Probability Distribution

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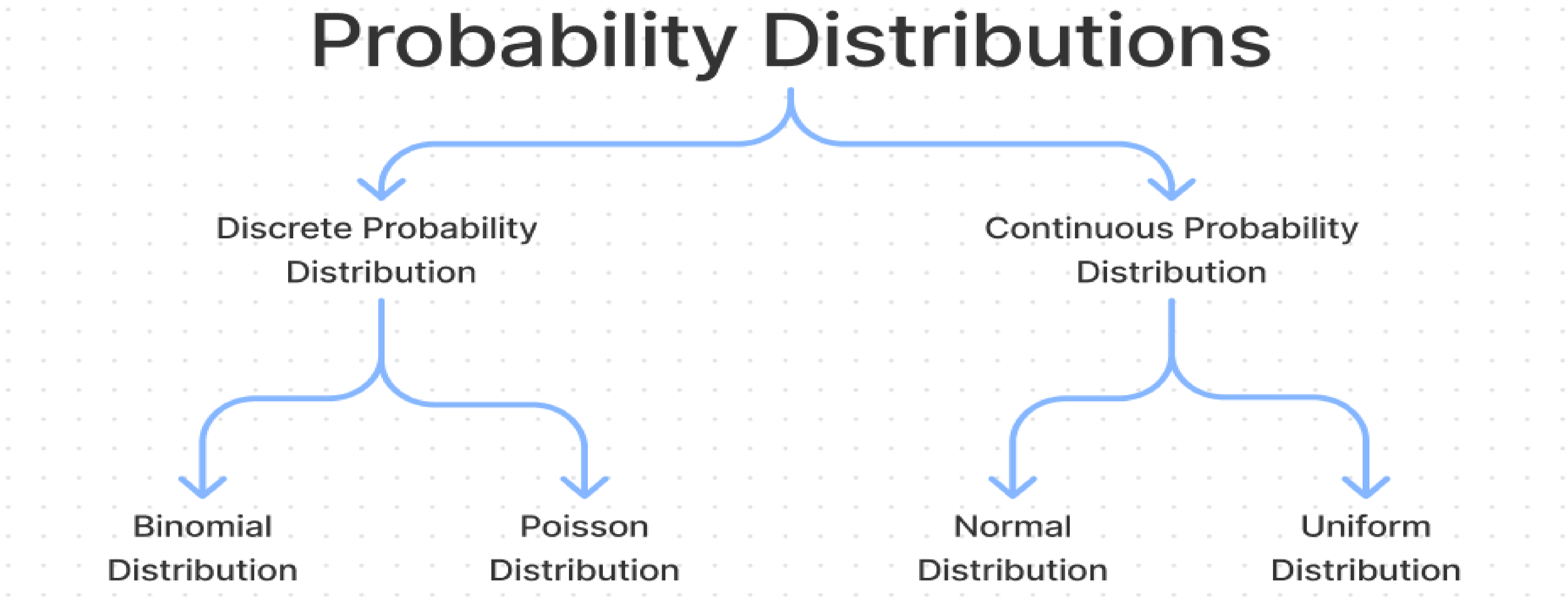
Lecturer

NFE, DIU

Probability distribution

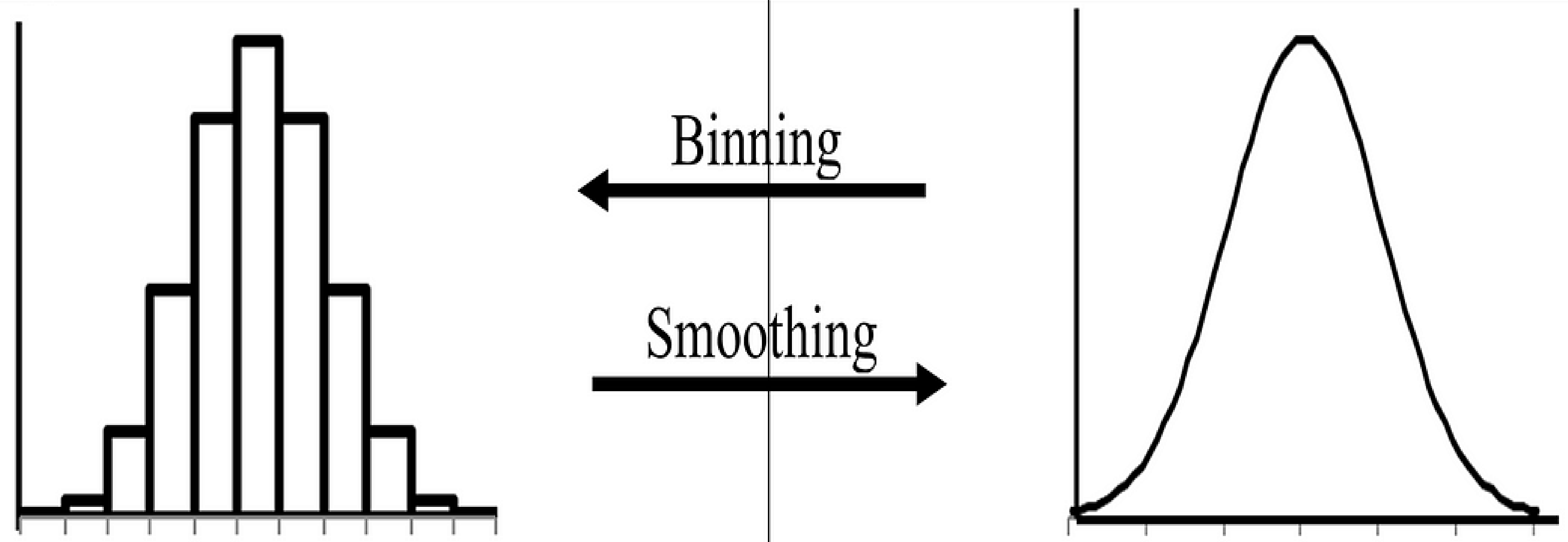
It describes the **likelihood** of different **outcomes or values** in a random **experiment**, random **process**, or statistical **data** set.

