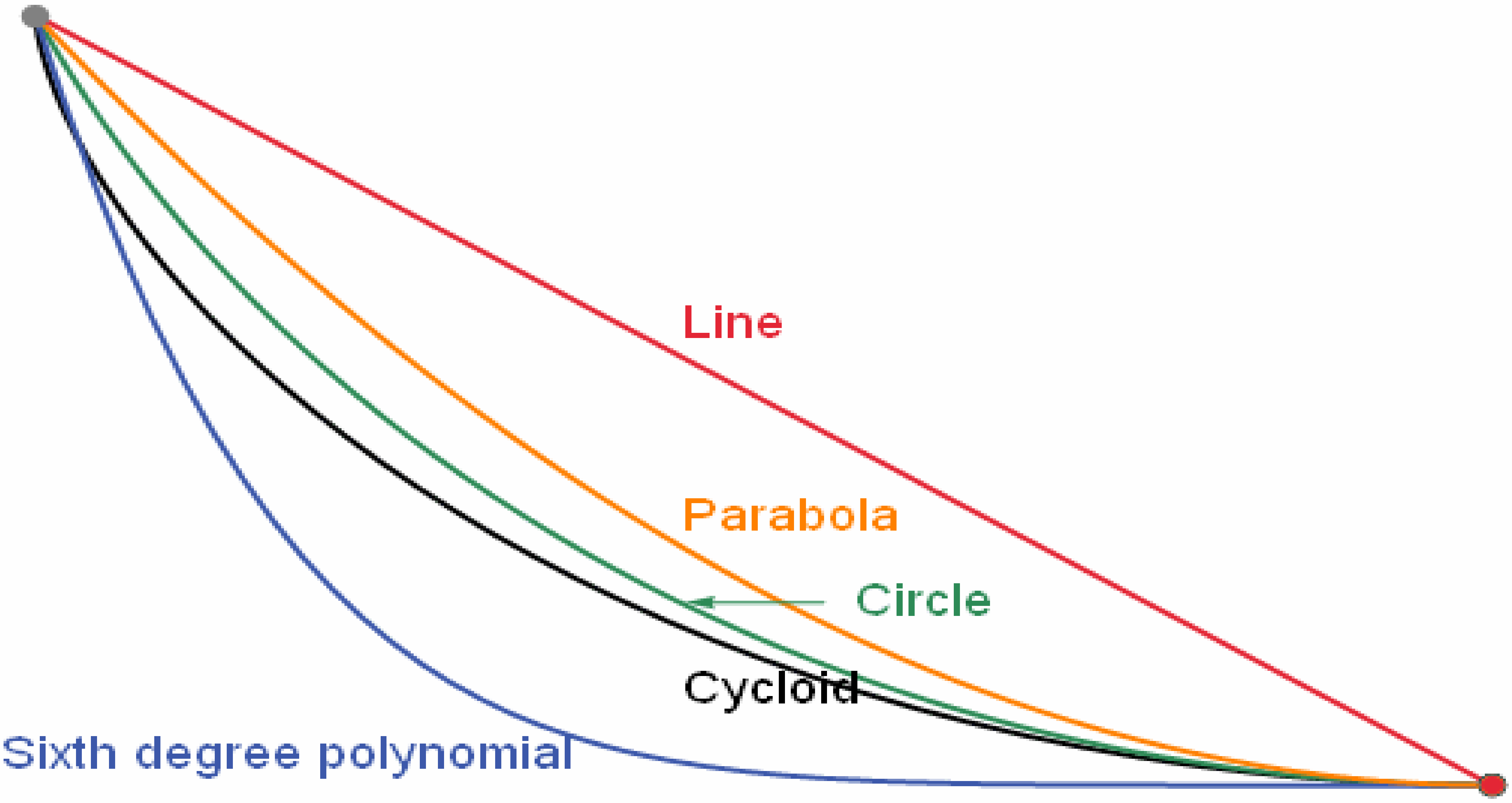
