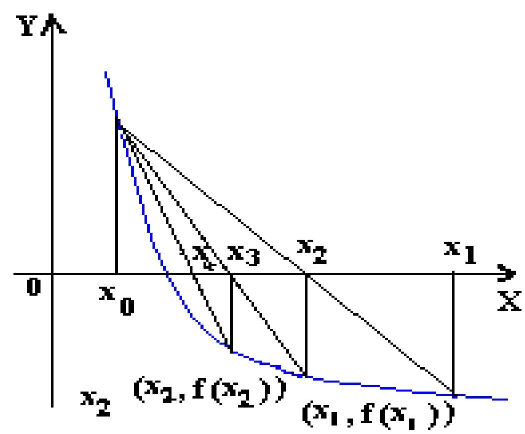
REGULA-FALSI METHOD

The converge process in the bisection method is very slow. It depends only on the choice of end points of the interval [a, b]. The function f(x) does not have any role in finding the point c (which is just the mid-point of a and b). It is used only to decide the next smaller interval [a, c] or [c, b]. A better approximation to c can be obtained by taking the straight line L joining the points (a, f(a)) and (b, f(b)) intersecting the x-axis. To obtain the value of c we can equate the two expressions of the slope m of the line L.

$$m = \frac{f(b) - f(a)}{(b-a)} = \frac{0 - f(b)}{(c-b)}$$

$$\Rightarrow (c-b) * (f(b)-f(a)) = -(b-a) * f(b)$$



$$c = b - \frac{f(b) * (b-a)}{f(b) - f(a)}$$

Now the next smaller interval which brackets the root can be obtained by checking

f(a) * f(b) < 0 then b = c
> 0 then a = c
= 0 then c is the root.

Selecting **c** by the above expression is called Regula-Falsi method or False position method.

Algorithm - False Position Scheme

Given a function f (x) continuos on an interval [a,b] such that f (a) * f (b) < 0
Do

$$c = \frac{a*f(b) - b*f(a)}{f(b) - f(a)}$$

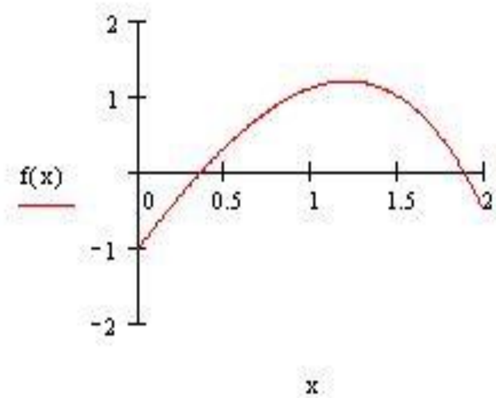
if f (a) * f (c) < 0 then b = c
else a = c
while (none of the convergence criterion C1, C2 or C3 is satisfied)

The false position method is again bound to converge because it brackets the root in the whole of its convergence process.

Numerical Example:

Find a root of 3x + sin(x) - exp(x) = 0.

The graph of this equation is given in the figure.



From this it's clear that there is a root between 0 and 0.5 and also another root between 1.5 and 2.0. Now let us consider the function f (x) in the interval [0, 0.5] where f (0) * f (0.5) is less than zero and use the regula-falsi scheme to obtain the zero of f (x) = 0.

Iteration No.	a	b	c	f(a) * f(c)
1	0	0.5	0.376	1.38 (+ve)
2	0.376	0.5	0.36	-0.102 (-ve)
3	0.376	0.36	**0.36**	-0.085 (-ve)

So one of the roots of 3x + sin(x) - exp(x) = 0 is approximately **0.36**. Note : Although the length of the interval is getting smaller in each iteration, it is possible that it may not go to zero. If the graph y = f(x) is concave near the root 's', one of the endpoints becomes fixed and the other end marches towards the root.