# CE 414: Prestressed Concrete
# Lecture 6
# Flexural Analysis

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# Contents

- Transfer of concentric prestress
- Shortening of concrete
- Pretension member and bulk head
- Pretensioned member with duct

of application of the prestress, then, by St. Venant's principle, the unit stress in concrete is uniform across that section and is given by the usual formula,

$$f = \frac{F}{A}$$

where $A$ is the area of that concrete section.

For a pretensioned member, when the prestress in the steel is transferred from the bulkheads to the concrete, Fig. 5-1, the force that was resisted by the bulkheads is now transferred to both the steel and the concrete in the member. The release of the resistance from the bulkheads is equivalent to the application of an opposite force $F_i$ to the member. Using the transformed section method, and with $A_c$ = net sectional area of concrete, the compressive stress produced in the concrete is

$$f_c = \frac{F_i}{A_c + nA_s} = \frac{F_i}{A_t} \tag{5-1}$$

while that induced in the steel is

$$\Delta f_s = nf_c = \frac{nF_i}{A_c + nA_s} = \frac{nF_i}{A_t} \tag{5-2}$$

which represents the immediate reduction of the prestress in the steel as a result of the transfer.

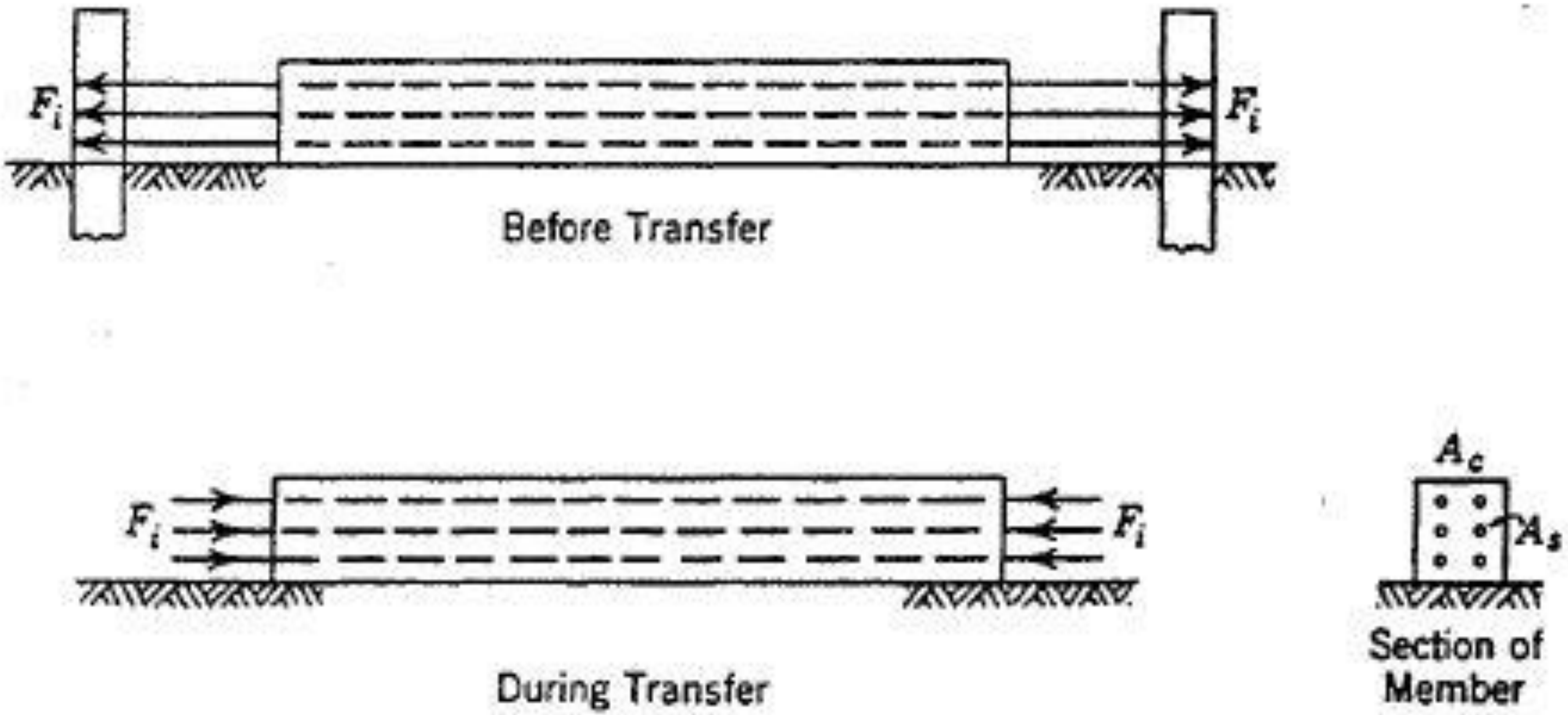


**Fig. 5-1.** Transfer of concentric prestress in a pretensioned member.

shortening of concrete and approximated by

$$\Delta f_s = \frac{nF_i}{A_c} \quad \text{or} \quad \frac{nF_i}{A_g} \tag{5-3}$$

which differs a little from formula 5-2 but is close enough for all practical purposes, since the total amount of reduction is only about 2 or 3% and the value of $n$ cannot be accurately known anyway. The high-strength steel used for

