CE 414
Prestressed Concrete

Lesson 6
Design of Section for Flexure

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REFERENCE

6

DESIGN OF SECTIONS FOR FLEXURE

<u>Book Chapter</u>

- Prestressed Concrete- by T.Y. Lin

PRELIMINARY DESIGN

Preliminary Design

Preliminary design of prestressed-concrete sections for flexure can be performed by a very simple procedure, based on a knowledge of the internal C-T couple acting in the section. In practice the depth h of the section is either given, known, or assumed, as is the total moment M_T on the section. Under the working load, the lever arm for the internal couple could vary between 30 to 80% of the overall height h and averages about $0.65h$. Hence the required effective prestress F can be computed from the equation

$$F = T = \frac{M_T}{0.65h} \tag{6-1}$$

if we assume the lever arm to be $0.65h$, Fig. 6-1. If the effective unit prestress is f_s for the steel, then the area of steel required is

$$A_{ps} = \frac{F}{f_{se}} = \frac{M_T}{0.65hf_{se}} \tag{6-2}$$

PRELIMINARY DESIGN

The total prestress $A_{ps}f_{se}$ is also the force C on the section. This force will produce an average unit stress on the concrete of

$$\frac{C}{A_c} = \frac{T}{A_c} = \frac{A_{ps}f_{se}}{A_c}$$

The top fiber stress, f_c, under working loads following ACI Code is $0.45f'_c$, Fig. 6-1. Table 1-2, Chapter 1, summarizes permissible stresses in steel and concrete for prestressed concrete members. For preliminary design, the average stress can be assumed to be about 50% of the maximum allowable stress f_c, under the working load. Hence,

$$\frac{A_{ps}f_{se}}{A_c} = 0.50f_c$$

$$A_c = \frac{A_{ps}f_{se}}{0.50f_c} \tag{6-3}$$

Note that in the above procedure the only approximations made are the

PRELIMINARY DESIGN

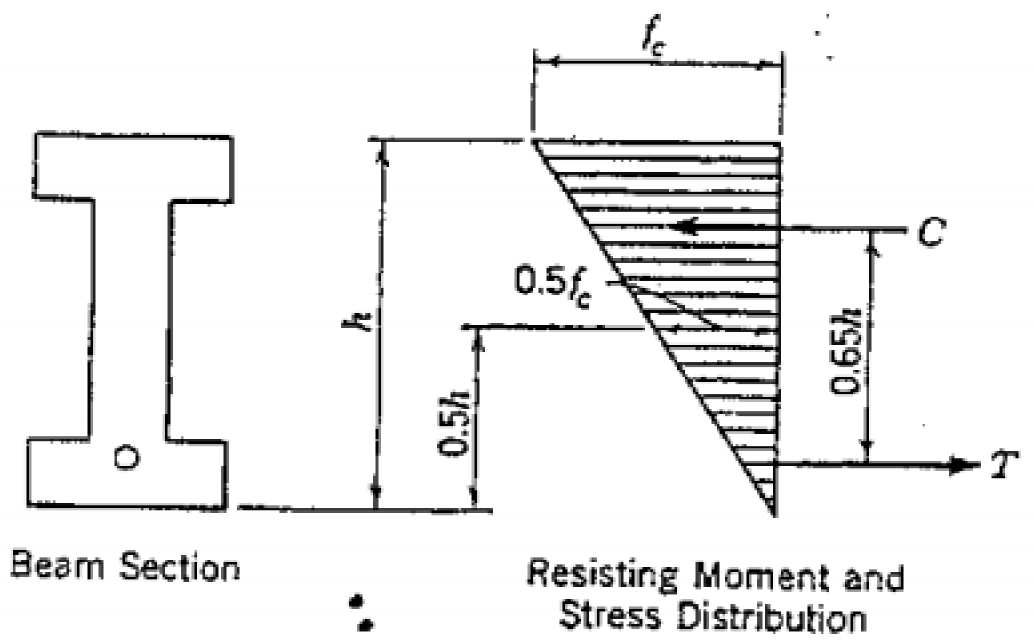


Fig. 6-1. Preliminary design of a beam section.

coefficients of 0.65 and 0.50. These coefficients vary widely, depending on the shape of the section. However, with experience and knowledge, they can be closely approximated for each particular section, and the preliminary design can be made rather accurately.

The above procedure is based on the design for working load, with little or no tension in the concrete. Preliminary designs can also be made on the basis of ultimate strength theories with proper load factors. Such an alternative procedure will be discussed in section 6-7.

PRELIMINARY DESIGN

<u>Design Criteria</u>

If $M_G/M_T > 30\%$ use equation of F for M_T and design T section

If $M_G/M_T < 20\%$ use equation of F for M_L and design I section

If $20\% < M_G/M_T < 30\%$ use both equation of F for M_T and M_L and consider largest value of F and design section accordingly

if F is the largest for M_T then design T section

if F is the largest for M_L then design I section

PRELIMINARY DESIGN

Example 6.1 (T.Y. Lin)

Make a preliminary design for section of a prestressed-concrete beam to resist a total moment of 320 k-ft (434 kN-m). The overall depth of the section is given as 36 in (914.4 mm). The effective prestress for steel is 125,000 psi (862 N/mm^2), and allowable stress for concrete under working load is -1600 psi (-11.03 N/mm^2).

Solution From equations 6-1, 6-2, and 6-3,

$$F = T = M_T/0.65h$$

$$= (320 \times 12)/(0.65 \times 36) = 164 \text{ k } (729.5 \text{ kN})$$

$$A_{ps} = F/f_{se} = 164/125 = 1.31 \text{ sq in } (845 \text{ mm}^2)$$

$$A_c = 164/(0.5 \times 1.60) = 205 \text{ sq in. } (132 \times 10^3 \text{ mm}^2)$$

Now a preliminary section can be sketched with a total concrete area of about 205 sq in. (132×10^3 mm^2), a height of 36 in. (914.4 mm), and a steel area of 1.31 sq in. (845 mm^2) Such a section is shown in Fig. 6-2. A T-section is chosen here because it is an economical shape when M_G/M_T ratio is large.

