Alternating Current

An alternating current is a current that is periodic function of time. In other words, an alternating current is one which passes through a cycle of regular intervals. In an alternating current circuit, we are connected with a steady state current and voltage which are oscillating simultaneously without change in amplitude.

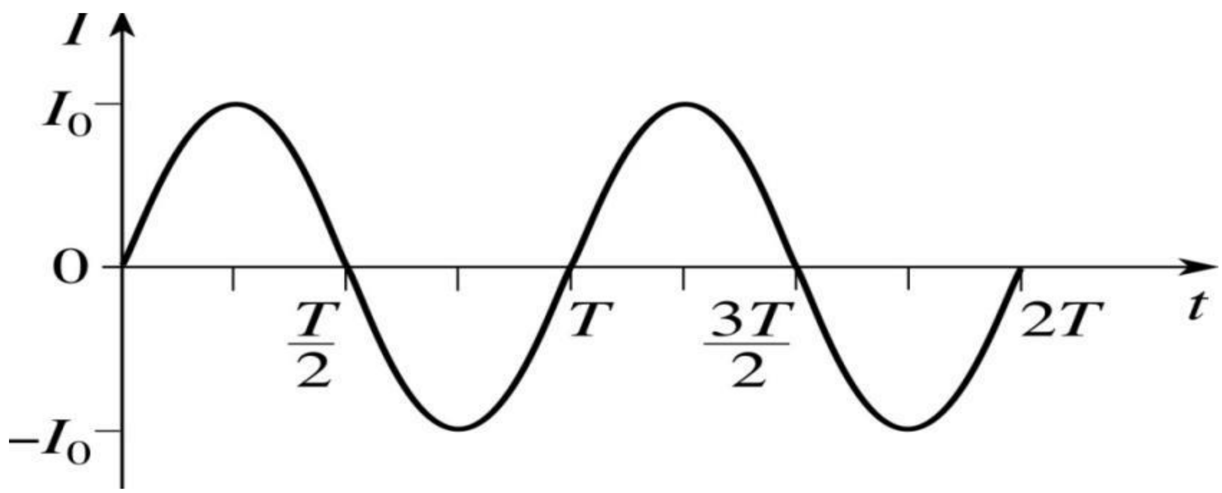
The variation of e.m.f is

$\mathcal{E} = \mathcal{E}_0 \sin \omega t$

And, the corresponding current is

$I = I_0 \sin \omega t$

Where, $\mathcal{E}_0$ and I_0 are the peak values of voltage and current respectively at any instant of time.



There are two methods in general to represent an AC circuit
i) The vector method
ii) The complex number method

The A.C network analysis becomes simple and convenient by the complex number representation.

A complex number can be written as

Z = x + i y Where, x is the real part and iy is the imaginary part

Average value (mean value) of alternating current over one cycle:

The value of current at any time t is given by

$I = I_0 \sin \omega t$

The average value of a sinusoidal wave over one complete cycle is given by

$$\bar{I} = \frac{\int_0^T I_0 \sin \omega t \, dt}{\int_0^T dt}$$

$$= \frac{I_0}{\omega} \left(-\cos \omega t \right)_0^T \times \frac{1}{T}$$

$$= -\frac{I_0}{\omega} \left(\cos \omega t \right)_0^T \times \frac{1}{T}$$

$$= -\frac{I_0 T}{2\pi} \left(\cos \omega t \right)_0^T \times \frac{1}{T}$$

$$= -\frac{I_0}{2\pi} \left[\cos \frac{2\pi}{T} T - \cos 0 \right]$$

$$= -\frac{I_0}{2\pi} \left[\cos 2\pi - \cos 0 \right]$$

$$= 0$$

Thus the average value of alternating current over complete cycle is zero.

Average value (mean value) of alternating current during half cycle:

The value of current at any time t is given by

I = I0 sin ωt .

