v_i}{\sum_{i=1}^{n} q_i}, \tag{2}$$

where q_i is the number of vehicles having speed v_i, and n is the number of such speed categories.

Space mean speed (v_s)

The space mean speed also averages the spot speed, but spatial weightage is given instead of temporal. This is derived as below. Consider unit length of a road, and let v_i is the spot speed of i^{th} vehicle. Let t_i is the time the vehicle takes to complete unit distance and is given by $\frac{1}{v_i}$. If there are n such vehicles, then the average travel time t_s is given by,

$$t_s = \frac{\Sigma t_i}{n} = \frac{1}{n}\Sigma\frac{1}{v_i}. \qquad (1)$$

If t_{av} is the average travel time, then average speed $v_s = \frac{1}{t_s}$. Therefore, from the above equation,

$$v_s = \frac{n}{\sum_{i=1}^{n}\frac{1}{v_i}}. \qquad (2)$$

This is simply the harmonic mean of the spot speed. If the spot speeds are expressed as a frequency table, then,

$$v_s = \frac{\sum_{i=1}^{n} q_i}{\sum_{i=1}^{n}\frac{q_i}{v_i}} \qquad (3)$$

Numerical Example

If the spot speeds are 50, 40, 60, 54 and 45, then find the time mean speed and space mean speed.

Solution

Time mean speed v_t is the average of spot speed. Therefore, $v_t = \frac{\Sigma v_i}{n} = \frac{50+40+60+54+45}{5} = \frac{249}{5} =$ 49.8. Space mean speed is the harmonic mean of spot speed. Therefore, $v_s = \frac{n}{\Sigma \frac{1}{v_i}} = \frac{5}{\frac{1}{50}+\frac{1}{40}+\frac{1}{60}+\frac{1}{54}+\frac{1}{45}} = \frac{5}{0.12} = 48.82.$

The results of a speed study is given in the form of a frequency distribution table. Find the time mean speed and space mean speed.

speed range	frequency
2-5	1
6-9	4
10-13	0
14-17	7

Solution

No.	speed range	average speed (v_i)	volume of flow (q_i)	$v_i q_i$	$\frac{q_i}{v_i}$
1	2-5	3.5	1	3.5	2.29
2	6-9	7.5	4	30.0	0.54
3	10-13	11.5	0	0	0
4	14-17	15.5	7	108.5	0.45
	total		12	142	3.28

The time mean speed and space mean speed can be found out from the frequency table given below. First, the average speed is computed, which is the mean of the speed range. For example, for the first speed range, average speed, $v_i = \frac{2+5}{2}$ = 3.5 seconds. The volume of flow q_i for that speed range is same as the frequency. The terms $v_i . q_i$ and $\frac{q_i}{v_i}$ are also tabulated, and their summations given in the last row. Time mean speed can be computed as, $v_t = \frac{\Sigma q_i v_i}{\Sigma q_i} = \frac{142}{12} = 11.83$. Similarly, space mean speed can be computed as, $v_s = \frac{\Sigma q_i}{\Sigma \frac{q_i}{v_i}} = \frac{12}{3.28} = 3.65$.

Relation between time mean speed and space mean speed

The relation between time mean speed(v_t) and space mean speed(v_s) is given by the following relation:

$$v_t = v_s + \frac{\sigma^2}{v_s} \qquad (1)$$

where, σ^2 is the standard deviation of the spot speed. The derivation of the formula is given in the next subsection. The standard deviation(σ^2) can be computed in the following equation:

$$\sigma^2 = \frac{\Sigma q_i v_i^2}{\Sigma q_i} - (v_t)^2 \qquad (2)$$

where, q_i is the frequency of the vehicle having v_i speed.

Numerical Example

For the data given below,compute the time mean speed and space mean speed. Also verify the relationship between them. Finally compute the density of the stream.

speed range	frequency
0-10	5
10-20	15
20-30	20
30-40	25
40-50	30

	speed	mid interval	flow				
No.	range	$v_i = \frac{v_l + v_u}{2}$	q_i	$q_i v_i$	v_i^2	$q_i v_i^2$	q_i / v_i
	$v^l < v < v^u$						
1	0-10	5	6	30	25	150	6/5
2	10-20	15	16	240	225	3600	16/15
3	20-30	20	24	600	625	15000	24/25
4	30-40	25	25	875	1225	30625	25/35
5	40-50	30	17	765	2025	34425	17/45
	total		88	2510		83800	4.3187

The solution of this problem consist of computing the time mean speed $v_t = \frac{\Sigma q_i v_i}{\Sigma q_i}$,space mean speed $v_s = \frac{\Sigma q_i}{\frac{\Sigma q_i}{v_i}}$,verifying their relation by the equation $v_t = v_s + \frac{\sigma^2}{v_s}$,and using this to compute the density. To verify their relation, the standard deviation also need to be computed $\sigma^2 = \frac{\Sigma q v^2}{\Sigma q} - v_t^2$. For convenience,the calculation can be done in a tabular form as shown in table 0.1.1.

The time mean speed(v_t) is computed as:

$$v_t = \frac{\Sigma q_i v_i}{\Sigma q_i}$$

$$= \frac{2510}{88} = 28.52$$

The space mean speed can be computed as:

$$v_s = \frac{\Sigma q_i}{\frac{\Sigma q_i}{v_i}}$$

$$= \frac{88}{4.3187} = 20.38$$

The standard deviation can be computed as:

$$\sigma^2 = \frac{\Sigma q v^2}{\Sigma q} - v_t^2$$

$$= \frac{83800}{88} - 28.52^2 = 138.727$$

The time mean speed can also v_t can also be computed as:

$$v_t = v_s + \frac{\sigma^2}{v_s} = 20.38 + \frac{138.727}{20.38} = 27.184$$

The density can be found as:

$$k = \frac{q}{v} = \frac{88}{20.38} = 4.3 \text{ vehicle/km}$$

Fundamental diagrams of traffic flow

The relation between flow and density, density and speed, speed and flow, can be represented with the help of some curves. They are referred to as the fundamental diagrams of traffic flow. They will be explained in detail one by one below.

Flow-density curve

The flow and density varies with time and location. The relation between the density and the corresponding flow on a given stretch of road is referred to as one of the fundamental diagram of traffic flow. Some characteristics of an ideal flow-density relationship is listed below:

1. When the density is zero, flow will also be zero,since there is no vehicles on the road.
2. When the number of vehicles gradually increases the density as well as flow increases.
3. When more and more vehicles are added, it reaches a situation where vehicles can't move. This is referred to as the jam density or the maximum density. At jam density, flow will be zero because the vehicles are not moving.
4. There will be some density between zero density and jam density, when the flow is maximum. The relationship is normally represented by a parabolic curve as shown in figure 1

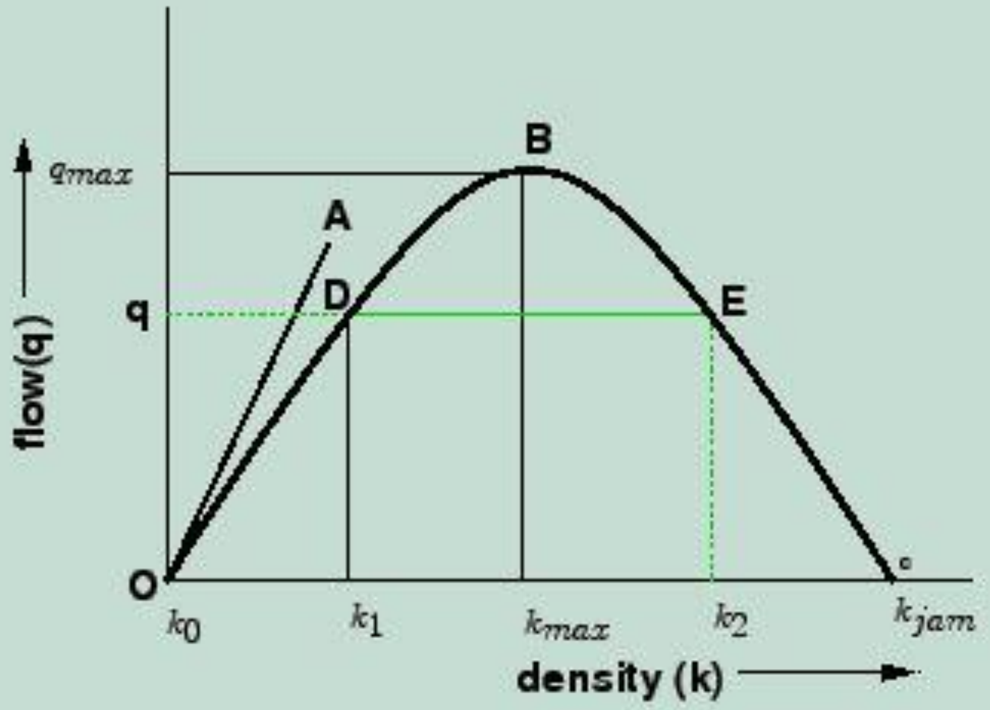


Figure 1: Flow density curve

The point O refers to the case with zero density and zero flow. The point B refers to the maximum flow and the corresponding density is k_{max}. The point C refers to the maximum density k_{jam} and the corresponding flow is zero. OA is the tangent drawn to the parabola at O, and the slope of the line OA gives the mean free flow speed, ie the speed with which a vehicle can travel when there is no flow. It can also be noted that points D and E correspond to same flow but has two different densities. Further, the slope of the line OD gives the mean speed at density k_1 and slope of the line OE will give mean speed at density k_2. Clearly the speed at density k_1 will be higher since there are less number of vehicles on the road.

Speed-density diagram

Similar to the flow-density relationship, speed will be maximum, referred to as the free flow speed, and when the density is maximum, the speed will be zero. The most simple assumption is that this variation of speed with density is linear as shown by the solid line in figure 1. Corresponding to the zero density, vehicles will be flowing with their desire speed, or free flow speed. When the density is jam density, the speed of the vehicles becomes zero.