EXAMPLE 5-1

A pretensioned member, similar to that shown in Fig. 5-1, has a section of 8 in. by 12 in. (203 mm by 305 mm). It is concentrically prestressed with 0.8 sq in. (516 mm$^2$) of high-tensile steel wire, which is anchored to the bulkheads of a unit stress of 150,000 psi (1034 N/mm$^2$). Assuming that $n=6$, compute the stresses in the concrete and steel immediately after transfer.

*Solution*

1. An exact theoretical solution. Using the elastic theory, we have

$$f_c = \frac{F_i}{A_c + nA_s} = \frac{F_i}{A_g + (n-1)A_s}$$

$$= \frac{0.8 \times 150{,}000}{12 \times 8 + 5 \times 0.8} = 1200 \text{ psi } (8.3 \text{ N/mm}^2)$$

$$nf_c = 6 \times 1200 = 7200 \text{ psi } (49.6 \text{ N/mm}^2)$$

Stress in steel after transfer = 150,000 − 7200 = 142,800 psi (985 $N/mm^2$).

*Solution*

2. An approximate solution. The loss of prestress in steel due to elastic shortening of

concrete is estimated by

$$= n\frac{F_i}{A_g}$$

$$= 6\frac{120{,}000}{8 \times 12} = 7500 \text{ psi } (51.7 \text{ N/mm}^2)$$

Stress in steel after loss = 150,000 − 7500 = 142,500 psi (983 N/mm²). Stress in concrete is

$$f_c = \frac{142{,}500 \times 0.8}{96} = 1190 \text{ psi } (8.2 \text{ N/mm}^2)$$

Note that, in this second solution, the approximations introduced are: (1) using gross area of concrete instead of net area, (2) using the initial stress in steel instead of the reduced stress. But the answers are very nearly the same for both solutions. The second method is more convenient and is usually followed.

