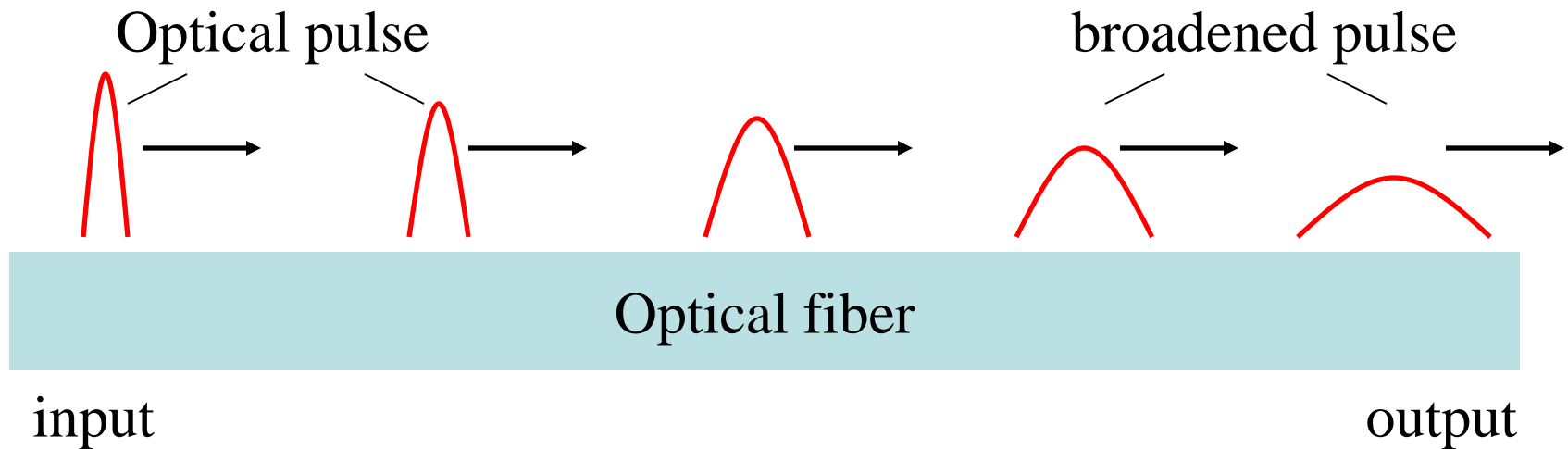


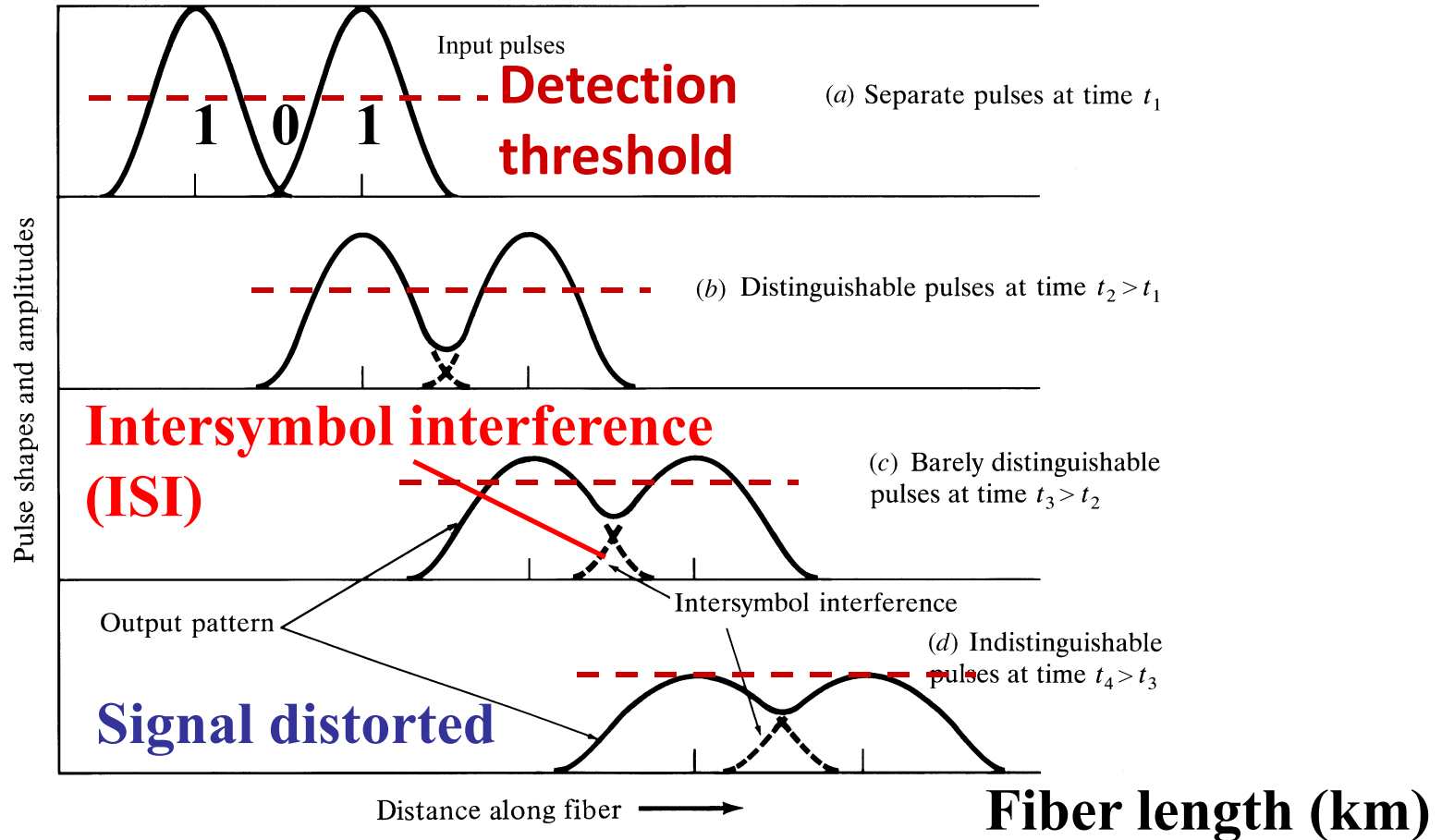
Fiber dispersion

- Fiber dispersion results in *optical pulse broadening* and hence *digital signal degradation*.



- Dispersion mechanisms:**
1. **Modal** (or *intermodal*) **dispersion**
 2. **Chromatic dispersion** (CD)
 3. **Polarization mode dispersion** (PMD)

Pulse broadening limits fiber bandwidth (data rate)

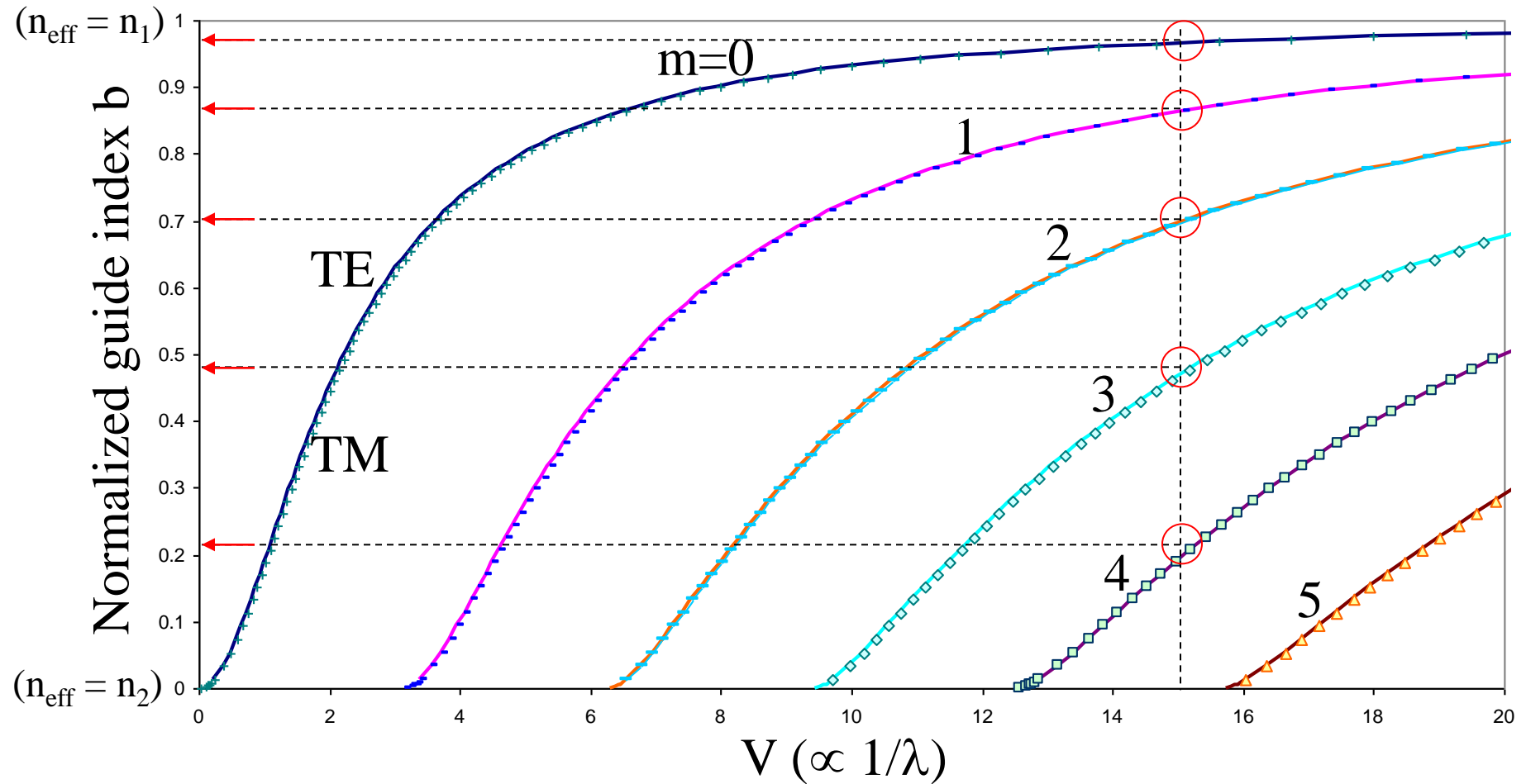


- An *increasing number of errors* may be encountered on the digital optical channel as the ISI becomes more pronounced.