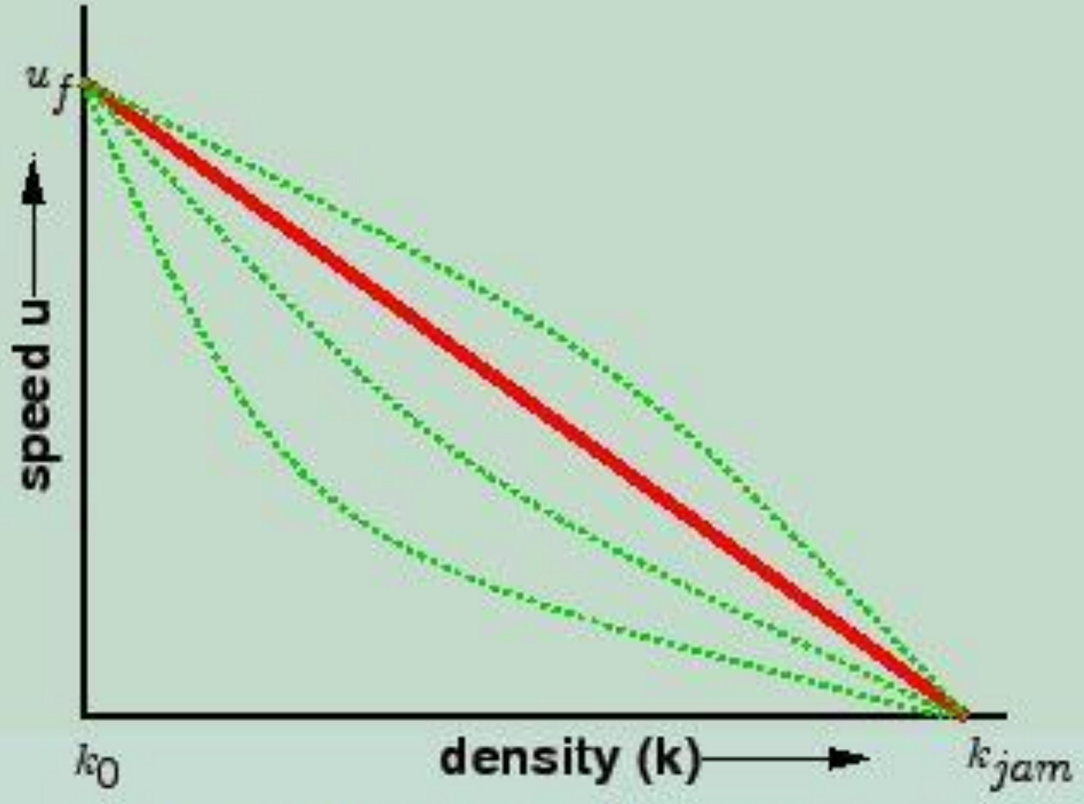


Figure 1: Speed-density diagram

It is also possible to have non-linear relationships as shown by the dotted lines. These will be discussed later.

Speed flow relation

The relationship between the speed and flow can be postulated as follows. The flow is zero either because there is no vehicles or there are too many vehicles so that they cannot move. At maximum flow, the speed will be in between zero and free flow speed. This relationship is shown in figure 1.

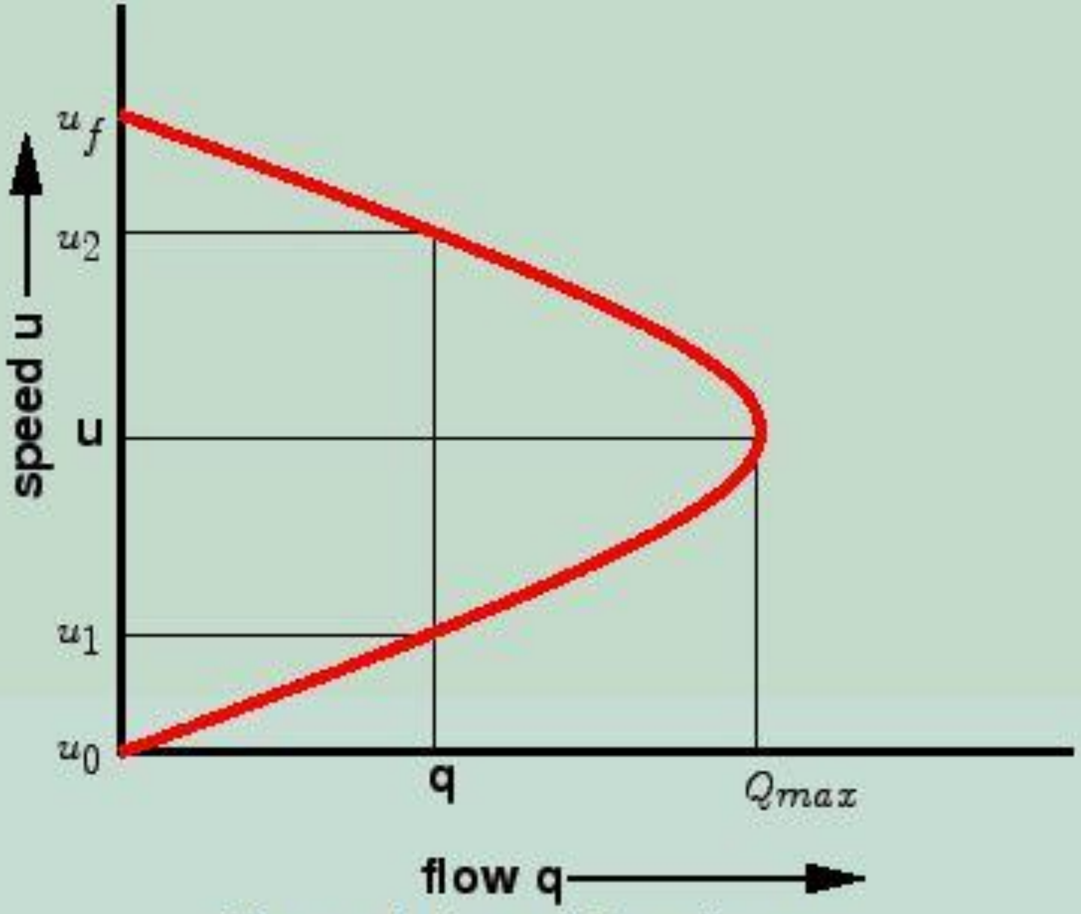


Figure 1: Speed-flow diagram

The maximum flow q_{max} occurs at speed u. It is possible to have two different speeds for a given flow.

Combined diagrams

The diagrams shown in the relationship between speed-flow, speed-density, and flow-density are called the fundamental diagrams of traffic flow. These are as shown in figure 2. One could observe the inter-relationship of these diagrams.

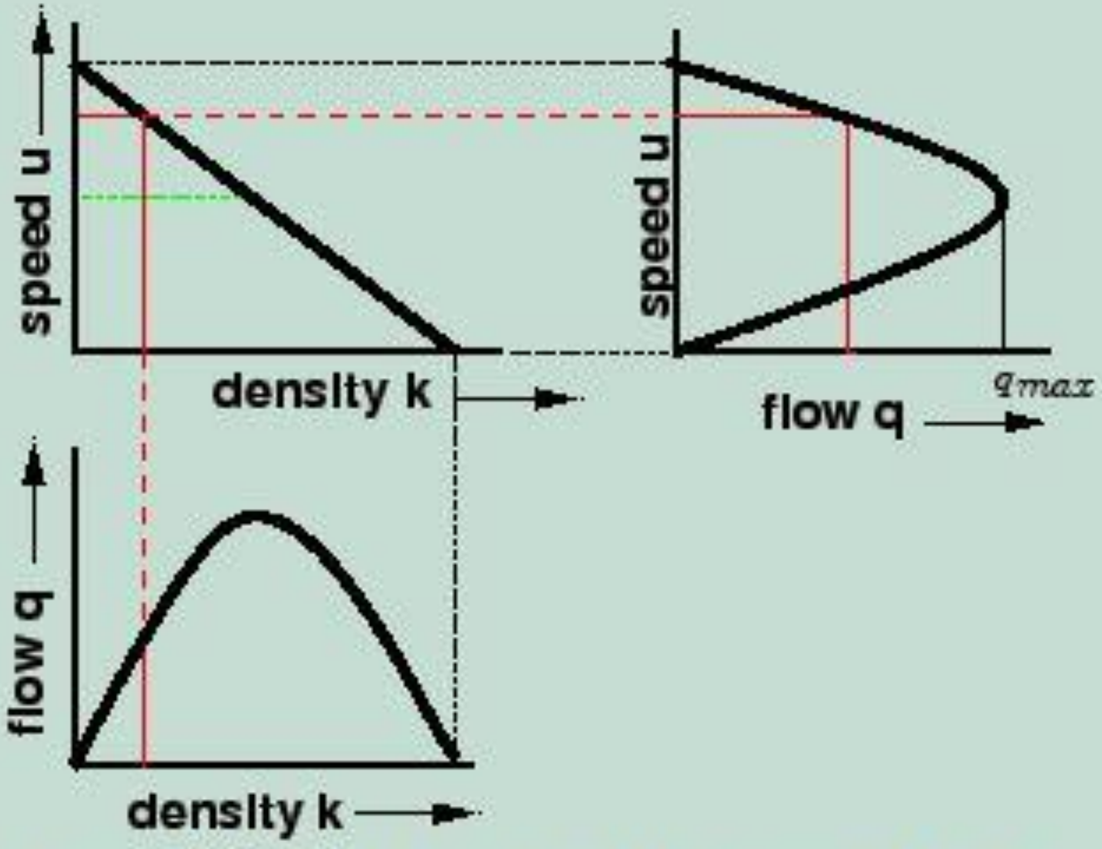


Figure 2: Fundamental diagram of traffic flow

Greenshield's macroscopic stream model

Macroscopic stream models represent how the behaviour of one parameter of traffic flow changes with respect to another. Most important among them is the relation between speed and density. The first and most simple relation between them is proposed by Greenshield. Greenshield assumed a linear speed-density relationship as illustrated in figure 1 to derive the model.