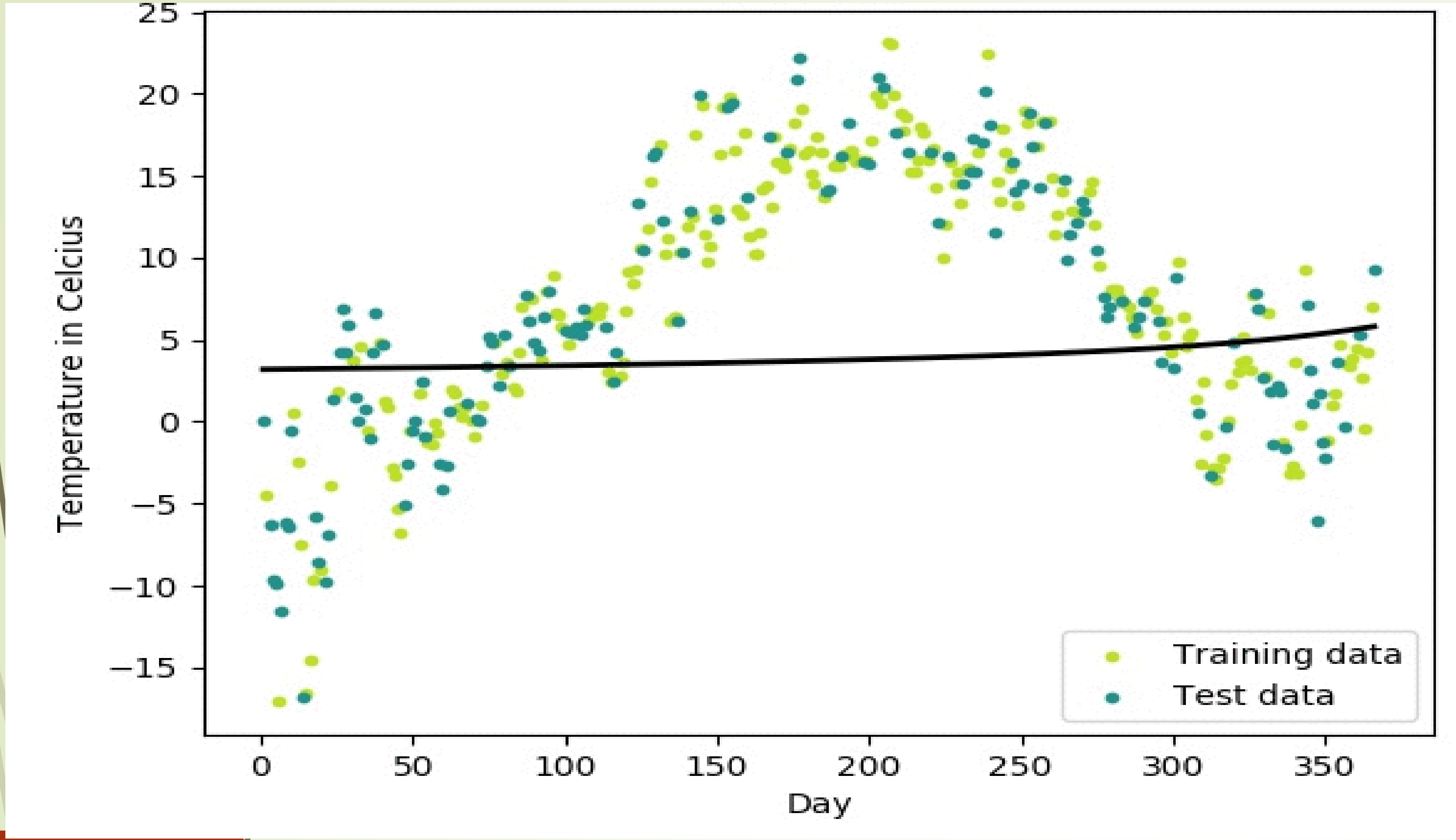