The average value of a sinusoidal wave over half cycle is given by

$$\overline{I}_{first} = \frac{\int_0^{T/2} Io \sin \omega t \, dt}{\int_0^{T/2} dt}$$

$$= \frac{I_0}{\omega} \left(-\cos \omega t\right)_0^{T/2} \times \frac{1}{\frac{T}{2}}$$

$$= -\frac{I_0}{\omega} \left(\cos \omega t\right)_0^{T/2} \times \frac{2}{T}$$

$$= -\frac{2 \, Io}{T \, \frac{2\pi}{T}} \left(\cos \omega t\right)_0^{T/2}$$

$$= -\frac{Io}{\pi} \left(\cos \frac{2\pi}{T} \frac{T}{2} - \cos 0\right)$$

$$= -\frac{Io}{\pi} \left[\cos \pi - \cos 0\right]$$

$$= -\frac{Io}{\pi} \left[-1 - 1\right]$$

$$= 0.636 \, I_0$$

The average value for second half cycle is given by,

$$\overline{I} = \frac{\int_{T/2}^{T} Io\ \sin \omega t\, dt}{\int_{T/2}^{T} dt}$$

$$\overline{I}_{second} = - 0.636\ I_0$$

The average value of alternating current (Voltage) during the 1st and 2nd half cycles are equal but opposite in sign. i.e. they are alternatively positive and negative.

Root mean square value of A.C.:

The value of current at any time t is given by
I = I0 sin ωt

The average value of I^2

$$\overline{I^2} = \frac{\int_0^T I_o^2\ \sin^2 \omega t\, dt}{\int_0^T dt}$$

$$2 \sin^2 A = 1 - \cos 2A$$

$$= I_o^2 \times \frac{1}{T} \times \int_0^T \sin^2 \omega t\, dt$$

$$= I_o^2 \times \frac{\omega}{2\pi} \times \int_0^T 2\ \frac{\sin^2 \omega t}{2}\, dt$$

$$= \frac{I_0^2}{2} \times \frac{\omega}{2\pi} \times \int_0^T (1 - \cos 2\omega t)\, dt$$

$$= \frac{I_0^2}{2} \times \frac{\omega}{2\pi} \times \left[\ t\ -\ \frac{\sin 2\omega t}{2\omega}\ \right]_0^T$$

$$= \frac{I_0^2}{2} \times \frac{\omega}{2\pi} \times \left[T - \frac{\sin 2.\omega T}{2\omega}\right]$$

$$= \frac{I_0^2}{2} \times \frac{\omega}{2\pi} \times \left[\frac{2\Pi}{\omega} - \frac{\sin 2.\omega 2\Pi/\omega}{2\omega}\right]$$

$$= \frac{I_0^2}{2} \times \frac{\omega}{2\pi} \times \left(\frac{2\Pi}{\omega} - 0\right)$$

$$= \frac{I_0^2}{2}$$

Root mean square value of the alternating current

$$I_{r.m.s} = \sqrt{\frac{I_o^2}{2}}$$

$$= \frac{1}{\sqrt{2}} \times \text{Peak Value}$$

$I_{r.m.s}$ is also called the virtual value. It is represented by I_v

Circuit containing resistor and capacitor only:

Figure shows a circuit containing series combination of a resistor and a capacitor C with an alternating emf

$\mathcal{E} = \mathcal{E}_0 e^{i\omega t}$ …………… (1)

Instantaneous current I is given by

$$I = \frac{\text{Applied emf}}{\text{Vector impedance of the circuit}}$$

$$I = \frac{\varepsilon_0 e^{i\omega t}}{Z}$$

$$I = \frac{\varepsilon_0 e^{i\omega t}}{R + \frac{1}{i\omega c}}$$

$$I = \frac{\varepsilon_0 e^{i\omega t}}{R - \frac{i}{\omega c}} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

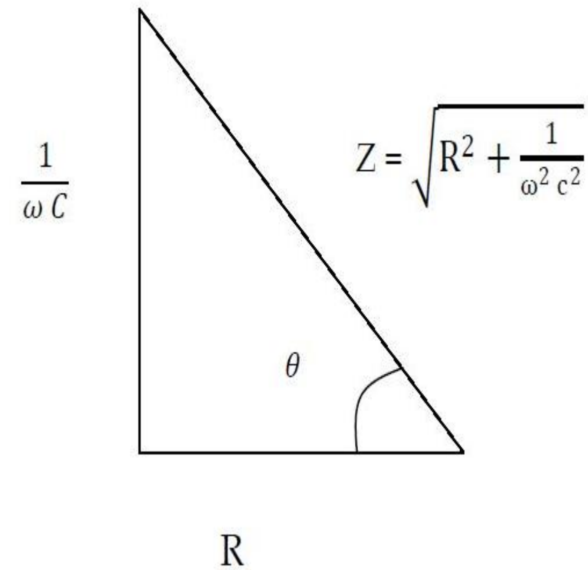
Where, R is the impedance due to the resistance and 1/i ω c = is the impedance due to the capacitance.