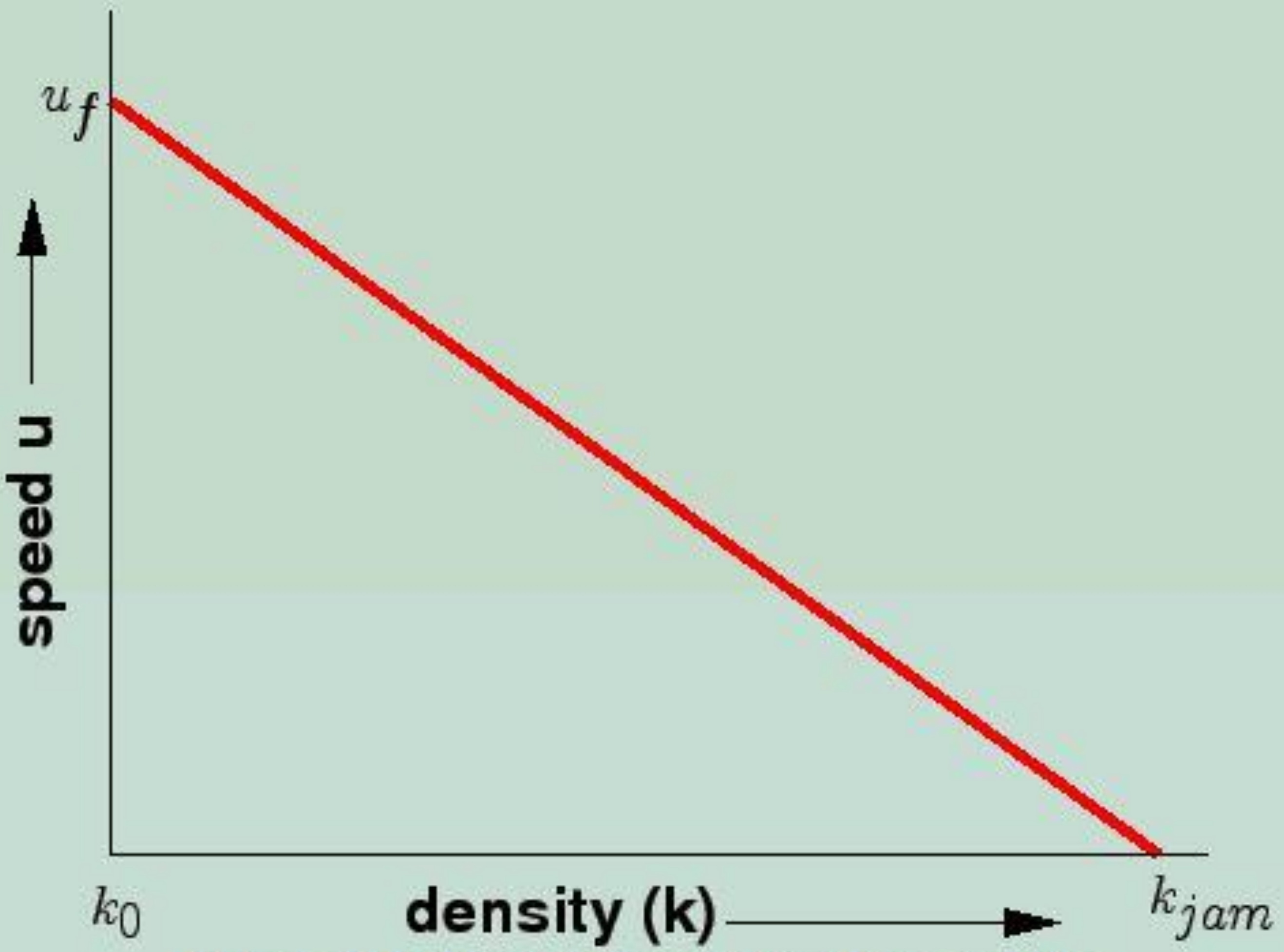


Figure 1: Relation between speed and density

The equation for this relationship is shown below.

$$v = v_f - \left[\frac{v_f}{k_j}\right].k \qquad (1)$$

where v is the mean speed at density k, v_f is the free speed and k_j is the jam density. This equation (1) is often referred to as the Greenshield's model. It indicates that when density becomes zero, speed approaches free flow speed (ie. $v \longrightarrow v_f$ when $k \rightarrow 0$).

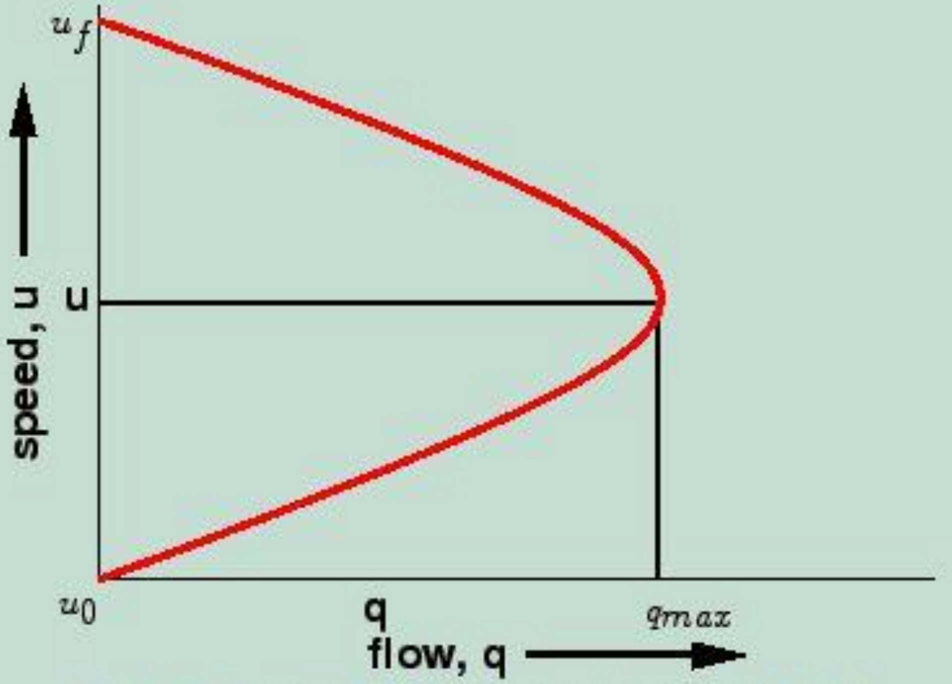


Figure 2: Relation between speed and flow

Once the relation between speed and flow is established, the relation with flow can be derived. This relation between flow and density is parabolic in shape and is shown in figure 3. Also, we know that

$$q = k.v \tag{2}$$

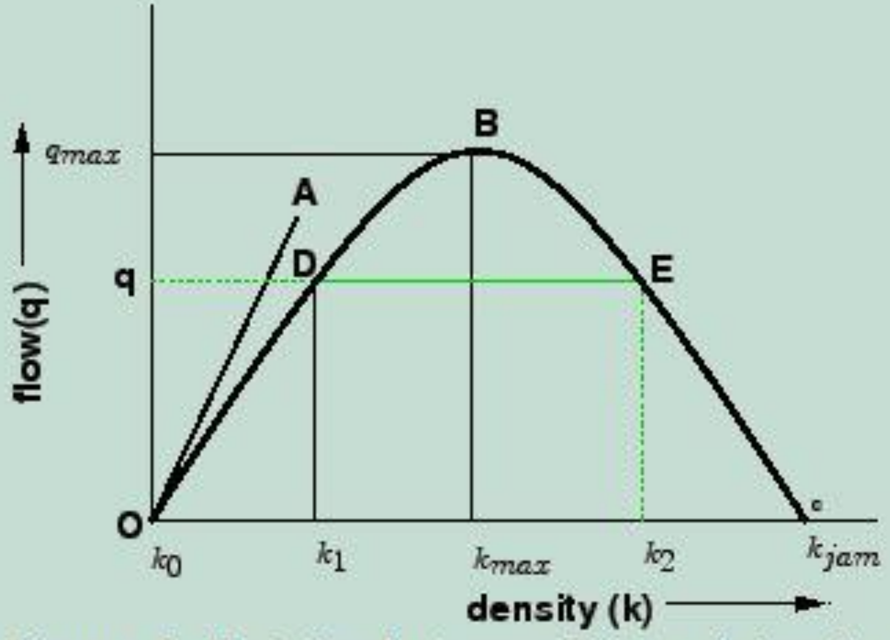


Figure 3: Relation between flow and density
1

Now substituting equation 1 in equation 2, we get

$$q = v_f.k - \left[\frac{v_f}{k_j}\right] k^2 \tag{3}$$

Similarly we can find the relation between speed and flow. For this, put $k = \frac{q}{v}$ in equation 1 and solving, we get

$$q = k_j.v - \left[\frac{k_j}{v_f}\right] v^2 \tag{4}$$

This relationship is again parabolic and is shown in figure 2. Once the relationship between the fundamental variables of traffic flow is established, the boundary conditions can be derived. The boundary conditions that are of interest are jam density, free-flow speed, and maximum flow. To find density at maximum flow, differentiate equation 3 with respect to k and equate it to zero. ie.,

$$\frac{dq}{dk} = 0$$

$$v_f - \frac{v_f}{k_j}.2k = 0$$

$$k = \frac{k_j}{2}$$

Denoting the density corresponding to maximum flow as k_0,

$$k_0 = \frac{k_j}{2} \tag{5}$$

Therefore, density corresponding to maximum flow is half the jam density. Once we get k_0, we can derive for maximum flow, q_{max}. Substituting equation 5 in equation 3

$$q_{max} = v_f.\frac{k_j}{2} - \frac{v_f}{k_j}.\left[\frac{k_j}{2}\right]^2$$

$$= v_f.\frac{k_j}{2} - v_f.\frac{k_j}{4}$$

$$= \frac{v_f.k_j}{4}$$

Thus the maximum flow is one fourth the product of free flow and jam density. Finally to get the speed at maximum flow, v_0, substitute equation 5 in equation 1 and solving we get,

$$v_0 = v_f - \frac{v_f}{k_j}.\frac{k_j}{2}$$

$$v_0 = \frac{v_f}{2} \quad (6)$$

Therefore, speed at maximum flow is half of the free speed.

Other macroscopic stream models

In Greenshield's model, linear relationship between speed and density was assumed. But in field we can hardly find such a relationship between speed and density. Therefore, the validity of Greenshield's model was questioned and many other models came up. Prominent among them are Greenberg's logarithmic model, Underwood's exponential model, Pipe's generalized model, and multi-regime models. These are briefly discussed below.