1. Modal dispersion

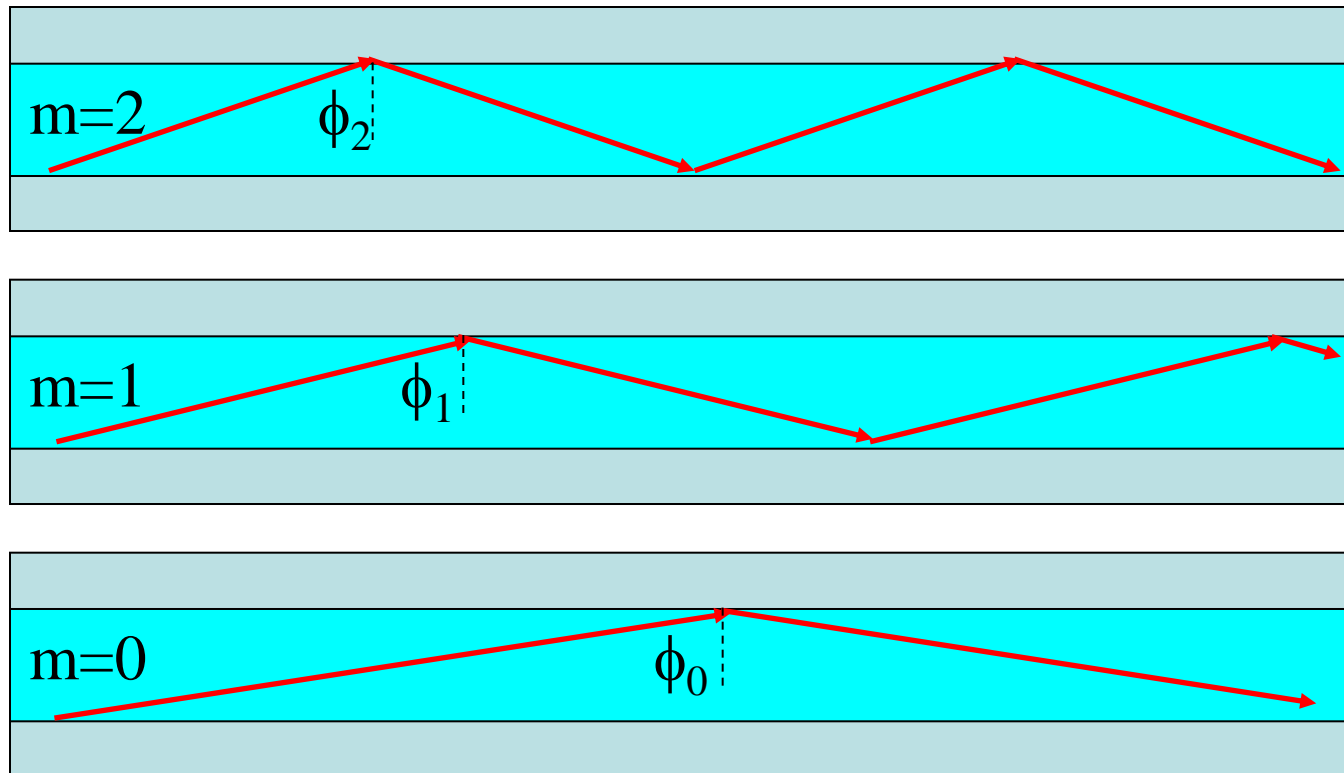
- When numerous waveguide modes are propagating, they all travel with different net velocities with respect to the waveguide axis.
- An input waveform distorts during propagation because its energy is distributed among several modes, each traveling at a different speed.
- Parts of the wave arrive at the output before other parts, spreading out the waveform. This is thus known as **multimode (modal) dispersion**.
- **Multimode dispersion does *not* depend on the source linewidth** (even a *single* wavelength can be simultaneously carried by *multiple modes* in a waveguide).
- **Multimode dispersion would *not* occur if the waveguide allows *only one mode to propagate* - the advantage of *single*-mode waveguides!**²⁴

Modal dispersion as shown from the mode chart of a symmetric slab waveguide



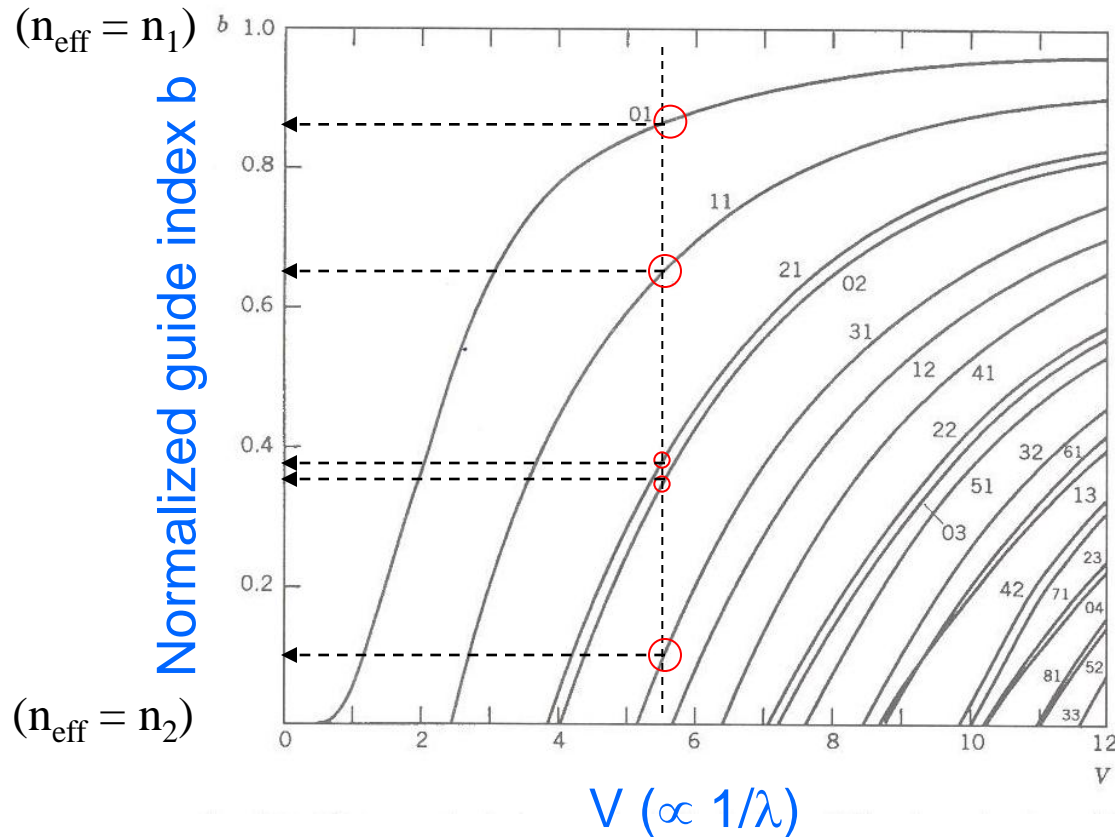
- Phase velocity for mode $m = \omega/\beta_m = \omega/(n_{\text{eff}}(m) k_0)$
(note that $m = 0$ mode is the *slowest* mode)

Modal dispersion in multimode waveguides



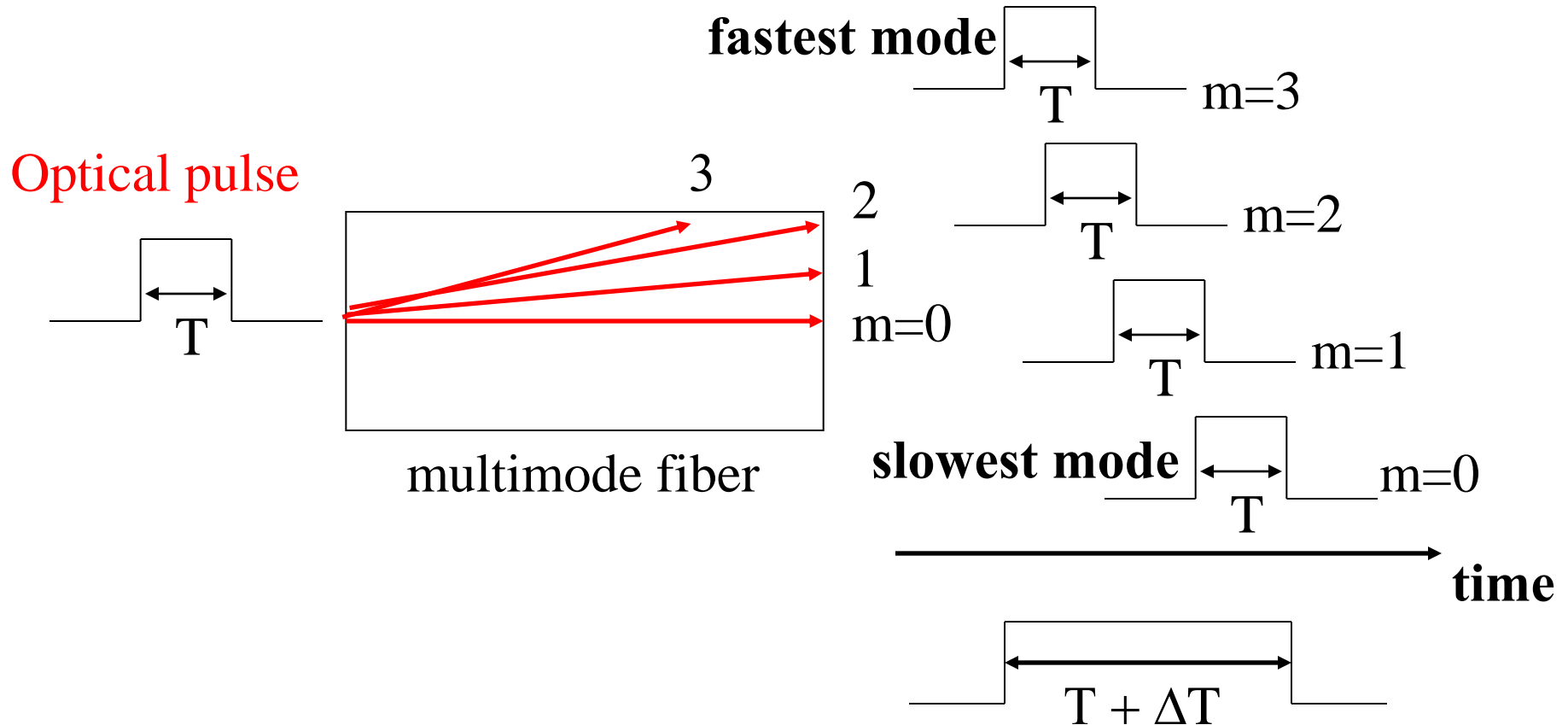
The carrier wave can propagate along all these different “zig-zag” ray paths of *different path lengths*.