It specifies the **probabilities** associated with each **possible** outcome, event, or value of a random variable.



	Discrete Distributions	Continuous Distribution
Range of possible values	only take on a specific set of values	any value within a given range
Probability of specific values	probability of a specific value occurring in a discrete probability distribution is non-zero	probability of a specific value occurring in a continuous probability distribution is always zero.
Probability function	probability mass function	probability density function
Sum of probabilities	sum of the probabilities of all possible outcomes must equal to 1	sum of the probabilities cannot be calculated

Discrete	**Continuous**



Probability Mass Function	Probability Density Function
Count,Sum,Proportion	Integration
$P(X = x) = f(x)$	$P(X=x) = \int f(x).\, dx$
CMF,PMF = Sum, Difference	CDF,PDF = Integrate, Differentiate

Two key characteristics of a probability distribution are:

1. Probability Mass Function (PMF) or Probability Density Function (PDF): It specifies the **probability associated** with **each possible outcome** or value of the random variable.

2. Cumulative Distribution Function (CDF): It provides the **probability** that the **random variable** is **less than or equal** to a particular value. It is a **cumulative measure** of the probabilities as you move along the values of the random variable.

Binomial distribution

It **models** the number of **successful outcomes** (usually denoted as "**x**") in a **fixed number** of **independent** and **identical Bernoulli trials**. Each Bernoulli trial has **two** possible outcomes: "**success**" and "**failure**."

Binomial Distribution Formula

$$P(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

where

n = the number of trials (or the number being sampled)

x = the number of successes desired

p = probability of getting a success in one trial

$q = 1 - p$ = the probability of getting a failure in one trial

Poisson distribution

is a probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space.

It is characterized by λ (lambda), which represents the average rate of occurrence of the events in the given interval.

The mean (μ) and variance ($\sigma 2$) of the distribution are both equal to λ.

$$\boldsymbol{\mu = \sigma 2 = \lambda}$$

Poisson Distribution Formula

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

where

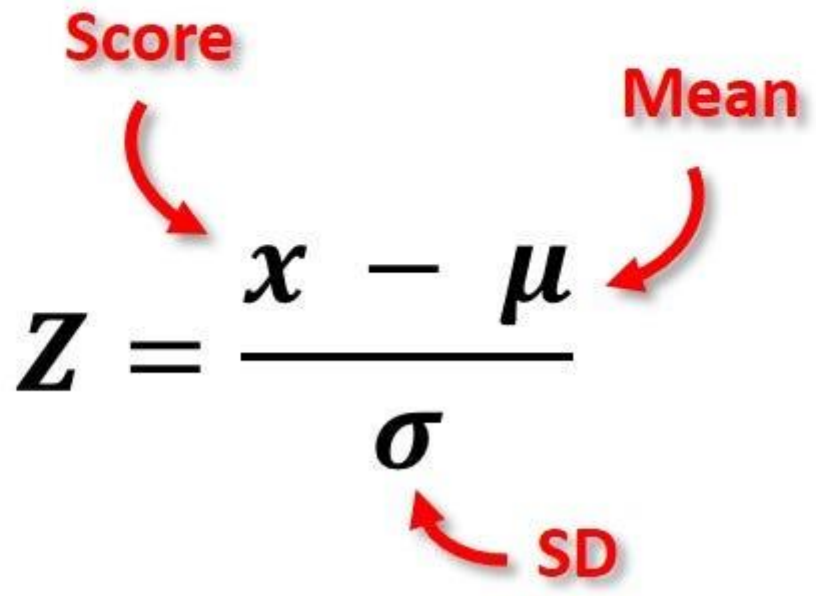
x = 0, 1, 2, 3, ...

