CE 414: Prestressed Concrete
Lecture 10
Flexural Analysis (Composite)

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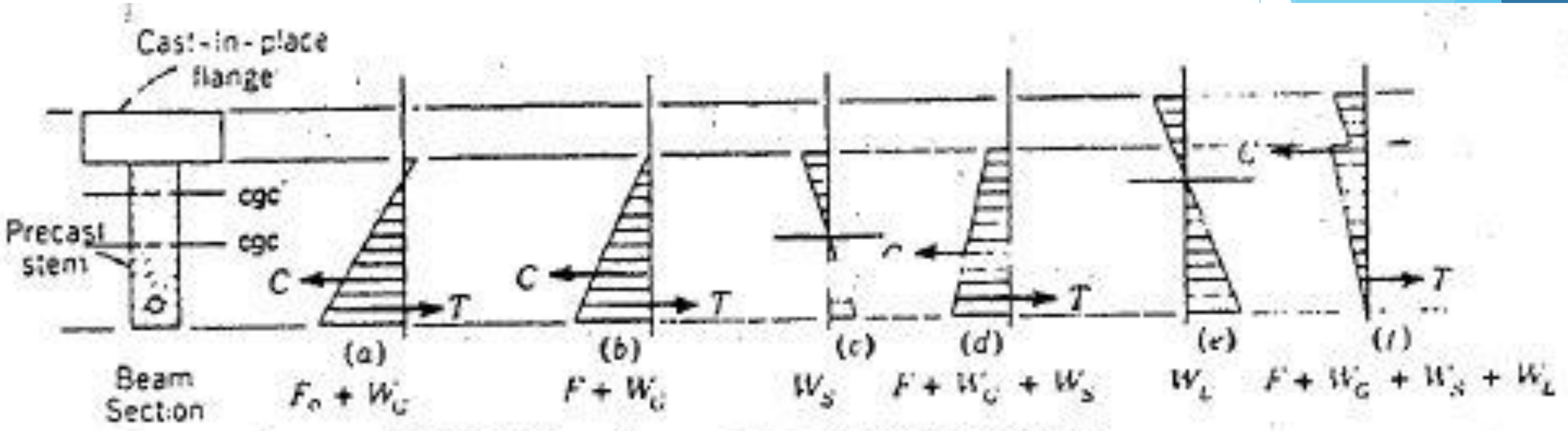

Fig. 5-33. Stress distribution for a composite section.

In the same figure, stress distributions are shown for various stages of loading. These are discussed as follows.

(*a*) Owning to the initial prestress and the weight of the stem, there will be heavy compression in the lower fibers and possibly some small tension in the top fibers. The tensile force *T* in the steel and the compressive force *C* in the concrete form a resisting couple with a small lever arm between them.

(*b*) After losses have taken place in the prestress, the effective prestress together with the weight of the stem will result in a slightly lower compression in the bottom fibers and some small tension or compression in the top fibers. The *C*-*T* couple will act with a slightly greater lever arm.

(*c*) Owning to the addition of the slab, its weight produces additional moment and stress as shown.

(*d*) Owning to the effective prestress plus the weight of the stem and slab, we can add (*b*) to (*c*) and a somewhat smaller compression is found to exist at the bottom fibers and some compression at the top fibers. The lever arm for the *C*-*T* couple further increases.

(*e*) Stresses resulting from live load moment are shown, the moment being resisted by the composite section.

(*f*) Adding (*d*) to (*e*), we have stress block as in (*f*), with slight tension or compression in the bottom fibers, but with high compressive stresses in the top fibers of the stem and the slab. The couple *T* and *C* now acts with an appreciable lever arm.