Modal dispersion as shown from the LP mode chart of a silica optical fiber



- Phase velocity for LP mode = $\omega/\beta_{lm} = \omega/(n_{\text{eff}}(lm) k_0)$
(note that LP_{01} mode is the *slowest* mode)

Modal dispersion results in pulse broadening

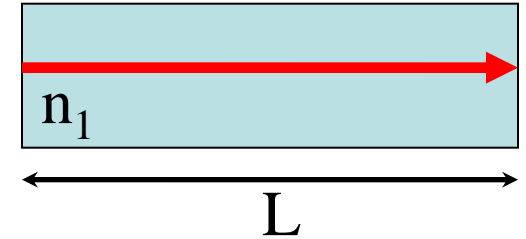


modal dispersion: different modes arrive at the receiver with different delays \Rightarrow pulse broadening

Estimated modal dispersion pulse broadening using phase velocity

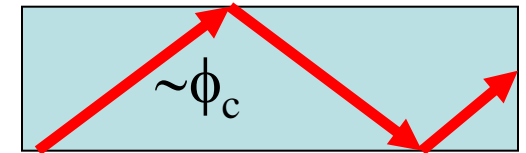
- A zero-order mode traveling near the waveguide axis needs time:

$$t_0 = L/v_{m=0} \approx Ln_1/c \quad (v_{m=0} \approx c/n_1)$$



- The highest-order mode traveling near the critical angle needs time:

$$t_m = L/v_m \approx Ln_2/c \quad (v_m \approx c/n_2)$$



=> the *pulse broadening* due to modal dispersion:

$$\Delta T \approx t_0 - t_m \approx (L/c) (n_1 - n_2)$$

$$\approx (L/2cn_1) NA^2$$

$$(n_1 \sim n_2)$$

e.g. How much will a light pulse spread after traveling along 1 km of a step-index fiber whose $NA = 0.275$ and $n_{\text{core}} = 1.487$?

How does modal dispersion restricts fiber bit rate?

Suppose we transmit at a low bit rate of 10 Mb/s

$$\Rightarrow \text{Pulse duration} = 1 / 10^7 \text{ s} = 100 \text{ ns}$$

Using the above e.g., each pulse will spread up to $\approx 100 \text{ ns}$ (i.e. \approx pulse duration !) every km

\Rightarrow The broadened pulses overlap! (**Intersymbol interference (ISI)**)

*Modal dispersion limits the bit rate of a fiber-optic link to $\sim 10 \text{ Mb/s}$.
(a coaxial cable supports this bit rate easily!)

Bit-rate distance product

- We can relate the pulse broadening ΔT to the *information-carrying capacity* of the fiber measured through the bit rate B .
- Although a precise relation between B and ΔT depends on many details, such as the pulse shape, it is intuitively clear that ΔT *should be less than the allocated bit time slot* given by $1/B$.

\Rightarrow An *order-of-magnitude* estimate of the supported bit rate is obtained from the condition $B\Delta T < 1$.

\Rightarrow ***Bit-rate distance product*** (limited by modal dispersion)

$$BL < 2c n_{\text{core}} / NA^2$$

This condition provides a rough estimate of a fundamental limitation of step-index multimode fibers.

(the *smaller is the NA, the larger is the bit-rate distance product*)

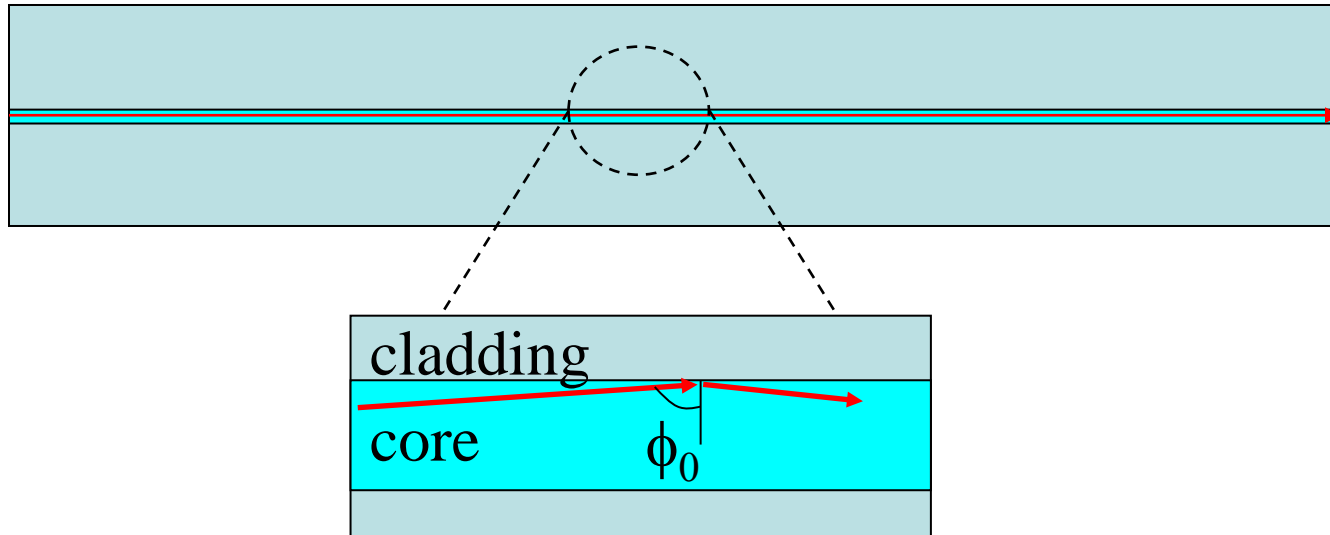
The capacity of optical communications systems is frequently measured in terms of the **bit rate-distance product**.

e.g. If a system is capable of transmitting **10 Mb/s** over a distance of **1 km**, it is said to have a *bit rate-distance* product of **10 (Mb/s)-km**.

This may be suitable for some *local-area networks (LANs)*.

Note that the same system can transmit **100 Mb/s** along **100 m**, or **1 Gb/s** along **10 m**, or **10 Gb/s** along **1 m**, or **100 Gb/s** along **10 cm**, or **1 Tb/s** along **1 cm**

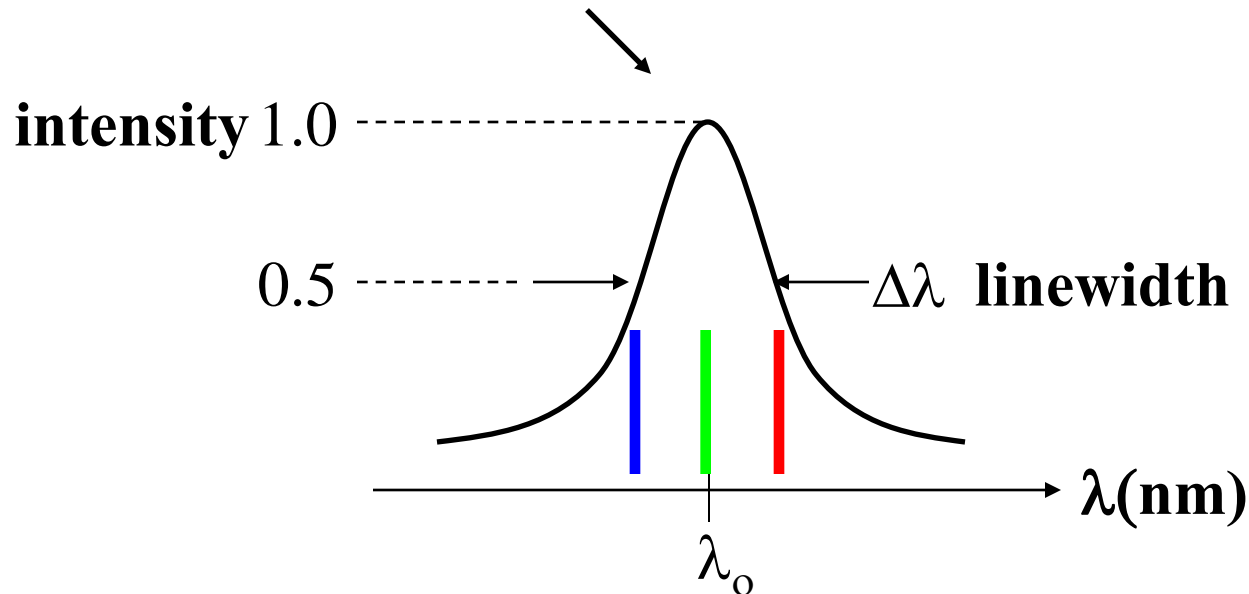
Single-mode fiber eliminates modal dispersion



- The main advantage of *single*-mode fibers is to propagate *only one mode* so that *modal dispersion is absent*.
- However, *pulse broadening does not disappear altogether*. The *group velocity* associated with the fundamental mode is *frequency dependent* within the pulse *spectral linewidth* because of chromatic dispersion.

2. Chromatic dispersion

- Chromatic dispersion (CD) may occur in *all* types of optical fiber. The optical pulse broadening results from the *finite spectral linewidth of the optical source and the modulated carrier*.