Greenberg's logarithmic model

Greenberg assumed a logarithmic relation between speed and density. He proposed,

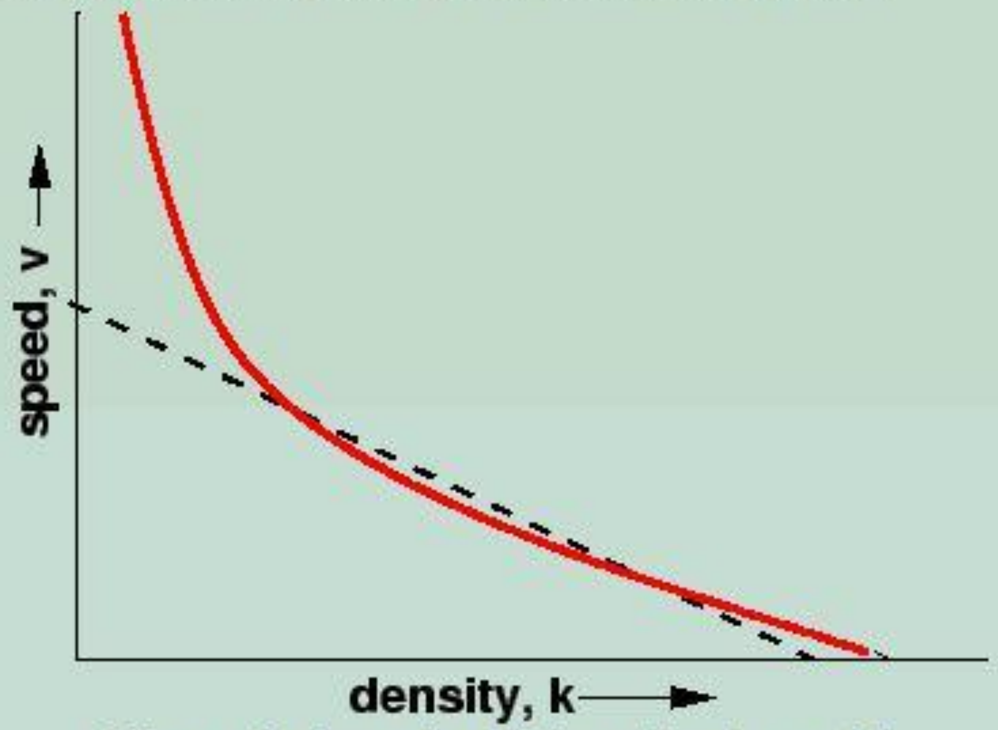


Figure 1: Greenberg's logarithmic model

$$v = v_0 \ln \frac{k_j}{k} \tag{1}$$

This model has gained very good popularity because this model can be derived analytically. (This derivation is beyond the scope of this notes). However, main drawbacks of this model is that as density tends to zero, speed tends to infinity. This shows the inability of the model to predict the speeds at lower densities.

Underwood's exponential model

Trying to overcome the limitation of Greenberg's model, Underwood put forward an exponential model as shown below.

$$v = v_f . e^{\frac{-k}{k_0}} \qquad (1)$$

where v_f The model can be graphically expressed as in figure 1. is the free flow speed and k_o is the optimum density, i.e. the density corresponding to the maximum flow.

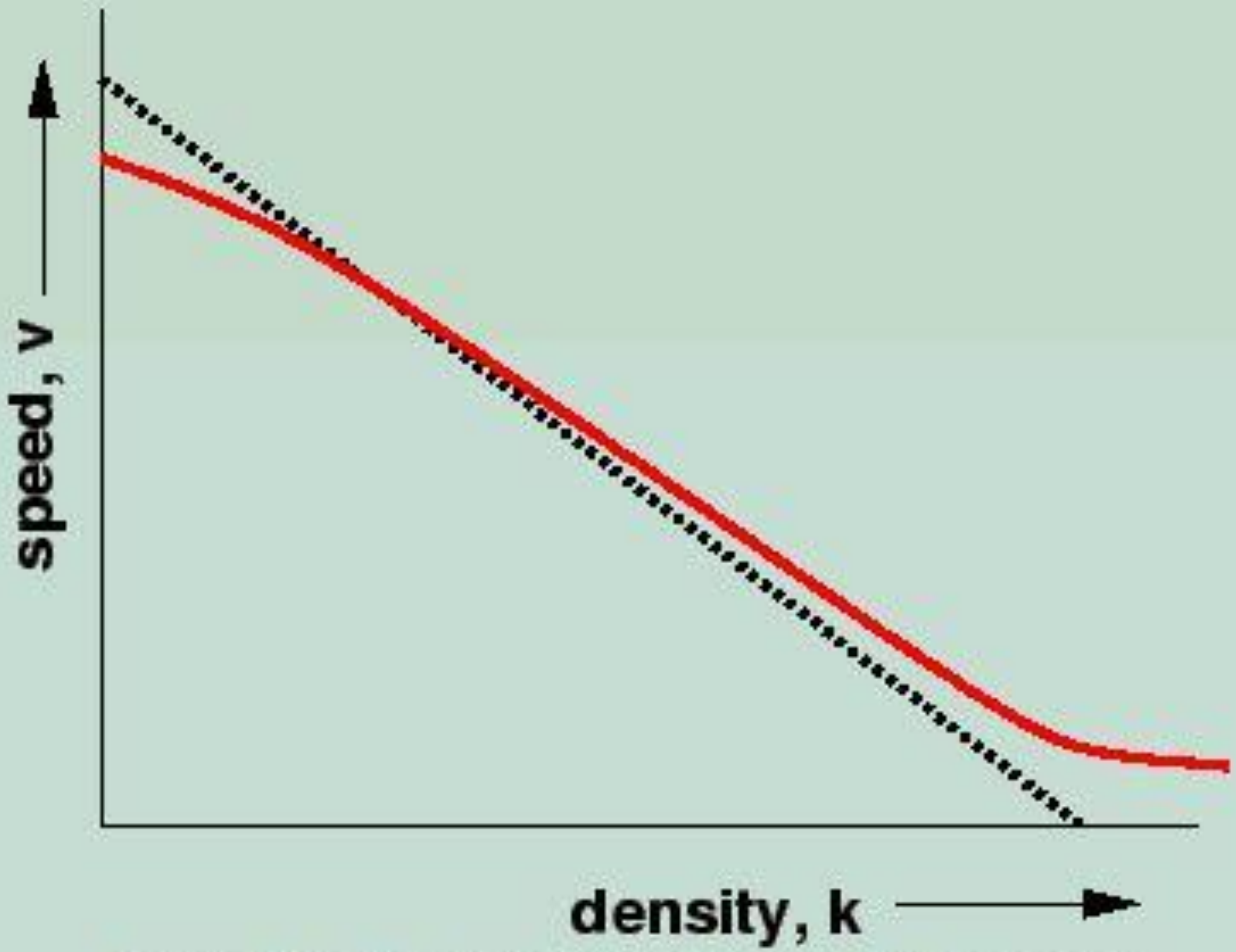


Figure 1: Underwood's exponential model

In this model, speed becomes zero only when density reaches infinity which is the drawback of this model. Hence this cannot be used for predicting speeds at high densities.

Pipes' generalized model

Further developments were made with the introduction of a new parameter (n) to provide for a more generalized modeling approach. Pipes proposed a model shown by the following equation.

$$v = v_f \left[1 - \left(\frac{k}{k_j} \right)^n \right] \quad (1)$$

When n is set to one, Pipe's model resembles Greenshield's model. Thus by varying the values of n, a family of models can be developed.

Multi-regime models

All the above models are based on the assumption that the same speed-density relation is valid for the entire range of densities seen in traffic streams. Therefore, these models are called single-regime models. However, human behaviour will be different at different densities. This is corroborated with field observations which shows different relations at different range of densities. Therefore, the speed-density relation will also be different in different zones of densities. Based on this concept, many models were proposed generally called multi-regime models. The most simple one is called a two-regime model, where separate equations are used to represent the speed-density relation at congested and uncongested traffic.

Macroscopic flow models

If one looks into traffic flow from a very long distance, the flow of fairly heavy traffic appears like a stream of a fluid. Therefore, a *macroscopic* theory of traffic can be developed with the help of hydrodynamic theory of fluids by considering traffic as an effectively one-dimensional compressible fluid. The behaviour of individual vehicle is ignored and one is concerned only with the behaviour of sizable aggregate of vehicles. The earliest traffic flow models began by writing the balance equation to address vehicle number conservation on a road. In fact, all traffic flow models and theories must satisfy the law of conservation of the number of vehicles on the road. Assuming that the vehicles are flowing from left to right, the continuity equation can be written as

$$\frac{\partial k(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0 \tag{1}$$

where x denotes the spatial coordinate in the direction of traffic flow, t is the time, k is the density and q denotes the flow. However, one cannot get two unknowns, namely $k(x,t)$ by and $q(x,t)$ by solving one equation. One possible solution is to write two equations from two regimes of the flow, say before and after a bottleneck. In this system the flow rate before and after will be same, or

$$k_1 v_1 = k_2 v_2 \tag{2}$$

From this the shock wave velocity can be derived as

$$v(t_o)_p = \frac{q_2 - q_1}{k_2 - k_1} \tag{3}$$

This is normally referred to as Stock's shock wave formula. An alternate possibility which Lighthill and Whitham adopted in their landmark study is to assume that the flow rate q is determined primarily by the local density k, so that flow q can be treated as a function of only density k. Therefore the number of unknown variables will be reduced to one. Essentially this assumption states that k(x,t) and q (x,t) are not independent of each other. Therefore the continuity equation takes the form

$$\frac{\partial k(x,t)}{\partial t} + \frac{\partial q(k(x,t))}{\partial x} = 0 \tag{4}$$

However, the functional relationship between flow q and density k cannot be calculated from fluid-dynamical theory. This has to be either taken as a phenomenological relation derived from the empirical observation or from microscopic theories. Therefore, the flow rate q is a function of the vehicular density k; $q = q(k)$. Thus, the balance equation takes the form

$$\frac{\partial k(x,t)}{\partial t} + \frac{\partial q(k(x,t))}{\partial x} = 0 \tag{5}$$

Now there is only one independent variable in the balance equation, the vehicle density k. If initial and boundary conditions are known, this can be solved. Solution to LWR models are kinematic waves moving with velocity

$$\frac{dq(k)}{dk} \tag{6}$$

This velocity v_k is positive when the flow rate increases with density, and it is negative when the flow rate decreases with density. In some cases, this function may shift from one regime to the other, and then a shock is said to be formed. This shock wave propagate at the velocity

$$v_s = \frac{q(k_2) - q(k_1)}{k_2 - k_1} \tag{7}$$

where $q(k_2)$ and $q(k_1)$ are the flow rates corresponding to the upstream density k_2 and downstream density k_1 of the shock wave. Unlike Stock's shock wave formula there is only one variable here.

Shock waves

The flow of traffic along a stream can be considered similar to a fluid flow. Consider a stream of traffic flowing with steady state conditions, i.e., all the vehicles in the stream are moving with a constant speed, density and flow. Let this be denoted as state A (refer figure 1. Suddenly due to some obstructions in the stream (like an accident or traffic block) the steady state characteristics changes and they acquire another state of flow, say state B. The speed, density and flow of state A is denoted as v_A, k_A, and q_A, and state B as v_B, k_B, and q_B respectively.