PRELIMINARY DESIGN

Example 6.1 (T.Y. Lin)

For T section

Ac = area of flange + area of web

=bxt + (h-t)x t

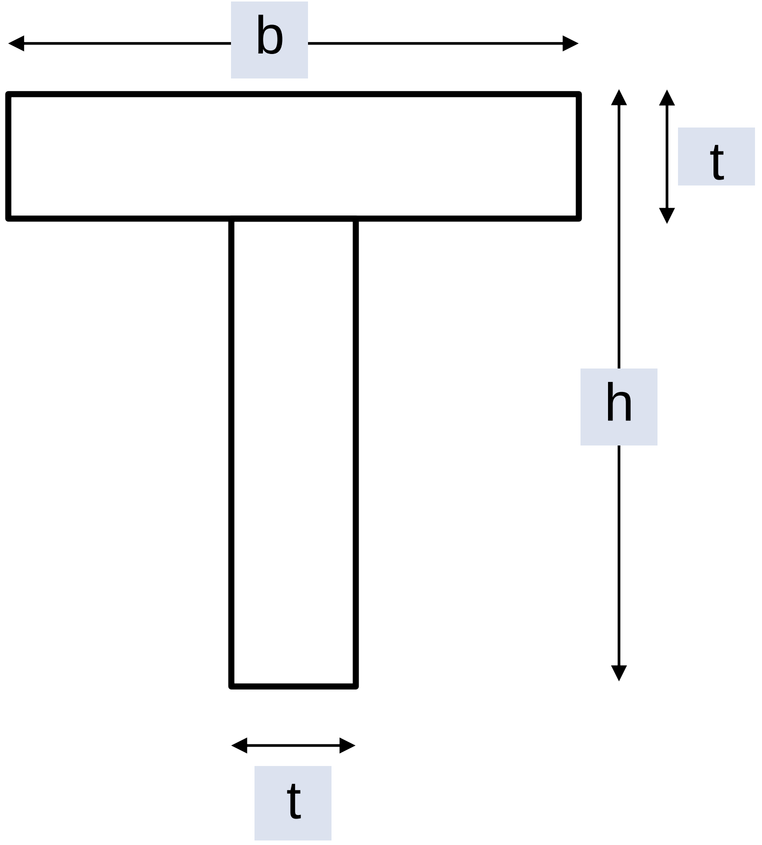
For T section

Ac =bxt + (h-t)x t

=> 205 = bx4+ (36-4)x4   [Let, t=4 in.]

$\Rightarrow$ b =19.25 in

$\Rightarrow$ Say, b = 19in or 19.5 in



PRELIMINARY DESIGN

Example 6.1 (T.Y. Lin)

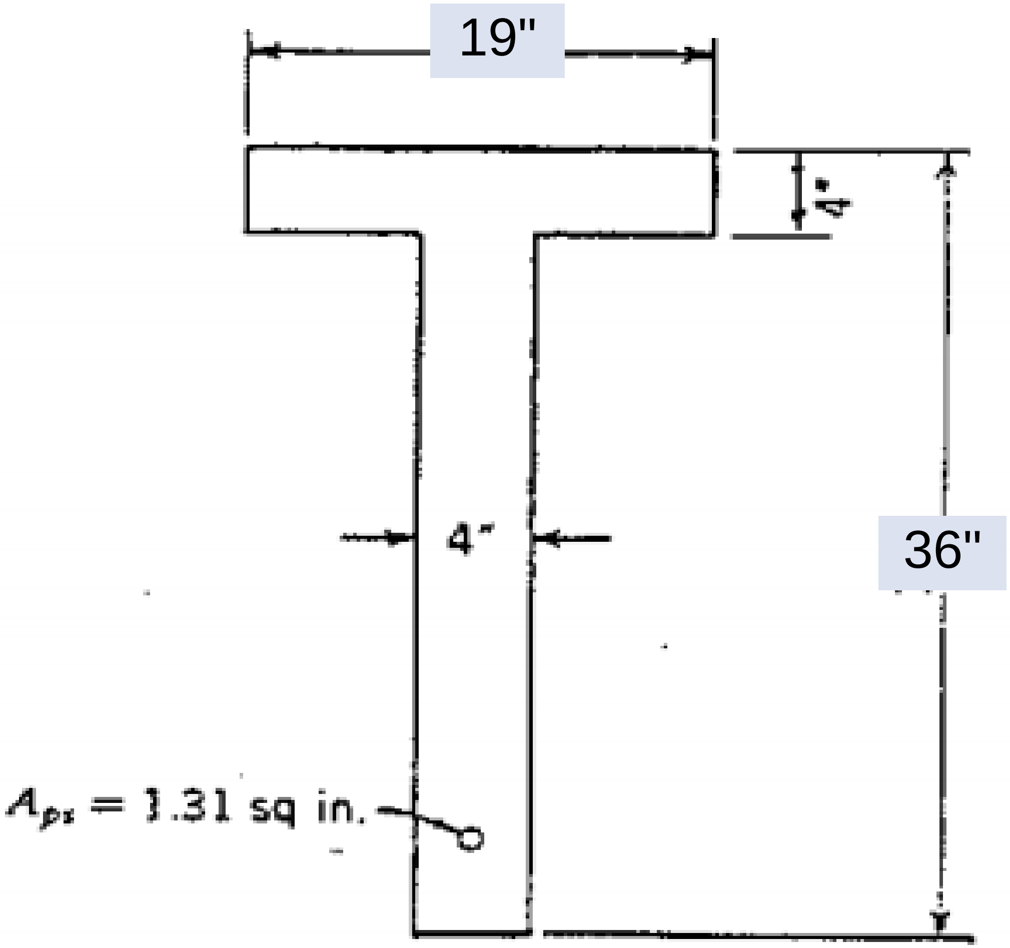


Fig. 6-2. Example 6-1.

