Newton Forward and Backward Interpolation

Interpolation is the technique of estimating the value of a function for any intermediate value of the independent variable, while the process of computing the value of the function outside the given range is called **extrapolation**.
Forward Differences: The differences y1 – y0, y2 – y1, y3 – y2, ……, yn – y(n–1) when denoted by dy0, dy1, dy2, ……, dy(n–1) are respectively, called the first forward differences. Thus the first forward differences are:

Forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_0	y_0					
		Δy_0				
x_1	y_1		$\Delta^2 y_0$			
$(= x_0 + h)$		Δy_1		$\Delta^3 y_0$		
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$	
$(= x_0 + 2h)$		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$	
$= (x_0 + 3h)$		Δy_3		$\Delta^3 y_2$		
x_4	y_4		$\Delta^2 y_3$			
$= (x_0 + 4h)$		Δy_4				
x_5	y_5					
$= (x_0 + 5h)$						

NEWTON'S GREGORY FORWARD INTERPOLATION FORMULA:

$$f(a+hu) = f(a)+u\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+...+\frac{u(u-1)(u-2)...(u-n+1)}{n!}\Delta^n f(a)$$

This formula is particularly useful for interpolating the values of f(x) near the beginning of the set of values given. h is called the interval of difference and **u = (x – a) / h**, Here a is first term.

Example :

```
Input : Value of Sin 52
```

θ°	45°	50°	55°	60°
sin θ	0.7071	0.7660	0.8192	0.8660

```
Output :
```

x°	Differences			
	$10^4 y$	$10^4 \Delta y$	$10^4 \Delta^2 y$	$10^4 \Delta^3 y$
45°	7071	589	−57	−7
50°	7660	532	−64	
55°	8192	468		
60°	8660			

Value at Sin 52 is 0.788003

Below is the implementation of newton forward interpolation method.