λ = mean number of occurrences in the interval

e = Euler's constant $\approx$ 2.71828

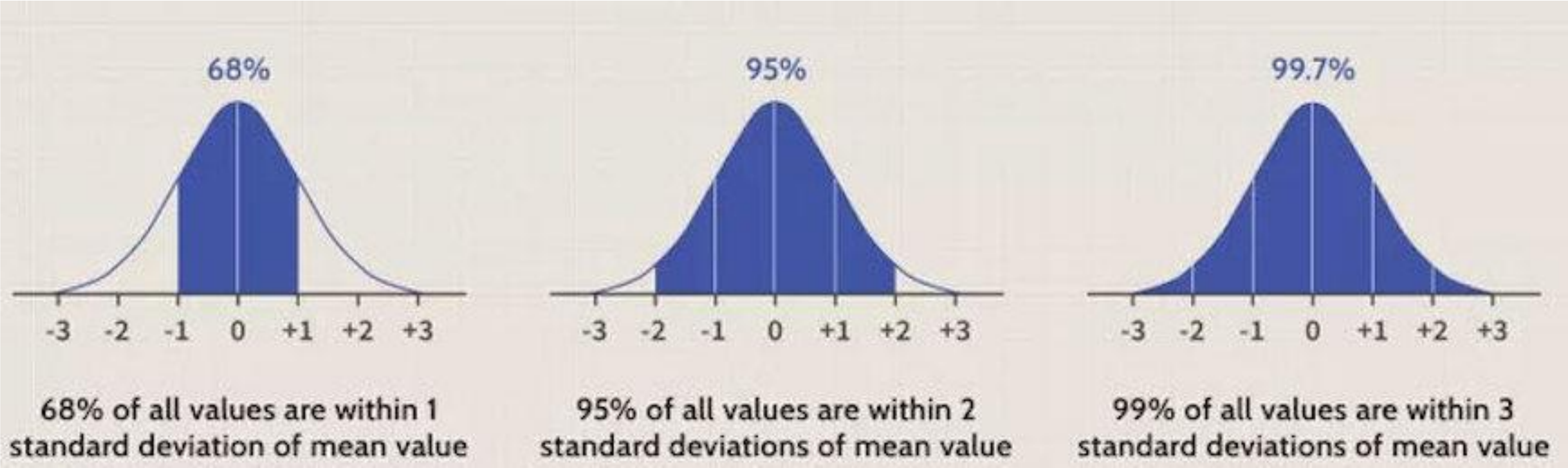
Normal distribution:

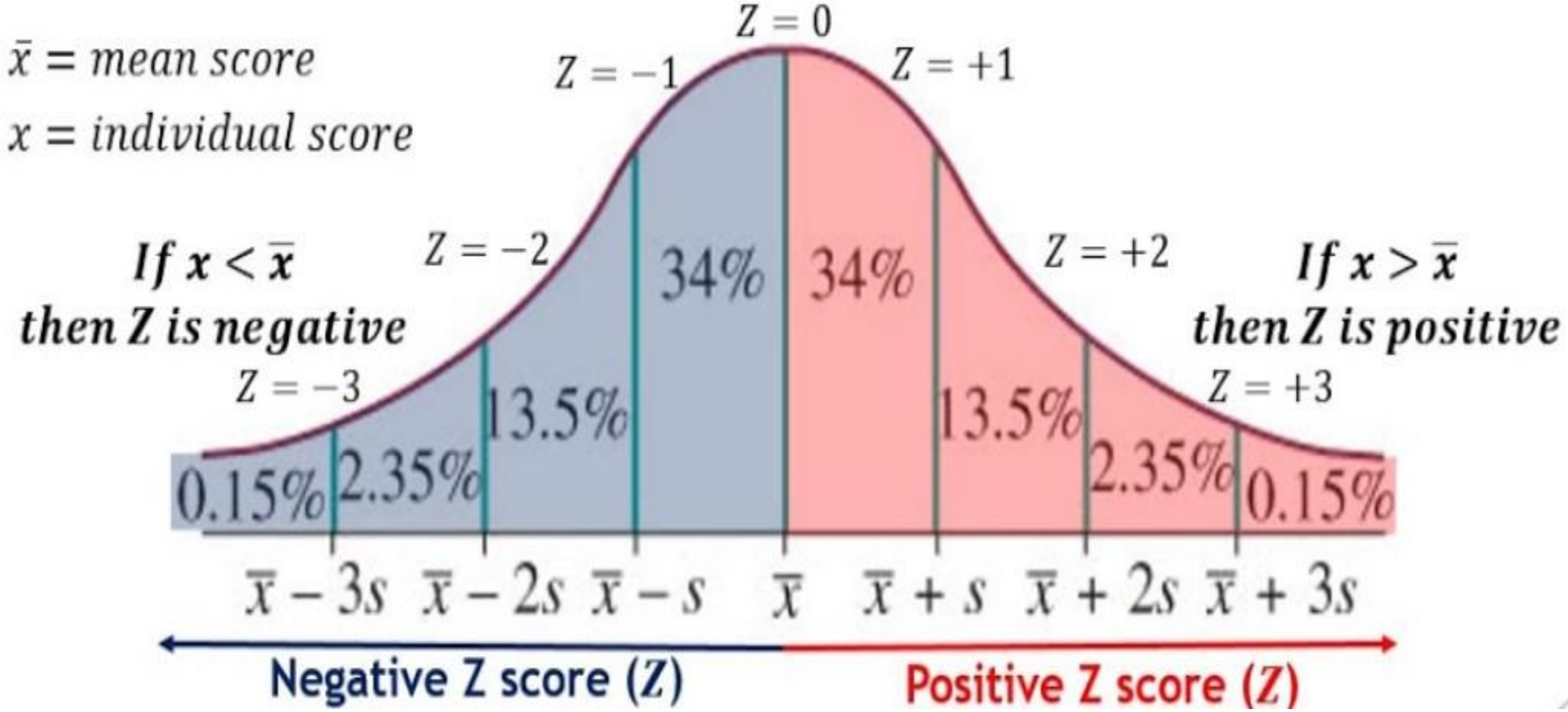
also known as the Gaussian distribution or bell curve, is a continuous probability distribution that is symmetric around its mean, which represents the central tendency of the distribution. The shape of the normal distribution is characterized by a bell-shaped curve, and it is completely defined by its mean (μ) and standard deviation (σ).



