CE 414: Prestressed Concrete
Lecture 7
Flexural Analysis (Contd. I)

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- Post-tensioned bonded concrete
- Variation of steel stress with load
- Bonded and unboned tendon stress
- Modulus of rupture

EXAMPLE 5-4

A posttensioned bonded concrete beam, Fig. 5-8, has a prestress of 350 kips (1,557 kN) in the steel immediately after prestressing, which eventually reduces to 300 kips (1334 kN)

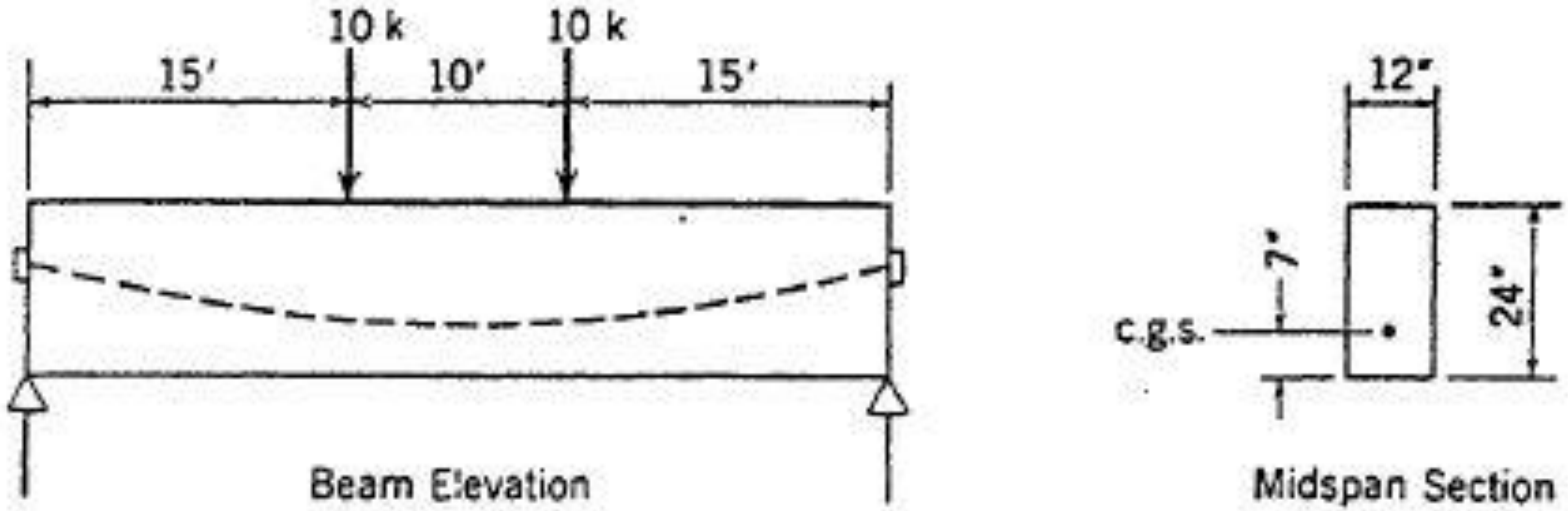

Fig. 5-8. Example 5-4.

due to losses. The beam carries two live loads of 10 kips (44.48 kN) each in addition to its own weight of 300 plf (4.377 kN/m). Compute the extreme fiber stresses at midspan, (*a*) under the initial condition with full prestress and no live load, and (*b*) under the final condition, after the losses have taken place, and with full live load.

Solution To be theoretically exact, the net concrete section should be used up to the time of grouting, after which the transformed section should be considered. This is not deemed necessary, and an approximate but sufficiently exact solution is given below, using the gross section of concrete at all times that is,

$$I = 12 \times 24^3/12 = 13{,}800 \text{ in.}^4\ (5744 \times 10^6 \text{ mm}^4)$$

1. *Initial condition*. Dead-load moment at midspan, assuming that the beam is simply supported after prestressing:

$$M = \frac{wL^2}{8} = \frac{300 \times 40^2}{8} = 60{,}000 \text{ ft-lb}\ (81{,}360 \text{ N} - \text{m})$$

$$f = \frac{F}{A} \pm \frac{Fey}{I} \pm \frac{My}{I}$$

$$= \frac{-350{,}000}{288} \pm \frac{350{,}000 \times 5 \times 12}{13{,}800} \pm \frac{60{,}000 \times 12 \times 12}{13{,}800}$$

$$= -1215 + 1520 - 625 = -320 \text{ psi } (-2.21 \text{ N/mm}^2), \text{ top fiber}$$

$$= -1215 - 1520 + 625 = -2110 \text{ psi } (-14.55 \text{ N/mm}^2), \text{ bottom fiber}$$