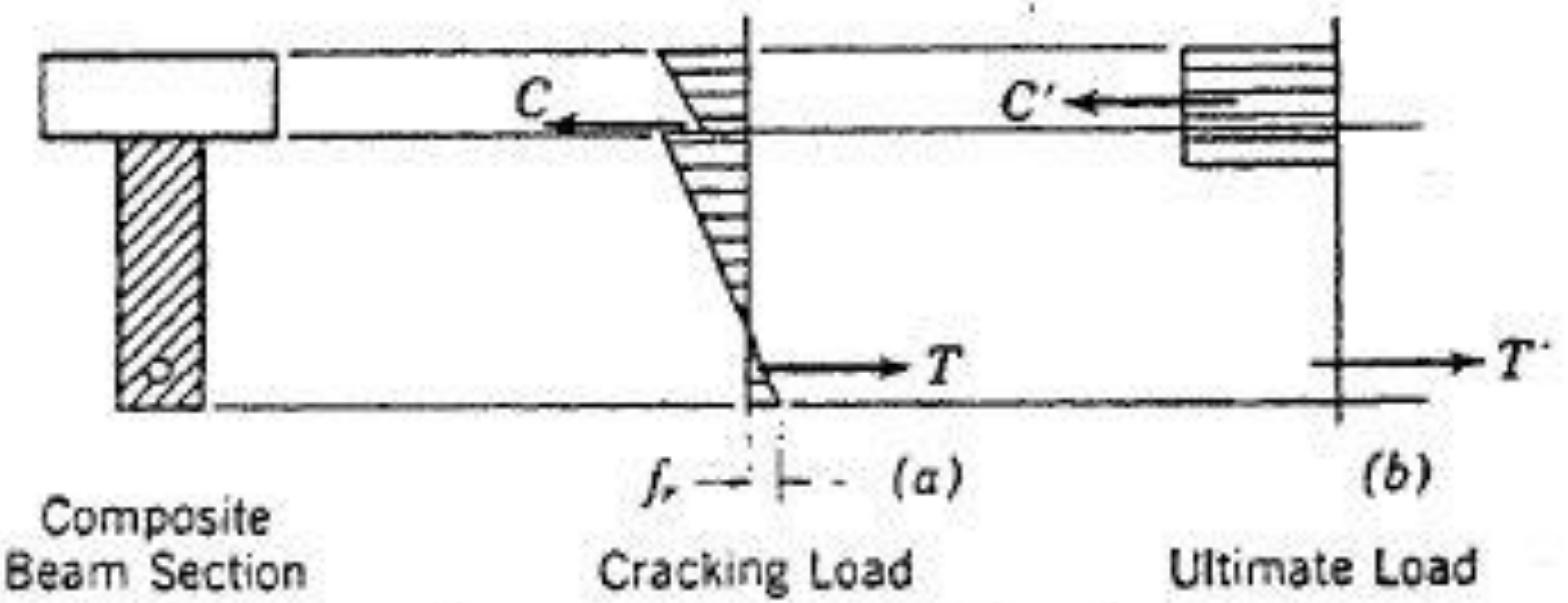

Fig. 5-34. Stress distributions for cracking and ultimate loads.