$$\frac{1}{R - \frac{i}{\omega c}} = \frac{R + \frac{i}{\omega c}}{\left(R - \frac{i}{\omega c}\right)\left(R + \frac{i}{\omega c}\right)}$$

$$= \frac{R + \frac{i}{\omega c}}{R^2 + \frac{1}{\omega^2 c^2}}$$

$$= \frac{R}{R^2 + \frac{1}{\omega^2 c^2}} + \frac{\frac{i}{\omega c}}{R^2 + \frac{1}{\omega^2 c^2}}$$

From the figure



$$\tan \Theta = \frac{1}{R \, \omega \, C}$$

$$\cos \Theta = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 c^2}}}$$

$$\sin \Theta = \frac{\frac{1}{\omega c}}{\sqrt{R^2 + \frac{1}{\omega^2 c^2}}}$$

Now,

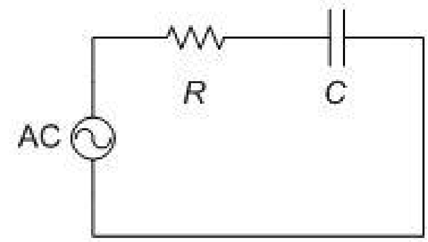
$$\frac{1}{R - \frac{i}{\omega c}} = \frac{1}{\sqrt{R^2 + \frac{1}{\omega^2 c^2}}} (\cos \Theta + i \sin \Theta)$$

$$= \frac{1}{\sqrt{R^2 + \frac{1}{\omega^2 c^2}}} e^{i\theta}$$

From equation number (1)

$$I = \frac{\varepsilon_o \, e^{\, i(\omega t + \Theta)}}{\sqrt{R^2 + \frac{1}{\omega^2 c^2}}}$$

Finally, the equation becomes



$$I = I_0 e^{i(\omega t + \theta)} \quad \ldots\ldots\ldots\ldots (2)$$

$$\text{Where, } I_0 = \frac{\varepsilon_o}{\sqrt{R^2 + \frac{1}{\omega^2 c^2}}}$$

Equation number (2) represents the variation of current with time and shows that current leads the applied voltage in phase by an angle

$$\theta = \tan^{-1} \frac{1}{R\,\omega\,C}$$

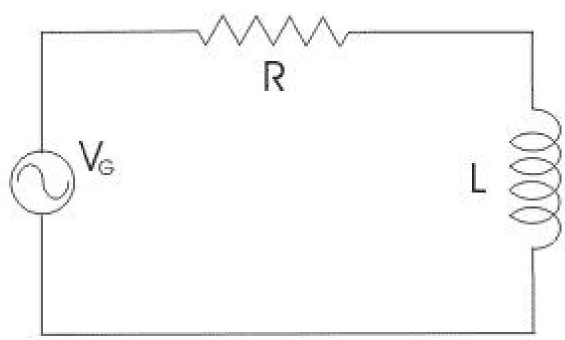
Circuit containing inductor and resistor only:
Instantaneous current in the circuit is given by

$$I = \frac{\text{Applied emf}}{\text{Vector impedance of the circuit}}$$

Vector impedance

$$Z = R + i\,\omega\,L$$

$$|Z| = \sqrt{R^2 + \omega^2 L^2}$$



Thus

$$I = \frac{\varepsilon}{Z} = \frac{\varepsilon_0 e^{i\omega t}}{R + i\,\omega\,L} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

$$\frac{1}{R + i\,\omega\,L} = \frac{(R - i\omega L)}{(R + i\omega L)\,(R - i\omega L)}$$