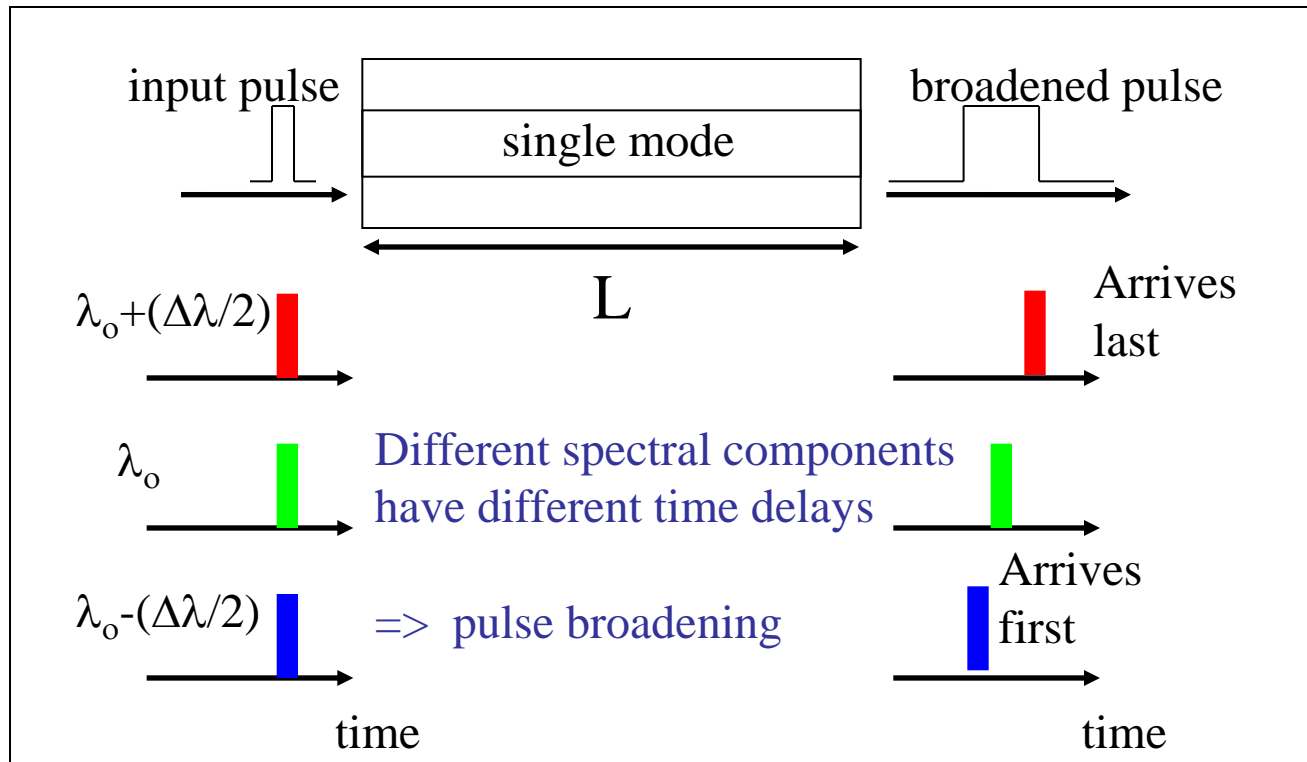
*In the case of the semiconductor laser $\Delta\lambda$ corresponds to only a fraction of % of the centre wavelength λ_o . For LEDs, $\Delta\lambda$ is likely to be a significant percentage of λ_o .

Spectral linewidth

- Real sources emit over a range of wavelengths. This range is the *source linewidth* or *spectral width*.
- The smaller is the linewidth, the smaller is the spread in wavelengths or frequencies, the more *coherent* is the source.
- An *ideal perfectly coherent* source emits light at a single wavelength. It has *zero* linewidth and is perfectly monochromatic.

Light sources	Linewidth (nm)
Light-emitting diodes	20 nm – 100 nm
Semiconductor laser diodes	1 nm – 5 nm
Nd:YAG solid-state lasers	0.1 nm
HeNe gas lasers	0.002 nm

- Pulse broadening occurs because there may be *propagation delay differences* among the *spectral components* of the transmitted signal.



Chromatic dispersion (CD): Different spectral components of a *pulse* travel at different *group velocities*. This is known as *group velocity dispersion* (GVD).

Dispersion in Single-mode Fibres

Single-mode fibres are far superior to multimode fibres in dispersion characteristics, because they avoid multipath effects, ie. energy of injected pulse is transported by a single-mode only.

However, some dispersion still exists.

Group velocity of fundamental mode is frequency dependent – hence different spectral components of the pulse travel at slightly different group velocities.

Group Velocity Dispersion (GVD)

Contribution: (i) material dispersion (ii) waveguide dispersion

Time delay for a specific spectral component at frequency ω arriving at output of fibre of length L is

$$T = \frac{L}{v_g}$$

where group velocity $v_g = \left(\frac{d\beta}{d\omega} \right)^{-1}$

Since $\beta = \bar{n}k_0 = \bar{n} \frac{\omega}{c}$

$$\Rightarrow v_g = \frac{c}{\bar{n}_g}$$

where Group Index $\bar{n}_g = \bar{n} + \omega \left(\frac{d\bar{n}}{d\omega} \right)$

Mode index

Frequency dependence of v_g causes pulse broadening because different spectral components of the pulse disperse

during propagation and do not arrive simultaneously at the fibre output.

If $\Delta\omega$ is spectral width of the pulse \Rightarrow pulse broadening for fibre length L is

$$\Delta T = \frac{dT}{d\omega} \Delta\omega = \frac{d}{d\omega} \left(\frac{L}{v_g} \right) \Delta\omega = L \frac{d^2\beta}{d\omega^2} \Delta\omega$$

In optical fibre communications, $\Delta\lambda$ is used instead of $\Delta\omega$.

If the frequency spread $\Delta\omega$ is determined by range of wavelengths $\Delta\lambda$ emitted by source, using $\omega = \frac{2\pi c}{\lambda}$

$$\Delta\omega = -\frac{2\pi c}{\lambda^2} \Delta\lambda$$

$$\Rightarrow \Delta T = -\frac{2\pi c}{\lambda^2} \left(\frac{d^2\beta}{d\omega^2} \right) L \Delta\lambda$$

$$\Rightarrow \boxed{\Delta T = D L \Delta\lambda}$$

~~$f = \frac{c}{\lambda}$~~
 $2\pi f_1 - 2\pi f_2 \Delta\lambda = -\frac{2\pi c}{\lambda^2} (\lambda_1 - \lambda_2)$
 $v = \frac{c}{v_g}$

where Dispersion parameter $D \equiv -\frac{2\pi c}{\lambda^2} \left(\frac{d^2\beta}{d\omega^2} \right)$

D is expressed in ps/km-nm

Effect of dispersion on bit rate

Using rough criterion that pulse broadening ΔT must be less than time allocated to a bit ie. $\frac{1}{B}$

Require $\Delta T < \frac{1}{B}$

$$\Rightarrow \boxed{B |D| L \Delta\lambda < 1}$$

- gives order of magnitude estimate of $B \cdot L$ product offered by single-mode fibre.

eg. For operation near $\lambda = 1.3 \mu m$, $D \sim 1 \text{ ps/km} \cdot \text{nm}$

For semiconductor (multimode) laser of spectral width $\Delta\lambda = 2 - 4 \text{ nm}$

$$\Rightarrow B L \sim 100 \text{ (Gb/s)} \cdot \text{km}$$

For semiconductor (single-mode) laser of spectral width $\Delta\lambda < 1 \text{ nm}$

$$\Rightarrow B L \sim 1 \text{ (Tb/s)} \cdot \text{km} \quad \text{for fibre.}$$

Behaviour of dispersion parameter D

D varies with wavelength

$$D = -\frac{2\pi c}{\lambda^2} \left(\frac{d^2 \beta}{d\omega^2} \right)$$

and $\beta = -\frac{\bar{n}\omega}{c}$ where \bar{n} is the mode index

$$\Rightarrow D = -\frac{2\pi}{\lambda^2} \left(2 \frac{d \bar{n}}{d\omega} + \omega \frac{d^2 \bar{n}}{d\omega^2} \right)$$

This can be decomposed into two additive terms.

D_M - material dispersion

due to refractive index of silica changing with optical freq. ω

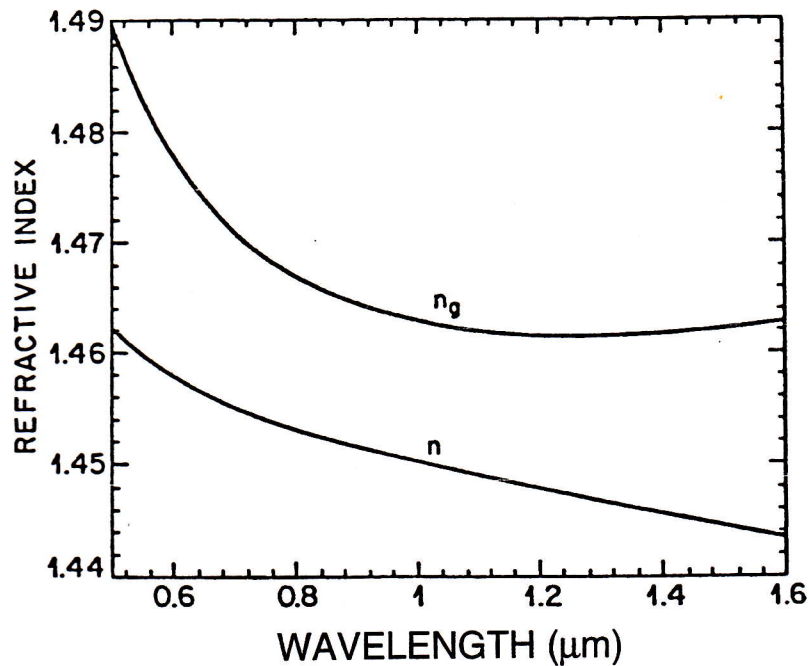
D_W - waveguide dispersion

due to normalised propagation constant b changing with optical freq ω

(Proportion of field in core & cladding changes with ω & V)