2. *Final condition.* Live-load moment at midspan = 150,000 ft-lb (203,400 N-m); therefore, total external moment = 210,000 ft-lb (284,760 N-m), while the prestress is reduced to 300,000 lb (1,334 kN); hence,

$$f = \frac{-300,000}{288} \pm \frac{300,000 \times 5 \times 12}{13,800} \pm \frac{210,000 \times 12 \times 12}{13,800}$$

$$= -1040 + 1300 - 2190 = -1930 \text{ psi } (-13.31 \text{ N/mm}^2), \text{ top fiber}$$

$$= -1040 - 1300 + 2190 = -150 \text{ psi } (-1.03 \text{ N/mm}^2), \text{ bottom fiber}$$

EXAMPLE 5-5

For the same problem as in example 5-4, compute the concrete stresses under the final loading conditions by locating the center of pressure C for the concrete section.

Solution Referring to Fig. 5-10, a is computed by

$$a = (210 \times 12)/300 = 8.4 \text{ in. } (213 \text{ mm})$$

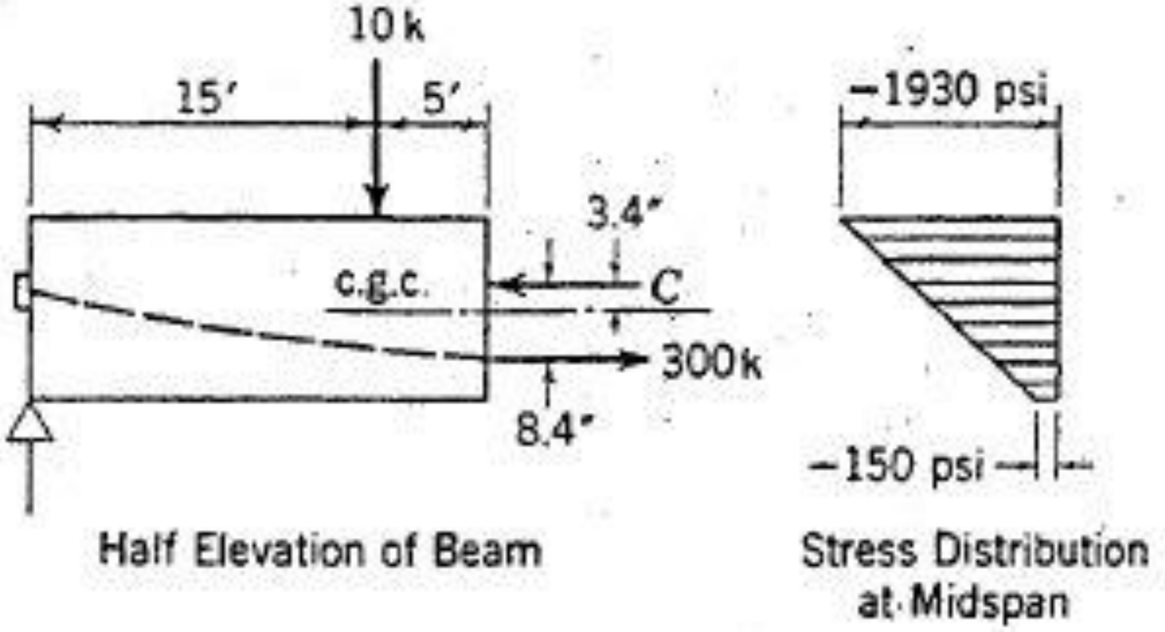

Fig. 5-10. Example 5-5.

Hence e for C is $8.4-5=3.4$ in. Since $C=F=300{,}000$ lb (1,334 kN).

