# Amplitude Modulation

**Ref:** Modern Digital and Analog Communication Systems, B. P. Lathi, 3rd Edition

**Modulation:** Modulation is a process that causes a shift in the range of frequencies in a signal.

➢**Baseband Communication:** In baseband communication, baseband signals are transmitted without modulation, that is, without any shift in the range of frequencies of the signal.

➢**Carrier Communication:** Communication that uses modulation to shift the frequency spectrum of a signal is known as carrier communication.

•In this mode, one of the basic parameters (amplitude, frequency, or phase) of a **sinusoidal carrier** of high frequency $\omega_c$ is varied in proportion to the baseband signal m(t). This results in:

⇨ Amplitude Modulation (AM)
⇨ Frequency Modulation (FM)
⇨ Phase Modulation (PM)

} Angle Modulation (FM, PM)

•FM and PM are similar types of modulation and belong to the class of modulation known as **angle modulation**.

## Amplitude Modulation: Double Sideband (DSB)

➢In amplitude modulation, the amplitude of the **carrier** is varied in proportion to the baseband (message) or **modulating signal**. The frequency and the phase are constant.

Let, the carrier signal: $A \cos(\omega_c t + \theta_c)$

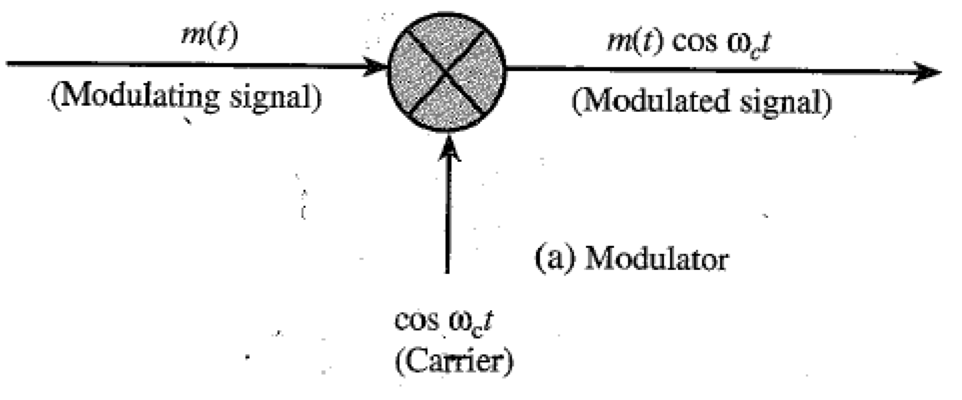
If the carrier amplitude A is made directly proportional to the modulating signal m(t), the **modulated signal** is $m(t) \cos \omega_c t$ [assuming $\theta_c = 0$].

➢This type of modulation simply shifts the spectrum of m(t) to the carrier frequency.

Thus if $m(t) \Longleftrightarrow M(\omega)$

then $m(t) \cos \omega_c t \Longleftrightarrow \frac{1}{2}[M(\omega + \omega_c) + M(\omega - \omega_c)]$

$M(\omega - \omega_c)$ is *M(ω)* shifted to the right by $\omega_c$ and $M(\omega + \omega_c)$ is *M(ω)* shifted to the left by $\omega_c$.



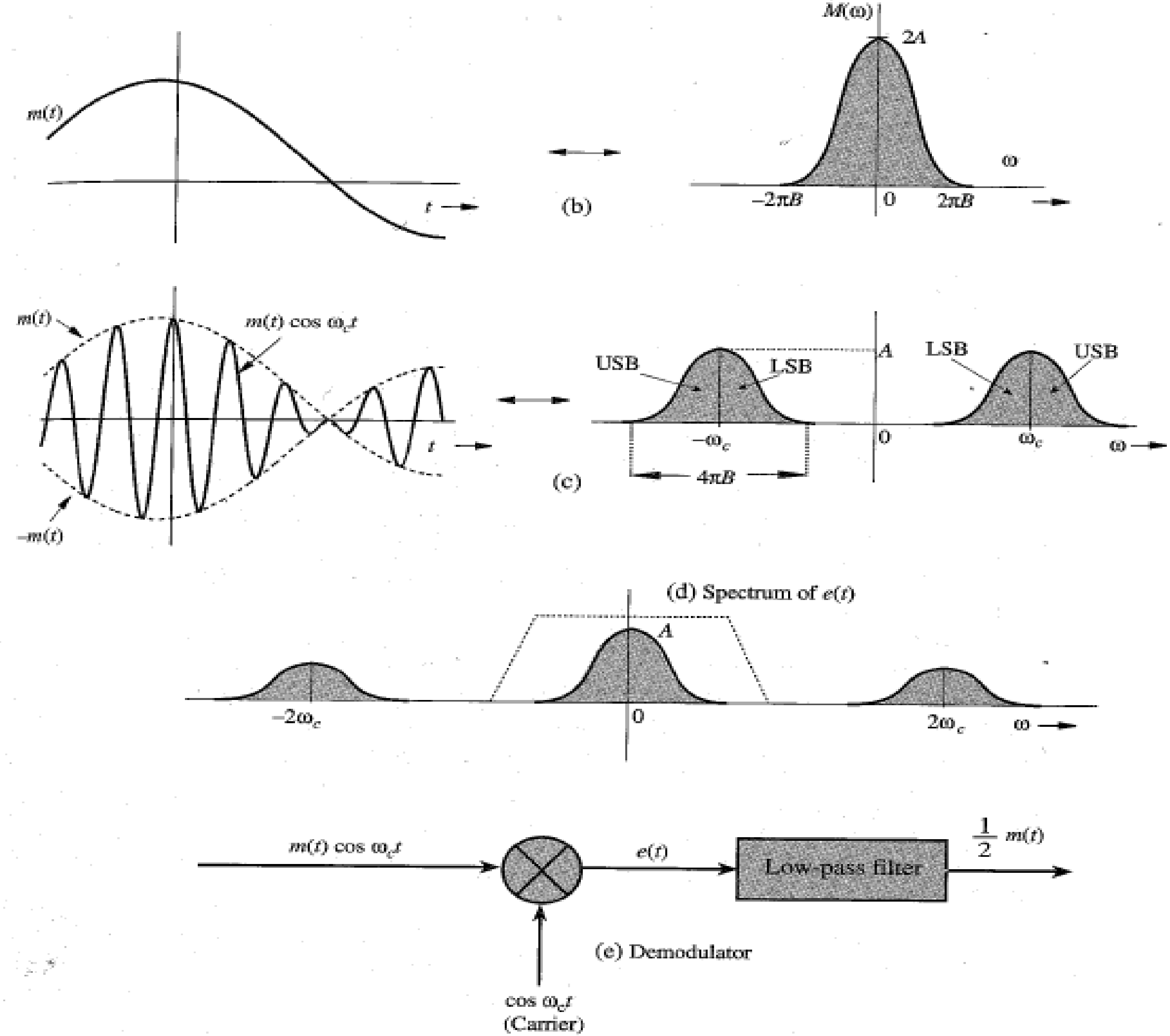
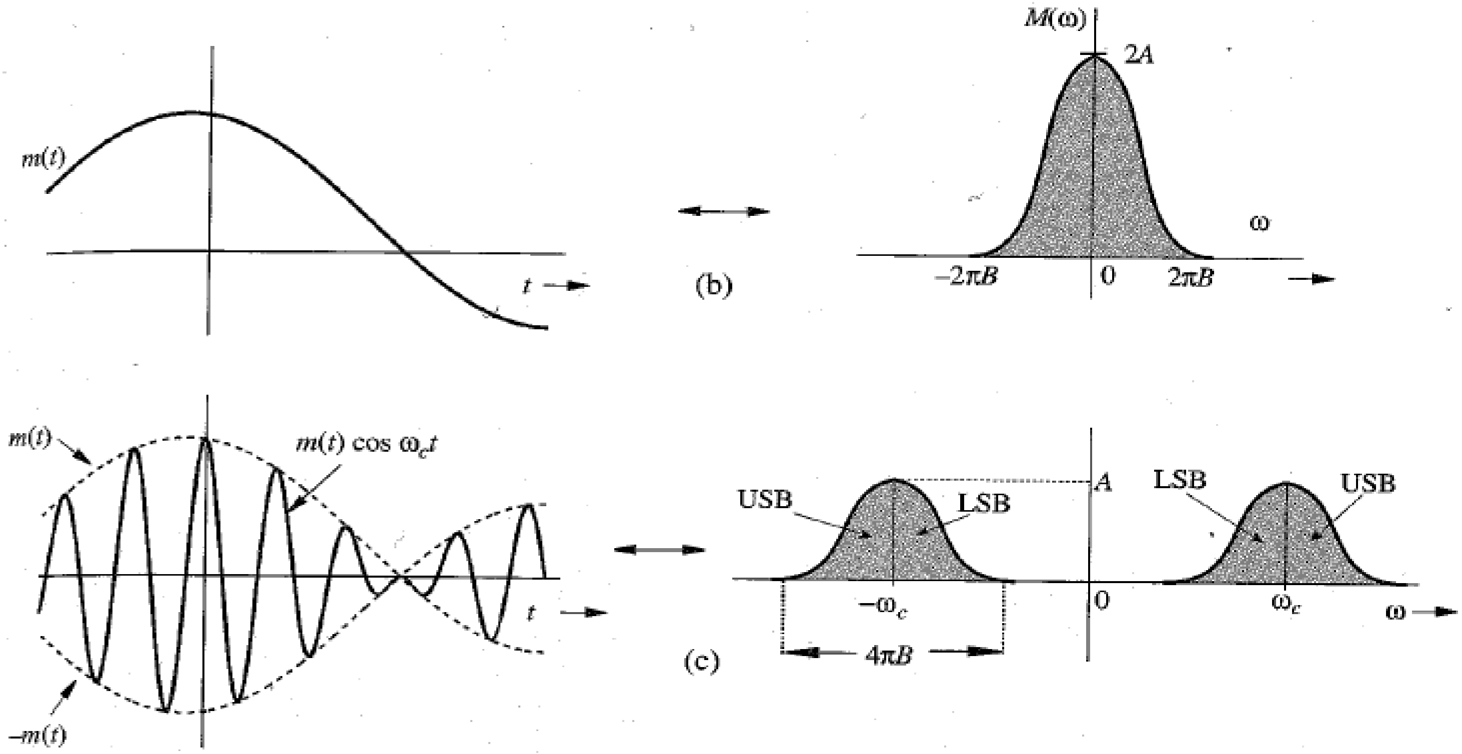