Properties of the normal distribution include:

1. Approximately 68% of the data falls within one standard deviation of the mean.
2. Approximately 95% falls within two standard deviations.
3. Approximately 99.7% falls within three standard deviations.





x̄ = mean score
x = individual score
Z = 0
Z = −1
Z = +1
Z = −2
Z = +2
Z = −3
Z = +3
If x < x̄ then Z is negative
If x > x̄ then Z is positive
34%
34%
13.5%
13.5%
2.35%
2.35%
0.15%
0.15%
x̄ − 3s
x̄ − 2s
x̄ − s
x̄
x̄ + s
x̄ + 2s
x̄ + 3s
Negative Z score (Z)
Positive Z score (Z)

Poisson Distribution - Example

At a Major League Baseball park, five customers arrive at a concession stand on average in a ten-minute period.

A) What is the probability that in a ten-minute period exactly three people will arrive at the concession stand?

B) Find the probability of exactly 3 customers arriving in 20 minutes?

C) Find the probability of 3 or fewer customers arriving in 20 minutes?

D) Find the probability of more than 3 customers arriving in 20 minutes?

Solution

A) $\lambda = 5, \ t = 1 \quad \mu = \lambda t = 5 \qquad x = 3$

$$P(X = x) = P(x) = \frac{e^{-\mu}(\mu)^x}{x!}$$

$$P(X = 3) = P(3) = \frac{e^{-5}(5)^3}{3!} = 0.1404$$

Poisson Distribution - Example

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B) Find the probability of exactly 3 customers arriving in 20 minutes?

C) Find the probability of 3 or fewer customers arriving in 20 minutes?

D) Find the probability of more than 3 customers arriving in 20 minutes?

Solution

B) $\lambda = 5, \quad t = 2 \quad \mu = \lambda t = 10 \qquad x = 3$

$$P(X = x) = P(x) = \frac{e^{-\mu}(\mu)^x}{x!}$$

$$P(X = 3) = P(3) = \frac{e^{-10}(10)^3}{3!} = 0.0076$$

Poisson Distribution - Example

At a Major League Baseball park, five customers arrive at a concession stand on average in a ten-minute period.

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D) Find the probability of more than 3 customers arriving in 20 minutes?

Solution

$$P(X = x) = P(x) = \frac{e^{-\mu}(\mu)^x}{x!}$$

C) $\lambda = 5, \quad t = 2 \quad \mu = \lambda t = 10$

$$P(X \leq 3) = P(0) + P(1) + P(2) + P(3) = \frac{e^{-10}(10)^0}{0!} + \frac{e^{-10}(10)^1}{1!} + \frac{e^{-10}(10)^2}{2!} + \frac{e^{-10}(10)^3}{3!}$$

$$= e^{-10}\left(\frac{1}{0!} + \frac{10}{1!} + \frac{10^2}{2!} + \frac{10^3}{3!}\right) = 0.0103$$

Poisson Distribution - Example

At a Major League Baseball park, five customers arrive at a concession stand on average in a ten-minute period.

A) What is the probability that in a ten-minute period exactly three people will arrive at the concession stand?

