Newton Backward Interpolation

Interpolation is the technique of estimating the value of a function for any intermediate value of the independent variable, while the process of computing the value of the function outside the given range is called **extrapolation**.

Backward Differences : The differences y1 – y0, y2 – y1, ……, yn – y(n–1) when denoted by dy1, dy2, ……, dy(n), respectively, are called first backward difference. Thus the first backward differences are :
Backward differences are :

Backward difference table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
x_0	y_0					
		∇y_1				
x_1 $(= x_0 + h)$	y_1		$\nabla^2 y_2$			
		∇y_2		$\nabla^3 y_3$		
x_2 $(= x_0 + 2h)$	y_2		$\nabla^2 y_3$		$\nabla^4 y_4$	
		∇y_3		$\nabla^3 y_4$		$\nabla^5 y_5$
x_3 $(= x_0 + 3h)$	y_3		$\nabla^2 y_4$		$\nabla^4 y_5$	
		∇y_4		$\nabla^3 y_5$		
x_4 $(= x_0 + 4h)$	y_4		$\nabla^2 y_5$			
		∇y_5				
x_5 $(= x_0 + 5h)$	y_5					

NEWTON'S GREGORY BACKWARD INTERPOLATION FORMULA:

$$f(a+nh+uh) = f(a+nh) + u\nabla f(a+nh) + \frac{u(u+1)}{2!}\nabla^2 f(a+nh) + \ldots + \frac{u(u+1)\ldots(u+\overline{n-1})}{n!}\nabla^n f(a+nh)$$

This formula is useful when the value of f(x) is required near the end of the table. h is called the interval of difference and **u = (x – an) / h**, Here is a last term.

Example :

```
Input : Population in 1925
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Year (x):	*1891*	*1901*	*1911*	*1921*	*1931*
Population (y): (in thousands)	*46*	*66*	*81*	*93*	*101*

```
Output :
```

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1891	46				
		20			
1901	66		– 5		
		15		2	
1911	81		– 3		– 3
		12		– 1	
1921	93		– 4		
		8			
1931	101				

```
Value in 1925 is 96.8368
```

Below is the implementation of newton backward interpolation method.