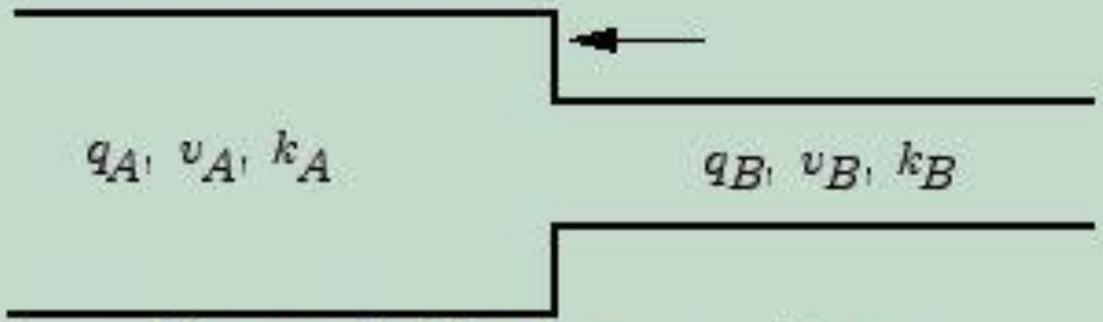


Figure 1: Shock wave: Stream characteristics

The flow-density curve is shown in figure 2.

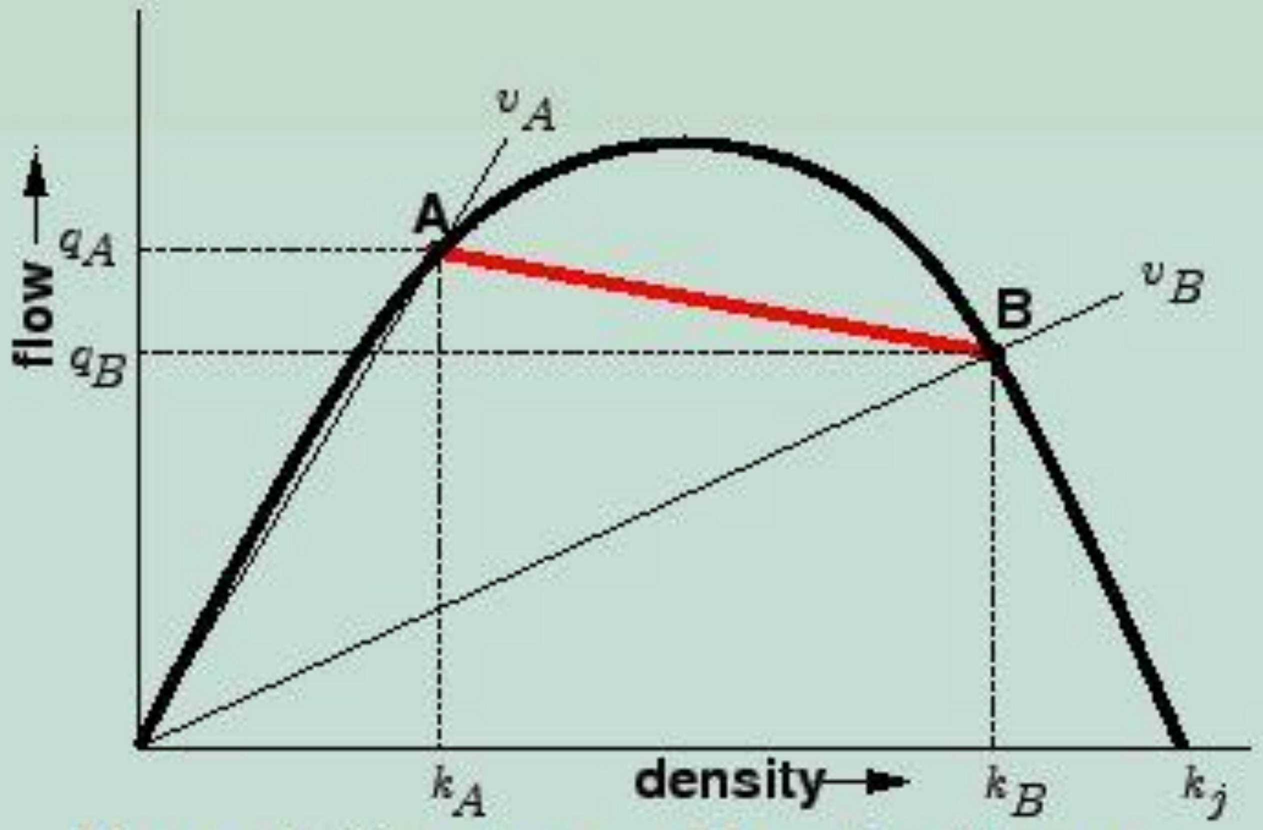


Figure 2: Shock wave: Flow-density curve

The speed of the vehicles at state A is given by the line joining the origin and point A in the graph. The time-space diagram of the traffic stream is also plotted in figure 3.

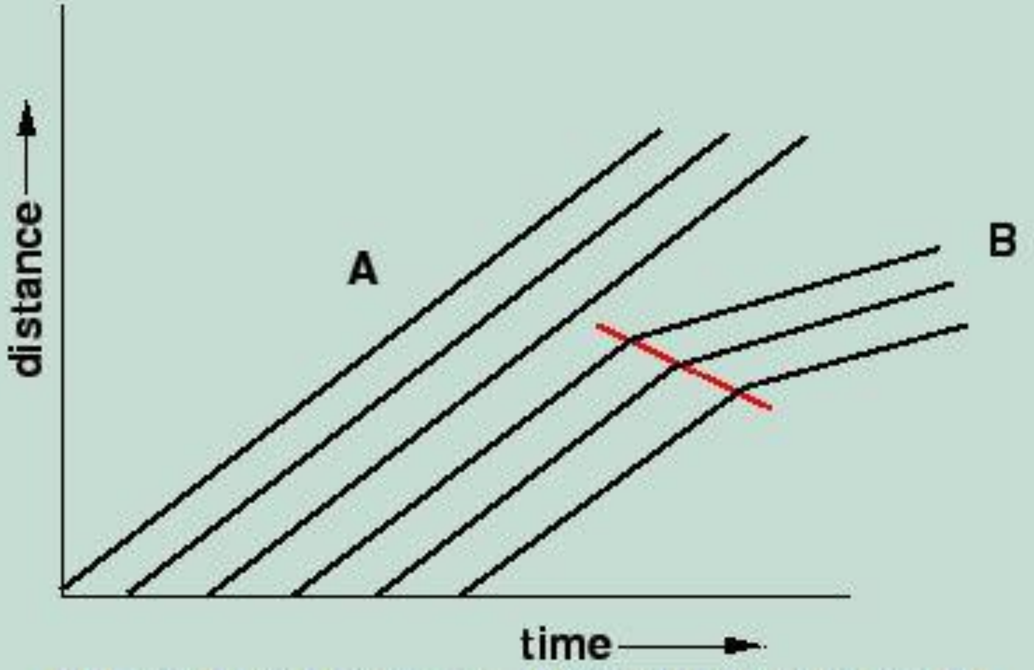


Figure 3: Shock wave : time-distance diagram

All the lines are having the same slope which implies that they are moving with constant speed. The sudden change in the characteristics of the stream leads to the formation of a shock wave. There will be a cascading effect of the vehicles in the upstream direction. Thus shock wave is basically the movement of the point that demarcates the two stream conditions. This is clearly marked in the figure 2. Thus the shock waves produced at state B are propagated in the backward direction. The speed of the vehicles at state B is the line joining the origin and point B of the flow-density curve. Slope of the line AB gives the speed of the shock wave (refer figure 2). If speed of the shock-wave is represented as ω_{AB} , then

$$\omega_{AB} = \frac{q_A - q_B}{k_A - k_B} \qquad (1)$$

The above result can be analytically solved by equating the expressions for the number vehicles leaving the upstream and joining the downstream of the shock wave boundary (this assumption is true since the vehicles cannot be created or destroyed. Let N_A be the number of vehicles leaving the section A. Then, $N_A = q_B\ t$. The relative speed of these vehicles with respect to the shock wave will be $v_A - \omega_{AB}$. Hence,

$$N_A = k_A\ (v_A - \omega_{AB})\ t \tag{2}$$

Similarly, the vehicles entering the state B is given as

$$N_B = k_A\ (v_B - \omega_{AB})\ t \tag{3}$$

Equating equations 2 and 3, and solving for ω_{AB} as follows will yield to:

$$N_A = N_B$$

$$k_A\ (v_A - \omega_{AB})\ t = k_B\ (v_B - \omega_{AB})\ t$$

$$k_A\ v_A\ t - k_A\ \omega_{AB}\ t = k_B\ v_B\ t - k_B \omega_{AB}\ t$$

$$k_A \omega_{AB}\ t - k_B \omega_{AB}\ t = k_A\ v_A - k_B\ v_B$$

$$\omega_{AB}\ (k_A - k_B) = q_A - q_B$$

This will yield the following expression for the shock-wave speed.

$$\omega_{AB} = \frac{q_A - q_B}{k_A - k_B} \tag{4}$$

In this case, the shock wave move against the direction of traffic and is therefore called a backward moving shock wave. There are other possibilities of shock waves such as forward moving shock waves and stationary shock waves. The forward moving shock waves are formed when a stream with higher density and higher flow meets a stream with relatively lesser density and flow. For example, when the width of the road increases suddenly, there are chances for a forward moving shock wave. Stationary shock waves will occur when two streams having the same flow value but different densities meet.

Calculations for Moving Observer method

Numerical Example

The length of a road stretch used for conducting the moving observer test is 0.5 km and the speed with which the test vehicle moved is 20 km/hr. Given that the number of vehicles encountered in the stream while the test vehicle was moving against the traffic stream is 107, number of vehicles that had overtaken the test vehicle is 10, and the number of vehicles overtaken by the test vehicle is 74, find the flow, density and average speed of the stream.

Solution

Time taken by the test vehicle to reach the other end of the stream while it is moving along with the traffic is $t_w = \frac{0.5}{20}$ = 0.025 hr

Time taken by the observer to reach the other end of the stream while it is moving against the traffic is $t_a = t_w$ = 0.025 hr

Flow is given by equation, q = $\frac{107+(10-74)}{0.025+0.025}$ = 860 veh/hr

Stream speed v_s can be found out from equation $v_s = \frac{0.5}{0.025-\frac{10.74}{860}}$ = 5 km/hr

Density can be found out from equation as k = $\frac{860}{5}$ = 172veh/km

Calculations for Moving Observer method

Numerical Example

The data from four moving observer test methods are shown in the table. Column 1 gives the sample number, column 2 gives the number of vehicles moving against the stream, column 3 gives the number of vehicles that had overtaken the test vehicle, and last column gives the number of vehicles overtaken by the test vehicle. Find the three fundamental stream parameters for each set of data. Also plot the fundamental diagrams of traffic flow.