**Figure 4.1** DSB-SC modulation and demodulation.




➢If the bandwidth of m(t) is B Hz, then the bandwidth of the modulated signal is 2B Hz.

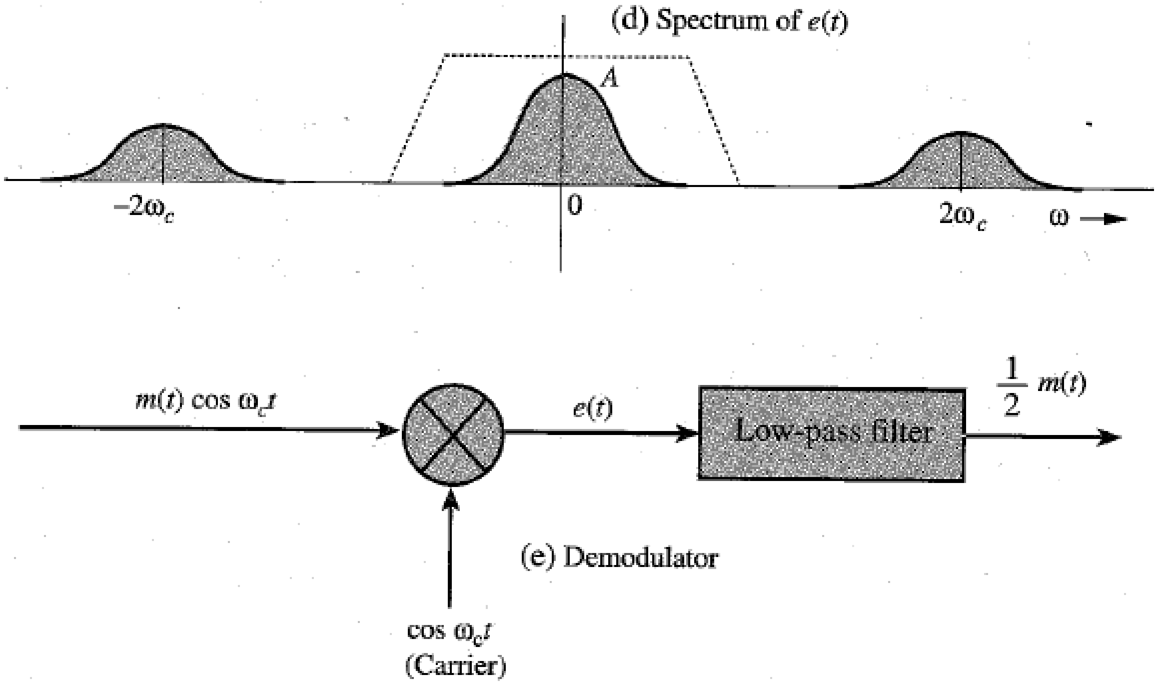
➢The modulated signal spectrum is composed of two parts: **Upper sideband (USB)** and **Lower sideband (LSB)**.

➢The modulated signal in this scheme does not contain a discrete component of the carrier frequency $\omega_c$ . For this reason, it is called **double side-band suppressed carrier (DSB-SC) modulation**.

**Demodulation:**

➢The process of recovering the signal from the modulated signal is referred to as **demodulation**, or **detection**.

➢Demodulation consists of multiplication of the incoming modulated signal $m(t)\cos\omega_c t$ by a carrier $\cos\omega_c t$ followed by a low pass filter.



In time domain,

$$e(t) = m(t)\cos^2 \omega_c t$$
$$= \frac{1}{2}[m(t) + m(t)\cos 2\omega_c t]$$

Therefore, the Fourier transform of the signal e(t) is

$$E(\omega) = \frac{1}{2}M(\omega) + \underbrace{\frac{1}{4}[M(\omega + 2\omega_c) + M(\omega - 2\omega_c)]}_{\text{suppressed by low pass filter}}$$

➢This method of recovering the baseband signal is called **synchronous detection** or **coherent detection** where we need to generate a local carrier at the receiver in frequency and phase coherence (synchronism) with the carrier used at the modulator.