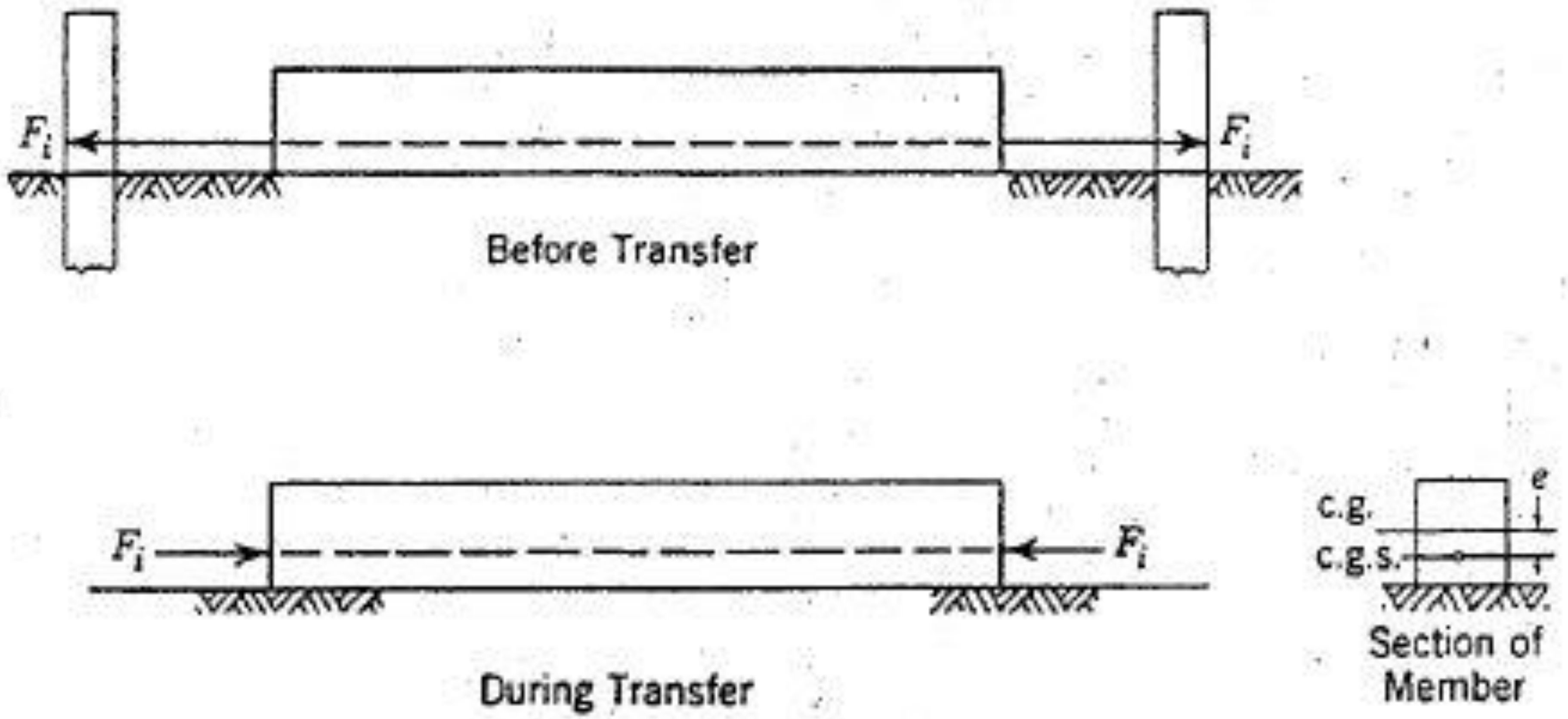


Fig. 5-3. Transfer of eccentric prestress in a pretensioned member.

EXAMPLE 5-2

A pretensioned member similar to that shown in Fig. 5-3 has a section of 8 in. by 12 in. (203 mm by 305 mm) deep. It is eccentrically prestressed with 0.8 sq in. (516 $mm^2$) of high-tensile steel wire which is anchored to the bulkheads at a unit stress of 150,000 psi (1034 $N/mm^2$). The c.g.s. is 4 in. (101.6 mm) above the bottom fiber. Assuming that $n=6$, compute the stresses in the concrete immediately after transfer due to the prestress only.

