Problem 1: The table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface:

x(height)	100	150	200	250	300	350	400
y(distance)	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the values of y when x = 218 ft and 410 ft.

Solⁿ: Forward difference table is as follows:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
100	10.63	2.40	-0.39	0.15	-0.07
150	13.03	2.01	-0.24	0.08	-0.05
200	15.04	1.77	-0.16	0.03	-0.01
250	16.81	1.61	-0.13	0.02	
300	18.42	1.48	-0.11		
350	19.90	1.37			
400	21.27				

(1) For $x = 218$ ft

Let $x_0 = 200$, then $y_0 = 15.04$, $\Delta y_0 = 1.77$, $\Delta^2 y_0 = -0.16$, $\Delta^3 y_0 = 0.03$, $\Delta^4 y_0 = -0.01$ and so on

$p = \frac{x - x_0}{h} = \frac{218 - 200}{50}$ $\quad x = 218$, $h = 50$

$p = 0.36$

By Using Newton's forward interpolation formula,

$$y_{218} = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \cdots$$

$y_{218} = 15.8$ nautical miles.

(ii) For $x = 410$ ft.

Let $x_n = 400$, $y_n = 21.27$, $\nabla y_n = 1.37$, $\nabla^2 y_n = -0.11$, $\nabla^3 y_n = 0.02$, $\nabla^4 y_n = -0.01$ and so on

$$p = \frac{x - x_n}{h} = \frac{410 - 400}{50} = 0.2$$

By using Newton's backward ~~interpolation~~ extrapolation formula

$$y_{410} = y_{400} + p\nabla y_{400} + \frac{p(p+1)}{\lfloor 2}\nabla^2 y_{400} + \frac{p(p+1)(p+2)}{\lfloor 3}\nabla^3 y_{400} + \frac{p(p+1)(p+2)(p+3)}{\lfloor 4}\nabla^4 y_{400} + \cdots$$

$y_{410} = 21.7$ nautical miles.