**Example 4.1:** For a baseband signal $m(t) = \cos\omega_m t$ , find the DSB-SC signal. And sketch its spectrum. Identify the USB and LSB. Verify that the DSB-SC modulated signal can be demodulated by the synchronous demodulator.

**Solution:** This is a case of tone modulation because the modulating signal is a pure sinusoid, or tone, $\cos\omega_m t$.

The spectrum of the baseband signal $m(t) = \cos\omega_m t$ is given by

$$M(\omega) = \pi[\delta(\omega - \omega_m) + \delta(\omega + \omega_m)]$$

In time domain, for the baseband signal $m(t) = \cos\omega_m t$ , the DSB-SC signal $\varphi_{DSB-SC}(t)$ is

$$\begin{aligned}\varphi_{DSB-SC}(t) &= \mathrm{m(t)}\cos\omega_c t \\ &= \cos\omega_m t\cos\omega_c t \\ &= \frac{1}{2}[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t]\end{aligned}$$

For synchronous demodulation, $\varphi_{DSB-SC}(t)$ is multiplied by a carrier $\cos\omega_c t$ to get e(t) followed by a low pass filter.

Here, $e(t) = \cos \omega_m t \cos \omega_c t \cos \omega_c t$

$$= \cos \omega_m t \, cos^2 \omega_c t$$

$$= \frac{1}{2} \cos \omega_m t \; (1 + \cos 2\omega_c \mathrm{t})$$

$$= \frac{1}{2} \cos \omega_m t + \frac{1}{2} \cos \omega_m t \cos 2\omega_c \mathrm{t}$$

The spectrum of the term $\cos \omega_m t \cos 2\omega_c \mathrm{t}$ is centered at $2\omega_c$, and will be suppressed by the low-pass filter, yielding $\frac{1}{2} \cos \omega_m t$ as the output.

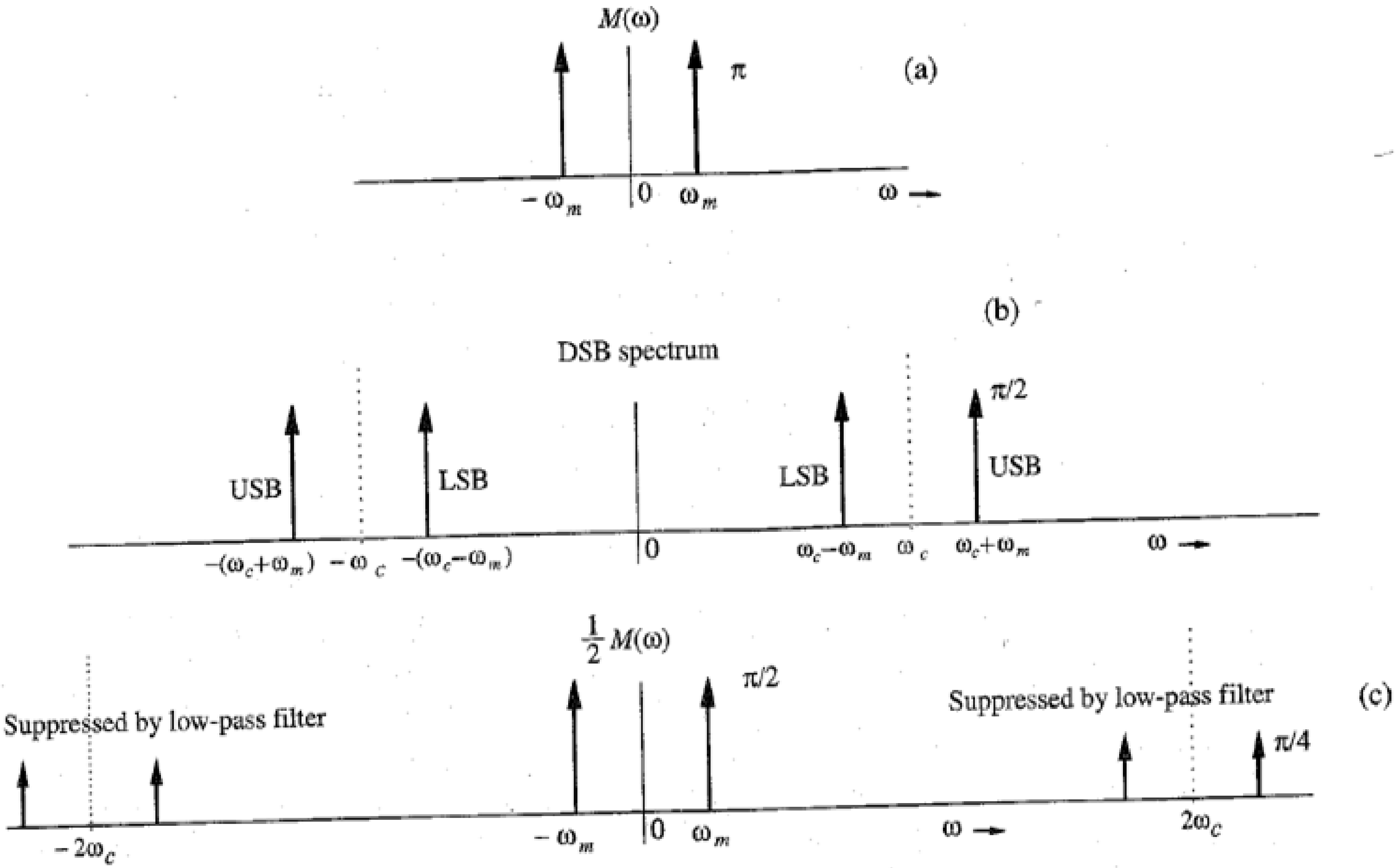


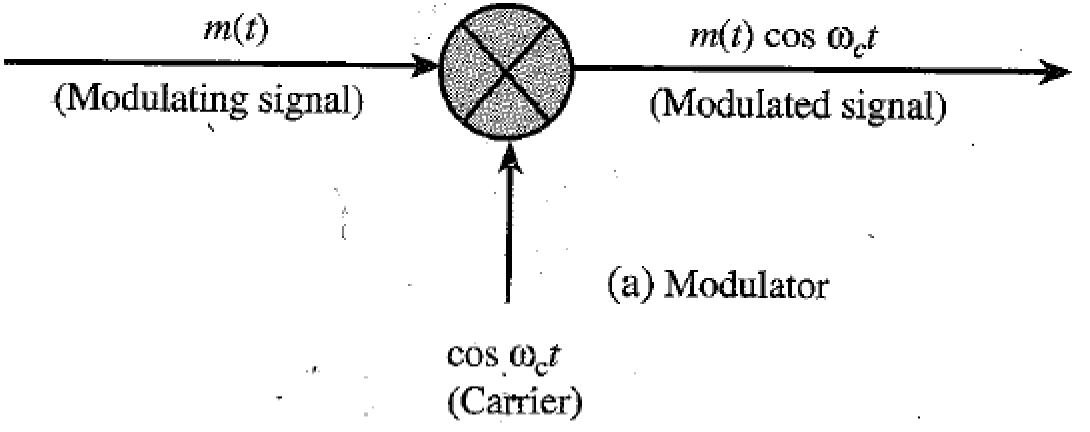

**Figure 4.2** Example of DSB-SC modulation.

**Modulators:**

**Multiplier Modulators:**

➢Here modulation is achieved directly by multiplying m(t) by $\cos \omega_c t$ using an analog multiplier whose output is proportional to the product of two input signals.