Material Dispersion

Occurs because refractive index of silica changes with optical frequency ω



Wavelength dependence of n and group index n_g for fused silica

$$D_M \propto \frac{dn_g}{d\lambda}$$

It turns out that $\frac{dn_g}{d\lambda} = 0$ at $\lambda = 1.276 \mu m \equiv \lambda_{ZD}$

ie. zero material dispersion wavelength $D_M = 0$ at $\lambda = \lambda_{MZD}$

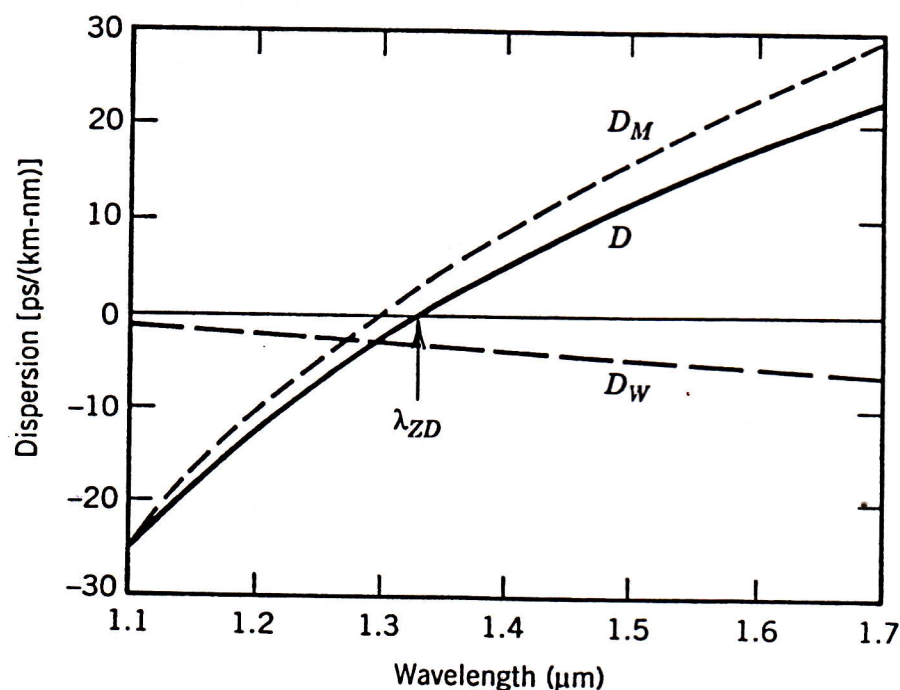
Note $D_M < 0$ below λ_{MZD} and $D_M > 0$ above λ_{MZD}

Waveguide Dispersion

Occurs because propagation constant b depends on optical frequency ω and on the V parameter of the fibre.

It turns out that $D_W < 0$ in the entire range 0-1.6 μm .

Since $D_M > 0$ for $\lambda > \lambda_{MZD}$, it is possible to cancel the waveguide & material dispersion at a specific λ to get total dispersion $D = D_M + D_W = 0$



Total dispersion $D = 0$ at $\lambda_{ZD} = 1.31 \mu\text{m}$.

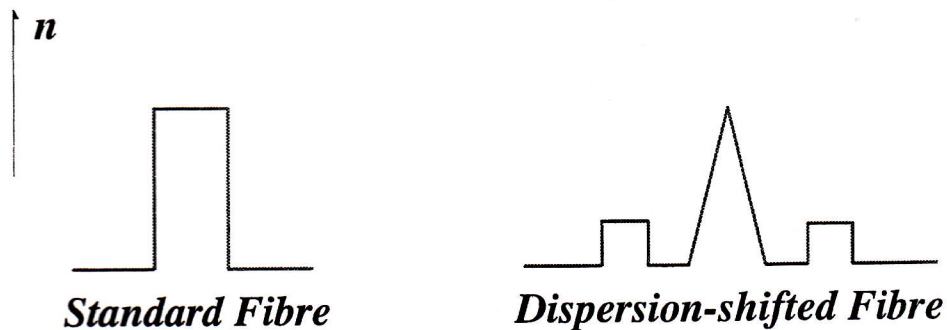
At the low-fibre-loss wavelength of $\lambda = 1.55 \mu\text{m}$,
 $D = 15 - 18 \text{ ps}/(\text{km} \cdot \text{nm})$

This is relatively high and limits the performance of 1.55 μm systems.

Dispersion-shifted Fibres

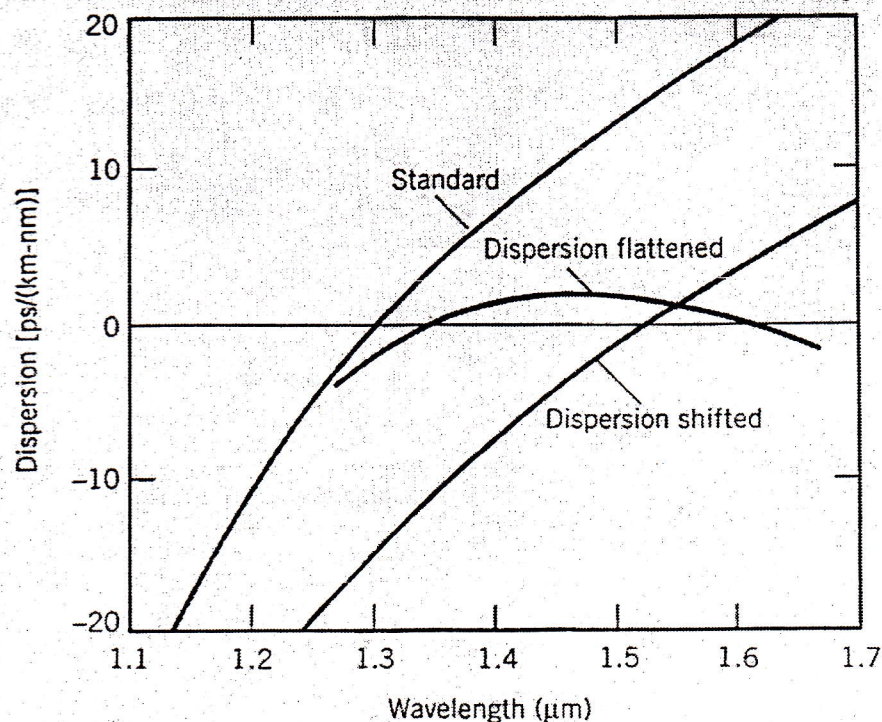
The waveguide contribution D_W depends on fibre parameters such as core radius a , index difference Δ .

Hence can design fibre index profile so that λ_{ZD} is shifted to $1.55 \mu\text{m}$.



It is also possible to tailor the waveguide contribution so that the total dispersion is reduced over a range $1.3\text{-}1.6 \mu\text{m} \rightarrow$ dispersion flattened fibres.

The design of dispersion modified fibres uses multiple cladding layers & tailored refractive index profile.



Typical fibre characteristics

Single-mode fibre

Corning SMF-28

$$NA = 0.13$$

$$\Delta = 0.36\%$$

$$\text{Spot size diameter} = 9.3 \mu\text{m}$$

$$\lambda_{\text{ZD}} = 1.312 \mu\text{m}$$

Dispersion-shifted fibre

Corning SMF-DS

$$NA = 0.17$$

$$\Delta = 0.90\%$$

$$\text{Spot size diameter} = 8.1 \mu\text{m}$$

$$\lambda_{\text{ZD}} = 1.550 \mu\text{m}$$

Chromatic Dispersion Compensation

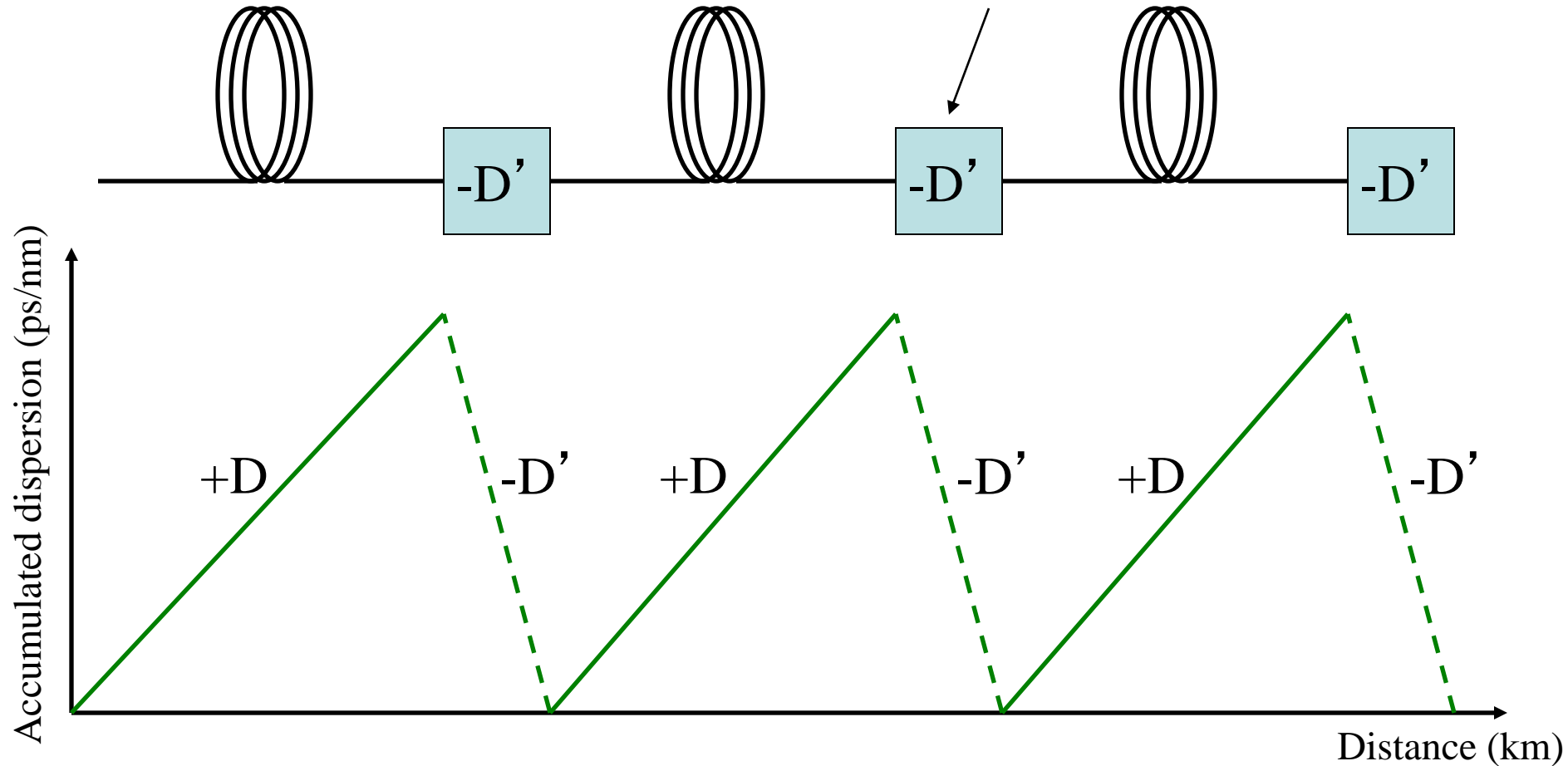
- Chromatic dispersion is ***time independent*** in a *passive* optical link
⇒ allow compensation along the entire fiber span
(**Note** that recent developments focus on *reconfigurable* optical link, which makes chromatic dispersion *time dependent*!)