FINAL DESIGN (ELASTIC DESIGN, NO TENSION IN CONCRETE)

Large Ratios of M_G/M_T. When the ratio of M_G/M_T is large, the value of $e - k_b$ computed from equation 6-6 may place c.g.s. outside of the practical limit, for example, below the section of the beam. Then it is necessary to place the c.g.s. only as low as practicable and design accordingly.

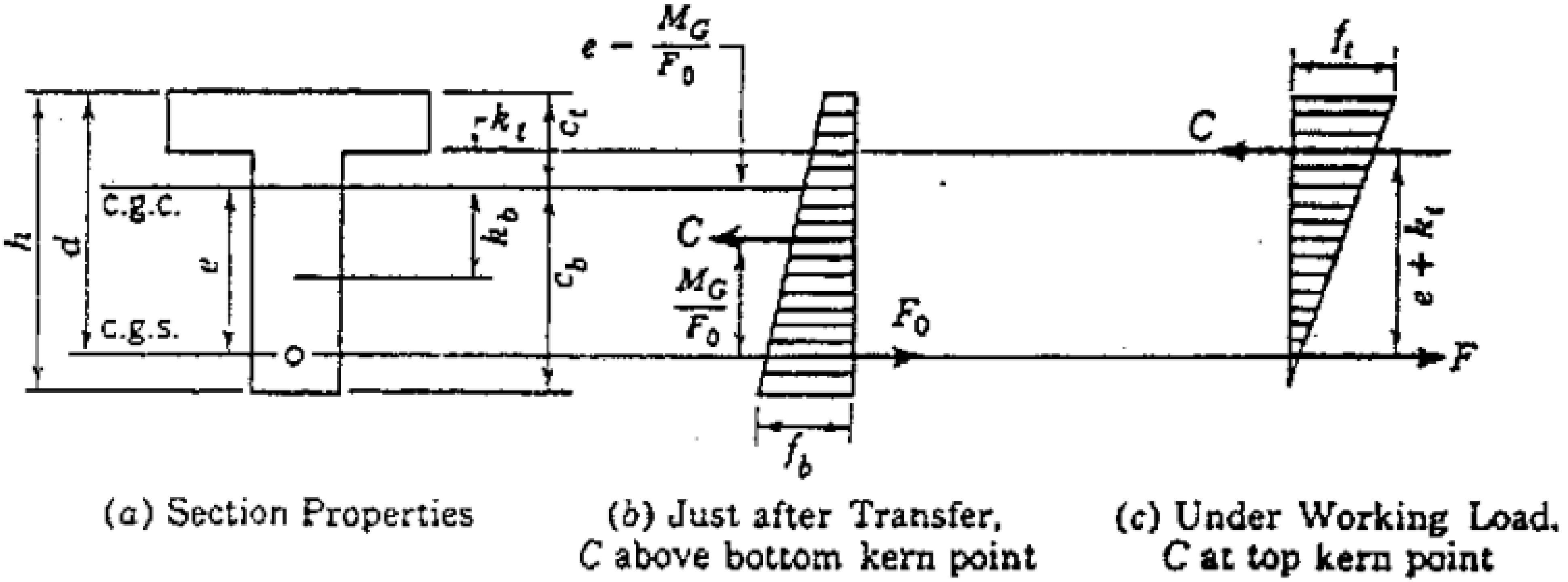


Fig. 6-9. Stress distribution, no tension in concrete (large ratios of M_G/M_t).

FINAL DESIGN (ELASTIC DESIGN, NO TENSION IN CONCRETE)

Step 1. From the preliminary section, compute the theoretical location for c.g.s. by

$$e - k_b = M_G / F_0$$

If it is feasible to locate c.g.s. as indicated by this equation, follow the first procedure. If not, locate c.g.s. at the practical lower limit and proceed as follows.

Step 2. Compute F (and then F_0) by

$$F = \frac{M_T}{e + k_t}$$

FINAL DESIGN (ELASTIC DESIGN, NO TENSION IN CONCRETE)

Step 3. Compute the required area by equations 6-9*a* and 6-10.

$$A_c = Fh/f_t c_b$$

$$A_c = \frac{F_0}{f_b}\left(1 + \frac{e-(M_G/F_0)}{k_t}\right)$$

Step 4. Use the greater of the two A_c's and the new value of F, and revise the preliminary section. Repeat steps 1 through 4 if necessary.

FINAL DESIGN (ELASTIC DESIGN, NO TENSION IN CONCRETE)

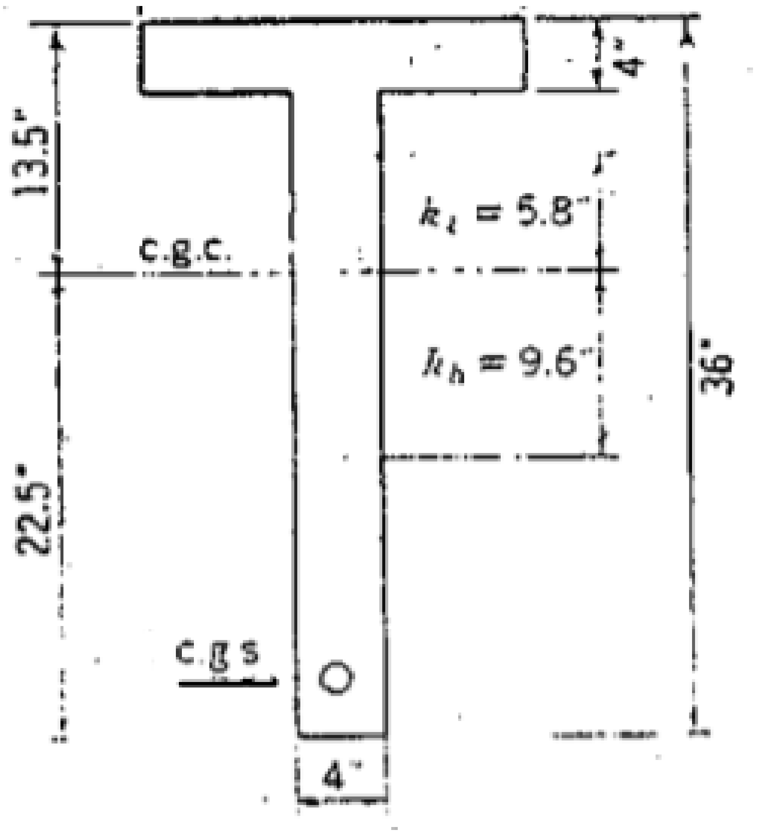
Example 6.4 (T.Y. Lin)