➢It is rather difficult to maintain linearity in this kind of amplifier, and they tend to be rather expensive.




(a) Modulator

## Nonlinear Modulators:

➢Modulation can be achieved by using non linear devices, such as a semiconductor diode or a transistor.

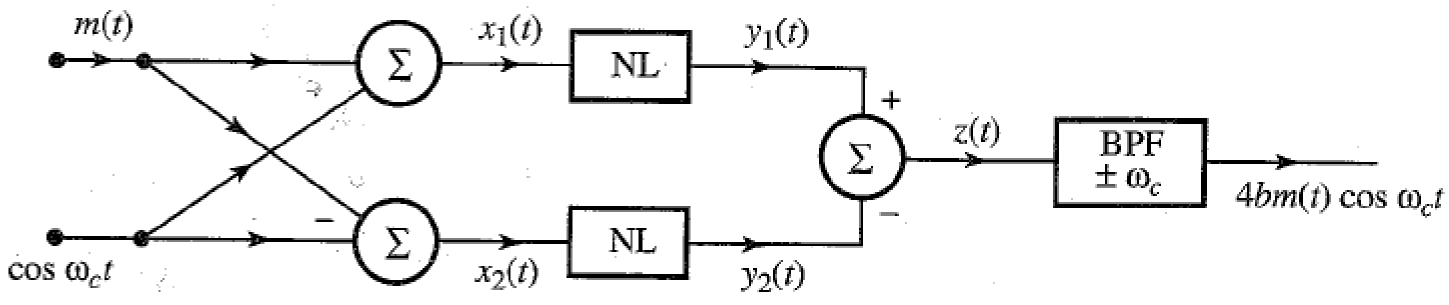



**Figure 4.3** Nonlinear DSB-SC modulator.

➢Here, NL= Nonlinear Elements.

Let the input-output characteristics of either of the nonlinear elements be approximated by a power series: $y(t) = ax(t) + bx^2(t)$

Where x(t) and y(t) are the input and output, respectively, of the nonlinear element. The summer output z(t) is given by

$$z(t) = y_1(t) - y_2(t)$$

$$= [ax_1(t) + b{x_1}^2(t)] - [ax_2(t) + b{x_2}^2(t)]$$

Here, $x_1(t) = \cos\omega_c t + m(t)$
and $x_2(t) = \cos\omega_c t - m(t)$

Substituting these two input values into the equation of z(t) yields

$$z(t) = 2am(t) + 4bm(t)\ \cos\omega_c t$$

➢When z(t) is passed through a bandpass filter tuned to $\omega_c$ , the signal *am(t)* is suppressed and the desired modulated signal $4bm(t)\cos\omega_c t$ passes through unharmed.

➢In this circuit, the carrier signal does not appear at the input of the final bandpass filter. For this reason, it is called a **single balanced modulator**.

## Switching Modulators:

➢The multiplication operation required for modulation can be replaced by a simpler switching operation if we realize that a modulated signal can be obtained by multiplying m(t) not only by a pure sinusoid but by any periodic signal ϕ(t) of the fundamental radian frequency $\omega_c$ .

➢Such a periodic signal can be expressed by a trigonometric Fourier series as

$$\phi(t) = \sum_{n=0}^{\infty} C_n \cos(n\omega_c t + \theta_n)$$

Hence, $$m(t)\phi(t) = \sum_{n=0}^{\infty} C_n\, m(t) \cos(n\omega_c t + \theta_n)$$

This shows that the spectrum of the product m(t)ϕ(t) is the spectrum M(ω) shifted to $\pm\omega_c,\ \pm 2\omega_c,\ \ldots,\ \pm n\omega_c, \ldots$. If this signal is passed through a bandpass filter of bandwidth 2B Hz and tuned to $\omega_c$, then we get the desired modulated signal $c_1 m(t) \cos(\omega_c t + \theta_1)$.

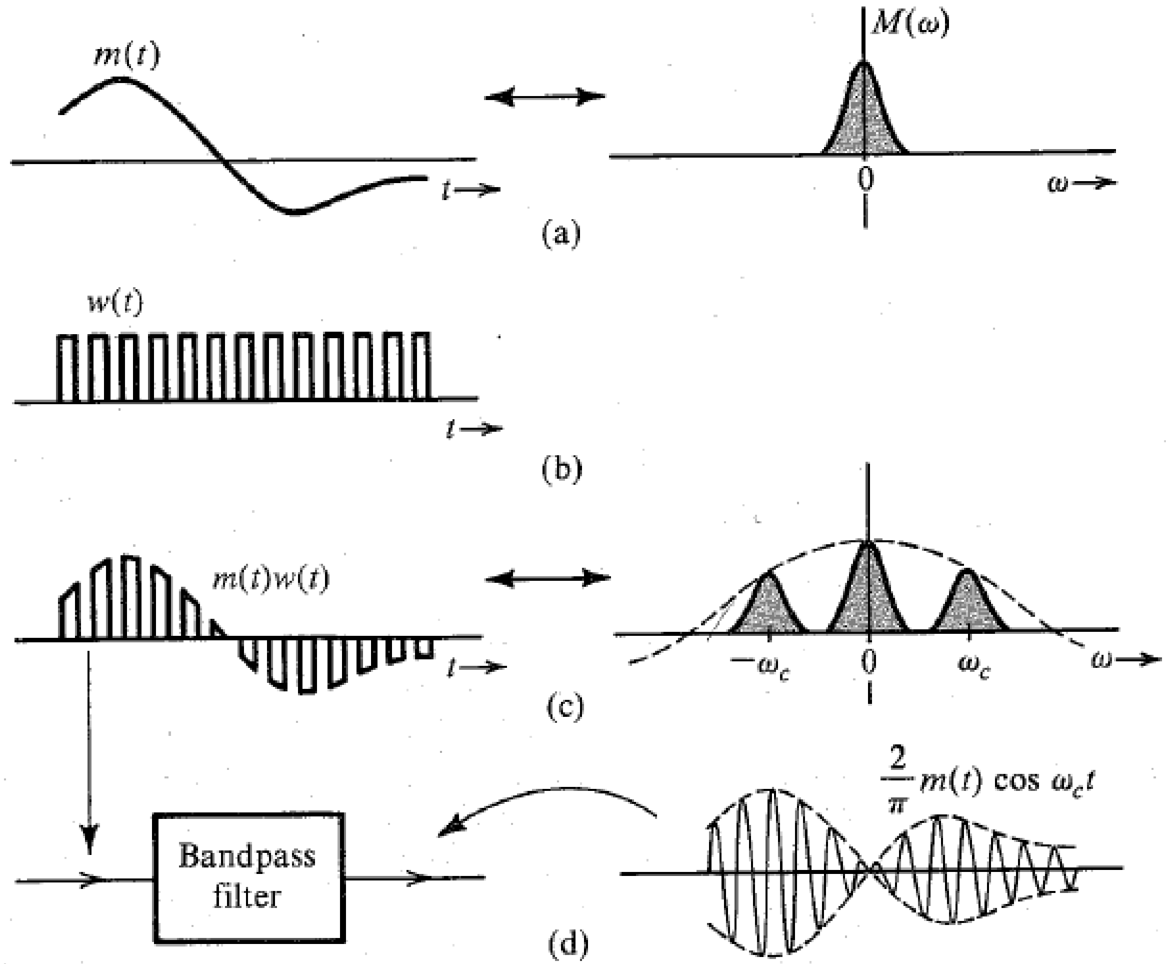



**Figure 4.4** Switching modulator for DSB-SC.

The square pulse train w(t) in Fig. 4.4(b) is a periodic signal whose Fourier series can be expressed as

$$w(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos \omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right)$$

The signal m(t)w(t) is given by

$$m(t)w(t) = \frac{1}{2}m(t) + \frac{2}{\pi}\left[m(t)\cos \omega_c t - \frac{1}{3}m(t)\cos 3\omega_c t + \frac{1}{5}m(t)\cos 5\omega_c t - \cdots\right]$$

➢Multiplication of a signal by a square pulse train is in reality a switching operation.