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Solution

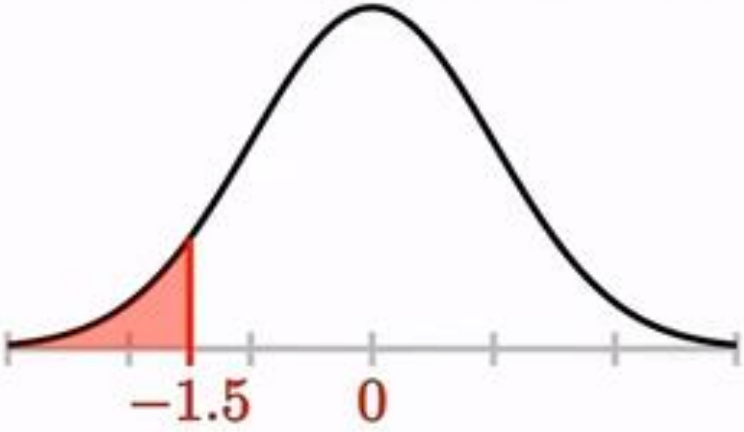
$$P(X = x) = P(x) = \frac{e^{-\mu}(\mu)^x}{x!}$$

D) $\lambda = 5, \quad t = 2 \quad \mu = \lambda t = 10$

$$P(X > 3) = P(4) + P(5) + P(6) + \cdots = 1 - P(X \leq 3) = 1 - 0.0103 = 0.9897$$

Normal Distribution

(Ex) Your local pizza shop claims their large is at least 16 in. or its free. Their pizza is normally distributed with $\mu = 16.3$ in. and $\sigma = 0.2$ in. What's the prob. of getting free pizza? What's the prob. of getting lucky with pizza over 16.5 in.? What's prob. of getting pizza between 15.95 and 16.63 in.?



$$P(X < 16)$$
$$\Downarrow$$
$$P(Z < -1.5)$$

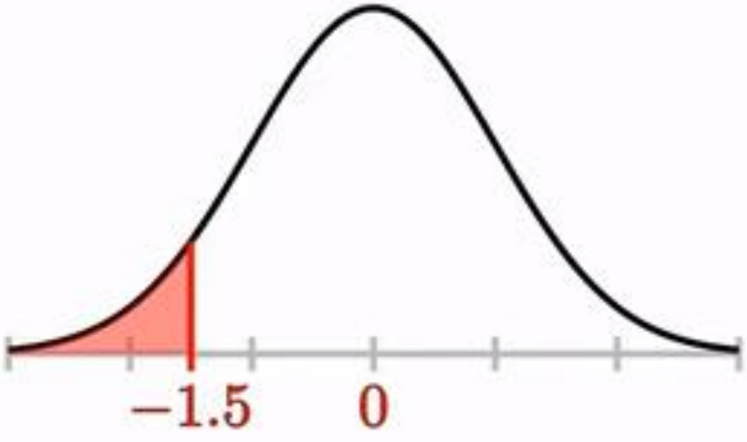
$$z = \frac{x - \mu}{\sigma} = \frac{16 - 16.3}{0.2} = \frac{-0.3}{0.2} = -1.5$$

Normal Distribution

Ex

Your local pizza shop claims their larg[e]
pizza is normally distributed with $\mu =$
prob. of getting free pizza? What's the
over 16.5 in.? What's prob. of getting

z	.00	.01	.02	.03	.04
−1.5	.0668	.0655	.0643	.0630	.0618
−1.4	.0808	.0793	.0778	.0764	.0749
−1.3	.0968	.0951	.0934	.0918	.0901
−1.2	.1151	.1131	.1112	.1093	.1075
−1.1	.1357	.1335	.1314	.1292	.1271
−1.0	.1587	.1562	.1539	.1515	.1492
−0.9	.1841	.1814	.1788	.1762	.1736
−0.8	.2119	.2090	.2061	.2033	.2005



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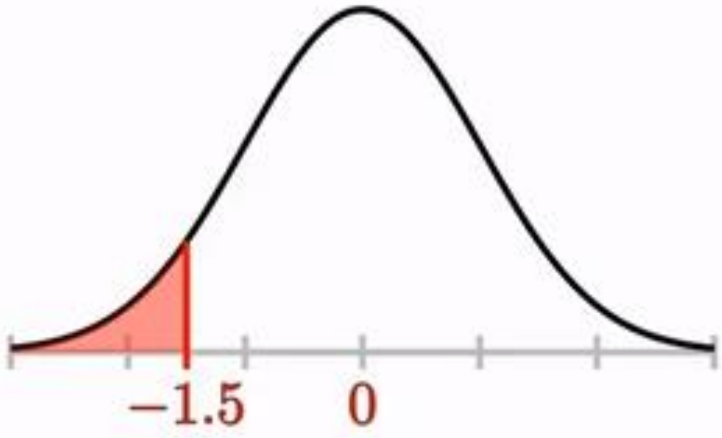
$$\Downarrow$$
$$P(Z < -1.5)$$

$$z = \frac{x-\mu}{\sigma} = \frac{16-16.3}{0.2} = \frac{-0.3}{0.2} = -1.5$$

Normal Distribution

(Ex) Your local pizza shop claims their large is at least 16 in. or its free. Their pizza is normally distributed with $\mu = 16.3$ in. and $\sigma = 0.2$ in. What's the prob. of getting free pizza? What's the prob. of getting lucky with pizza over 16.5 in.? What's prob. of getting pizza between 15.95 and 16.63 in.?