Two basic techniques: (1) **dispersion-compensating fiber DCF**
(2) **dispersion-compensating fiber grating**

- **The basic idea for DCF:** the *positive dispersion* in a conventional fiber (say ~ 17 ps/(km-nm) in the 1550 nm window) can be compensated for by inserting a ***fiber with negative dispersion*** (*i.e.* with large -ve D_{wg}).

Positive dispersion transmission fiber

Negative dispersion element



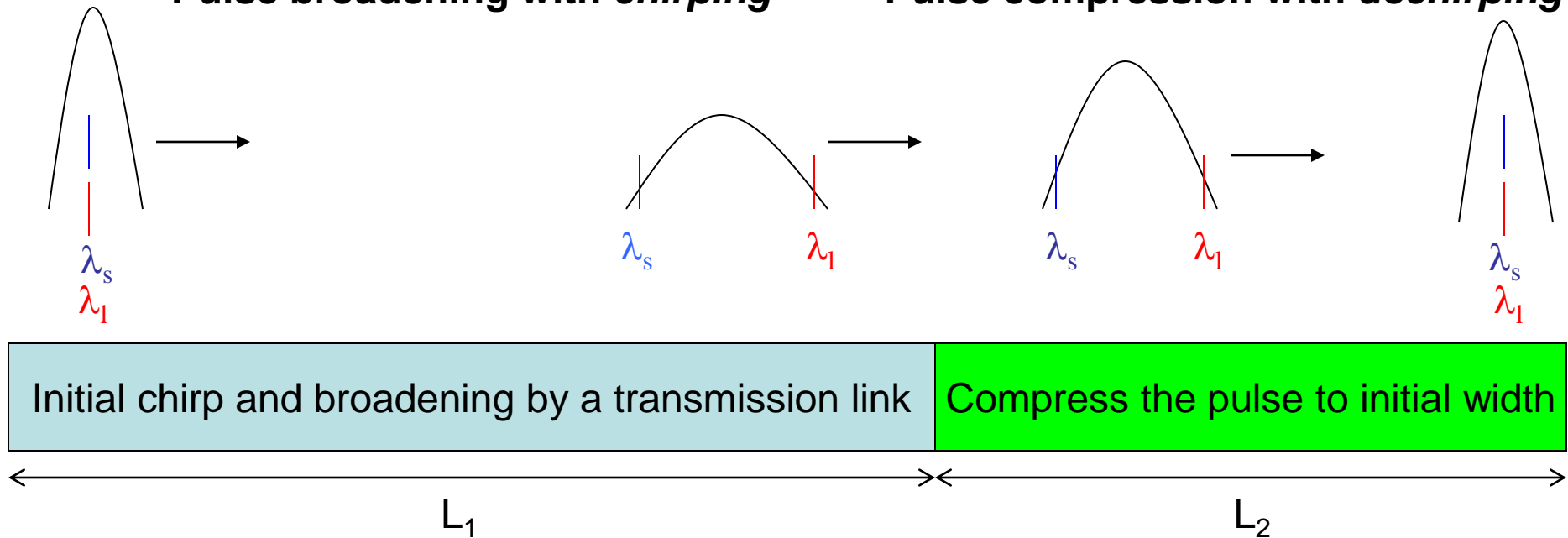
- In a *dispersion-managed* system, positive dispersion transmission fiber *alternates with negative dispersion compensation elements*, such that the total dispersion is zero end-to-end.

Dispersion-Compensating Fiber

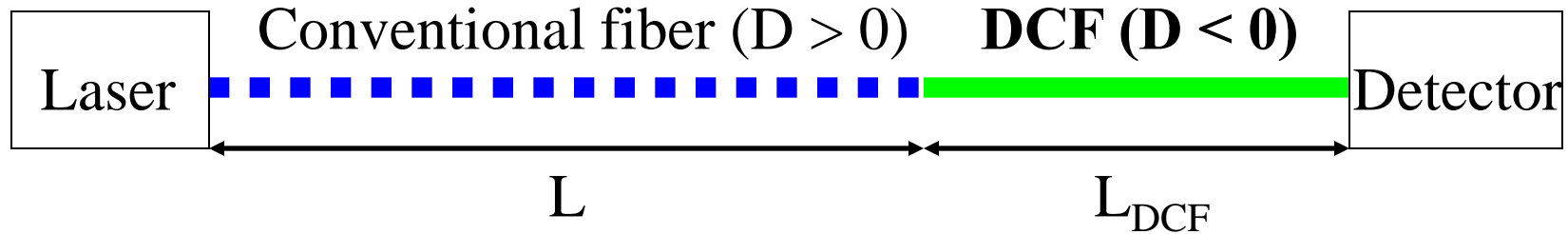
The concept: using a span of fiber to *compress* an initially chirped pulse.

Pulse broadening with *chirping*

Pulse compression with *dechirping*



Dispersion compensated channel: $D_2 L_2 = - D_1 L_1$



e.g. What DCF is needed in order to compensate for dispersion in a conventional single-mode fiber link of 100 km?

Suppose we are using Corning SMF-28 fiber,

=> the dispersion parameter $D(1550 \text{ nm}) \sim 17 \text{ ps}/(\text{km}\cdot\text{nm})$.

=> Pulse broadening $\Delta T_{\text{chrom}} = D(\lambda) \Delta\lambda L \sim 17 \times 1 \times 100 = 1700 \text{ ps}$.

assume the semiconductor (diode) laser linewidth $\Delta\lambda \sim 1 \text{ nm}$.

⇒ The DCF needed to compensate for 1700 ps with a large *negative-dispersion* parameter

i.e. we need $\Delta T_{\text{chrom}} + \Delta T_{\text{DCF}} = 0$

$$\Rightarrow \Delta T_{\text{DCF}} = D_{\text{DCF}}(\lambda) \Delta\lambda L_{\text{DCF}}$$

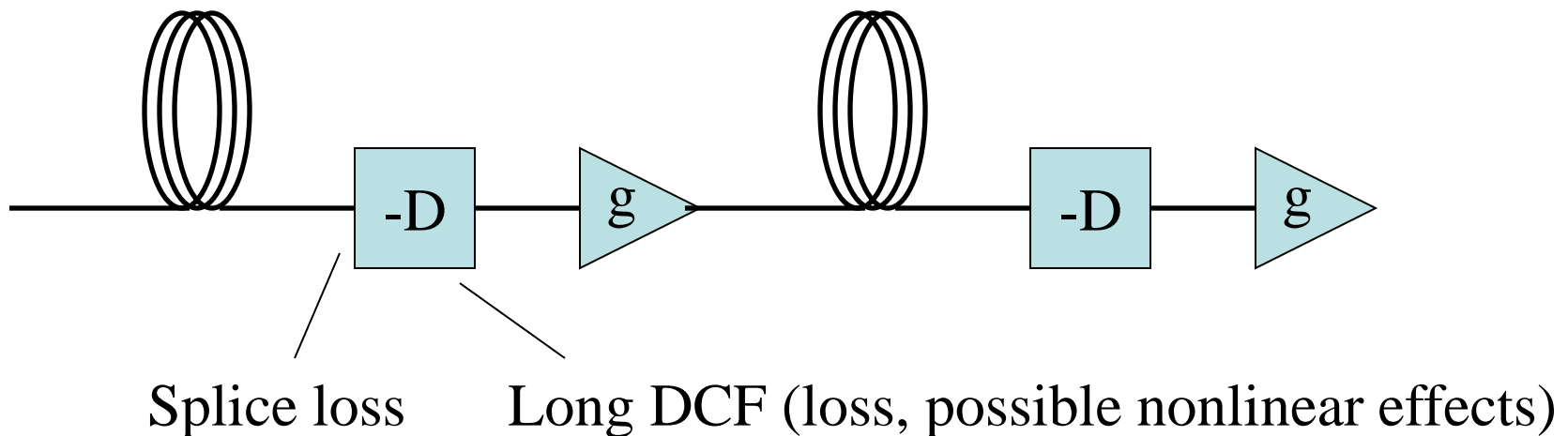
suppose typical ratio of $L/L_{\text{DCF}} \sim 6 - 7$, we assume $L_{\text{DCF}} = 15 \text{ km}$

$$\Rightarrow D_{\text{DCF}}(\lambda) \sim -113 \text{ ps}/(\text{km-nm})$$

*Typically, **only one wavelength can be compensated exactly.**
Better CD compensation requires **both *dispersion* and *dispersion slope* compensation.**

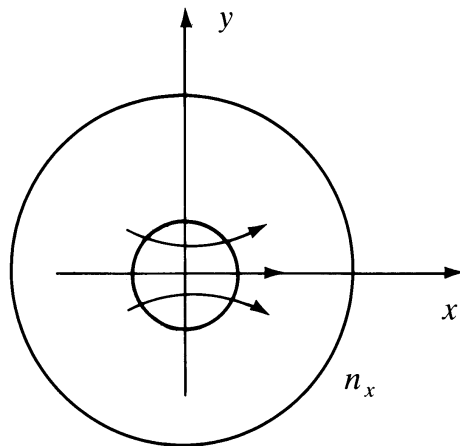
Disadvantages in using DCF

- *Added loss* associated with the increased fiber span
- *Nonlinear effects* may degrade the signal over the long length of the fiber if the signal is of sufficient intensity.
- Links that use DCF often require an *amplifier* stage to compensate the added loss.

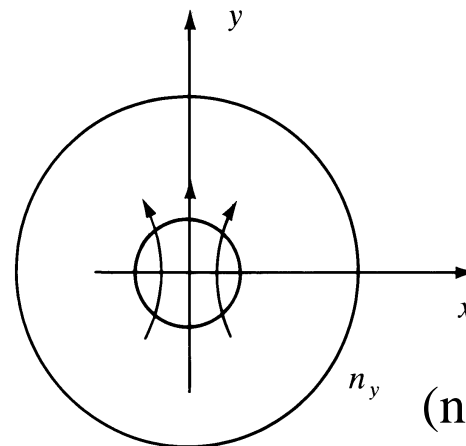


3. Polarization Mode Dispersion (PMD)

- In a single-mode optical fiber, the optical signal is carried by the *linearly polarized* “fundamental mode” LP_{01} , which has *two polarization components that are orthogonal*.