➢Electronic switch like **Diode-Bridge Modulator** can be used to accomplish the switching operation.

➢The switching action is controlled by $A \cos \omega_c t$ .

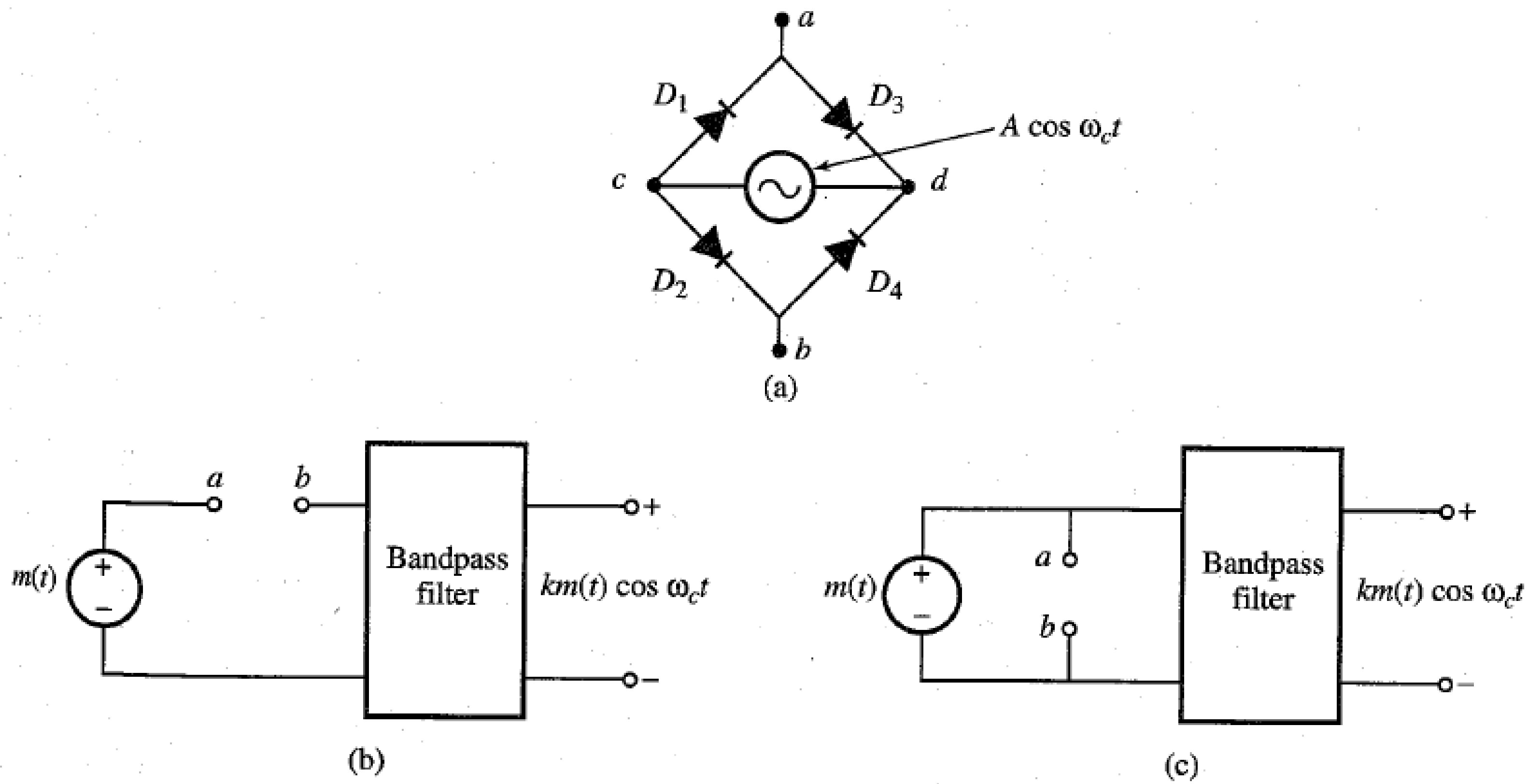



**Figure 4.5** (a) Diode-bridge electronic switch. (b) Series-bridge diode modulator. (c) Shunt-bridge diode modulator.

- Diodes $D_1, D_2$ *and* $D_3, D_4$ are matched pairs.
- To obtain the signal m(t)w(t), the electronic switch can be placed in series or across (in parallel) m(t).
- These modulators are known as the series- bridge diode modulator and the shunt-bridge diode modulator, respectively.

**Amplitude Modulation:**

➢In DSBSC, a receiver must generate a carrier in frequency and phase synchronism with the carrier at the transmitter. This calls for a sophisticated receiver and could be quite costly.

➢In AM, the transmitter transmit a carrier $A \cos \omega_c t$ [along with the modulated signal $m(t) \cos \omega_c t$ ] so that there is no need to generate a carrier at the receiver.

➢In this case, the transmitter needs to transmit much larger power, which makes it rather expensive.

In AM (amplitude modulation), the transmitted signal $\phi_{AM}(t)$ is given by

$$\begin{aligned}\varphi_{\mathrm{AM}}(t) &= A\cos\omega_c t + m(t)\cos\omega_c t \\ &= [A + m(t)]\cos\omega_c t\end{aligned}$$

The spectrum of $\phi_{AM}(t)$ is the same as that of $m(t)\cos\omega_c t$ plus two additional impulses at $\pm\omega_c$.

$$\varphi_{\mathrm{AM}}(t) \iff \frac{1}{2}[M(\omega+\omega_c) + M(\omega-\omega_c)] + \pi A[\delta(\omega+\omega_c) + \delta(\omega-\omega_c)]$$

- Envelope of the modulated wave: [A+ m(t)]

- Case 1: A is large enough so that A+ m(t) ≥ 0 (is non-negative) for all values of t. In this case, A+ m(t) is the envelope of $\phi_{AM}(t)$. Envelope detection is possible in this case.

- Case 2: A is not large enough so that $A + m(t) \ngtr 0$ for all t. In this case, the envelope is not A+ m(t), but rectified A+ m(t). m(t) cannot be recovered from the envelope.