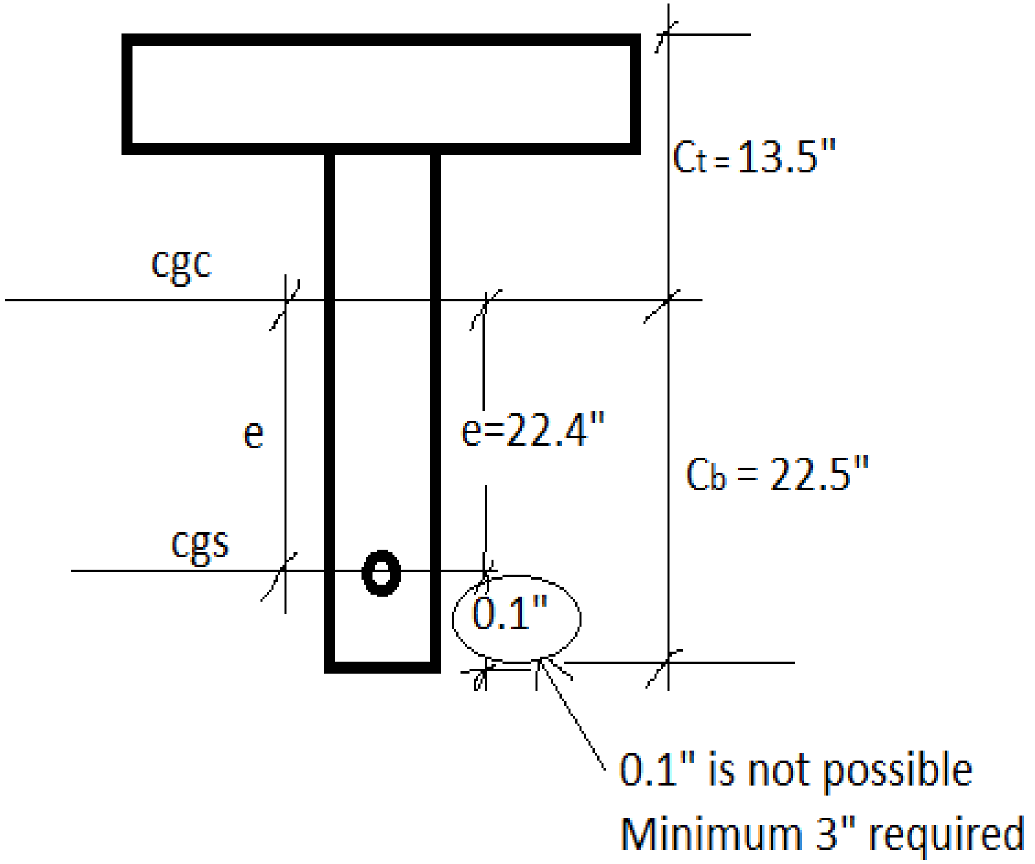
Make final design for the preliminary section obtained in example 6-1, $M_G = 210$ k-ft, allowing $f_b = -1.80$ ksi, $f_0 = 150$ ksi. Other values given were $M_T = 320$ k-ft; $h = 36$ in.; $f_{se} = 125$ ksi; $f_t = -1.60$ ksi.

The preliminary section is shown in Fig. 6-10, with $A_c = 200$ sq in., $c_t = 13.5$ in., $c_b = 22.5$ in., $I = 26{,}000$ in.^{4}, $k_t = 5.8$ in., $k_b = 9.6$ in., $F = 164$ k, $F_0 = 164(150/125) = 197$ k ($M_G = 285$ kN-m, $f_b = -12.41$ N/mm^2, $f_0 = 1034$ N/mm^2; $M_T = 434$ kN-m, $h = 914.4$ mm, $f_{se} = 862$ N/mm^2; $f_t = -11.03$ N/mm^2, $A_c = 129 \times 10^3$ mm, $c_t = 343$ mm; $c_b = 572$ mm, $I = 10.82 \times 10^9$ mm^4, $k_t = 147$ mm; $k_b = 244$ mm, $F = 730$ kN, and $F_0 = 876$ kN).

FINAL DESIGN (ELASTIC DESIGN, NO TENSION IN CONCRETE)

Example 6.4 (T.Y. Lin)





FINAL DESIGN (ELASTIC DESIGN, NO TENSION IN CONCRETE)

Example 6.4 (T.Y. Lin) **Solution:**

M_G/M_T = 210/320 = 0.656 = 65.6%, which is $> 30\%$, *So* design T section

Fo = F (fo / fse) = 164 x (150/125) = 197 k

Step 1. Theoretical lowest location for c.g.s. is given by

$$e - k_b = M_G/F_0$$

$$= (210 \times 12)/197$$

$$= 12.8 \text{ in. } (325 \text{ mm})$$ So, e= 12.8+9.6 =22.4 in

indicating 12.8 in. (325 mm) below the bottom kern, or 0.1 in. (2.54 mm) above the bottom fiber, which is obviously impossible. Suppose that for practical reasons the c.g.s. has to be kept 3 in. (76.2 mm) above the bottom fiber to provide sufficient concrete protection. This problem then belongs to the second case, and we proceed as below.