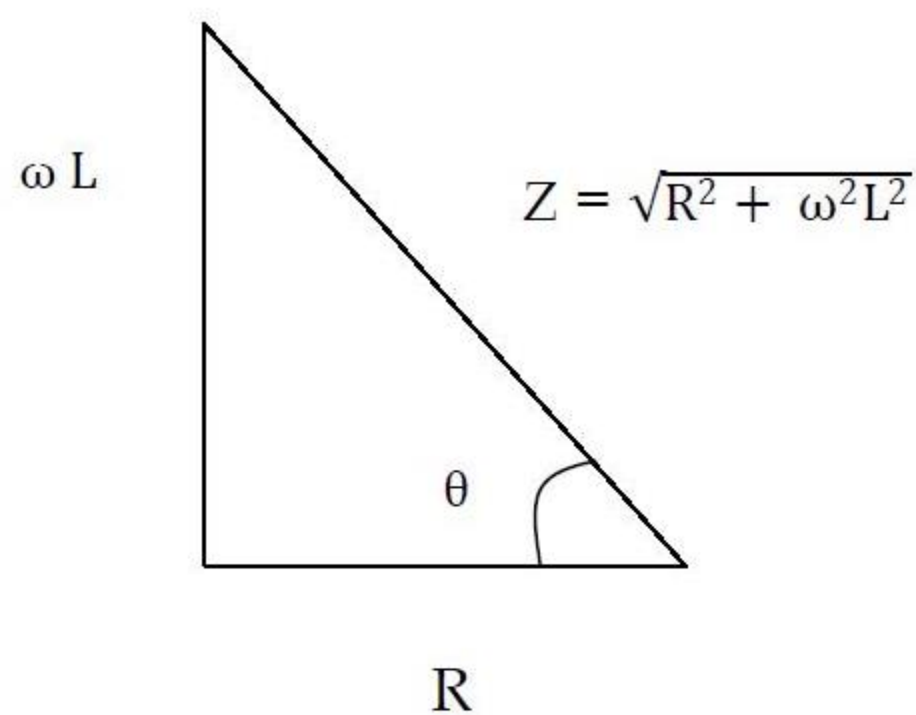
$$= \frac{(R - i\omega L)}{R^2 + \omega^2 L^2}$$

$$= \frac{R}{R^2 + \omega^2 L^2} - \frac{i\omega L}{R^2 + \omega^2 L^2}$$

From the figure

$$\tan \Theta = \frac{\omega L}{R}$$

$$\cos \Theta = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$



$$\sin \Theta = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}$$

Then

$$\frac{1}{R + i\,\omega\, L} = \frac{1}{\sqrt{R^2 + \omega^2 L^2}} (\cos \Theta - i \sin \Theta)$$

Finally, this equation becomes

$$I = \frac{\varepsilon_0\, e^{i(\omega t - \theta)}}{\sqrt{R^2 + \omega^2 L^2}} = I_0\, e^{i\,(\omega t - \theta)} \;\ldots\ldots\ldots\; (3)$$

Where, impedance, $Z = \sqrt{R^2 + \omega^2 L^2}$ and $I_0 = \frac{\varepsilon_0}{\sqrt{R^2 + \omega^2 L^2}}$

Equation (3) represents the variation of current in the circuit with time as shown in fig. Also indicates that the current lags behind the applied emf in phase by an angle,

$$\text{where } \theta = \tan^{-1} \frac{\omega L}{R}$$

A.C. Circuit containing inductor, resistor and capacitor:

Instantaneous emf,

$\mathcal{E} = \mathcal{E}_0\, e^{\,i\omega t}$ …………… (1)