Sample no.	1	2	3
1	107	10	74
2	113	25	41
3	30	15	5
4	79	18	9

Moving Observer method

Sample no.	m_a	m_o	m_p	$m_w = (m_o - m_p)$	t_a	t_w	$q = \frac{m_a+m_w}{t_a+t_w}$	$u = \frac{l}{t_w - \frac{m_w}{q}}$	$k = \frac{q}{v}$
1	107	10	74	-64	0.025	0.025	860	5.03	171
2	113	25	41	-16	0.025	0.025	1940	15.04	129
3	30	15	5	10	0.025	0.025	800	40	20
4	79	18	9	9	0.025	0.025	1760	25.14	70

From the calculated values of flow, density and speed, the three fundamental diagrams can be plotted as shown in figure 1.

Moving Observer method

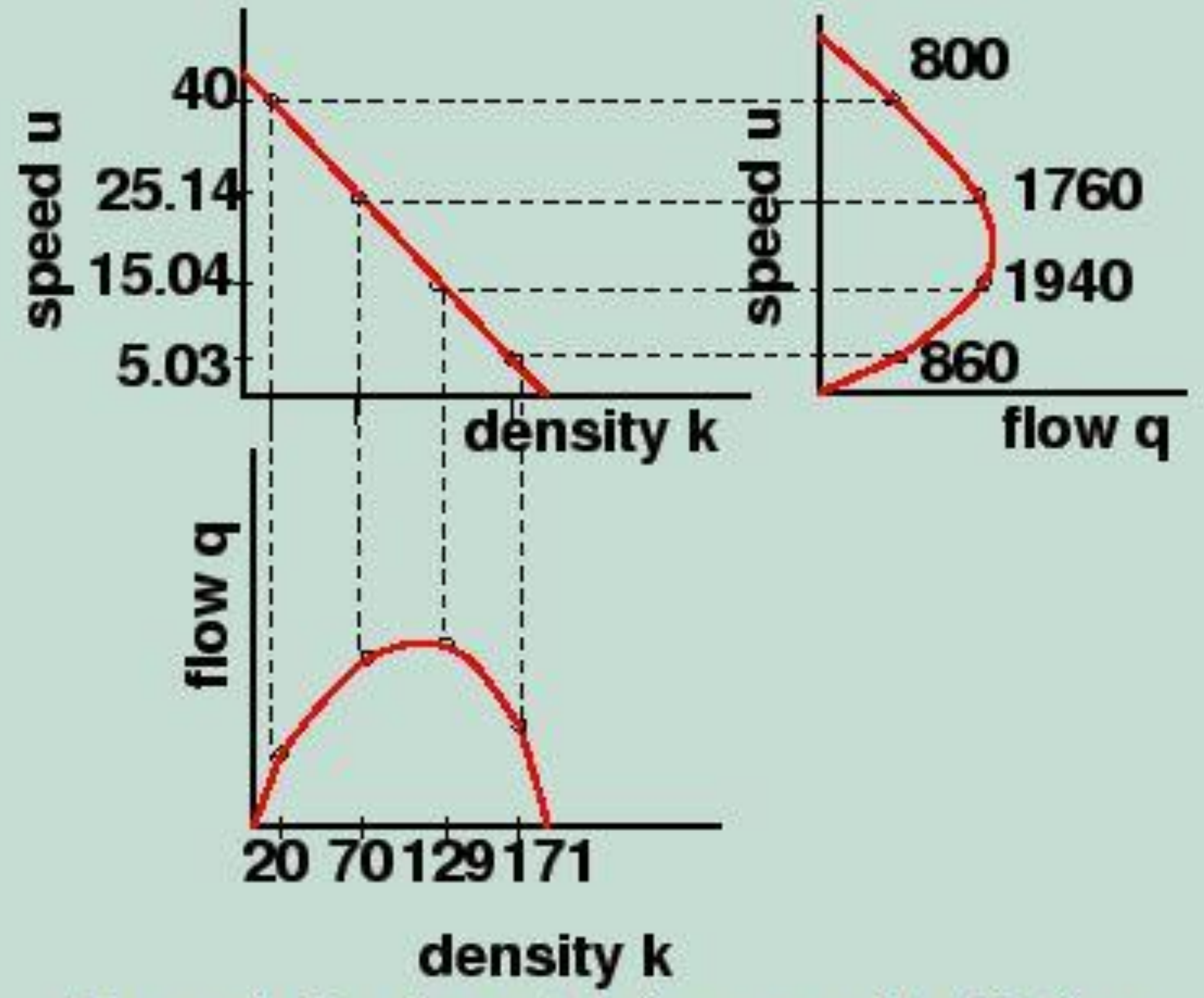


Figure 1: Fundamental diagrams of traffic flow

Peak hour Factor Calculation

The table below shows the volumetric data observed at an intersection. Calculate the peak hour volume, peak hour factor (PHF), and the actual (design) flow rate for this approach.

Table 1: Volumetric data

Time interval	Cars
4:00 - 4:15	30
4:15 - 4:30	26
4:30 - 4:45	35
4:45 - 5:00	40
5:00 - 5:15	49
5:15 - 5:30	55
5:30 - 5:45	65
5:45 - 6:00	50
6:00 - 6:15	39
6:15 - 6:30	30

Peak hour Factor Calculation

We can locate the hour with the highest volume and the 15 minute interval with the highest volume. The peak hour is shown in blue below with the peak 15 minute period shown in bold font.

Table 2: Solution of the problem

Time interval	Cars
4:00 - 4:15	30
4:15 - 4:30	26
4:30 - 4:45	35
4:45 - 5:00	40
5:00 - 5:15	49
5:15 - 5:30	55
5:30 - 5:45	**65**
5:45 - 6:00	50
6:00 - 6:15	39
6:15 - 6:30	30

Peak hour Factor Calculation

The peak hour volume is just the sum of the volumes of the four 15 minute intervals within the peak hour (219). The peak 15 minute volume is 65 in this case. The peak hour factor (PHF) is found by dividing the peak hour volume by four times the peak 15 minute volume. $PHF = \frac{219}{4\times65} = 0.84$ The actual (design) flow rate can be calculated by dividing the peak hour volume by the PHF, 219/0.84 = 260 vehicles/hr, or by multiplying the peak 15 minute volume by four, $4 \times 65 = 260$ vehicles per hour.

Peak hour Factor Calculation

Table 1: Values of PCU

Car	1.0
Motorcycle	0.5
Bicycle	0.2
LCV	2.2
Bus, Truck	3.5
3-wheeler	0.8

Highway capacity is measured in PCU/hour daily.

Peak hour Factor Calculation

The table below shows the volumetric data collected at an intersection: Calculate the peak hour volume, peak hour factor (PHF), and the actual (design) flow rate for this approach.

Table 2: Volumetric data collected

From	To	HCV	LCV	CAR	3W	2W
2.30	2.40	4	10	6	38	24
2.40	2.50	8	12	9	63	33
2.50	3.00	7	13	8	42	27
3.00	3.10	6	13	15	37	32
3.10	3.20	7	14	10	51	28
3.20	3.30	6	10	9	63	41
3.30	3.40	8	11	8	48	38
3.40	3.50	10	6	15	47	21
3.50	4.00	9	7	9	54	26
4.00	4.10	10	9	11	62	35
4.10	4.20	12	11	12	61	39
4.20	4.30	8	8	10	54	42

Peak hour Factor Calculation

The first step in this solution is to find the total traffic volume for each 15 minute period in terms of passenger car units. For this purpose the PCU values given in the table are used. Once we have this, we can locate the hour with the highest volume and the 15 minute interval with the highest volume. The peak hour is shown in blue below with the peak 15 minute period shown in a darker shade of blue.

Table 3: Solution of the problem

From	To	Flow in PCU
2.30	2.40	84.4
2.40	2.50	130.3
2.50	3.00	108.2
3.00	3.10	110.2
3.10	3.20	120.1
3.20	3.30	122.9
3.30	3.40	117.6
3.40	3.50	111.3
3.50	4.00	112.1
4.00	4.10	132.9
4.10	4.20	146.5
4.20	4.30	119.8

Peak hour Factor Calculation

The peak hour volume is just the sum of the volumes of the six 10 minute intervals within the peak hour (743.6 PCU). The peak 10 minute volume is 146.5 PCU in this case. The peak hour factor (PHF) is found by dividing the peak hour volume by four times the peak 10 minute volume.

$$PHF = \frac{743.6}{6 \times 146.5} = 0.85$$

The actual (design) flow rate can be calculated by dividing the peak hour volume by the PHF, $743.6/0.85 = 879\ PCU/hr$, or by multiplying the peak 10 minute volume by six, $6 \times 146.5 = 879\ PCU/hr$.

Determination of PCU

Traffic in many parts of the world is heterogeneous, where road space is shared among many traffic modes with different physical dimensions. Loose lane discipline prevails; car following is not the norm. This complicates computing of PCU. Some of the methods for determining passenger car units (PCU) are following:

- Modified Density Method
- Chandra's method
- Method Based on Relative Delay
- Headway method
- Multiple linear regression method
- Simulation method

It may be appropriate to use different values for the same vehicle type according to circumstances like volume of traffic, speed of vehicle, lane width and several external factors.

Method based on headway

Realizing one of the primary effects of heavy vehicles in the traffic stream is that they take up more space, headways have been used for some of the most popular methods to calculate PCUs. In 1976, Werner and Morrall suggested that the headway method is best suited to determine PCUs on level terrain at low levels of service. The PCU is calculated as

$$E_t = \frac{\left(\frac{H_m}{H_b}\right) - P_c}{P_t} \qquad (1)$$

where H_M is the average headway for a sample including all vehicle types, H_B is the average headway for a sample of passenger cars only, P_C is the proportion of cars, and P_T is the proportion of trucks.

The table given below show headway data for a number of traffic conditions. It is assumed that the traffic contains only car and truck. Compute the PCU value for each traffic condition Note that h_m, h_c, p_c, p_t respectively denote the average headway for mixed traffic, average headway for traffic consisting of cars only, the percentage of cars and percentage of trucks of the traffic stream.