Line
Parabola
Circle
Cycloid
Sixth degree polynomial



Curve Fitting

Chapter 4

Curve fitting

Curve fitting is the process of constructing a curve,

or

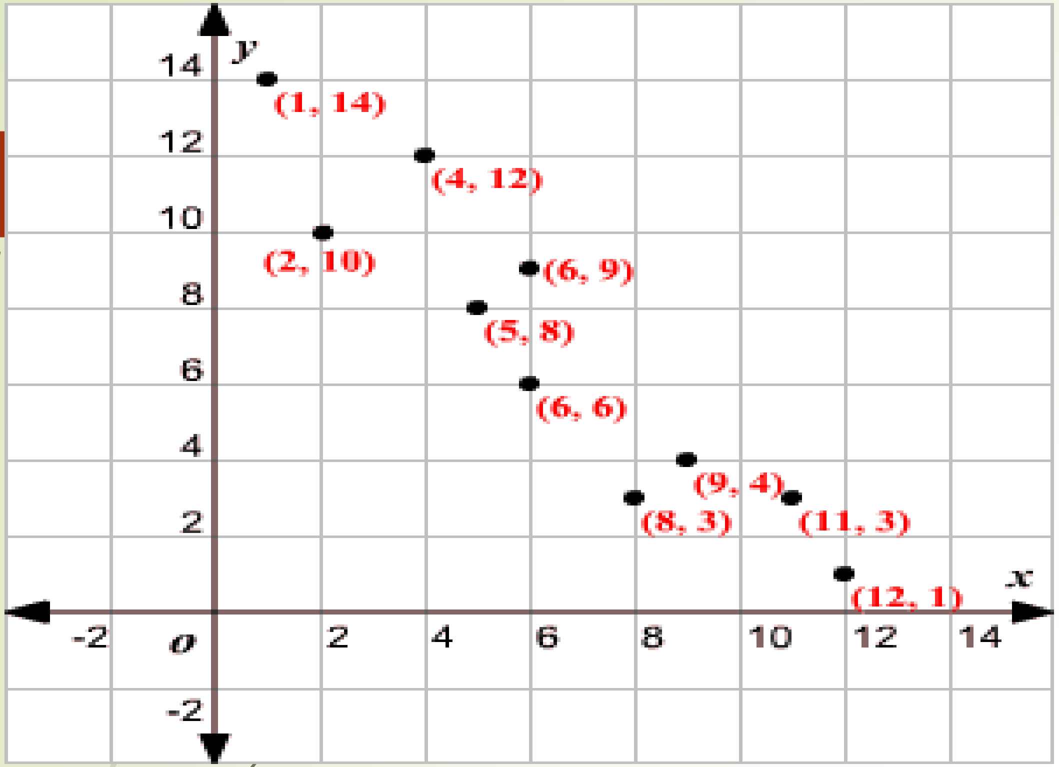
The procedure in finding a curve which matches a series of data points , possibly subject to constraints.