Step 2. The effective prestress required is, corresponding to a lever arm of $e + k_t = 22.5 - 3 + 5.8 = 25.3$ in. (643 mm),

$$F = (320 \times 12)/25.3 = 152 \text{ k } (676 \text{ kN})$$

$$F_0 = 152(150/125) = 182 \text{ k } (810 \text{ kN})$$

FINAL DESIGN (ELASTIC DESIGN, NO TENSION IN CONCRETE)

Example 6.4 (T.Y. Lin)

Step 3. Compute the area required by

$$A_c = \frac{Fh}{f_t c_b}$$

$$= \frac{152 \times 36}{1.60 \times 22.5}$$

$$= 152 \text{ sq in. } (98 \times 10^3 \text{ mm}^2)$$

$$A_c = \frac{F_0}{f_b}\left(1 + \frac{e - (M_G/F_0)}{k_t}\right)$$

$$= \frac{182}{1.80}\left(1 + \frac{19.5 - 210 \times 12/182}{5.8}\right)$$

$$= 199 \text{ sq in. } (128 \times 10^3 \text{ mm}^2)$$

FINAL DESIGN (ELASTIC DESIGN, NO TENSION IN CONCRETE)

Example 6.4 (T.Y. Lin)

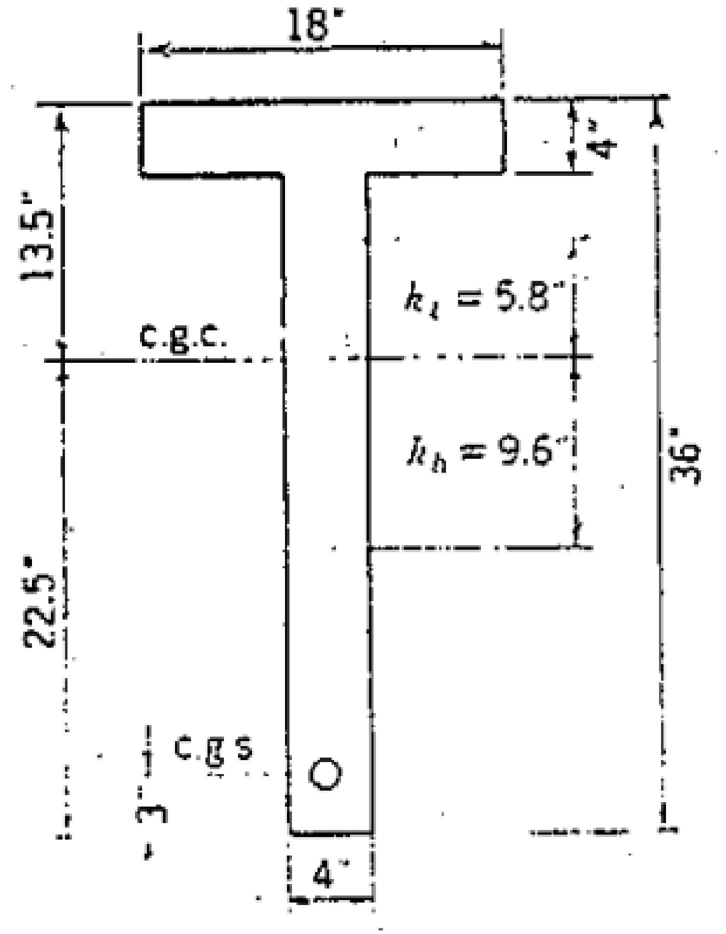
For T section

$$Ac = bxt + (h-t)x\ t$$

=> 199 = bx4+ (36-4)x4 [Let, t=4 in.]

$\Rightarrow$ b = 17.75 in

$\Rightarrow$ Say, b = 18 in



PRELIMINARY DESIGN

In estimating the depth of a prestressed section, an approximate rule is to use 70% of the corresponding depth for conventional reinforced-concrete construction. Some other empirical rules are also available. For example, the thickness of prestressed slabs may vary from $L/35$ for heavy loads to $L/55$ for light loads. The depth of beams of the usual proportions can be approximated by the following formula.

$$h = k\sqrt{M}$$

where h = depth of beam in inches

M = maximum bending moment in k-ft

k = a coefficient varying from 1.5 to 2.0

PRELIMINARY DESIGN

It is needless to add that such empirical rules apply only under the average conditions and should be used merely as a preliminary guide.

A more accurate preliminary design can be made if the girder moment M_G is known in addition to the total moment M_T. When M_G is much greater than 20 to 30% of M_T, the initial condition under M_G generally will not control the design, and the preliminary design needs be made only for M_T. When M_G is small relative to M_T, then the c.g.s. cannot be located too far outside the kern point, and the design is controlled by $M_L = M_T - M_G$. In this case, the resisting lever arm for M_L is given approximately by $k_t + k_b$, which averages about $0.50h$. Hence the total effective prestress required is

$$F = \frac{M_L}{0.50h} \tag{6-4}$$

When M_G/M_T is small, this equation should be used instead of equation 6-1. Equation 6-3 is still applicable.

PRELIMINARY DESIGN

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A more accurate preliminary design can be made if the girder moment M_G is known in addition to the total moment M_T. When M_G is much greater than 20 to 30% of M_T, the initial condition under M_G generally will not control the design, and the preliminary design needs be made only for M_T. When M_G is small relative to M_T, then the c.g.s. cannot be located too far outside the kern point, and the design is controlled by $M_L = M_T - M_G$. In this case, the resisting lever arm for M_L is given approximately by $k_t + k_b$, which averages about $0.50h$. Hence the total effective prestress required is

$$F = \frac{M_L}{0.50h} \tag{6-4}$$

When M_G/M_T is small, this equation should be used instead of equation 6-1. Equation 6-3 is still applicable.

PRELIMINARY DESIGN

Example 6.2 (T.Y. Lin)

Make a preliminary design for the beam section in example 6-1, with $M_T = 320$ k-ft, $M_G = 40$ k-ft, $h = 36$ in., $f_{se} = 125{,}000$ psi, and $f_c = -1600$ psi ($M_T = 434$ kN-m, $M_G = 54$ kN-m, $h = 914.4$ mm, $f_{se} = 862\ \mathrm{N/mm^2}$, and $f_c = -11.03\ \mathrm{N/mm^2}$).