Table 1: Headway data for a number of traffic conditions

h_m	h_c	p_c	p_t
2.70	2.5	0.90	0.10
2.80	2.5	0.85	0.15
2.94	2.5	0.80	0.20
3.10	2.5	0.75	0.25
3.25	2.5	0.70	0.30
3.35	2.5	0.65	0.35
3.70	2.5	0.50	0.50
3.80	2.5	0.45	0.55
3.95	2.5	0.40	0.60
4.20	2.5	0.30	0.70

Use the formula given above to find the value of PCU.

Table 2: Table for the value of PCU

h_m	h_c	p_c	p_t	E_t
2.70	2.5	0.90	0.10	1.80
2.80	2.5	0.85	0.15	1.80
2.94	2.5	0.80	0.20	1.88
3.10	2.5	0.75	0.25	1.96
3.25	2.5	0.70	0.30	2.00
3.35	2.5	0.65	0.35	1.97
3.70	2.5	0.50	0.50	1.96
3.80	2.5	0.45	0.55	1.95
3.95	2.5	0.40	0.60	1.97
4.20	2.5	0.30	0.70	1.97

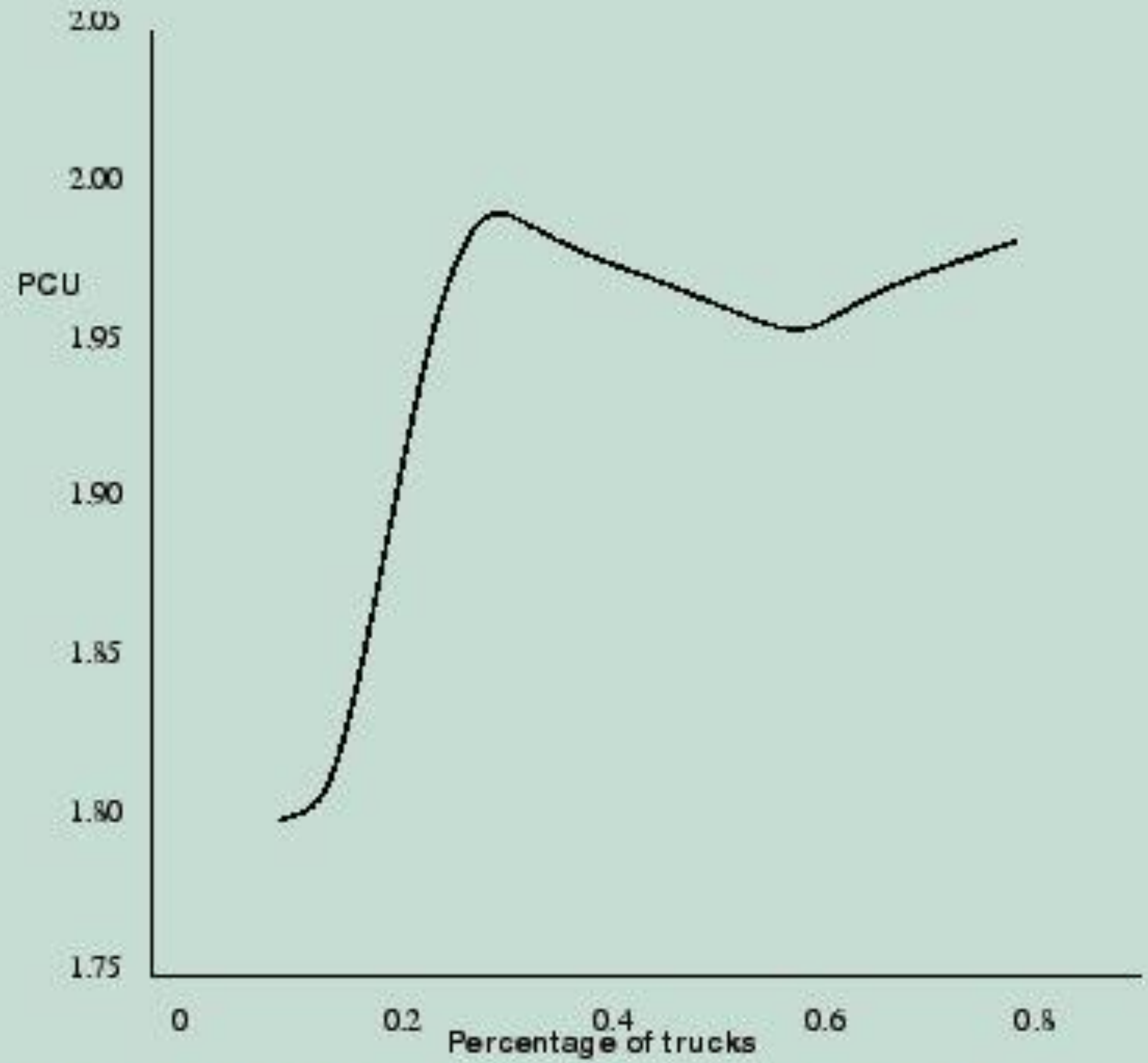


Figure 1: Graph showing the variation of PCU with percentage of truck using the data of the problem given above

Calculation for Control delay

A test was conducted to determine the delay in an intersection. Table 1 presents a sample computation on direct observation of vehicle-in-queue counts at the intersection. The traffic signal at the intersection operates with a cycle time of 115 sec. The test was conducted on the 2 lane road over a 15-min period, which is almost thirteen cycles . Count interval was 15-s. The total number of vehicle is 530 and the total number of stopped vehicle is 223. Assume the free flow speed to be 65 km/h and the empirical adjustment factor 0.9

General Information

Analyst ______
Agency or Company ______
Date Performed ______
Analysis time period ______

Site Information

Intersection [illegible]
Area Type ☑ CBD ☐ Others
Jurisdiction ______
Analysis Year [illegible]

Input Initial Parameters

Number of lanes, N: 2
Free-flow speed, FFS (km/h): [illegible]
Survey count interval, I_s (s): [illegible]
Total vehicles arriving, V_{tot}: [illegible]
Stopped vehicle count, V_{stop}: [illegible]
Cycle length, C (s): ______

Input Field Data

Clock Time	Cycle Number	Number of vehicles in Queue – Count Interval 1	2	3	4	5	6	7	8	9	10
4:3[illegible]	1	3	8	[illegible]	10	12	2	0	2		
	2	6	12	10	16	6	0	0	2		
	3	7	[illegible]	14	14	2	0	0			
	4	5	7	10	13	12	2	0	1		
4:4[illegible]	5	4	6	10	12	3	0	0	1		
	6	5	7	8	13	4	0	0			
	7	3	5	8	12	12	0	0	0		
4:47	8	4	7	[illegible]	16	8	0				
Total		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4	0	[illegible]		

Figure 1: Example of the intersection control delay worksheet

Calculation for Control delay

1. Number of lane, N=2
2. Free-flow Speed, FFS =65 km/h
3. Survey count interval, Is =15 sec
4. Total vehicle in queue, $\Sigma V_{iq} = 371$
5. Total vehicles arriving, $V_{tot} = 530$
6. Stopped vehicles count, $V_{stop} = 223$
7. No of Cycle Surveyed, Nc=7.8
8. Acc./Dec. correction factor, CF=4 (from Table 7.1)
9. No. Of Vehicles stopped per lane each cycle

$$V_{stop} Nc \times N = \frac{223}{7.8 \times 2} = 14$$

10. Fraction of vehicles stopping,

$$FVS = \frac{V_{stop}}{V_{tot}} = \frac{223}{530} = 0.42$$

11. Time-in-queue per vehicle ,

$$d_{vq} = (Is \times \frac{\Sigma V_{iq}}{V_{tot}})0.9 = 9.5sec$$

Calculation for Control delay

Table 1: Acceleration-Deceleration Delay Correction Factor, CF (seconds)

Free-Flow Speed	$\leq$ 7 Vehicles	8-19 Vehicles	20-30 Vehicles
$\leq 60km/h$	5	2	1
60-71 km/h	7	4	2
$\geq$ 71 km/h	9	7	5

Calculation for Control delay

11. Time-in-queue per vehicle ,

$$d_{vq} = (Is \times \frac{\Sigma V_{iq}}{V_{tot}})0.9 = 9.5sec$$

12. Acc./Dec. correction delay,

$$dad = FVS \times CF = 0.42 \times 4 = 1.7sec$$

13. Control delay/vehicle,

$$d = dvq + dad = 11.2sec$$

Poisson vehicle arrival pattern computation

Numerical Example

The hourly flow rate in a road section is 120 vph. Use Poisson distribution to model this vehicle arrival.

Solution:

The flow rate is given as (μ) = 120 vph = $\frac{120}{60}$ = 2 vehicle per minute. Hence, the probability of zero vehicles arriving in one minute $p(0)$ can be computed as follows:

$$p(0) = \frac{\mu^x e^{-\mu}}{x!} = \frac{2^0.e^{-2}}{0!} = 0.135.$$

Similarly, the probability of one vehicles arriving in one minute $p(1)$ is given by,

$$p(1) = \frac{\mu^x e^{-\mu}}{x!} = \frac{2.e^{-2}}{1!} = 0.271.$$

Now, the probability that number of vehicles arriving is less than or equal to zero is given as

$$p(x \leq 0) = p(0) = 0.135.$$

Poisson vehicle arrival pattern computation

Similarly, probability that the number of vehicles arriving is less than or equal to 1 is given as:

$$p(x \leq 1) = p(0) + p(1) = 0.135 + 0.275 = 0.406.$$

Again, the probability that the number of vehicles arriving is between 2 to 4 is given as:

$$p(2 \leq x \leq 4) = p(2) + p(3) + p(4),$$

$$= .271 + .18 + .09 = 0.54.$$

Now, if the $p(0) = 0.135$, then the number of intervals in an hour where there is no vehicle arriving is

$$F(x) = p(0) \times 60 = 0.135 \times 60 = 8.12.$$

Poisson vehicle arrival pattern computation

The above calculations can be repeated for all the cases as tabulated in Table 1.

Table 1: Probability values of vehicle arrivals computed using Poisson distribution

n	$p(n)$	$p(x \leq n)$	$F(n)$
0	0.135	0.135	8.120
1	0.271	0.406	16.240
2	0.271	0.677	16.240
3	0.180	0.857	10.827
4	0.090	0.947	5.413
5	0.036	0.983	2.165
6	0.012	0.995	0.722
7	0.003	0.999	0.206
8	0.001	1.000	0.052
9	0.000	1.000	0.011
10	0.000	1.000	0.011

Poisson vehicle arrival pattern computation

The shape of this distribution can be seen from Figure 1 and the corresponding cumulative distribution is shown in Figure 2.