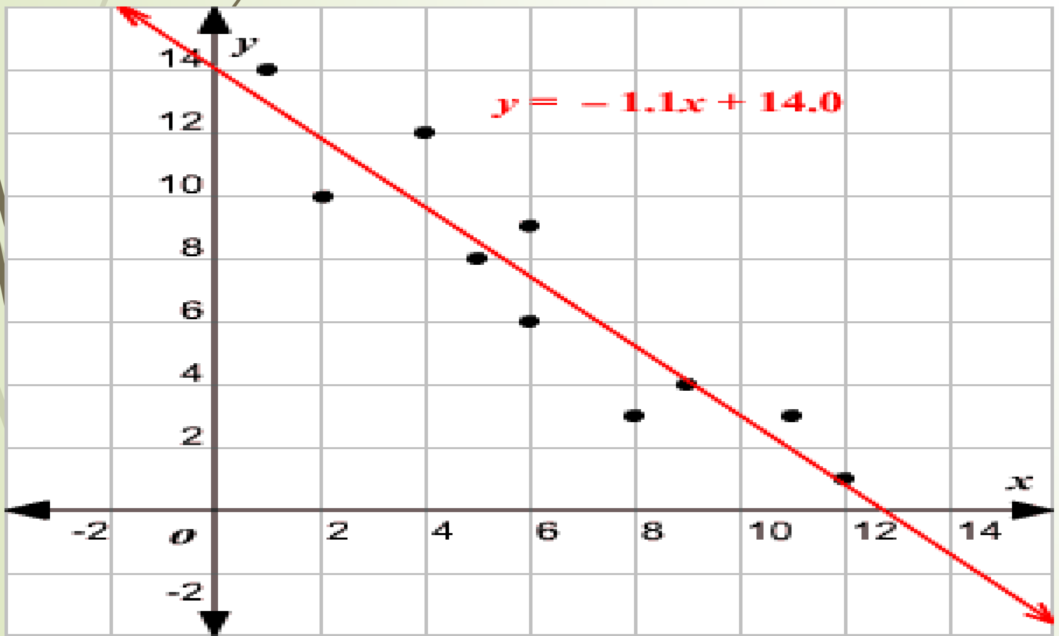
Example

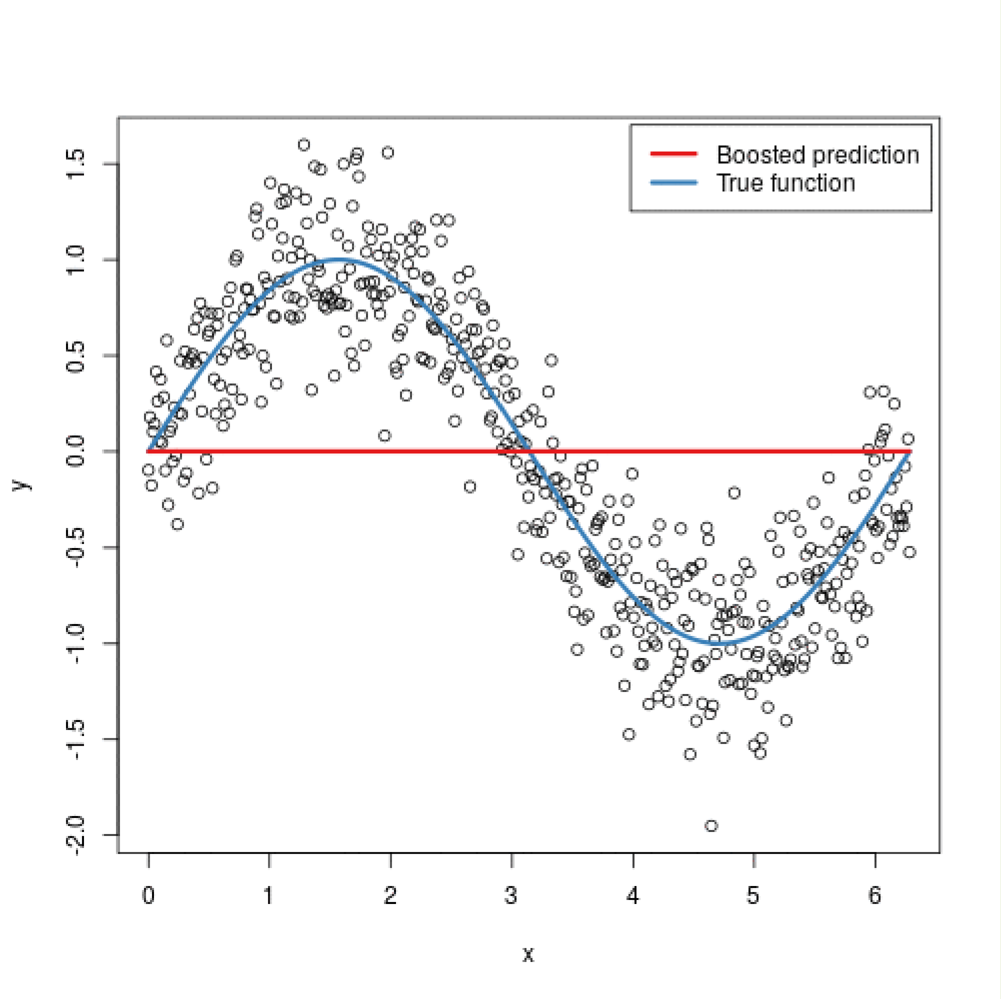
Plot the line.