$$P(X < 16) \quad \Rightarrow \quad \boxed{P(Z < -1.5) = 0.0668}$$

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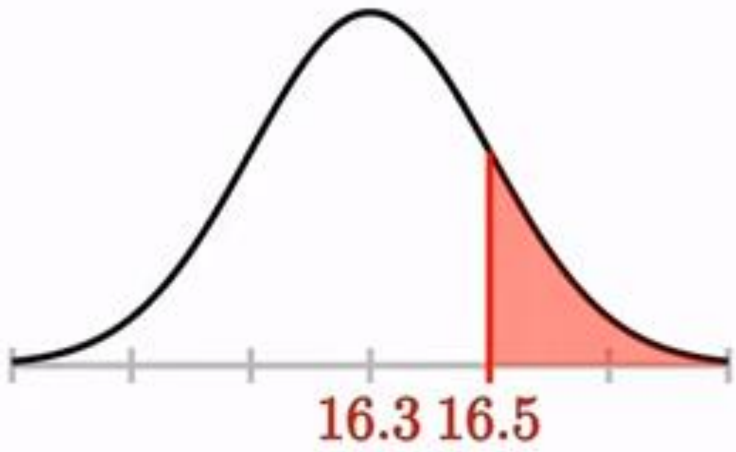


$$P(X < 16) \quad \Rightarrow \quad \boxed{P(Z < -1.5) = 0.0668}$$

$$P(X > 16.5)$$

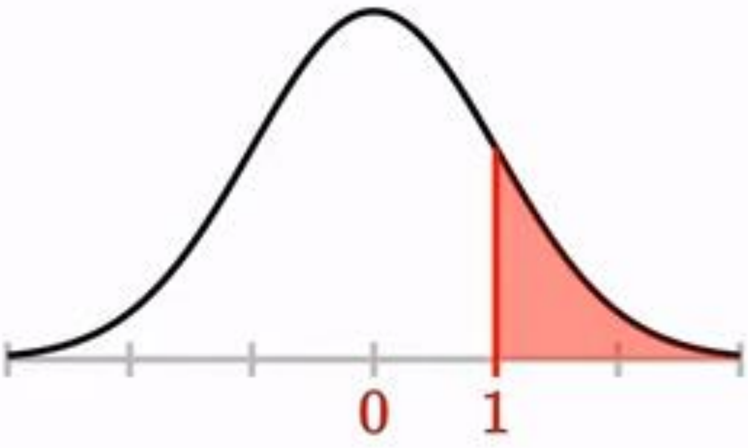
$$z = \frac{x-\mu}{\sigma} = \frac{16.5 - 16.3}{0.2} = \frac{0.2}{0.2} = 1$$

Normal Dist

Ex

Your local pizza shop claims their large is
pizza is normally distributed with $\mu = 16.$
prob. of getting free pizza? What's the p
over 16.5 in.? What's prob. of getting piz

z	.00	.01	.02	.03	.04
0.0	.5000	.5040	.5080	.5120	.5160
0.1	.5398	.5438	.5478	.5517	.5557
0.2	.5793	.5832	.5871	.5910	.5948
0.3	.6179	.6217	.6255	.6293	.6331
0.4	.6554	.6591	.6628	.6664	.6700
0.5	.6915	.6950	.6985	.7019	.7054
0.6	.7257	.7291	.7324	.7357	.7389
0.7	.7580	.7611	.7642	.7673	.7704
0.8	.7881	.7910	.7939	.7967	.7995
0.9	.8159	.8186	.8212	.8238	.8264
1.0	.8413	.8438	.8461	.8485	.8508
1.1	.8643	.8665	.8686	.8708	.8729
1.2	.8849	.8869	.8888	.8907	.8925



$$P(X < 16)$$

$$P(X > 16.5)$$
$$\Downarrow$$
$$P(Z > 1) = 1 - .8413$$

$$z = \frac{x - \mu}{\sigma} = \frac{16.5 - 16.3}{0.2} = \frac{0.2}{0.2} = 1$$

Normal Distribution

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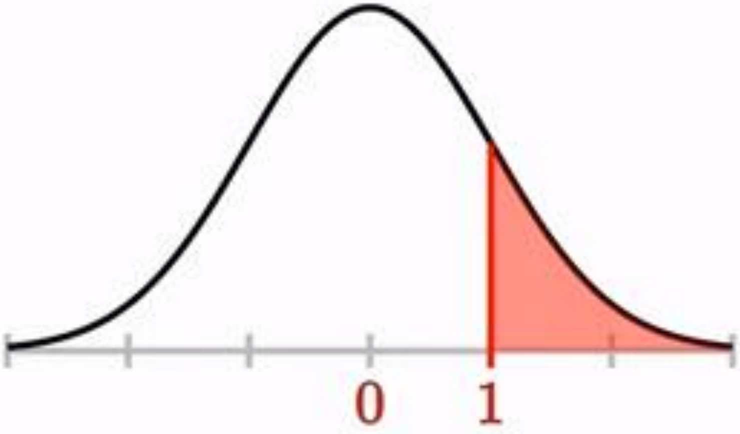

