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A CASE STUDY ON PATH OF PROJECTILE MOTION AND ITS APPLICATIONS

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Abstract: **In this paper, we are going to study about path of projectile and its applications. A projectile is any object that is cast, fired, flung, heaved, hurled, pitched, tossed, or thrown. The path of a projectile is called its trajectory. In this paper we derive the equation for the path of the projectile and the nature of the trajectory using initial velocity, acceleration, time, and distance travelled. The trajectory is a parabola. We use acceleration, velocity, time, range, height, mass, distance to find the formula and used it for solving the problems. It's applications in football and golf problems are also discussed.**

Keywords: **Projectile, Trajectory, Parabola, Velocity, Horizontal plan, Projection**

I. INTRODUCTION

Dynamics is the branch of applied mathematics which deals with the bodies in motion and a particle is said to be moving if it changes its position with respective to some fixed point. It is a classic discipline.

The dynamics of fluids is governed by the conservation laws of classical physics, namely conservation of mass, momentum and energy. Physical properties of the flow such as density and velocity can then be described as time- dependent scalar or vector fields.

A projectile is any object that is cast, fired, flung, heaved, hurled, pitched, tossed, or thrown. (This is an informal definition.) The path of a projectile is called its trajectory. Some examples of projectiles include…

- a baseball that has been pitched, batted, or thrown
- a bullet the instant it exits the barrel of a gun or rifle
- a bus driven off an uncompleted bridge
- a moving airplane in the air with its engines and wings disabled
- a runner in mid stride (since they momentarily lose contact with the ground)

The force of primary importance acting on a projectile is gravity. This is not to say that other forces do not exist, just that their effect is minimal in comparison. A tossed helium-filled balloon is not normally considered a projectile as the drag and buoyant forces on it are as significant as the weight. Helium-filled balloons can't be thrown long distances and don't normally fall.

II. DERIVATION

1. PATH OF PROJECTILE

SHOW THAT PATH OF PROJECTILE IS PARABOLA

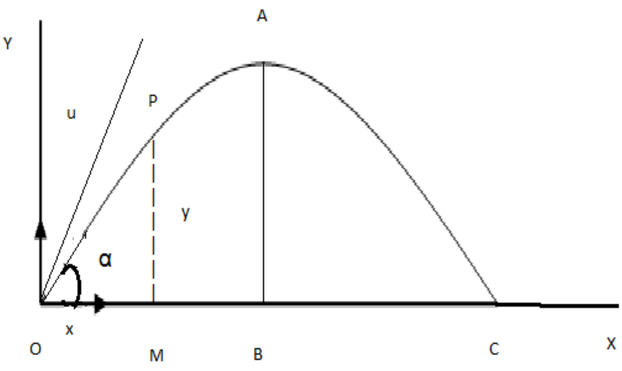

Fig. 1

Let the particle be projected from O, with a velocity U at an angle α to the horizon. Take O as the Origin, the horizontal and the upward vertical through O as axes of X and Y respectively. The initial velocity U can be spilt into two components, which are ucosα in the horizontal direction and usinα in the vertical direction. The horizontal component ucosα is constant. Through out the motion as there is no horizontal acceleration g downwards.

Let P(X, Y) be the position of the of the particle at time t sec after projection. Then

X= horizontal distance described in t sec= (u cos α) t --------- (1)

Y= Vertical distance described in t sec= (u sin α) t - $\frac{1}{2}gt^2$------- (2)

(1) And (2) can be taken as the parametric equations of the trajectory.

The equation to the trajectory. The path is got by eliminating t between them,

From (1),

$t=\frac{X}{u\cos\alpha}$ and putting this in (2) we get

$y= u\sin\alpha\frac{X}{u\cos\alpha}-\frac{1}{2}g(X/u\cos\alpha)^2$ ---------- (3)

Multiplying (3) by $2u^2\cos\alpha$,

$2u^2\cos^2\alpha\,(y) = 2u^2\cos^2\alpha\; X\frac{\sin\alpha}{\cos\alpha} - gX^2$

$$X^2-\frac{2u^2\sin\alpha\cos\alpha}{g}X = -\frac{2u^2\cos^2\alpha}{g}y$$

$$=\frac{-2u^2\cos^2\alpha}{g}\left(y-\frac{u^2\sin^2\alpha}{2g}\right)$$

Transfer the origin to the point,

$$\left(\frac{u^2\sin\alpha\cos\alpha}{g},\frac{u^2\sin^2\alpha}{2g}\right)$$

The above equation then becomes

$$X^2=-\frac{2u^2\cos^2\alpha}{g}Y \quad \text{------------------- (4)}$$

[4] is clearly the equation to a Parabola of Latus rectum $\frac{2u^2\cos^2\alpha}{g}$ whose axis is vertical and downwards and whose vertex is the point.

$$\left(\frac{u^2\sin\alpha\cos\alpha}{g},\frac{u^2\sin^2\alpha}{2g}\right)$$

Note:-

The Latus rectum of the above Parabola is

$=\frac{2u^2\cos^2\alpha}{g}$

$= \frac{2}{g}(u\cos\alpha)^2$

$=\frac{2}{g}$ * square of the horizontal velocity

So the Latus rectum (i.e. the size of the parabola) is independent of the initial
Vertical velocity and depends only on the horizontal velocity.