x	8	2	11	6	5	4	12	9	6
y	3	10	3	6	8	12	1	4	9

(8,3) ,(2,10),(11 , 3) , (6 ,6) , (5,8) , (4,12) , (12,1) , (9,4) , (6, 9) .


y
x
(1, 14)
(4, 12)
(2, 10)
(6, 9)
(5, 8)
(6, 6)
(9, 4)
(8, 3)
(11, 3)
(12, 1)


y = − 1.1x + 14.0


Boosted prediction
True function
y
x

Types of curves

Circle
Parabola
Hyperbola
Ellipse

Curve

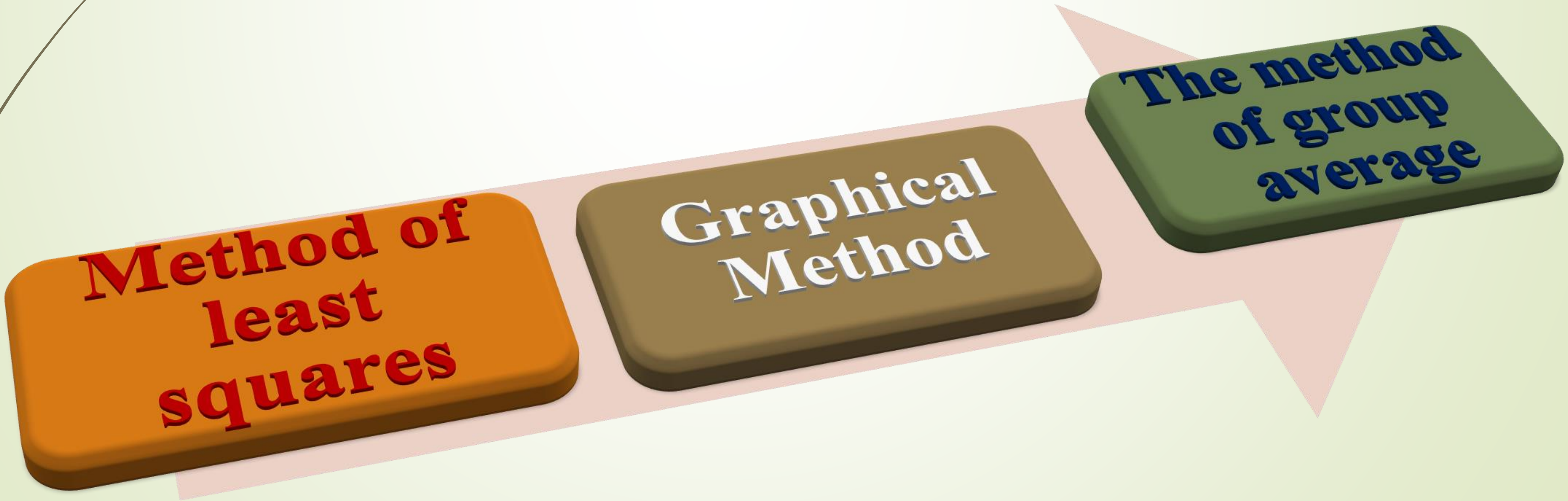
Linear | **Non Linear**

$$y = ax + b.$$

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

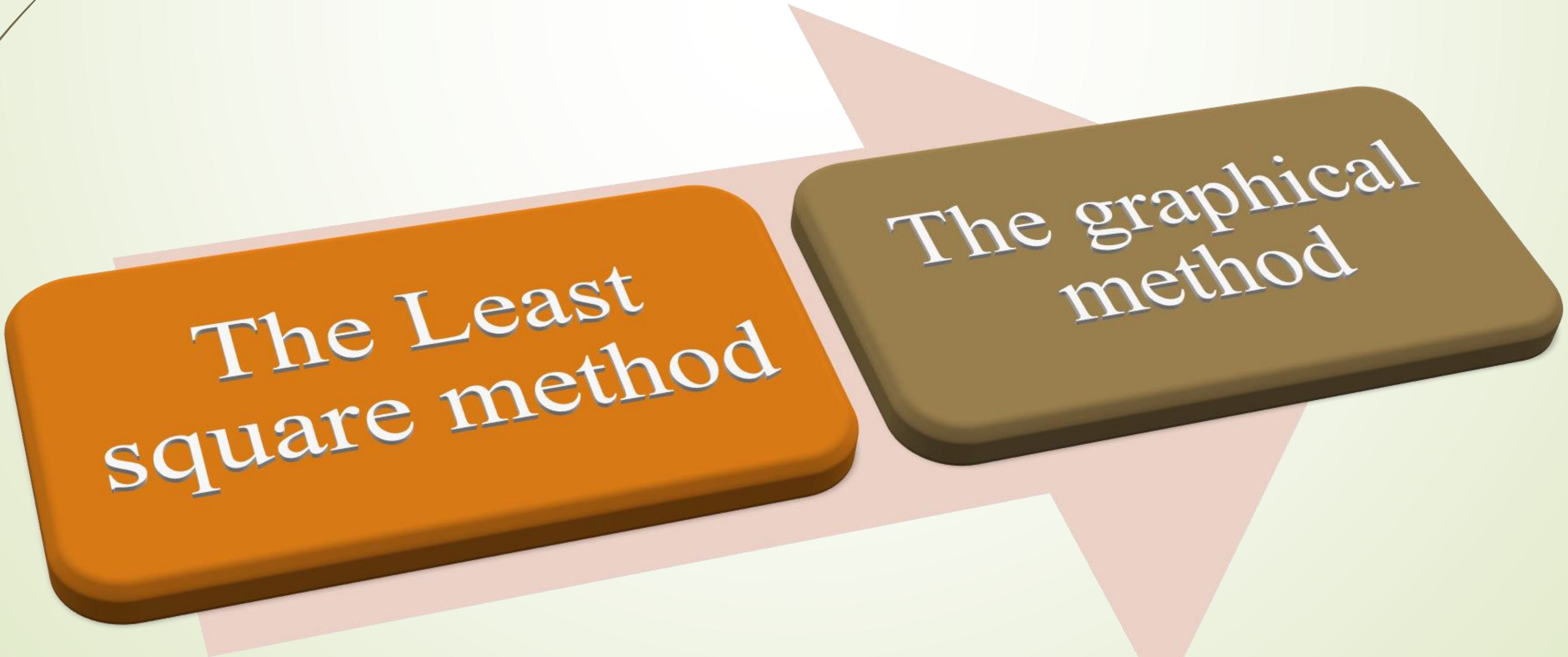
Types of Methods on curve

- The constant occurring in the equation $y = f(x)$ of the approximating curve can be found by several methods mentioned in the followings:



Linear Curve Fitting

There are two useful methods for finding a straight line.



Linear Curve Fitting

The Least Square method for finding a straight line.

Linear Curve Fitting

- **Least Square Formula for fitting the linear Curve :**

Normal Equations:

$$y = ax + b$$

$$\sum_{i=1}^{n} y_i = a\sum_{i=1}^{n} x_i + bn$$

$$\sum_{i=1}^{n} x_i\, y_i = a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i$$

Procedure of Linear Curve Fitting

So , Task:

(i) $y = ax + b$ …… ……(1)

(ii)

$\sum y = a\sum x + bn$ ……………….(2)

$\sum xy = a\sum x^2 + b\sum x$ ….(3)

……….(2)

Procedure of Linear Curve Fitting

Task:

(iii) Make a table to calculate the necessary summations.

(iv) Substituting those values in normal equations make two equations of a and b

(v) Solve two equations for a and b.

(vi) substitute the values of a and b in $y = ax + b$.

Problem

Problem 01: Use the method of least squares to fit a straight line to the following data:

x	0	5	10	15	20
y	7	11	16	20	26

Estimate the value of y when x = 25.

Solution :

Let the least square straight line to be fitted to the given data be $y = ax + b$……………………(1)

Then the normal Equations are:

$$\sum y = a\sum x + bn \text{ ………………….(2)}$$

$$\sum xy = a\sum x^2 + b\sum x \text{ ….(3)}$$

Here the number of data points of x , n = 5.