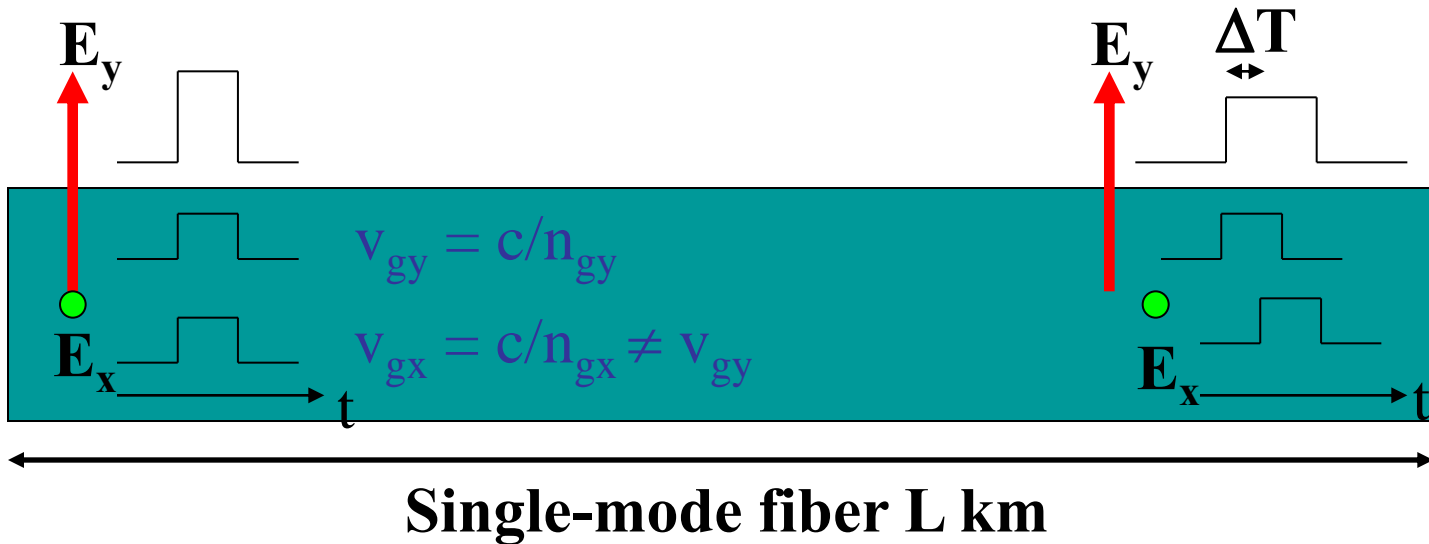
Horizontal mode



Vertical mode

(note that x and y are chosen arbitrarily)

- In a **real fiber (i.e. $n_{gx} \neq n_{gy}$)**, the two orthogonal polarization modes propagate at **different group velocities**, resulting in **pulse broadening** – **polarization mode dispersion**.



*1. **Pulse broadening** due to the orthogonal polarization modes
 (The time delay between the two polarization components is characterized as the **differential group delay (DGD)**.)

2. **Polarization varies** along the fiber length

- The *refractive index difference* is known as *birefringence*.

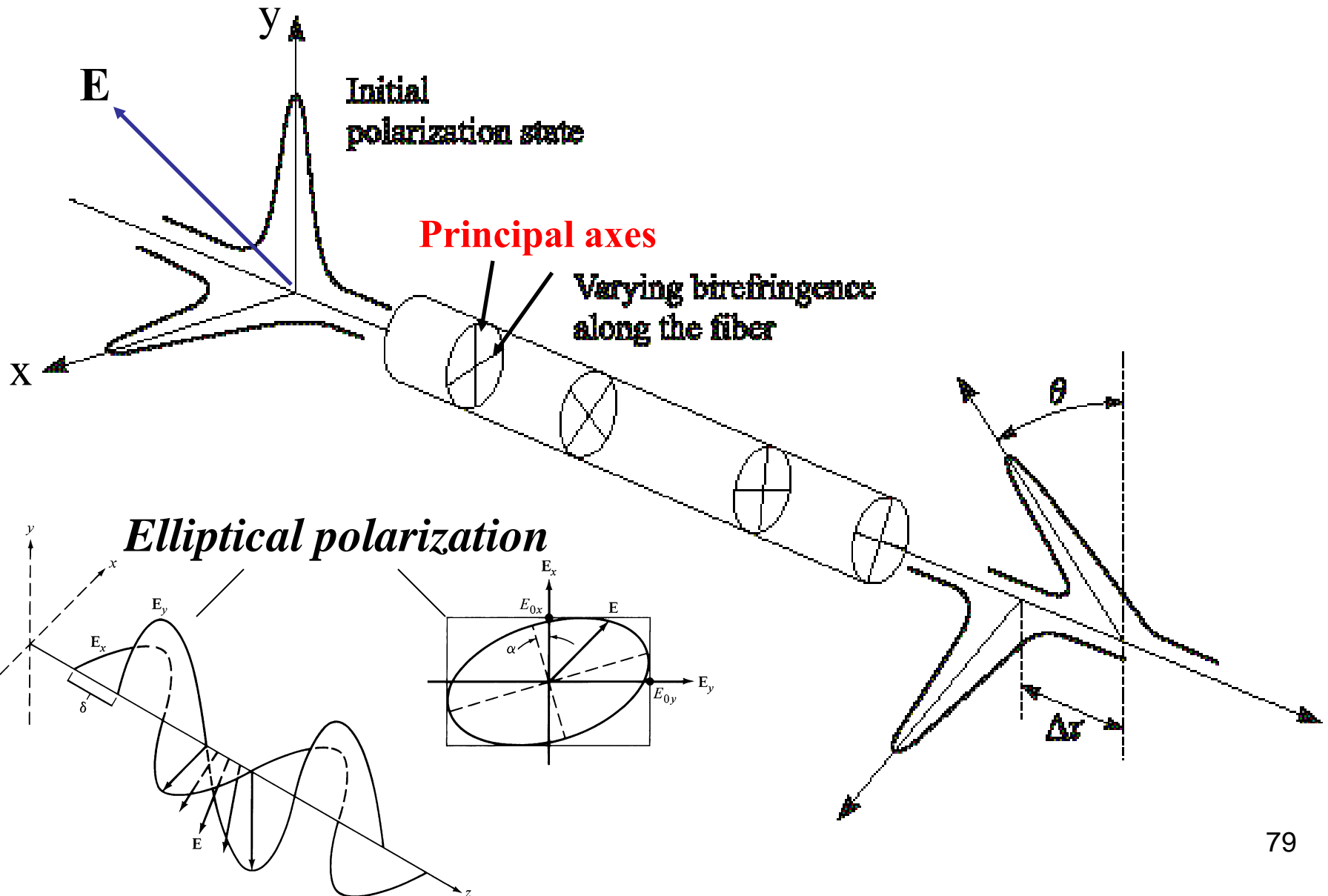
$$B = n_x - n_y \quad (\sim 10^{-6} - 10^{-5} \text{ for single-mode fibers})$$

assuming $n_x > n_y \Rightarrow$ y is the *fast axis*, x is the *slow axis*.

*B varies *randomly* because of *thermal and mechanical stresses over time* (due to *randomly varying* environmental factors in submarine, terrestrial, aerial, and buried fiber cables).

\Rightarrow *PMD is a statistical process !*

Randomly varying birefringence along the fiber



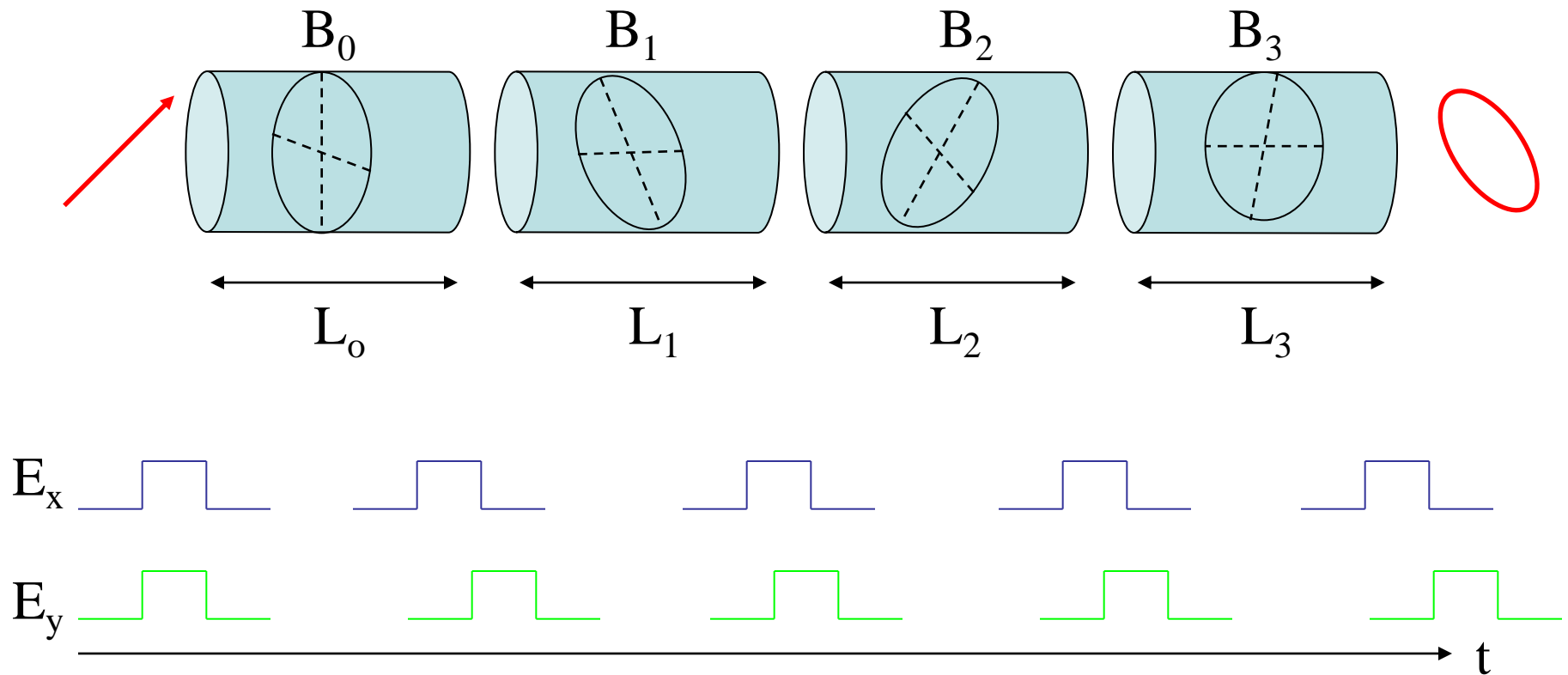
- The polarization state of light propagating in fibers with *randomly varying birefringence* will generally be *elliptical and would quickly reach a state of arbitrary polarization*.