2. NATURE OF TRAJECTORY

TO SHOW THAT THE PATH OF A PROJECTILE IS A PARABOLA.

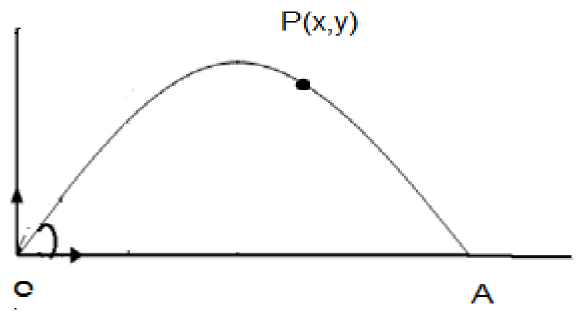

Fig. 2

Suppose a particle of mass m is projected from a point O with a speed u in a direction which makes angles α with the horizontal. Let the particle hit the horizontal plane through O, at A. choose OA as the x axis the and the upward vertical line through O as the y axis. Let
P(x,y)

Be the position of the particle at time t. then the horizontal and vertical dis placement6 of the particle at time t are
x,y.

Since earth's gravity is the only force on the projectile the horizontal and upwards vertical components of the acceleration are
O,-g.

Therefore the horizontal components of the velocity is always a which is the initial component u cos α. There fore the horizontal displacement in time t is

X= (velocity)*(time) = (u cos α) t. --------- (1)

For the upwards dis placement during the motion of the particle from O yo P, we have

Initial Velocity : u sin α
Acceleration : -g
Time : t
Distance travelled : y
Formula : $S = ut+1/2at^2$. --------------------- (2)

Now (1) and (2) are the parametric equations of the trajectory. Its xy equation is obtained by eliminating the parameter t as

$$y = u \sin\alpha \left\{\frac{x}{u\cos\alpha}\right\} - \frac{g}{2}\left\{\frac{x}{u\cos\alpha}\right\}^2 .$$

OR

$$y = x\tan\alpha - \frac{gx^2}{2u^2cos^2\alpha}$$

This is a seconds degree equation satisfying the relation " $h^2 - ab = 0$ ". So it represents a parabola . Hence the trajectory is a parabola .

REMARKS:

The above equation can be rewritten as

$$\left(x - \frac{u^2 sin\alpha cos\alpha}{g}\right) = \frac{2u^2cos^2\alpha}{g}\left(y - \frac{u^2sin^2\alpha}{2g}\right)$$

Which shows that the parabola has a downward vertical as its exits.

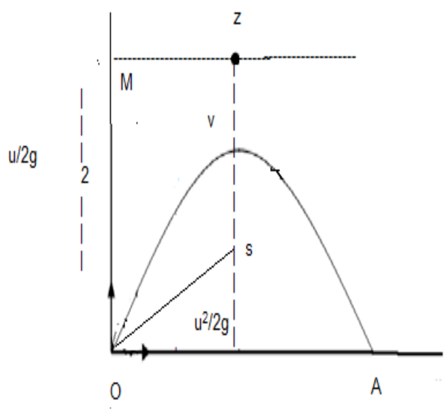

Fig. 3

$$\left(\frac{2u^2 sin\alpha cos\alpha}{g}, 0\right)$$

Its Latus rectum $(2u^2 - cos^2 \propto)/g$ and its vetex V is

$$\left(\frac{u^2 sin\alpha\ cos\alpha}{g}, \frac{u^2sin^2\alpha}{2g}\right)$$

III. FORMULA

- Acceleration : -g
- Velocity : v
- Time : t
- Range : R
- Height : h
- Mass : m
- Distance : d

Maximum height $=\frac{u^2 \sin^2\alpha}{2g}$

Maximum range $=\frac{u^2}{g}$

Greatest height :

$$h = \frac{u^2sin^2\alpha}{2g}$$

Time :

$$T = \frac{2usin\alpha}{g}$$

Range:

$$R = \frac{u^2 sin2\alpha}{g}$$

The time of flight = $\frac{2u\ sin\alpha}{g}$

Initial velocity :

U = V cos α

V = V sin α

IV. PROBLEMS

1] **A man can throw a stone with a velocity of 20 m/sec. Find the maximum distance he can throw it on a horizontal plane and to what height will it rise?**

Solution:

Given that u=20m/sec.