$$f=\frac{C}{A}\pm\frac{Cey}{I}$$

$$=\frac{-300{,}000}{288}\pm\frac{300{,}000\times 3.4\times 12}{13{,}800}$$

$$=-1040-890=-1930 \text{ psi } (-13.31\ \text{N/mm}^2), \text{ top fiber}$$

$$=-1040+890=-150 \text{ psi } (-1.03\ \text{N/mm}^2), \text{ bottom fiber}$$

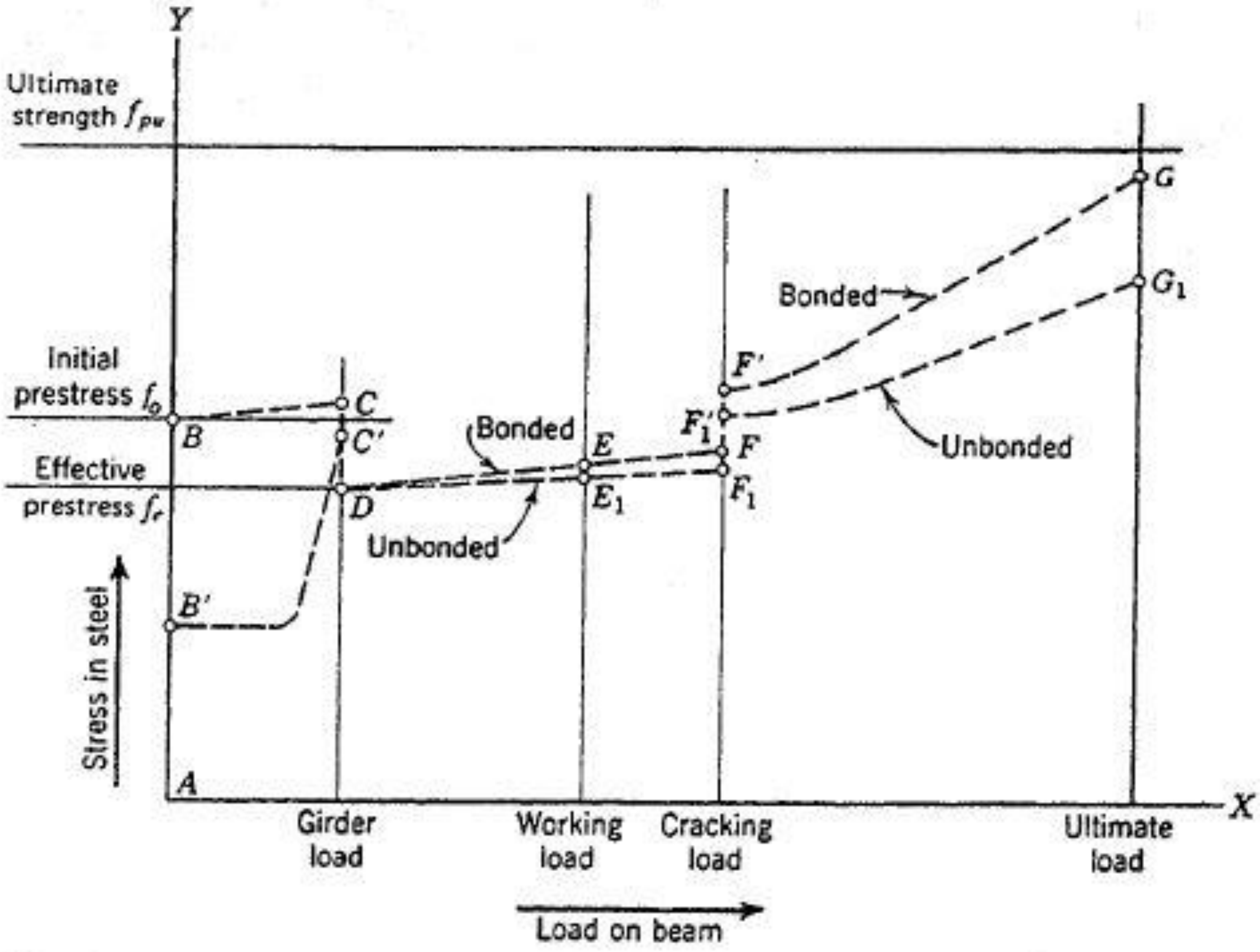

Fig. 5-11. Variation of steel stress with load.

EXAMPLE 5-6

A posttensioned simple beam on a span of 40 ft is shown in Fig. 5-13. It carries a superimposed load of 750 plf in addition to its own weight of 300 plf. The initial prestress in the steel is 138,000 psi, reducing to 120,000 psi after deducting all losses and assuming no bending of the beam. The parabolic cable has an area of 2.5 sq in., $n = 6$. Compute the stress in the steel at midspan, assuming: (1) the steel is bonded by grouting: (2) the steel is unbonded and entirely free to slip. (Span = 12.2 m, superimposed load = 10.94 kN/m, self-weight = 4377 kN/m, initial prestress = 951.5 N/mm^2, effective prestress = 827.4 N/mm^2, and cable area = 1613 mm^2.)