*Solution*

1. An exact theoretical solution. Using the elastic theory, the centroid of the transformed section and its moment of inertia are obtained as follows. Referring to Fig. 5-4, for $(n-1)A_s = 5 \times 0.8 = 4$ sq in.,

$$y_0 = \frac{4 \times 2}{96+4} = 0.08 \text{ in. } (2.032 \text{ mm})$$

$$I_t = \frac{8 \times 12^3}{12} + 96 \times 0.08^2 + 4 \times 1.92^2$$

$$= 1152 + 0.6 + 14.7$$

$$= 1167.3 \text{ in.}^4 \ (485.9 \times 10^6 \text{ mm}^4)$$

$$\text{Top fiber stress} = \frac{F_i}{A_t} + \frac{F_i e y}{I_t}$$

$$= \frac{-120{,}000}{100} + \frac{120{,}000 \times 1.92 \times 6.08}{1167.3}$$

$$= -1200 + 1200$$

$$= 0$$

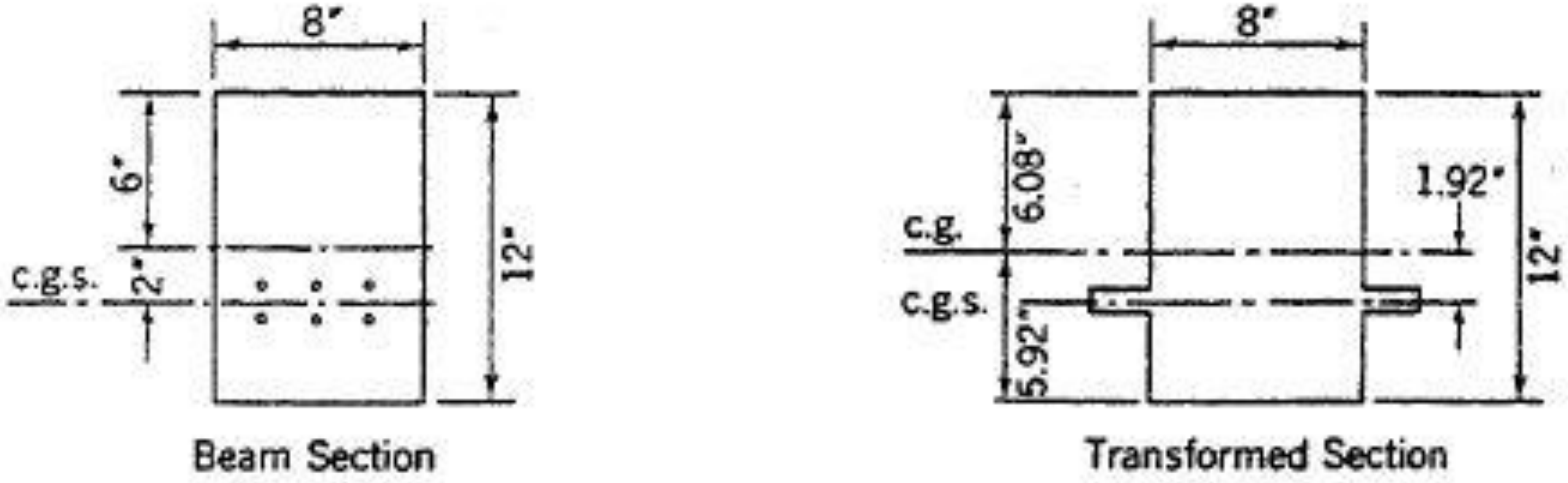


**Fig. 5-4. Example 5-2.**

$$\text{Bottom fiber stress} = \frac{-120{,}000}{100} - \frac{120{,}000 \times 1.92 \times 5.92}{1167.3}$$

$$= -1200 - 1170$$

$$= -2370 \text{ psi } (-16.34 \text{ N/mm}^2)$$

*Solution*

2. An approximate solution. The loss of prestress can be approximately computed, as in example 5-1, to be 7500 psi in the steel. Hence the reduced prestress is 142,500 psi or 114,000 lb. Extreme fiber stresses in the concrete can be computed to be

$$f_c = \frac{F}{A} \pm \frac{Fey}{I}$$

$$= \frac{-114{,}000}{96} \pm \frac{114{,}000 \times 2 \times 6}{(8 \times 12^3)/12}$$

$$= -1187 \pm 1187$$

$$= 0 \text{ in the top fiber}$$

$$= -2374 \text{ psi } (-16.37 \text{ N/mm}^2) \text{ in the bottom fiber}$$

The approximations here introduced are: using an approximate value of reduced prestress, and using the gross area of concrete. This second solution, although approximate, is more often used because of its simplicity.