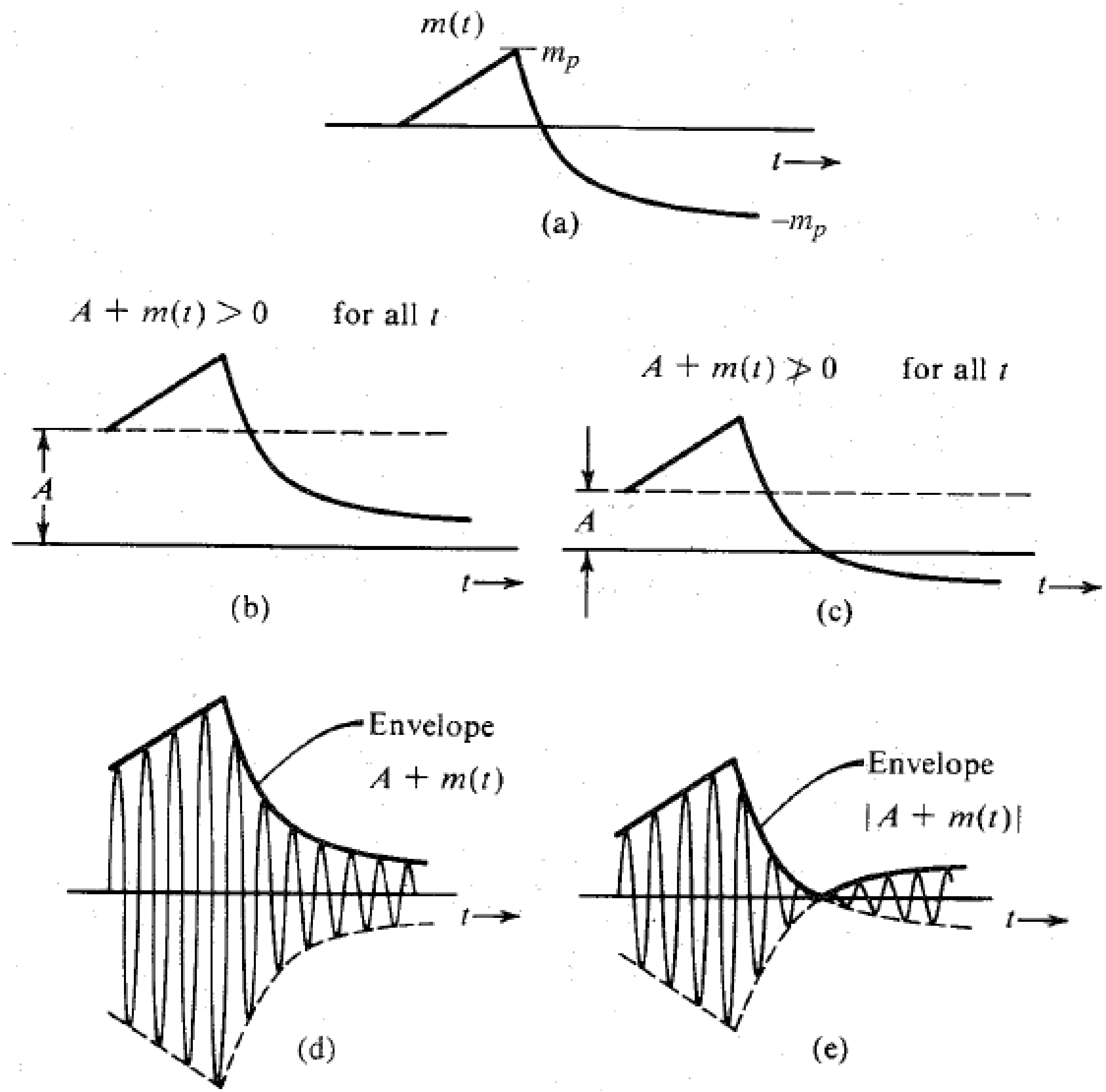



**Figure 4.8** AM signal and its envelope.

Condition for envelope detection of AM signal:

$$A + m(t) \geq 0 \qquad \text{for all } t$$

➢Let, $m_p$ be the peak amplitude (positive or negative) of m(t).
This means that $m(t) \geq -m_p$.
So, the condition is equivalent to $A \geq m_p$ .

**Modulation Index:** Modulation index μ is defined as

$$\mu = \frac{m_p}{A}$$

where A is the carrier amplitude.

➢Because $A \geq m_p$ and because there is no upper bound on A, it follows that the required condition for envelope detection is

$$0 \leq \mu \leq 1$$

➢When $A < m_p$ , $\mu > 1$ (over-modulation). In this case, envelope detection can no longer detect m(t) successfully. We need to use synchronous demodulation.

**Example 4.4:**

Sketch $\varphi_{AM}(t)$ for modulation indices of $\mu = 0.5$ and $\mu = 1$, when $m(t) = B\cos\omega_m t$. This case is referred to as **tone modulation** because the modulating signal is a pure sinusoid (or tone).

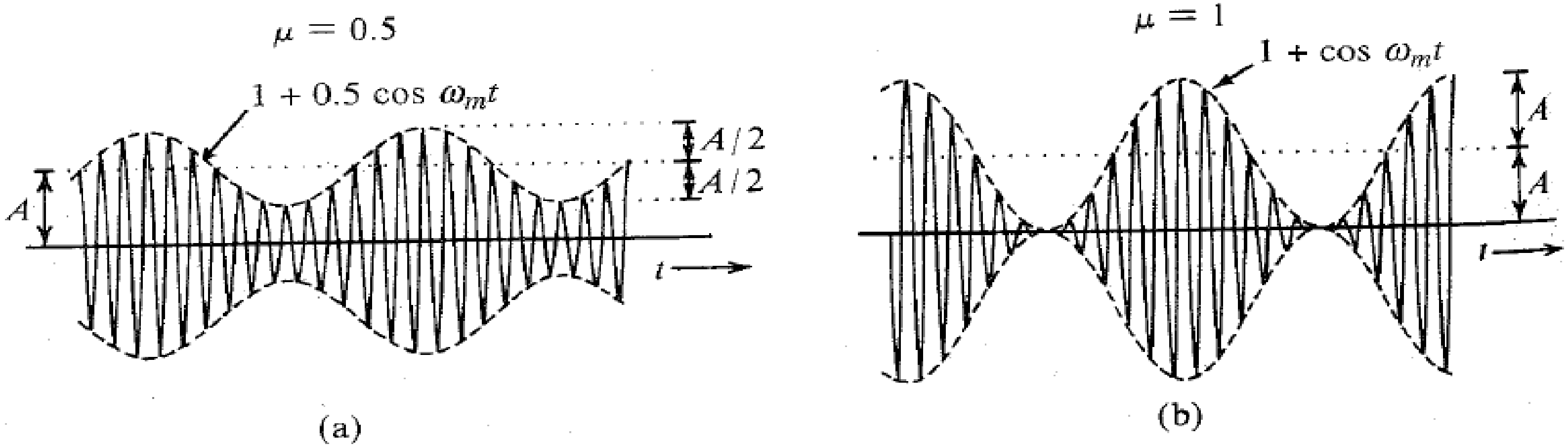



**Figure 4.9** Tone-modulated AM. (a) $\mu = 0.5$. (b) $\mu = 1$.

In this case, $m_p = B$ and the modulation index

$$\mu = \frac{B}{A}$$

Hence, $B = \mu A$ and $\quad m(t) = B\cos\omega_m t = \mu A\cos\omega_m t$

Therefore, $\quad \varphi_{AM}(t) = [A + m(t)]\cos\omega_c t = A[1 + \mu\cos\omega_m t]\cos\omega_c t$

Figure 4.9 shows the modulated signals corresponding to $\mu = 0.5$ and $\mu = 1$, respectively.