Solution

1. Moment at midspan due to dead and live loads is

$$\frac{wL^2}{8} = \frac{(300+750)40^2}{8}$$

$$= +210{,}000 \text{ ft-lb } (+284{,}760 \text{ N-m})$$

Moment at midspan due to prestress is

$$2.5 \times 120{,}000 \times \tfrac{5}{12} = -125{,}000 \text{ ft-lb } (-169{,}500 \text{ N-m})$$

Net moment at midspan is 210,000 − 125,000 = 85,000 ft-lb (115,260 N − m). Stress in concrete at the level of steel due to bending, using I of gross concrete section, is

$$= \frac{My}{I} = \frac{85,000 \times 12 \times 5}{13,800} = 370 \text{ psi } (2.55 \text{ N/mm}^2)$$

Stress in steel is thus increased by

$$f_s = nf_c = 6 \times 370 = 2220 \text{ psi } (15.31 \text{ N/mm}^2)$$

Resultant stress in steel = 122,220 psi (842.7 N/mm^2) at midspan.

Solution

2. If the cable is unbonded and free to slip, the average strain or stress must be obtained for the whole length of cable as given by formula 5-10,

$$f_s = \frac{n}{L} \int \frac{My}{I} dx$$

Using y_0 and M_0 for those at midspan and measuring x from the midspan, we can

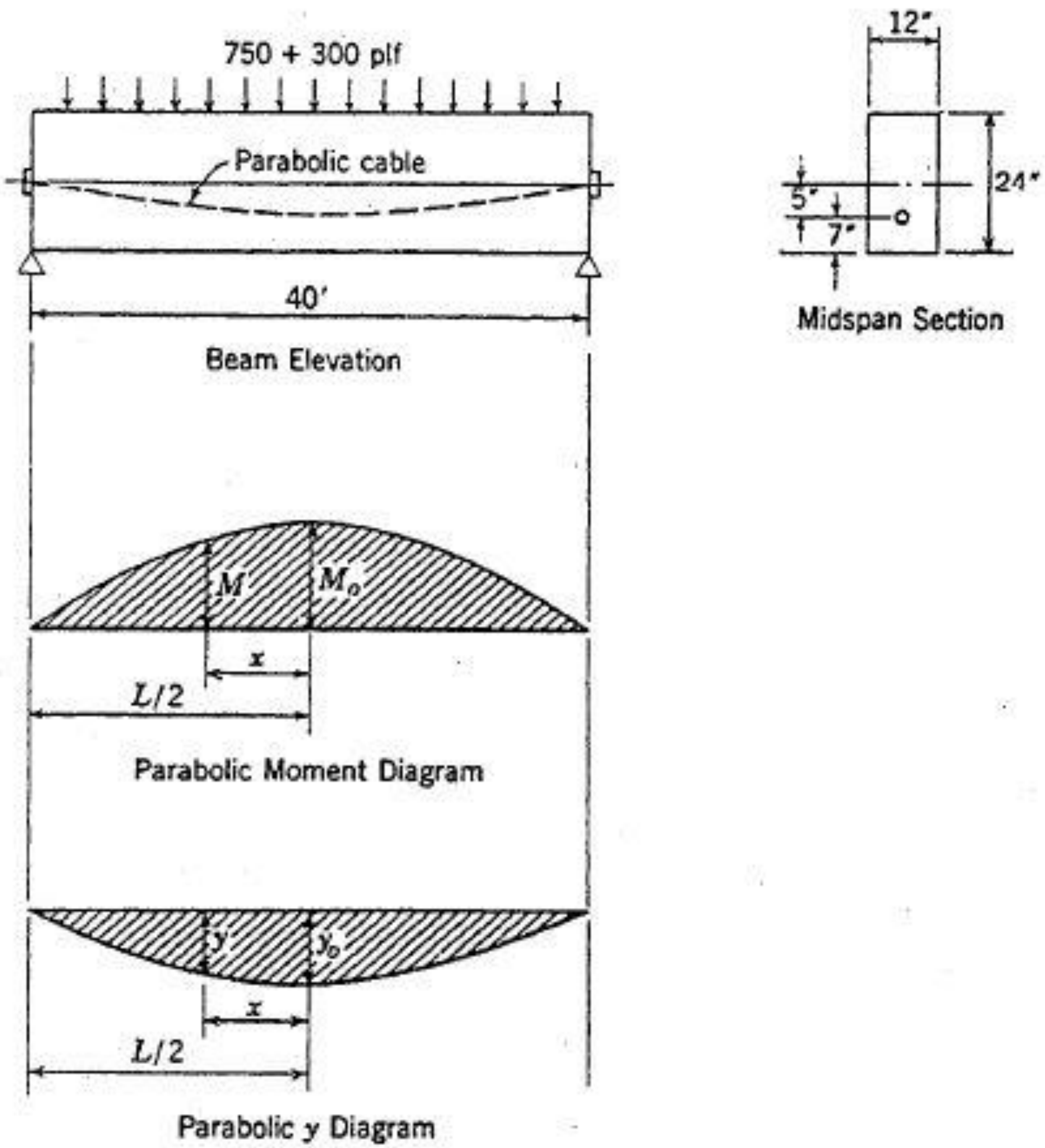

Fig. 5-13. Example 5-6.

express y and M in terms of x, thus,

$$M = M_0\left[1-\left(\frac{x}{L/2}\right)^2\right]$$

$$y = y_0\left[1-\left(\frac{x}{L/2}\right)^2\right]$$

$$f_s = \frac{n}{LI}\int_{-L/2}^{+L/2} M_0 y_0\left[1-\left(\frac{x}{L/2}\right)^2\right]^2 dx$$

$$= \frac{nM_0y_0}{LI}\left[x - \frac{2}{3}\frac{x^3}{(L/2)^2} + \frac{x^5}{5(L/2)^4}\right]_{-L/2}^{+L/2}$$

$$= \frac{8}{15}\left(\frac{nM_0y_0}{I}\right)$$

which is $\frac{8}{15}$ of the stress for midspan of the bonded beam, or $\frac{8}{15}(2220) = 1180$ psi (8.14 N/mm^2).

