## LAGRANGE'S INTERPOLATION FORMULA

This is again an N<sup>th</sup> degree polynomial approximation formula to the function f(x), which is known at discrete points  $x_i$ ,  $i = 0, 1, 2 \dots N^{th}$ . The formula can be derived from the Vandermonds determinant but a much simpler way of deriving this is from Newton's divided difference formula. If f(x) is approximated with an N<sup>th</sup> degree polynomial then the N<sup>th</sup> divided difference of f(x) constant and  $(N+1)^{th}$  divided difference is zero. That is

 $f[x_0, x_1, \ldots x_n, x] = 0$ 

From the second property of divided difference we can write

 $\frac{f_0}{(x_0 - x_1) \dots (x_0 - x_n)(x_0 - x)} + \frac{f_n}{(x_n - x_0) \dots (x_n - x_{n-1})(x_n - x)} + \dots + \frac{f_x}{(x - x_0) \dots (x - x_n)} = 0$ or

$$f(x) = \frac{(x - x_1) \dots (x - x_n)}{(x_0 - x_1) \dots (x_0 - x_n)} f_0 + \dots + \frac{(x - x_0) \dots (x - x_{n-1})}{(x_n - x_0) \dots (x_n - x_{n-1})} f_n$$

$$\sum_{i=0}^{n} \left(\begin{array}{cc} \frac{n}{||} & \underline{x} \cdot \underline{x}_{j} \\ j = 0 & (x_{i} \cdot x_{j}) \end{array}\right)_{f_{i}}$$

Since Lagrange's interpolation is also an  $N^{th}$  degree polynomial approximation to f(x) and the  $N^{th}$  degree polynomial passing through (N+1) points is unique hence the Lagrange's and Newton's divided difference approximations are one and the same. However, Lagrange's formula is more convinent to use in computer programming and Newton's divided difference formula is more suited for hand calculations.

**Example** : Compute **f**(**0.3**) for the data

using Lagrange's interpolation formula (Analytic value is 1.831)

$$f(x) = \frac{(x - x_1) (x - x_2)(x - x_3)(x - x_4)}{(x_0 - x_1) (x_0 - x_2)(x_0 - x_3)(x_0 - x_4)} f_0 + \ldots + \frac{(x - x_0)(x - x_1) (x - x_2)(x - x_3)}{(x_4 - x_0)(x_4 - x_1)(x_4 - x_2)(x_4 - x_3)} f_4$$

$$= \frac{(0.3 - 1)(0.3 - 3)(0.3 - 4)(0.3 - 7)}{(-1)(-3)(-4)(-7)} 1 + \frac{(0.3 - 0)(0.3 - 3)(0.3 - 4)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 4)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 4)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 4)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 4)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 4)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 4)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 4)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 4)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 4)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 4)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 4)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{(0.3 - 1)(0.3 - 7)}{1 \times (-2)(-6)} 3 + \frac{($$

(0.3 - 0)(0.3 - 1)(0.3 - 4)(0.3 - 7)	<b>49</b> +	(0.3 - 0)(0.3 - 1)(0.3 - 3)(0.3 - 7)	129 +
3 x 2 x (-1)(-4)	• •	4 x 3 x 1 (-3)	
		(0.3 - 0)(0.3 - 1)(0.3 - 3)(0.3 - 4)	)

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7 x 6 x 4 x 3

= 1.831