Maximum range $= \frac{u^2}{g}$

$= \frac{20*20}{9.8}$

= 40.8 m When $\alpha = \frac{\pi\pi}{4}$

There fore distance he can throw on a horizontal plane is 40.8m.

Maximum height $= \frac{u^2 \sin^2\alpha}{2g}$

$= \frac{u^2}{2g}\frac{1}{2}$ since $\alpha = \frac{\pi}{4}$

$= \frac{20*20*\frac{1}{2}}{2*9.8}$

= 10.2m

There fore Maximum height it can reach = 10.2 m

2] **With what minimum velocity a man can throw a ball to a maximum range of 80m, and to what height does it reach in this condition?**

Solution:

Maximum range = $\frac{u^2}{g}$ = 80

u^2 = g * 80

u^2 = 9.8*80

u^2 = 784

u = $\sqrt{784}$

= 28 m/ sec.

There fore Velocity required is 28 m/sec.

Maximum height $= \frac{u^2 \sin^2\alpha}{2g}$ When α = 45°

$= \frac{28^2 * \frac{1}{2}}{2*9.8}$

= 20 m.

3] **Find the velocity and direction of projection of a shot which passes in a horizontal direction just over the top of a wall which is 150 m off and 75 mm high.**

Solution:

Let u be the velocity of projection and α be the angle of projection.

Since it passes in a horizontal direction over a wall of 75m height,

Greatest height attained = $\frac{u^2 \sin^2\alpha}{2g}$ = 75

There fore $u^2 \sin^2 \alpha$ =150 g ---------------- (1)

Also the horizontal distance of the wall = Half of the horizontal range

$= \frac{1}{2}\frac{u^2 \sin 2\alpha}{g}$

i.e. $\frac{u^2 \sin\alpha\cos\alpha}{g} = 150$

u^2sinαcosα =150g

Dividing (1) by (2) tan α =1

There fore $\alpha = \frac{\pi}{4}$

From (1), $u^2 \sin^2\frac{\pi}{4}$ =150 g

u^2 = 300g

u = 300*9.8

u = $\sqrt{300 * 9.8}$

= 54.2 m/sec

4] **If h is the greatest height attained and R, the range on a horizontal plane of a projectile, show that the velocity of projection is given by $\left[2g\left(h + \frac{R^2}{16h}\right)\right]^{1/2}$.**

Solution:

Let u be the velocity of projection and α be the angle of projection. Greatest height is given by

$$h = \frac{u^2 sin^2\alpha}{2g}$$

$u^2 \sin^2 \alpha$ = 2gh --------------- (1)

Range $R = \frac{u^2 sin2\alpha}{g}$

$$= \frac{2u^2 sin\alpha cos\alpha}{g}$$

$$R^2g^2 = 4\,u^4 \sin^2\alpha(1 - sin^2\alpha)$$

$$= 4\,u^2 \sin^2\alpha\,(u^2 - u^2 \sin^2\alpha)$$

$$= 4\,2\,gh\,(u^2 - 2gh)$$

$$\frac{R^2g^2}{8gh} = u^2 - 2gh$$

$$u^2 = 2gh + \frac{R^2g^2}{8gh}$$

$$= 2gh + \left(h + \frac{R^2}{16h}\right)$$

$$u = \left[2g\left(h + \frac{R^2}{16h}\right)\right]^{1/2}$$

5] **Prove that if the time of flight of a bullet over a horizontal range R is T seconds, then the inclination of the direction of the projectile to the horizontal is tan$^{-1}\frac{gT^2}{2R}$.**

Solution:

$$R = \frac{u^2 sin2\alpha}{g}$$

$$T = \frac{2usin\alpha}{g}$$

$$\frac{gT^2}{2R} = \frac{g}{2}\frac{4u^2 sin^2\alpha}{g^2}\frac{g}{2u^2 sin\alpha cos\alpha}$$

$$= \tan\alpha$$

There fore $\alpha = \tan^{-1}\frac{gT^2}{2R}$

6] **If the focus of a trajectory lies as much below the horizontal plane through the poit of projection as the vertex is above, prove the angle of projection is given by sin α = $\frac{1}{\sqrt{3}}$.**

Solution:

Given that the focus and vertex of the projectile are equidistant from the x axis,

There fore Their ordinates are equal and opposite.

Ordinates of the focus = $\frac{u^2 \cos 2\alpha}{2g}$ (below x axis)

Ordinate of the vertex = Maximum height = $\frac{u^2 sin^2\alpha}{2g}$\

Given that ,

$$\frac{u^2 sin2\alpha}{2g} = \frac{u^2 sin^2\alpha}{2g}$$

There fore $\cos 2\alpha = \sin^2\alpha$

$1-2 \quad \sin^2\alpha = \sin^2\alpha$

$\sin^2\alpha = 1$

therefore $\sin\alpha = \frac{1}{\sqrt{3}}$.