$$P(X < 16) \Rightarrow \boxed{P(Z < -1.5) = 0.0668}$$

$$P(X > 16.5) \Rightarrow P(Z > 1) = 0.1587$$

$$z = \frac{x - \mu}{\sigma} = \frac{16.5 - 16.3}{0.2} = \frac{0.2}{0.2} = 1$$

Normal Distribution

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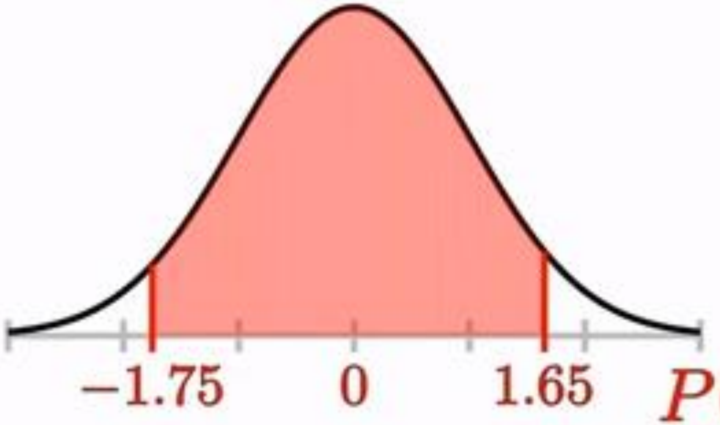

$$P(X < 16) \Rightarrow \boxed{P(Z < -1.5) = 0.0668}$$

$$P(X > 16.5) \Rightarrow \boxed{P(Z > 1) = 0.1587}$$

$$P(15.95 < X < 16.63)$$

$$z = \frac{x - \mu}{\sigma} = \frac{16.63 - 16.3}{0.2} = \frac{0.33}{0.2} = 1.65$$

Normal Distribution

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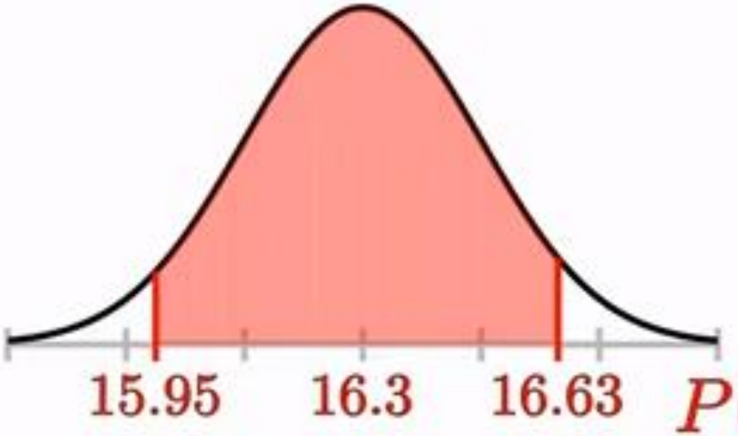

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$$P(15.95 < X < 16.63)$$

$$z = \frac{x - \mu}{\sigma} = \frac{15.95 - 16.3}{0.2} = \frac{-0.35}{0.2} = -1.75$$

Normal Distribution

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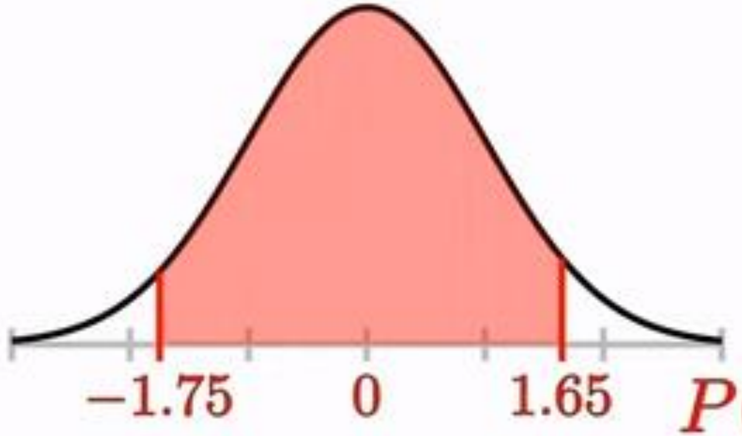