Solution Since M_G is only 12% of M_T, it is not likely that the c.g.s. can be located much outside the kern. Hence it will be more nearly correct to apply equation 6-4. Thus,

$$M_L = M_T - M_G = 320 - 40$$

$$= 280 \text{ k-ft } (380 \text{ kN-m})$$

$$F = M_L / 0.50h = 280 \times 12 / (0.50 \times 36)$$

$$= 187 \text{ k } (832 \text{ kN})$$

PRELIMINARY DESIGN

Example 6.2 (T.Y. Lin)

Applying the first part of formula 6-2 and also formula 6-3, we have

$$A_{ps} = F/f_{se} = 187/125$$

$$= 1.50 \text{ sq in. } (968 \text{ mm}^2)$$

$$A_c = A_{ps} f_{se}/0.50 f_c = 187/(0.50 \times 1.60)$$

$$= 234 \text{ sq in. } (151 \times 10^3 \text{ mm}^2)$$

Now a preliminary section can be sketched with a total concrete area of about 234 sq in. (151×10^3 mm^2), a height of 36 in. (914.4 mm), and a steel area of 1.50 sq in. (968 mm^2), as shown in Fig. 6-3. An I-section is chosen because it is a suitable form when the M_G/M_T ratio is small.

PRELIMINARY DESIGN

<u>Example 6.2 (T.Y. Lin)</u>

<u>For I section</u>

Ac = 2xarea of flange + area of web

=2xbxt + (h-2t)x t

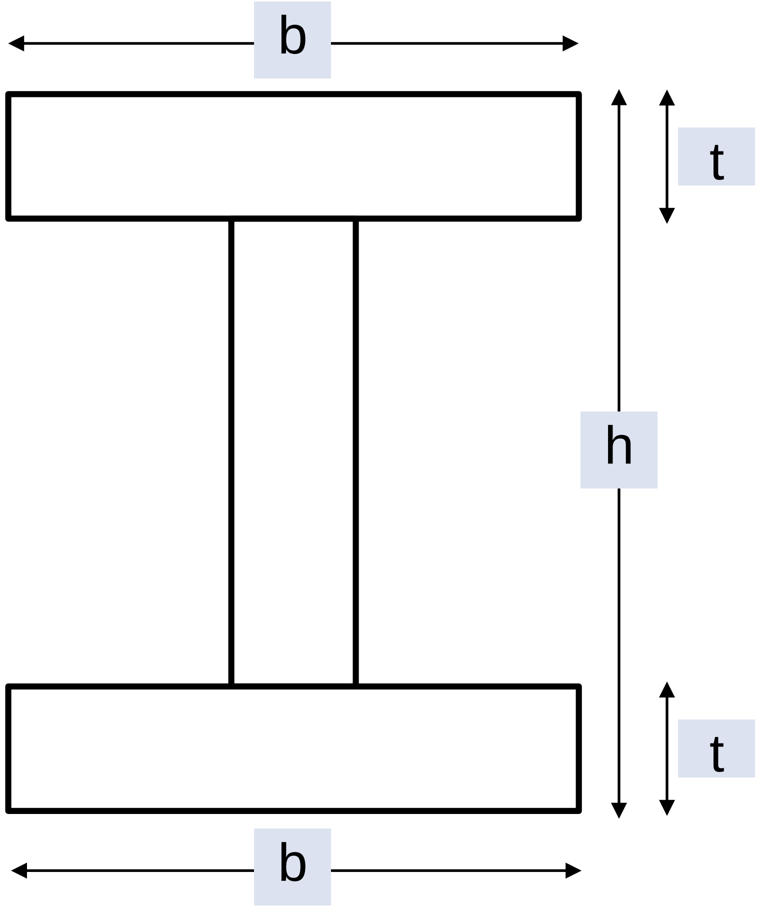
<u>For I section</u>

Ac =2xbxt + (h-2t)x t

=> 234 = 2xbx4+ (36-2x4)x4   [Let, t=4 in.]

⇒ b =15.25 in

⇒ Say, b = 15 in or 15.5 in



PRELIMINARY DESIGN

Example 6.2 (T.Y. Lin)

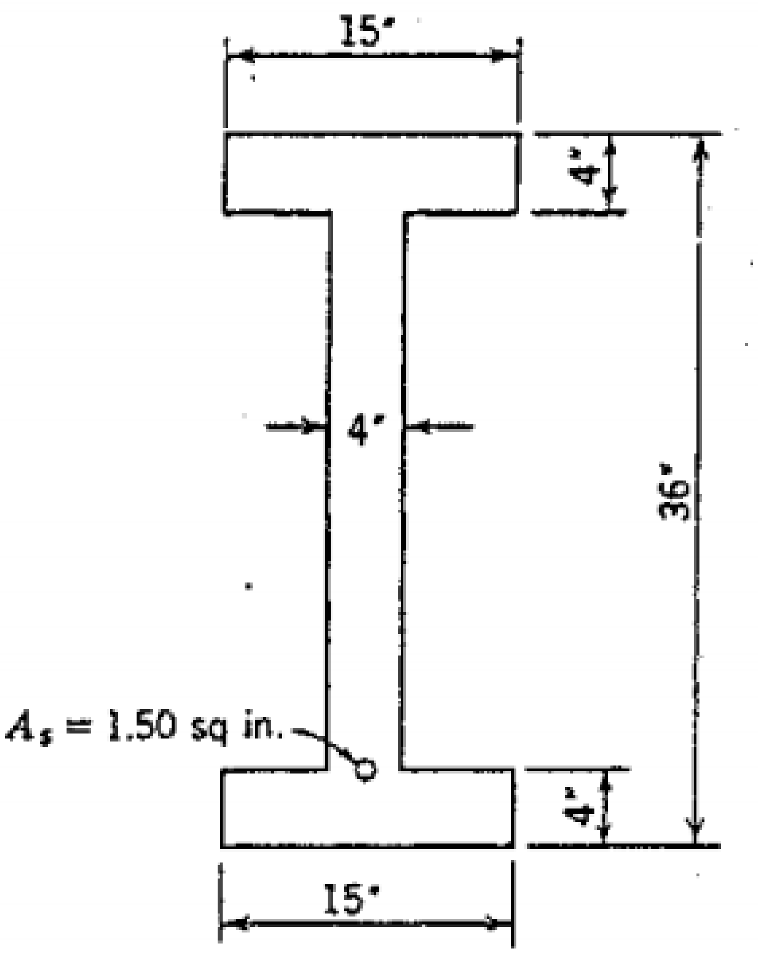


Fig. 6-3. Example 6-2.