## Sideband and Carrier Power:

In AM, the carrier term does not carry any information, and hence, the carrier power is wasted.

$$\varphi_{\text{AM}}(t) = \underbrace{A\cos\omega_c t}_{\text{carrier}} + \underbrace{m(t)\cos\omega_c t}_{\text{sidebands}}$$

The carrier power $P_c$ and sideband power $P_s$ are given by:

$$P_c = \frac{A^2}{2} \qquad \text{and} \qquad P_s = \frac{1}{2}\widetilde{m^2(t)}$$

➢The total power is the sum of the carrier (wasted) power and the sideband (useful) power.

Hence, the power efficiency,

$$\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_s}{P_c + P_s} = \frac{\widetilde{m^2(t)}}{A^2 + \widetilde{m^2(t)}} 100\%$$

For the special case of tone modulation,

$$m(t) = \mu A\cos\omega_m t \qquad \text{and} \qquad \widetilde{m^2(t)} = \frac{(\mu A)^2}{2}$$

Hence, $$\eta = \frac{\mu^2}{2+\mu^2} 100\%$$

with the condition that $0 \leq \mu \leq 1$ .It can be seen that $\eta$ increases monotonically with $\mu$ , and $\eta_{\max}$ occurs at $\mu = 1$ , for which $\eta_{\max} = 33\%$ .

- Thus, for tone modulation, under best conditions $(\mu = 1)$ , only one-third of the transmitted power is used for carrying message.

- For practical signals, the efficiency is even worse- on the order of 25% or lower- compared to DSB-SC case.

- Smaller values of $\mu$ degrade efficiency further.

## Example 4.5:

Determine $\eta$ and the percentage of the total power carried by the sidebands of the AM wave for tone modulation when **(a)** $\mu = 0.5$ and **(b)** $\mu = 0.3$.

For $\mu = 0.5$,

$$\eta = \frac{\mu^2}{2 + \mu^2} 100\% = \frac{(0.5)^2}{2 + (0.5)^2} 100\% = 11.11\%$$

Hence, only about 11% of the total power is in the sidebands. For $\mu = 0.3$,

$$\eta = \frac{(0.3)^2}{2 + (0.3)^2} 100\% = 4.3\%$$

Hence, only 4.3% of the total power is the useful power (power in sidebands).

## Generation of AM Signals:

### Switching Modulator:

➢In switching modulator, the switching action is provided by a single diode.

➢The input is $c\cos\omega_c t + m(t)$, with $c \gg m(t)$, so that the switching action of the diode is controlled by $c\cos\omega_c t$ .

➢The diode opens and shorts periodically with $\cos\omega_c t$ , in effect multiplying the input signal $[c\cos\omega_c t + m(t)]$ by w(t).

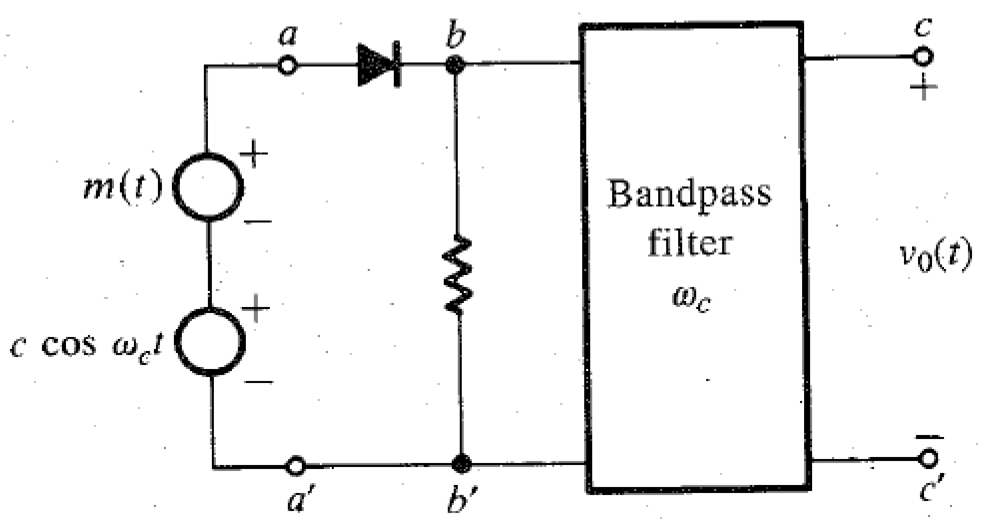




**Figure 4.10** AM generator.

The voltage across terminals $bb'$ is

$$v_{bb'}(t) = [c\cos\omega_c t + m(t)]w(t)$$

$$= [c\cos\omega_c t + m(t)]\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right)\right]$$

$$= \underbrace{\frac{c}{2}\cos\omega_c t + \frac{2}{\pi}m(t)\cos\omega_c t}_{\text{AM}} + \underbrace{\text{other terms}}_{\substack{\text{suppressed by}\\ \text{bandpass filter}}}$$

The bandpass filter tuned to $\omega_c$ suppresses all the other terms, yielding the desired AM signal at the output.

## Demodulation of AM Signals:

- Can be demodulated coherently/synchronously by a locally generated carrier.
- Two non-coherent methods of AM demodulation: 1) Rectifier Detection
  2) Envelope Detection

**Rectifier Detection:**

- If an AM signal is applied to a diode and a resistor circuit, the negative part of the AM wave will be suppressed.
- The output across the resistor is a half-wave rectified version of the AM signal.
- In essence, the AM signal is multiplied by w(t).

