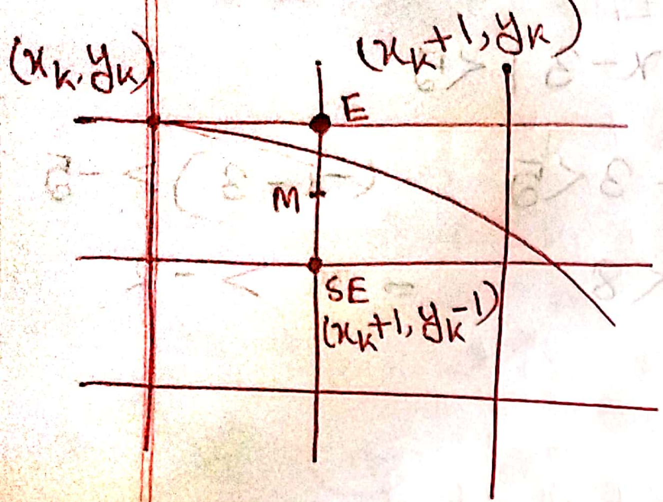
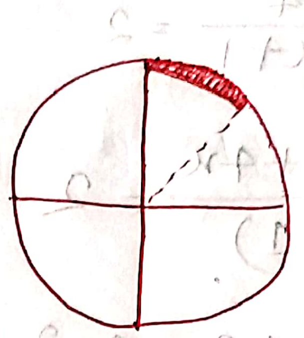
Mid Point Circle Algorithm Derivation





- M can be inside boundary
- outside boundary.
- on the boundary.

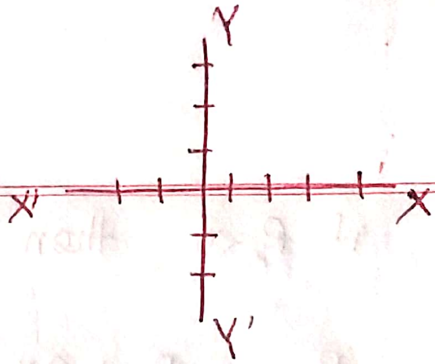
We draw circle depending on MID point (M). That's why we need to know, what are all the possible locations for the MID point.

We know, $x^2 + y^2 \cancel{= R} - r^2 = 0$

We define a circle function:

$$f(x,y) = x^2 + y^2 - r^2 \begin{cases} = 0 & \text{point lies on the circle boundary.} \\ < 0 & \text{lies inside the circle boundary} \\ > 0 & \text{lies outside the circle boundary} \end{cases}$$

We choose the pixel either E or SE depending on the distance of each pixel from the circle boundary.



Mid point = $\left\{ \dfrac{(x_k+1+x_k+1)}{2}, \dfrac{(y_k+y_k-1)}{2} \right\}$

$= (x_k+1), (y_k - \frac{1}{2})$.

For initial position,

$\therefore P_k = (x_k+1)^2 + (y_k - \frac{1}{2})^2 - r^2$

$\therefore P_{k+1} = (x_{k+1}+1)^2 + (y_{k+1} - \frac{1}{2})^2 - r^2$.

Now,

$P_{k+1} - P_k = -(x_k+1)^2 - (y_k - \frac{1}{2})^2 + (x_{k+1}+1)^2 + (y_{k+1} - \frac{1}{2})^2$

$= (x_k+2)^2 - (x_k+1)^2 + (y_{k+1} - \frac{1}{2})^2 - (y_k - \frac{1}{2})^2$

$= x_k^2 + 4x_k + 4 - (x_k^2 + 2x_k + 1) + y_{k+1}^2 - 2 \cdot \frac{1}{2} \cdot y_{k+1}$

$+ \frac{1}{4} - (y_k^2 - 2 \cdot y_k \cdot \frac{1}{2} + \frac{1}{4})$

$= \cancel{x_k^2} + 4x_k + 4 - \cancel{x_k^2} - 2x_k - 1 + y_{k+1}^2 - y_{k+1}$

$+ \cancel{\frac{1}{4}} - y_k^2 + y_k - \cancel{\frac{1}{4}}$

$P_{k+1} = P_k + 2x_k + 3 + y_{k+1}^2 - y_{k+1} - y_k^2 + y_k$ —— ①

if $P_k < 0$ then $y_{k+1} = y_k$.

$$P_{k+1} = P_k + 2x_k + 3 + y_k^2 - y_k - y_k^2 + y_k$$

$$= P_k + 2x_k + 3.$$

if $P_k \geq 0$ then $y_{k+1} = y_k - 1$.

$$P_{k+1} = P_k + 2x_k + 3 + (y_k - 1)^2 - (y_k - 1) - y_k^2 + y_k$$

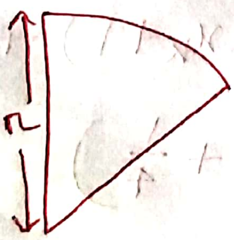
$$= P_k + 2x_k + 3 + y_k^2 - 2y_k + 1 - y_k + 1 - y_k^2 + y_k$$

$$= P_k + 2x_k + 5 - 2y_k.$$

Since the value of P_{k+1} is dependent on the value of P_k

o we need to initialize decision parameter P_k.

$(0, r)$ → initial point



$x_k = 0, \; y_k = r$.

Now, $P_0 = (0+1)^2 + (r - \frac{1}{2})^2 - r^2$

$$= 1 + r^2 - r - \frac{1}{4} - r^2$$

$$= 1 - r + \frac{1}{4}$$

$$= 5/4 - r$$

$P_0 = 5/4 - r$

if r = float.
$P_0 = 5/4 - r$.
if r = int.
$P_0 = 1 - r$.