PRELIMINARY DESIGN

Example 6.2 (T.Y. Lin)

When it is not known whether M_T or M_L should govern the design, one convenient way is to apply both equations 6-1 and 6-4, and use the greater of the two values of F. For example, if $M_G = 80$ k-ft (108 kN-m) in example 6-1, we have, from equation 6-1,

$$F = M_T/0.65h$$
$$= (320 \times 12)/(0.65 \times 36)$$
$$= 164 \text{ k } (730 \text{ kN})$$

Here, $M_G/M_T = 0.25$
$i.e.\ 25\%$

From equation 6-4, we have

$$F = M_L/0.50h$$
$$= [(320 - 80)12]/(0.50 \times 36)$$
$$= 160 \text{ k } (712 \text{ kN})$$

$F = 164$ k (730 kN) controls the design.

FINAL DESIGN (ELASTIC DESIGN, NO TENSION IN CONCRETE)

Small Ratios of M_G/M_T. For the section obtained from the preliminary design, the values of M_G, k_t, k_b, A_c are computed. When the ratio of M_G/M_T is small, c.g.s. is located outside the kern just as much as the M_G will allow. Since no tension is permitted in the concrete, c.g.s. will be located below the kern

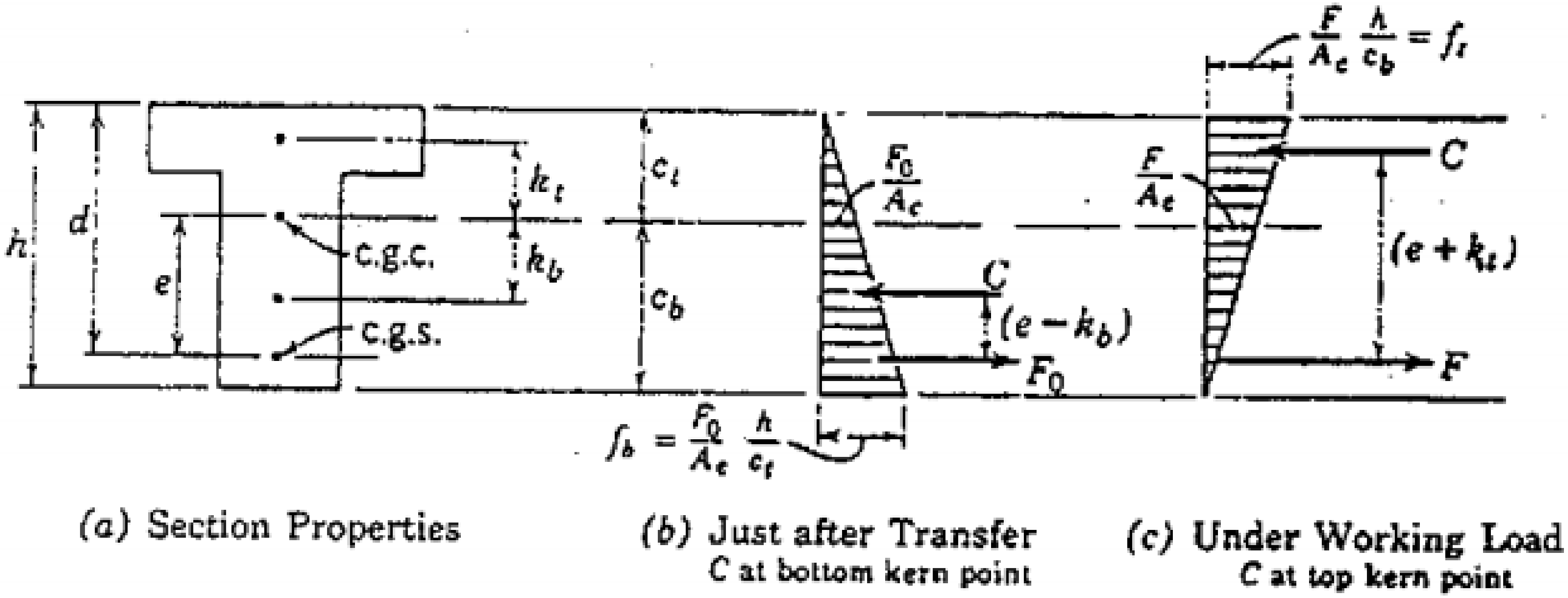


Fig. 6-7. Stress distribution, no tension in concrete (small ratios of M_G/M_T).

PRELIMINARY DESIGN (CONCENTRIC TENDON)

To summarize the procedure of design, we have:

Step 1. From the preliminary design section, locate c.g.s. by

$$e - k_b = M_G / F_0$$

Step 2. With the above location of c.g.s., compute the effective prestress F (and then the initial prestress F_0) by

$$F = \frac{M_T}{e + k_t}$$

Step 3. Compute the required A_c by

$$A_c = F_0 h / f_b c_t$$

and

$$A_c = F h / f_t c_b$$

Step 4. Revise the preliminary section to meet the above requirements for F and A_c. Repeat steps 1 through 4 if necessary.

FINAL DESIGN (ELASTIC DESIGN, NO TENSION IN CONCRETE)

From the above discussion, the following observations regarding the properties of a section can be made.