Resultant stress in steel is $120{,}000 + 1180 = 121{,}180$ psi (835.5 N/mm^2) throughout the entire cable. In this calculation, the I of the gross concrete section is used and the effect of the increase in the steel stress on the concrete stresses is also neglected. But these are errors of the second order. Since the change in steel stress is relatively small, exact computations are seldom required in an actual design problem.

5-5 Cracking Moment

The moment producing first hair cracks in a prestressed concrete beam is computed by the elastic theory, assuming that cracking starts when the tensile stress in the extreme fiber of concrete reaches its modulus of rupture. Questions have been raised as to the correctness of this method. First, some engineers

used. The ACI Code value for modulus of rupture, f_r, is $7.5\sqrt{f'_c}$ with units for both f_r and f'_c as psi.

bottom fiber. The resisting moment is given by the prestress F times its lever arm measured to the top kern point, (see Appendix A for definition of kern points k_t and k_b), Fig. 5-14, thus,

$$M_1 = F\left(e + \frac{r^2}{c}\right)$$

Additional moment resisted by the concrete up to its modulus of rupture is $M_2 = f_r I/c$. Hence the total moment at cracking is given by

$$M = M_1 + M_2 = F\left(e + \frac{r^2}{c}\right) + \frac{f_r I}{c} \tag{5-12}$$

which can be seen to be identical with formula 5-11.

In order to be theoretically correct when applying the above two formulas, care must be exercised in choosing the proper section for the computation of I, r, e, and c. For computing the term $f_r I/c$, the transformed section should be used for bonded beams, while the net concrete section should be used for unbonded beams (proper modification being made for the value of prestress due

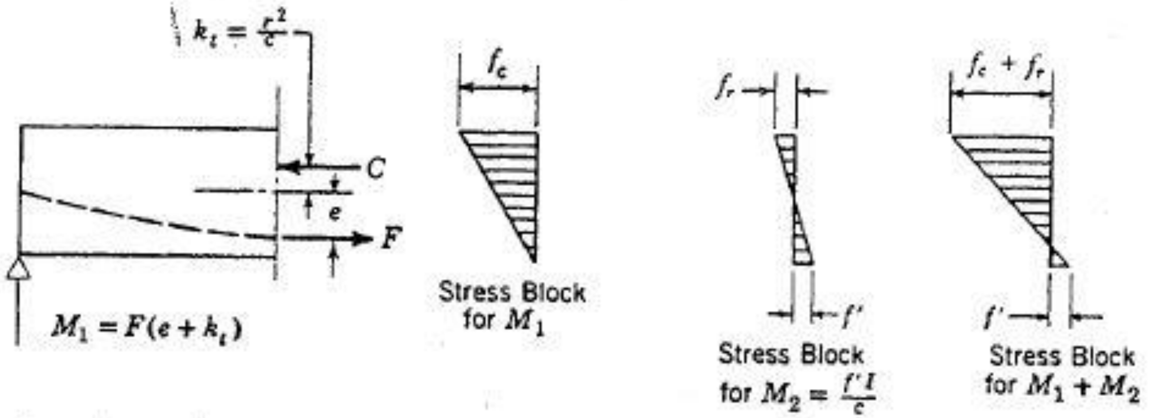

Fig. 5-14. Cracking moment.