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$$P(15.95 < X < 16.63)$$

$$\Downarrow$$

$$P(-1.75 < Z < 1.65)$$

$$z = \frac{x - \mu}{\sigma}$$

Normal Dis

Ex

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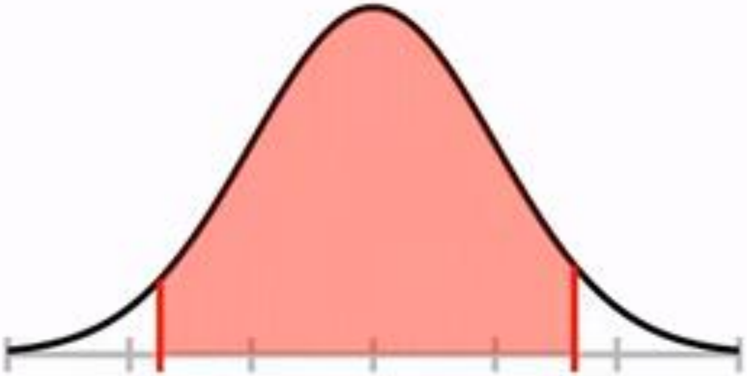
z	.00	.01	.02	.03	.04	.05	.06
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686



$P(X < 16)$

$P(X > 16.5$

$$P(15.95 < X < 16.63)$$

$$\Downarrow$$

$$P(-1.75 < Z < 1.65) = 0.9505$$

$$z = \frac{x - \mu}{\sigma}$$

Normal Dis

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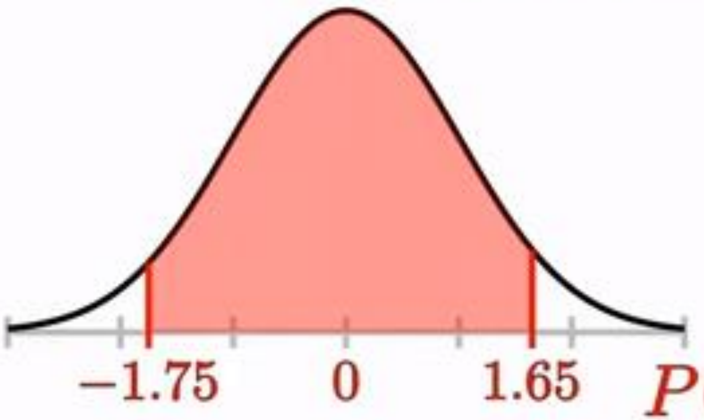
z	.00	.01	.02	.03	.04	.05	.06
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761



$P(X < 16)$

$P(X > 16.5$

$P(15.95 < X < 16.63)$

$\Downarrow$

$P(-1.75 < Z < 1.65) = 0.9505 - 0.0401$

$$z = \frac{x - \mu}{\sigma}$$

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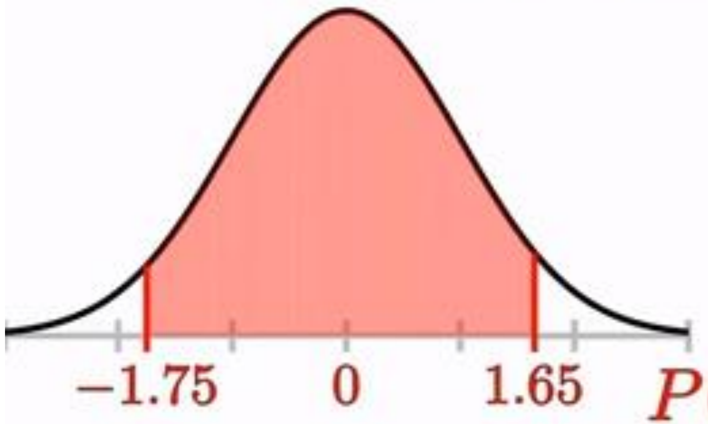
Normal Distribution

(Ex) Your local pizza shop claims their large is at least 16 in. or its free. Their pizza is normally distributed with $\mu = 16.3$ in. and $\sigma = 0.2$ in. What's the prob. of getting free pizza? What's the prob. of getting lucky with pizza over 16.5 in.? What's prob. of getting pizza between 15.95 and 16.63 in.?



$$P(X < 16) \Rightarrow \boxed{P(Z < -1.5) = 0.0668}$$

$$P(X > 16.5) \Rightarrow \boxed{P(Z > 1) = 0.1587}$$

$$P(15.95 < X < 16.63)$$

$$\Downarrow$$

$$\boxed{P(-1.75 < Z < 1.65) = 0.9104}$$

$$z = \frac{x - \mu}{\sigma}$$

Uniform distribution:

also known as a rectangular distribution, is a probability distribution where every possible outcome has an equal probability of occurring. In other words, all values in the distribution have the same likelihood of being observed