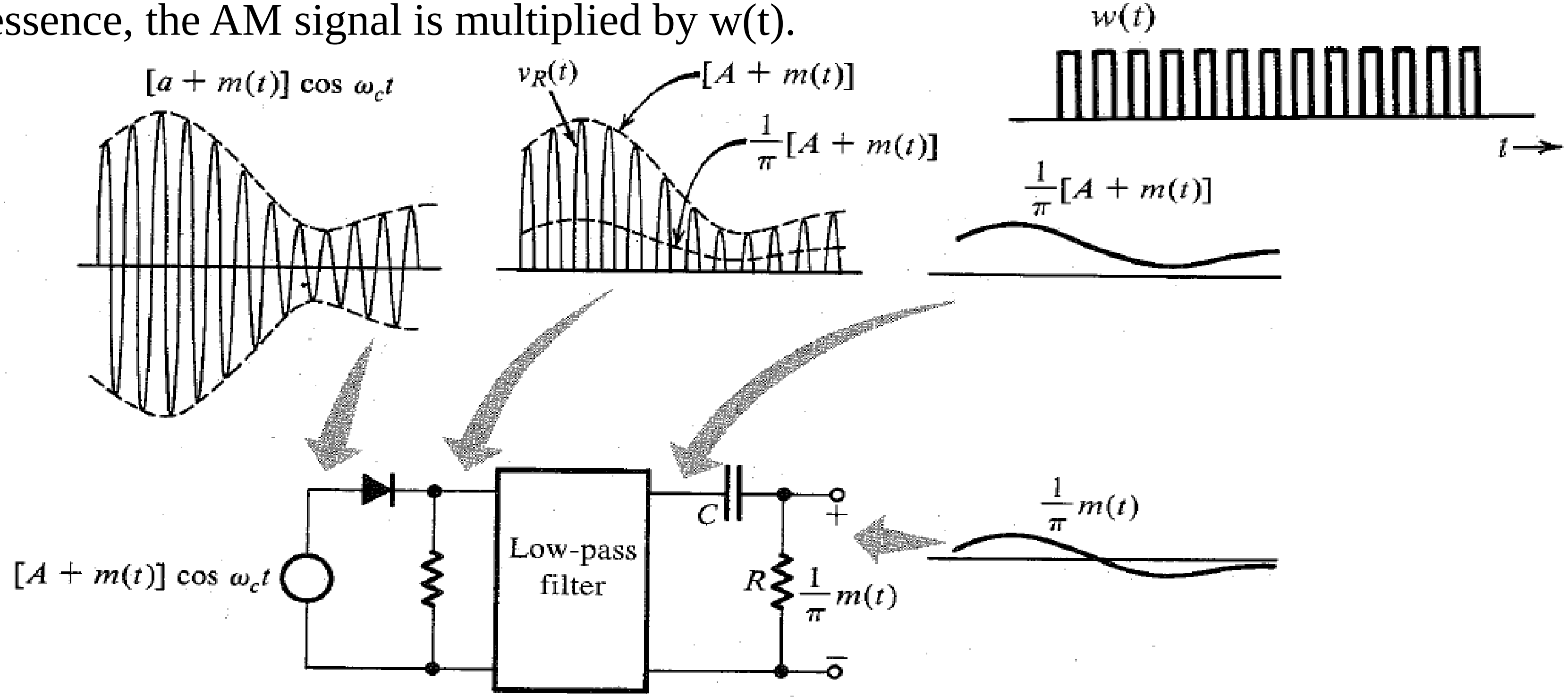



Figure 4.11 Rectifier detector for AM.

$$v_R = \{[A + m(t)]\cos\omega_c t\}\, w(t)$$

$$= [A + m(t)]\cos\omega_c t\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right)\right]$$

$$= \frac{1}{\pi}[A + m(t)] + \text{other terms of higher frequencies}$$

- When $v_R$ is applied to a low-pass filter of cutoff B Hz, the output is $[A + m(t)]/\pi$ , and all the other terms in $v_R$ of frequencies higher than B Hz are suppressed.

- The dc term $A/\pi$ may be blocked by a capacitor to give the desired output $m(t)/\pi$.

## Envelope Detector:

➢In an envelope detector, the output of the detector follows the envelope of the modulated signal.

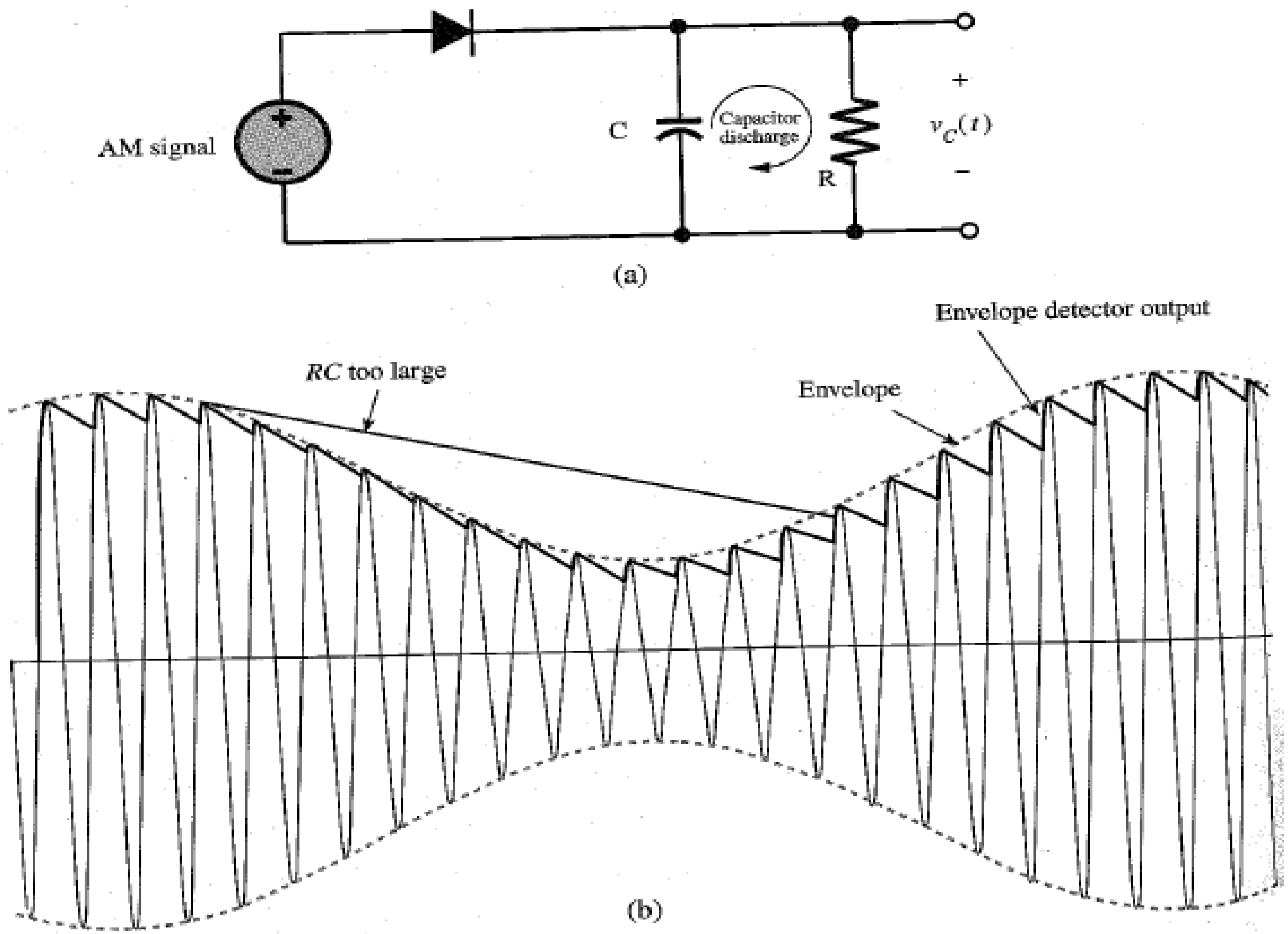



**Figure 4.12** Envelope detector for AM.

- During each positive cycle, the capacitor charges up to the peak voltage of the input signal and then decays slowly until the next positive cycle.
- The output voltage $v_C(t)$ closely follows the envelope of the input.

- The condition for reducing the ripple between positive peaks:

$$RC \gg \frac{1}{\omega_c}$$

- The condition for the capacitor voltage to follow the envelope:

$$RC \ll \frac{1}{2\pi B}$$

Where B is the highest frequency in m(t).

- The envelope detector output is $v_C(t) = A + m(t)$ with a ripple of frequency $\omega_c$ .
- The dc term A can be blocked out by a capacitor or a simple RC high-pass filter. The ripple may be reduced further by another RC low pass filter.