Calculation for finding the coefficients a and b of the least square line.

x	y	xy	x^2
0	7	0	0
5	11	55	25
10	16	160	100
15	20	300	225
20	26	520	400
$\sum x = 50$	$\sum y = 80$	$\sum xy = 1035$	$\sum x^2 = 750$

Now putting these values in the above equations (2) and (3) we get

$$50a + 5b = 80 \dots\dots(4)$$

$$750a + 50b = 1035 \quad \dots\dots.(5)$$

Solving above equations we get, $a = 0.94$ **and**

$$b = 6.6$$

Putting these values in the equation $y = ax + b$ we get the required line as

$$y = 0.94x + 6.6$$

Therefore, the expected value of y at $x = 25$ is

$$y(25) = 0.94 \times 25 + 6.6 = 30.1$$

as $x = 25.$

Problem

02: find the Least square line $y = ax + b$ for the data points (-1, 10), (0,9), (1,7), (2,5), (3,4), (4,3), (5,0), and (6, -1).

Solution

Let the least square straight line to be fitted to the given data be $y = ax + b$……………………(1)

Then the normal Equations are:

$$\sum y = a\sum x + bn \ \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\sum xy = a\sum x^2 + b\sum x \ \ldots.(3)$$

Here the number of data points of x , n = 8.

Calculation for finding the coefficients a and b of the least square line.

x	y	xy	x^2
-1	10	-10	1
0	9	0	0
1	7	7	1
2	5	10	4
3	4	12	9
4	3	12	16
5	0	0	25
6	-1	-6	36
$\sum x = 20$	$\sum y = 37$	$\sum xy = 25$	$\sum x^2 = 92$

Now putting these values in the above equations (2) and (3) we get

$$92a + 20b = 25 \text{ ....(4)}$$

$$20a + 8b = 37 \text{.............(5)}$$

Solving above equations we get, $a = -1.60714$ and $b = 8.64286$

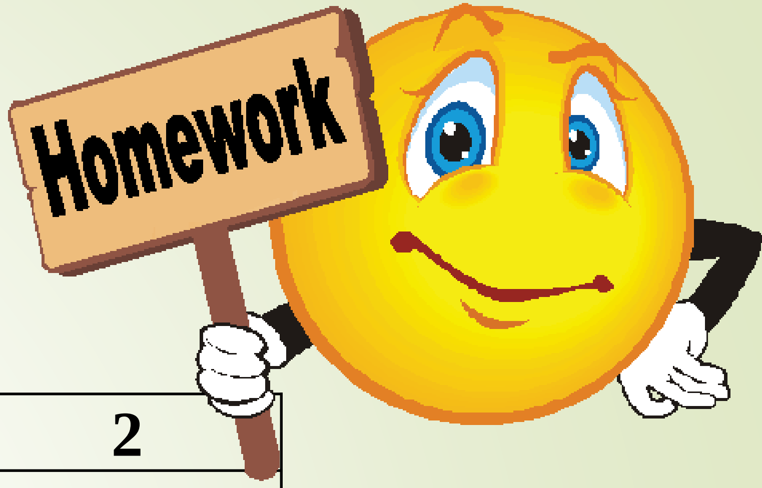
Putting these values in the equation

$$y = ax + b$$

we get the required line

as $y = -1.6071x + 8.64286.$

Home work



1.Find the least square line $y = ax + b$ for the data

x	-2	-1	0	1	2
y	1	2	3	3	4

2. Find the values of a_0 and a_1 so that $y = a_0 + a_1 x$ fits the data given in the table:

x	0	1	2	3	4
y	1	2.9	4.8	6.7	8.6

3. Fit a straight line of the form $y = a_0 + a_1 x$ to the data:

x	1	2	3	4	6	8
y	2.4	3.1	3.5	4.2	5	6

Exercise 4

4.The table below gives the temperature T (in 0^0 C) and length l(in mm) of a heated rod. If $l = a_0 + a_1 T$ find the values of a_0 and a_1 using linear least squares

T	40	50	60	70	80
l	600.5	600.6	600.8	600.9	601

5. Find the least square line $y = ax + b$ for the data

x	-4	-2	0	2	4
y	1.2	2.8	6.2	7.8	13.2