1. $e+k_t$ is a measure of the total moment-resisting capacity of the beam section. Hence the greater this value, the more desirable is the section.
2. $e-k_b$ locates the c.g.s. for the section, and is determined by the value of M_G. Thus, within certain limits, the amount of M_G does not seriously affect the capacity of the section for carrying M_L.
3. h/c_b is the ratio of the maximum top fiber stress to the average stress on the section under working load. Thus, the smaller this ratio, the lower will be the maximum top fiber stress.
4. h/c_t is the ratio of the maximum bottom fiber stress to the average stress on the section at transfer. Hence, the smaller this ratio, the lower will be the maximum bottom fiber stress.

PRELIMINARY DESIGN

Example 6.3 (T.Y. Lin)

For the preliminary section obtained in example 6-2, make a final design, allowing $f_b = -1.80$ ksi, $f_0 = 150$ ksi. Other given values were: $M_T = 320$ k-ft; $M_G = 40$ k-ft; $f_t = -1.60$ ksi; $f_{se} = 125$ ksi; $F = 187$ k. And the preliminary section is the same as in Fig. 6-3 ($f_b = -12.41$ N/mm², $f_0 = 1034$ N/mm², $M_T = 434$ kN-m, $M_G = 54$ kN-m, $f_t = -11.03$ N/mm², $f_{se} = 862$ N/mm², and $F = 832$ kN).

Solution For the trial preliminary section, compute the properties as follows

$$A_c = 2 \times 4 \times 15 + 4 \times 28 = 232 \text{ sq in. } (150 \times 10^3 \text{ mm}^2)$$

$$I = \frac{15 \times 36^3}{12} - \frac{11 \times 28^3}{12}$$

$$= 58{,}200 - 20{,}100$$

$$= 38{,}100 \text{ in.}^4 \ (15.86 \times 10^9 \text{ mm}^4)$$

$$r^2 = 38{,}100/232$$

$$= 164 \text{ in.}^2 \ (106 \times 10^3 \text{ mm}^2)$$

$$k_t = k_b = 164/18 = 9.1 \text{ in. } (231 \text{ mm})$$

Here, $r^2 = \frac{I}{A}$

$$k_t = \frac{r^2}{c_t}$$

$$k_b = \frac{r^2}{c_b}$$

PRELIMINARY DESIGN

Example 6.3 (T.Y. Lin)

Step 1. For an assumed

$$F = 187 \text{ k } (832 \text{ kN})$$

$$F_0 = \frac{150}{125} 187 = 225 \text{ k } (1001 \text{ kN})$$ Fo = F (fo / fse)

c.g.s. should be located at $e - k_b$ below the bottom kern, where

$$e - k_b = \frac{M_G}{F_0} = \frac{40 \times 12}{225} = 2.1 \text{ in. } (53 \text{ mm})$$

$$e = 9.1 + 2.1 = 11.2 \text{ in. } (285 \text{ mm})$$

Step 2. Effective prestress required is recomputed as

$$F = \frac{M_T}{e + k_t} = \frac{320 \times 12}{11.2 + 9.1}$$

$$= 189 \text{ k } (841 \text{ kN})$$

$$F_0 = \frac{150}{125} 189 = 227 \text{ k } (1010 \text{ kN})$$ Fo = F (fo / fse)

PRELIMINARY DESIGN

Example 6.3 (T.Y. Lin)

Step 3. A_c required is

$$A_c = \frac{F_0 h}{f_b c_t}$$

$$= \frac{227 \times 36}{1.80 \times 18}$$

$$= 252 \text{ sq in. } (163 \times 10^3 \text{ mm}^2) \text{ controlling}$$

$$A_c = \frac{F h}{f_t c_b}$$

$$= \frac{189 \times 36}{1.60 \times 18}$$

$$= 236 \text{ sq in. } (152 \times 10^3 \text{ mm}^2)$$

PRELIMINARY DESIGN

Example 6.3 (T.Y. Lin)

For I section

$$A_c = 2 \times b \times t + (h - 2t) \times t$$

$\Rightarrow 252 = 2 \times b \times 4 + (36 - 2 \times 4) \times 4$ [Let, t=4 in.]

$\Rightarrow b = 17.5$ in

$\Rightarrow$ Say, $b = 17.5$ in

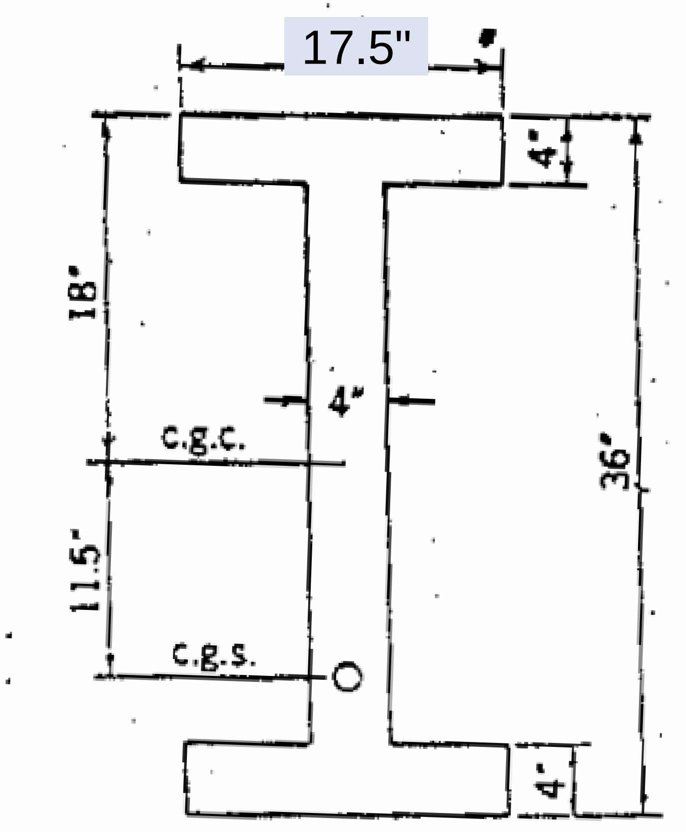


Fig. 6-8. Example 6-3.


Any
questions