*However, the final polarization state is *not* of concern for most lightwave systems as *photodetectors are insensitive to the state of polarization*.

(**Note:** recent technology developments in “*Coherent Optical Communications*” do require polarization state to be analyzed.)

- A simple model of PMD divides the fiber into a large number of segments. Both the *magnitude of birefringence B* and the *orientation of the principal axes* remain constant in each section but *changes randomly from section to section*.

A simple model of PMD



Randomly changing differential group delay (DGD)

- Pulse broadening caused by a *random* change of fiber polarization properties is known as polarization mode dispersion (PMD).

$$\text{PMD pulse broadening} \quad \Delta T_{\text{PMD}} = D_{\text{PMD}} \sqrt{L}$$

D_{PMD} is the PMD parameter (coefficient) measured in **ps/ $\sqrt{\text{km}}$** .

\sqrt{L} models the “random” nature (like “random walk”)

* D_{PMD} **does not depend on wavelength** (first order) ;

*Today’ s fiber (since 90’ s) PMD parameter is 0.1 - 0.5 ps/ $\sqrt{\text{km}}$.

(Legacy fibers deployed in the 80 ’s have $D_{\text{PMD}} > 0.8$ ps/ $\sqrt{\text{km}}$.)

e.g. Calculate the pulse broadening caused by PMD for a singlemode fiber with a PMD parameter $D_{\text{PMD}} \sim 0.5 \text{ ps}/\sqrt{\text{km}}$ and a fiber length of 100 km. (i.e. $\Delta T_{\text{PMD}} = 5 \text{ ps}$)

Recall that pulse broadening due to chromatic dispersion for a 1 nm linewidth light source was $\sim 15 \text{ ps/km}$, which resulted in 1500 ps for 100 km of fiber length.

=> **PMD** pulse broadening is *orders of magnitude less* than chromatic dispersion !

*PMD is relatively small compared with chromatic dispersion. But when one operates at **zero-dispersion** wavelength (or *dispersion compensated wavelengths*) with narrow spectral width, **PMD** can become a significant component of the total dispersion.

So why do we care about PMD?

Recall that chromatic dispersion can be compensated to ~ 0 ,
(at least for single wavelengths, namely, by designing proper
-ve waveguide dispersion)

but there is no simple way to eliminate PMD completely.

=> It is PMD that limits the fiber bandwidth after chromatic dispersion is compensated!

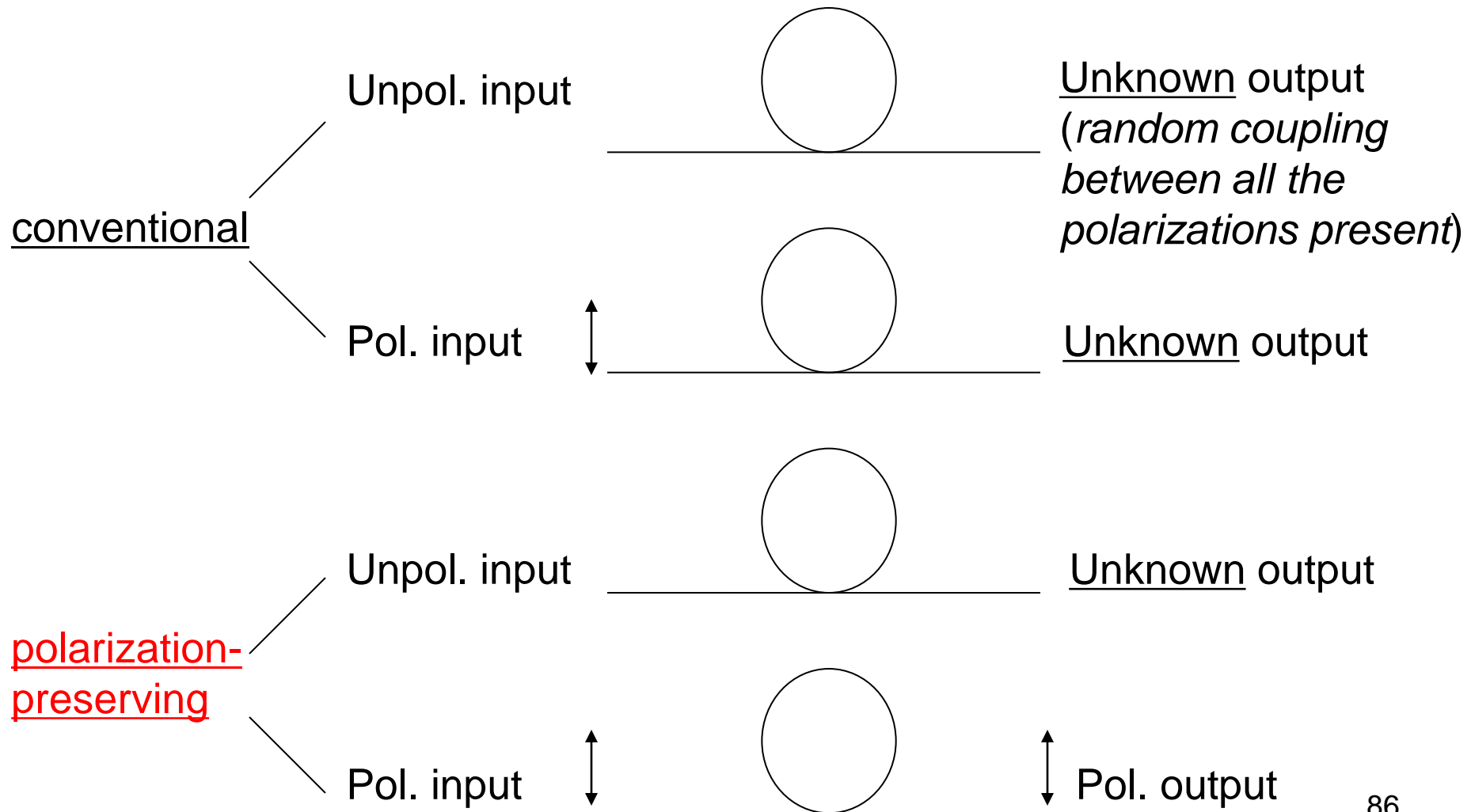
- PMD is of lesser concern in lower data rate systems. At lower transmission speeds (*up to and including 10 Gb/s*), networks have higher tolerances to all types of dispersion, including PMD.

As data rate increases, the dispersion tolerance reduces significantly, creating a need to control PMD as much as possible at the current **40 Gb/s** system.

e.g. The pulse broadening caused by PMD for a singlemode fiber with a PMD parameter of $0.5 \text{ ps}/\sqrt{\text{km}}$ and a fiber length of 100 km \Rightarrow **5 ps**.

However, this is comparable to **the 40G bit period = 25 ps !**

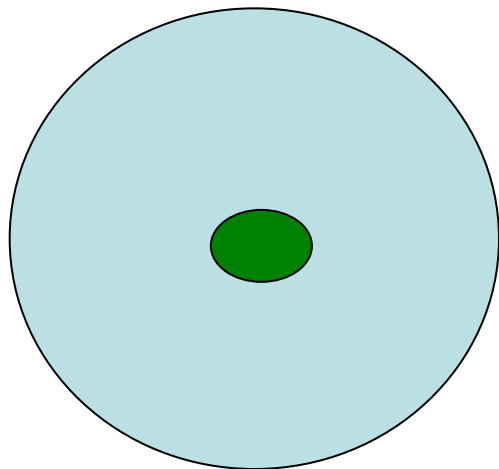
Polarizing effects of conventional / polarization-preserving fibers



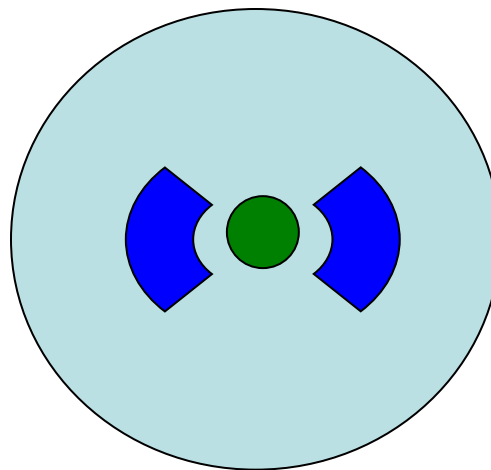
Polarization-preserving fibers

- The fiber birefringence is enhanced in single-mode *polarization-preserving* (*polarization-maintaining*) fibers, which are designed to maintain the polarization of the launched wave.
- *Polarization is preserved* because the two possible waves have significantly different propagation characteristics. This keeps them from exchanging energy as they propagate through the fiber.
- Polarization-preserving fibers are constructed by designing *asymmetries* into the fiber. Examples include fibers with elliptical cores (which cause waves polarized along the *major* and *minor* axes of the ellipse to have different effective refractive indices) and fibers that contain nonsymmetrical stress-producing parts.

Polarization-preserving fibers



Elliptical-core fiber



bow-tie fiber

- The shaded region in the bow-tie fiber is highly doped with a material such as boron. Because the thermal expansion of this doped region is so different from that of the pure silica cladding, a *nonsymmetrical stress* is exerted on the core. This produces a large *stress-induced birefringence*, which in turn *decouples the two orthogonal modes of the singlemode fiber*.

Multimode Fiber Transmission Distances

- The possible transmission distances when using fibers with different core sizes and bandwidths for Ethernet, Fibre Channel, and SONET/SDH applications.

Table 3.3 *Transmission distances in meters in multimode fibers using an 850-nm VCSEL*

Application	Data rate (Gb/s)	50- μ m core		62.5- μ m core	
		500 MHz.km	2000 MHz.km	160 MHz.km	200 MHz.km
Ethernet	1	550	860	220	275
	10	82	300	26	33
Fibre Channel	1	500	860	250	300
	2	300	500	120	150
	10	82	300	26	33
SONET/SDH	10	85	300	25	33