## EXAMPLE 5-3

A posttensioned beam has a midspan cross section with a duct of 2 in. by 3 in. (50.8 mm by 76.2 mm) to house the wires, as shown in Fig. 5-6. It is prestressed with 0.8 sq in. (516 $mm^2$) of steel to an initial stress of 150,000 psi (1034 $N/mm^2$). Immediately after transfer the stress is reduced by 5% owing to anchorage loss and elastic shortening of concrete. Compute the stresses in the concrete at transfer.

*Solution*

1. Using net section of concrete. The centroid and $I$ of the net concrete section are computed as follows

$$A_c = 96 - 6 = 90 \text{ sq in. } (58.1 \times 10^3 \text{ mm}^2)$$

$$y_0 = \frac{6 \times 3}{96 - 6} = 0.2 \text{ in. } (5.08 \text{ mm})$$

$$I = \frac{8 \times 12^3}{12} + 96 \times 0.2^2 - \frac{2 \times 3^3}{12} - 6 \times 3.2^2$$

$$= 1152 + 3.8 - 4.5 - 61.5$$

$$= 1090$$

Total prestress in steel $= 150{,}000 \times 0.8 \times 95\% = 114{,}000$ lb (507 kN)

$$f_c = \frac{-114{,}000}{90} \pm \frac{114{,}000 \times 3.2 \times 5.8}{1090}$$

$$= -1270 + 1940 = +670 \text{ psi } (+4.62 \text{ N/mm}^2) \text{ for top fiber}$$

$$f_c = -1270 - 2070 = -3340 \text{ psi } (-23.03 \text{ N/mm}^2) \text{ for bottom fiber}$$

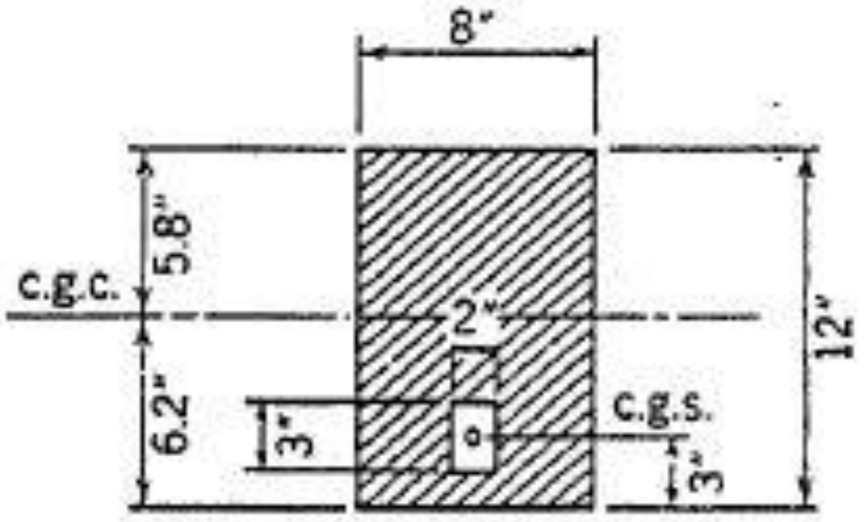


**Fig. 5-6. Example 5-3.**

*Solution*

2. Using the gross section of concrete. An approximate solution using the gross concrete section would give results not so close in this case (11% difference):

$$f_c = \frac{-114{,}000}{96} \pm \frac{114{,}000 \times 3 \times 6}{(8 \times 12^3)/12}$$

$$= -1270 + 1940 = +1783$$

$$= +596 \text{ psi } (+4.11 \text{ N/mm}^2) \text{ for top fiber}$$

$$= -2970 \text{ psi } (-20.48 \text{ N/mm}^2) \text{ for bottom fiber}$$