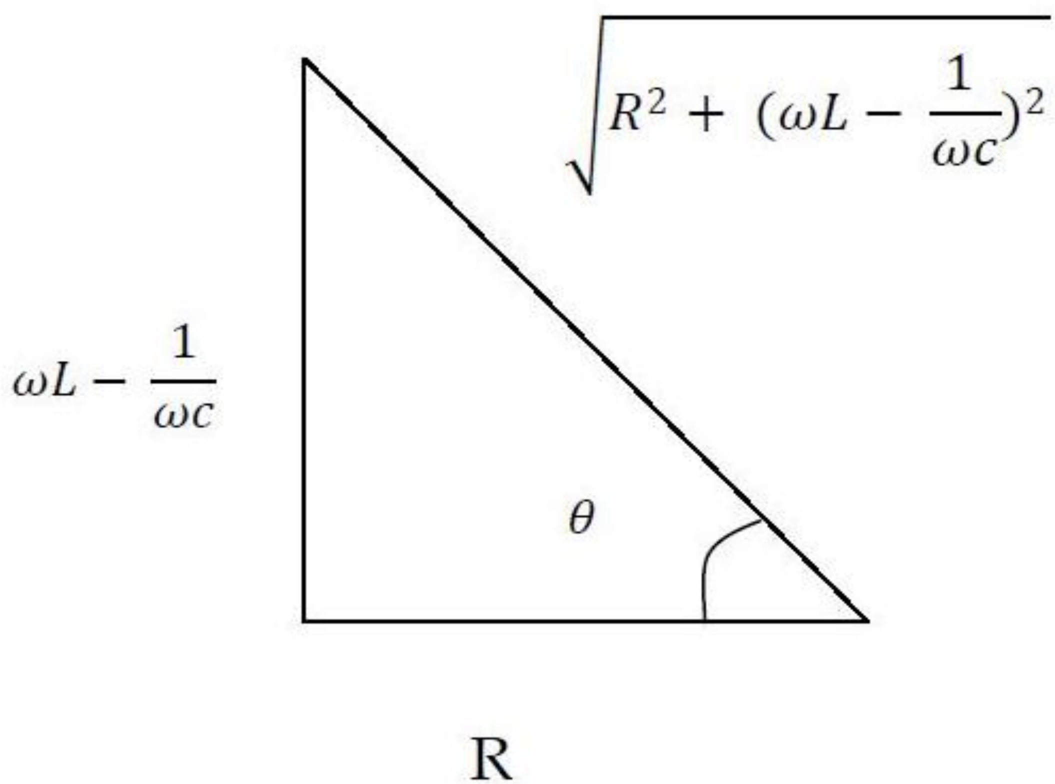
Instantaneous current

$$I = \frac{\varepsilon_0 e^{i\omega t}}{R + (i\omega L + \frac{1}{i\omega c})} = \frac{\varepsilon_0 e^{i\omega t}}{R + (i\omega L - \frac{i}{\omega c})}$$

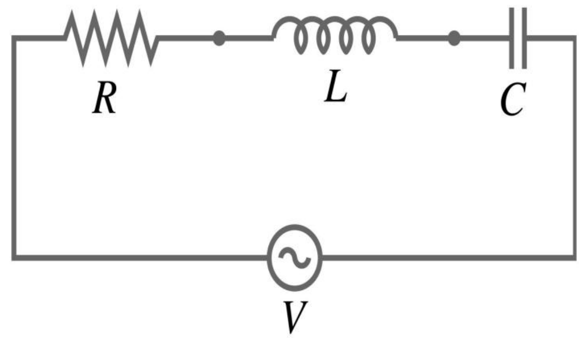
$$I = \frac{\varepsilon_o \; e^{i\omega t}}{R + i\,(\omega L - \frac{1}{\omega c})} \;\ldots\ldots\ldots\ldots\; (2)$$

$$\frac{1}{R + i\,(\omega L - \frac{1}{\omega c})} = \frac{R - i\,(\omega L - \frac{1}{\omega c})}{\left[R + i\left(\omega L - \frac{1}{\omega c}\right)\right]\left[R - i\left(\omega L - \frac{1}{\omega c}\right)\right]}$$

$$= \frac{R - i\,(\omega L - \frac{1}{\omega c})}{R^2 + (\omega L - \frac{1}{\omega c})^2}$$



From the figure



$$\tan\Theta = \frac{\omega L - \frac{1}{\omega c}}{R}$$

$$\cos\Theta = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega c})^2}}$$

$$\sin\Theta = \frac{\omega L - \frac{1}{\omega c}}{\sqrt{R^2 + (\omega L - \frac{1}{\omega c})^2}}$$

Hence

$$\frac{1}{R + i(\omega L - \frac{1}{\omega c})} = \frac{1}{\sqrt{R^2 + (\omega L - \frac{1}{\omega c})^2}} (\cos\Theta - i\sin\Theta)$$

$$= \frac{1}{\sqrt{R^2 + (\omega L - \frac{1}{\omega c})^2}} e^{-i\theta}$$

Equation number (2) becomes

$$I = \frac{\varepsilon_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega c})^2}} e^{i(\omega t - \theta)}$$

Finally, the equation becomes

$$I = I_0 e^{i(\omega t - \theta)} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

Where,

$$I_0 = \frac{\varepsilon_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega c})^2}}$$, I_0 is the peak value of the following current in the circuit.

Equation shows that, lags the applied voltage in phase by an angle θ given by

$$\theta = \tan^{-1} \frac{\omega L - \frac{1}{\omega c}}{R}$$