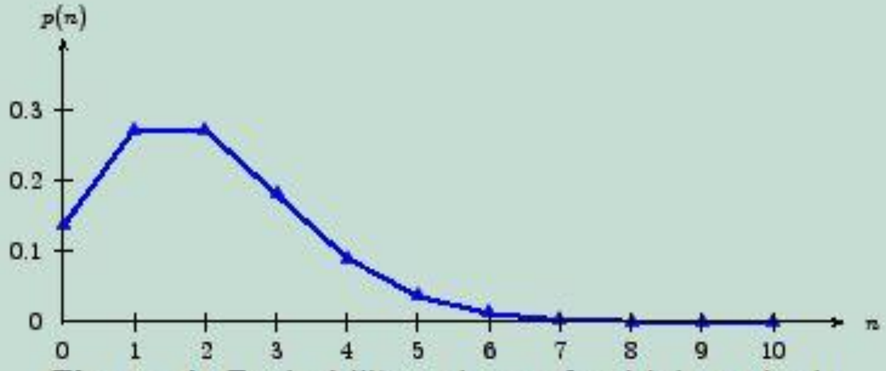


Figure 1: Probability values of vehicle arrivals computed using Poisson distribution

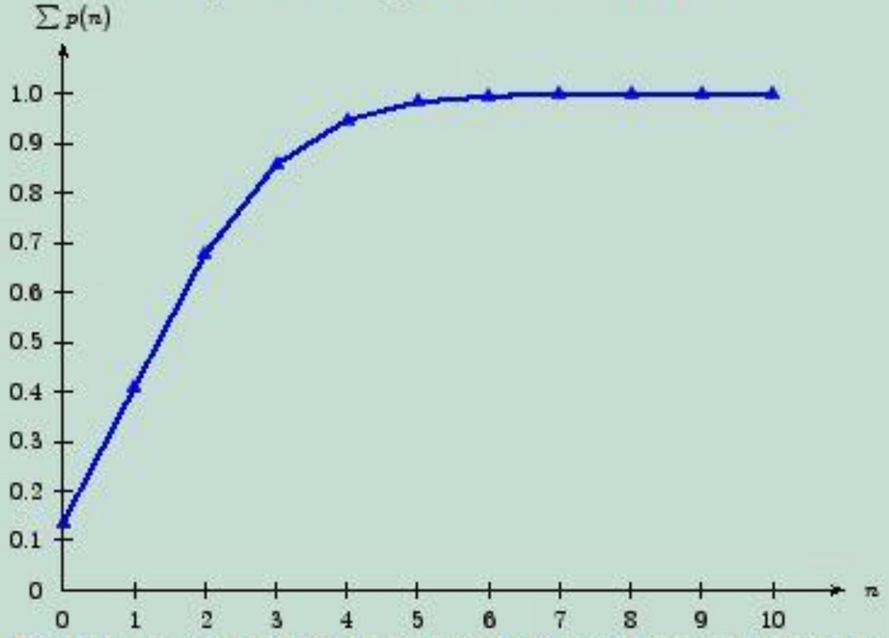


Figure 2: Cumulative probability values of vehicle arrivals computed using Poisson distribution

Acceptability of simulation models

Numerical example

The observed and simulated values obtained using Model 1 and Model 2 are given in Table below.

Table 1: Observed and Simulated values

	Simulated values, x	
Observed values, y	Model 1	Model 2
0.23	0.2	0.27
0.46	0.39	0.5
0.67	0.71	0.65
0.82	0.83	0.84

1. Comment on the performance of both the models based on the following error measures - RMSE, RMSNE, ME and MNE.
2. Using Theil's indicator, comment on the acceptability of the models.

Acceptability of simulation models

1. Using the formulas given below (Equations 16.4, 16.5, 16.6, 16.7), all the four errors can be calculated. Here N = 4.

$$RMSE = \sqrt{\frac{1}{N}\Sigma_{i=1}N(x_i - yy_i)^2}$$

$$RMSNE = \sqrt{\frac{1}{N}\Sigma_{i=1}N(\frac{x_i - y_i}{y_i})^2}$$

$$ME = \frac{1}{N}\Sigma_{i=1}N(x_i - y_i)$$

$$MNE = \frac{1}{N}\Sigma_{i=1}N(\frac{x_i - y_i}{y_i})$$

Tabulations required are given below.

Acceptability of simulation models

Table 2: Error calculations for Model 1

Model 1			
$(x-y)$	$\left(\frac{x-y}{y}\right)$	$(x-y)^2$	$\left(\frac{x-y}{y}\right)^2$
-0.030	-0.130	0.0009	0.0170
-0.070	-0.152	0.0049	0.0232
0.040	0.060	0.0016	0.0036
0.010	0.012	0.0001	0.0001
ϵ = -0.050	ϵ = -0.211	ϵ = 0.0075	ϵ = 0.0439
ME = 0.013	MNE = 0.053	RMSE = 0.043	RMSNE = 0.105

Acceptability of simulation models

Table 3: Error calculations for Model 2

Model 2			
$(x-y)$	$(\frac{x-y}{y})$	$(x-y)^2$	$(\frac{x-y}{y})^2$
0.040	0.174	0.0016	0.0302
0.040	0.087	0.0016	0.0076
-0.020	-0.030	0.0004	0.0009
0.020	0.024	0.0004	0.0006
ϵ = 0.080	ϵ = 0.255	ϵ= 0.0040	ϵ = 0.0393
ME = 0.020	MNE = 0.064	RMSE = 0.032	RMSNE = 0.099

Comparing Model 1 and Model 2 in terms of RMSE and RMSNE, Model 2 is better. But with respect to ME and MNE, Model 1 is better.

2. Theil's indicator

$$U = \frac{\sqrt{\frac{1}{N}\Sigma_{i=1}N(x_i - y_i)^2}}{\sqrt{\frac{1}{N}\Sigma_{i=1}N(x_i)^2} + \sqrt{\frac{1}{N}\Sigma_{i=1}N(y_i)^2}}$$

Acceptability of simulation models

The additional tabulations required are as follows:

Table 4: Theil's indicator calculation

x^2		
Model 1	Model 2	y^2
0.04	0.0729	0.0529
0.1521	0.25	0.2116
0.5041	0.4225	0.4489
0.6889	0.7056	0.6724
ϵ = 1.3851	ϵ = 1.451	ϵ = 1.3858

The value of Theil's indicator is obtained as: For Model 1, U = 0.037 which is $\leq$ 0.2, and For Model 2, U = 0.027 which is $\leq$ 0.2. Therefore both models are acceptable.

Acceptability of simulation models

The global index U is bounded, $0 \leq U \leq 1$, with U = 0 for a perfect fit and $x_i = y_i$ for i = 1 to N, between observed and simulated values. For $U \leq 0.2$, the simulated series can be accepted as replicating the observed series acceptably well. The closer the values are to 0, the better will be the model. For values greater than 0.2, the simulated series is rejected.

Parking survey

The parking survey data collected from a parking lot by license plate method is s shown in the table 1 below. Find the average occupancy, average turn over, parking load, parking capacity and efficiency of the parking lot.

Table 1: Licence plate parking survey data

Bay	Time			
	0-15	15-30	30-45	45-60
1	1456	9813	-	5678
2	1945	1945	1945	1945
3	3473	5463	5463	5463
4	3741	3741	9758	4825
5	1884	1884	-	7594
6	-	7357	-	7893
7	-	4895	4895	4895
8	8932	8932	8932	-
9	7653	7653	8998	4821
10	7321	-	2789	2789
11	1213	1213	3212	4778
12	5678	6678	7778	8888

Parking survey

See the following table for solution 2

Table 2: Licence plate parking survey solution

Bay	Time				Time				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	15	30	45	60	15	30	45	60	Turn over
1	1456	9813	-	5678	1	1	0	1	3
2	1945	1945	1945	1945	1	1	1	1	1
3	3473	5463	5463	5463	1	1	1	1	2
4	3741	3741	9758	4825	1	1	1	1	3
5	1884	1884	-	7594	1	1	0	1	2
6	-	7357	-	7893	0	1	0	1	2
7	-	4895	4895	4895	0	1	1	1	1
8	8932	8932	8932	-	1	1	1	0	1
9	7653	7653	8998	4821	1	1	1	1	3
10	7321	-	2789	2789	1	0	1	1	2
11	1213	1213	3212	4778	1	1	1	1	3
12	5678	6678	7778	8888	1	1	1	1	4
	Accumulation				10	11	9	11	
	Occupancy				0.83	0.92	0.75	0.92	2.25

Parking survey

Columns 1 to 5 is the input data. The parking status in every bay is coded first. If a vehicle occupies that bay for that time interval, then it has a code 1. This is shown in columns 6, 7, 8 and 9 of the table corresponding to the time intervals 15, 30, 45 and 60 seconds.

- Turn over is computed as the number of vehicles present in that bay for that particular hour. For the first bay, it is counted as 3. Similarly, for the second bay, one vehicle is present throughout that hour and hence turnout is 1 itself. This is being tabulated in column 10 of the table. Average turn over = $\frac{\text{Sum of turn-over}}{\text{Total number of bays}}$ = 2.25
- Accumulation for a time interval is the total of number of vehicles in the bays 1 to 12 for that time interval. Accumulation for first time interval of 15 minutes = 1+1+1+1+1+0+0+1+1+1+1+1 = 10
- Parking volume = Sum of the turn over in all the bays = 27 vehicles
- Average duration is the average time for which the parking lot was used by the vehicles. It can be calculated as sum of the accumulation for each time interval $\times$ time interval divided by the parking volume = $\frac{(10+11+9+11)\times 15}{27}$ = 22.78 minutes/vehicle.

Parking survey

- Occupancy for that time interval is accumulation in that particular interval divided by total number of bays. For first time interval of 15 minutes, occupancy = (10 $\times$ 100)/12 = 83% Average occupancy is found out as the average of total number of vehicles occupying the bay for each time interval. It is expressed in percentage. Average occupancy = $\frac{0.83+0.92+0.75+0.92}{4} \times 100$ = 85.42%.
- Parking capacity = number of bays $\times$ number of hours = 12 $\times$ 1 = 12 vehicle hours
- Parking load = total number of vehicles accumulated at the end of each time interval $\times$ time = $\frac{(10+11+9+11)\times 15}{60}$ = 10.25 vehicle hours
- Efficiency = $\frac{\text{Parking load}}{\text{Total number of bays}} = \frac{10.25}{12}$ = 85.42%.