to bending of the beam as explained in section 4-8). For the term $F\left(e+\frac{r^2}{c}\right)$, either the gross or the net section should be considered, depending on the computation of the effective prestress F. For a practical problem, these refine-

EXAMPLE 5-7

For the problem given in example 5-6, compute the total dead and live uniform load that can be carried by the beam, (1) for zero tensile stress in the bottom fibers, (2) for cracking in the bottom fibers at a modulus of rupture of 600 psi (4.14 N/mm^2), and assuming concrete to take tension up to that value.

Solution

1. Considering the critical midspan section and using the gross concrete section for all computations, k_t is readily computed to be at 4 in. (101.6 mm) above the middepth, Fig. 5-15. To obtain zero stress in the bottom fibers, the center of pressure must be located at the top kern point. Hence the resisting moment is given by the prestress multiplied by the lever arm, thus

$$F(e+k_t)=300(5+4)/12=225 \text{ k-ft } (305.1 \text{ kN-m})$$

Solution

2. Additional moment carried by the section up to beginning of cracks is

$$\frac{f_r I}{c} = \frac{600 \times 13{,}800}{12}$$

$$= 690{,}000 \text{ in.-lb}$$

$$= 57.6 \text{ k-ft } (78.1 \text{ kN} - \text{m})$$

Total moment at cracking is $225 + 57.6 = 282.6$ k-ft (383.2 kN-m), which can also be obtained directly by applying formula 5-11 or 5-12.

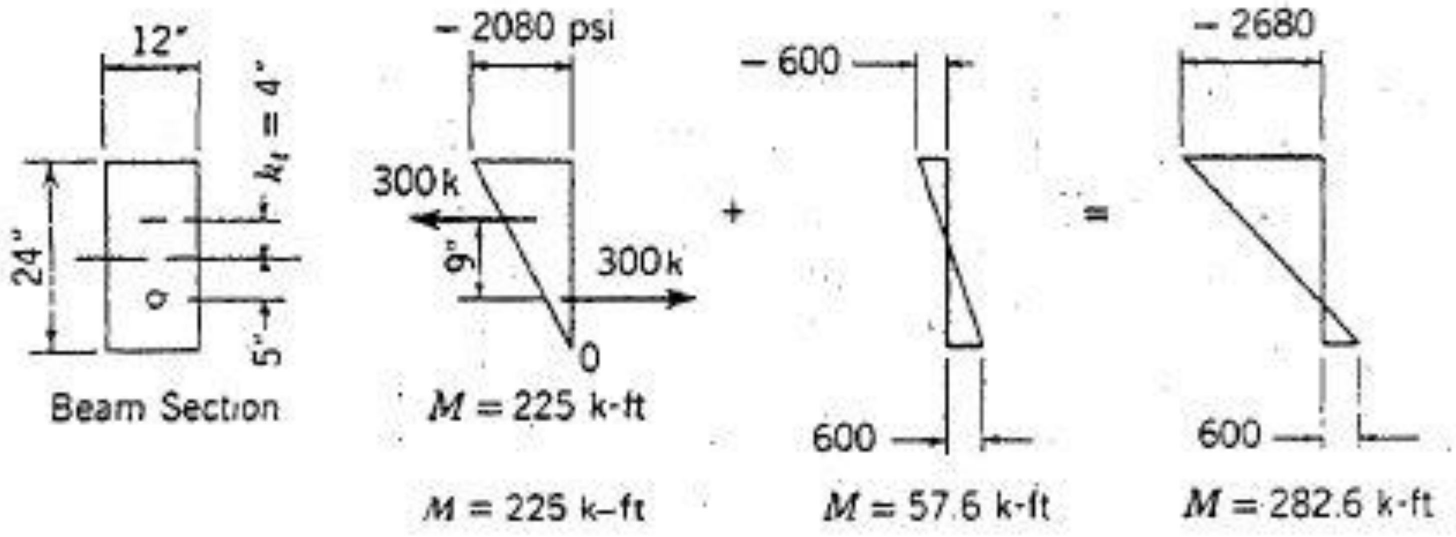

Fig. 5-15. Example 5-7.