V. APPLICATION OF PROBLEMS:

FOOT BALL & GOLF PROBLEMS:

1] **A quarterback throws a football to a stationary receiver 31.5m away from him. If the football is thrown at an initial angle of 40° to the ground, at what initial speed must the quarterback throw the ball for it to reach the receiver?**

Solution:

Given:- $d_x = 31.5m$,$\theta = 40°$

Missing: $v_i = ?$

You are given the range of the football and the angle it is thrown. Therefore to find the initial velocity needed for the football to reach the receiver, the easiest formula to use would be:

$d_x = (v_i^2 \sin2\theta) / g$

$31.5m = [v_i^2 \sin(2\times40°)]/ 9.81m/s^2$

$309\ m^2/s^2 = v_i^2 \sin80°$

$313.77\ m^2/s^2 = v_i^2$

$17.7\ m/s = v_i$

2] **A golf ball is hit a horizontal distance of exactly 300 m. What is the maximum height the golf ball reaches in the air if it is launched at an angle of 25° to the ground?**

Solution :

Given: $d_x = 300$ m , $\theta = 25°$

Missing: v_i =? , v_{iy} =? , d_y =?

You are given the range of the golf ball and the angle it is thrown and is looking for the height. This question will need a few more steps than the previous one.

It will be easier to find height with v_{iy}. The initial vertical velocity can be found if the initial velocity is found first. The easiest formula to use would be:

$d_x = (v_i^2 \sin2\theta) / g$

$300m = [v_i^2 \sin(2\times25°)]/ 9.81m/s^2$

$2943\ m^2/s^2 = v_i^2 \sin 50°$

$3841.81\ m^2/s^2 = v_i^2$

$61.98\ m/s = v_i$

Now that you have v_i, you can find v_{iy} using the formula:

$v_{iy} = v_i \sin\theta$

$v_{iy} = 61.98\ m/s \sin 25°$

$v_{iy} = 26.19\ m/s$

With v_{iy}, d_y can be determined using the following formula (remember that v_{fy} is 0 m/s at maximum height :

$v_{fy}^2 = v_{iy}^2 - 2gd_y$

$(0m/s)^2 = (26.19\ m/s)^2 - 2(9.81\ m/s^2)d_y$

$-686.17\ m^2/s^2 = (-19.62 m/s^2)d_y$

$34.97\ m = d_y$

3] A place kicker kicks a football with a velocity of 20.0 m/s and at an angle of 53 degrees.

(a) How long is the ball in the air?

(b) How far away does it land?

(c) How high does it travel?

Solution:

$$v_{ox} = v_o \cos\theta$$

$$v_{ox} = 20\cos 53 = 12.04\ m/s$$

$$v_{oy} = v_o \sin\theta$$

$$v_{oy} = 20\sin 53 = 15.97\ m/s$$

What i want to know?

T=? , X=? , Y_{max}=?

A] **How long is the ball in the air?**

$Y = V_{0y}\, t + ½\, gt^2$

$= (15.97)\, t - 4.9\, t^2$

$-15.97\, t = -4.9t^2$

T= 3.26 s

B] **How far away does it land?**

$X = V_{0x}\, t$

= (12.04) (3.26)

= 39.24 m

C] **How high does it travel?**

$Y = V_{0y}\, t + ½\, gt^2$

$= (15.97)(1.63) - 4.9(1.63)^2$

= 26.03 – 13.01

= 13.02

4] A golf ball ishitfromthe ground at35m/s at an angle of 55°.The ground is level.

1. **Howlong is theball in the air?**
2. **What is themaximum heightof theball?**

Solution:

$V_{ox} = v_0 \cos\theta$

$= 35 \cos 55^o$

= 20.07 m/s

$V_{0y} = v_{0y} \sin\theta$

$= 35 \sin 55^0$

= 28.67 m/s

1. Howlong is theball in the air?

$Y = v_o y\, t + ½\, gt^2$

$= 28.6\, t + ½\,(-9.8)\, t^2$

= 28.67 t+ ½ (-4.9)

$-28.67\, t = -4.9t^2$

$\frac{28.67}{4.9} = t$

T = 5.851

2. What is themaximum heightof theball?

Maximum height:

$H = \frac{u^2 sin^2\alpha}{2g}$

$0 = (u \sin\alpha)^2 - 2gh$

$= (35 \sin 55)^2 - 2g$

$= (28.67)^2 - 2(-9.8)$

= 821.9 – (-19.6)

- 821.9 H = - 19.6

H = 41.9 m/sec.

VI. CONCLUSION

In this paper, we study about path of projectile, trajectory and its applications. Different application problems are solved analytically with exact equation of path of the projectile. The initial speed, acceleration, height and speed could be calculated by using this methods.

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