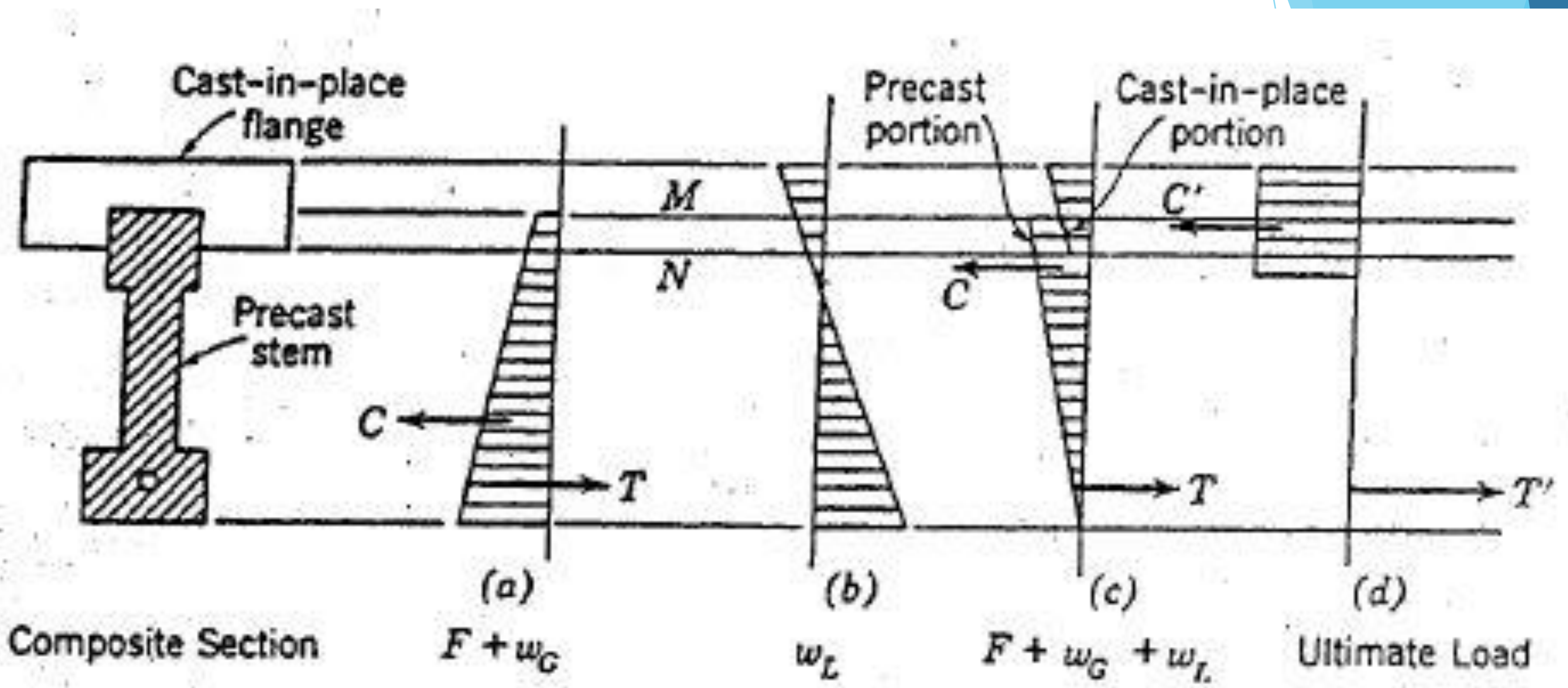

Fig. 5-35. Stress distribution for a special composite section.

EXAMPLE 5-12

The midspan section of a composite beam is shown in Fig. 5-37. The precast stem 12 in. by 36 in. (305 mm by 915 mm) deep is posttensioned with an initial force of 550 kips (2446 kN), Fig. 5-37(*a*). The effective prestress after losses is taken as 480 kips (2135 kN). Moment due to the weight of that precast section is 200 k-ft (271.2 kN-m) at midspan. After it is erected in place, the top slab of 6 in. by 36 in. (152 mm by 915 mm) wide is to be cast in place producing a moment of 100 k-ft (135.6 kN-m). After the slab concrete

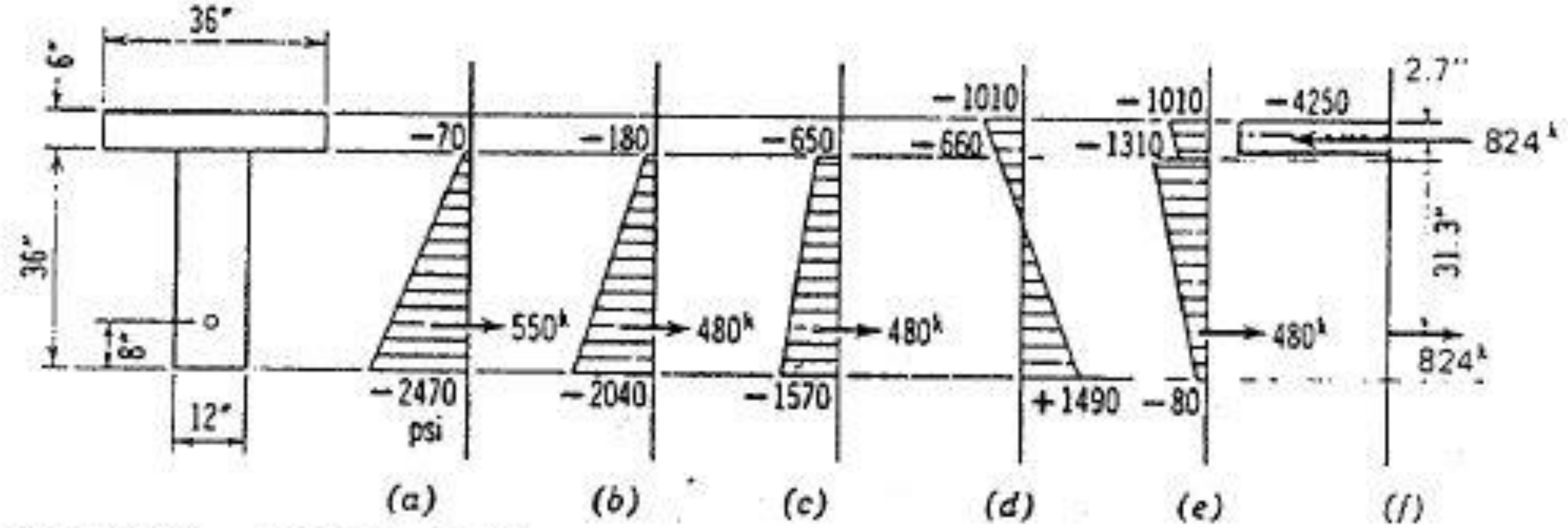

Fig. 5-37. Example 5-12.

750 kN-m. Compute stresses in the section at various stages. $A_{ps} = 2400$ mm², $f_{pu} = 1650$ MPa $f_c' = 34$ MPa Estimate the ultimate moment.

Solution C.g.c. of the composite section is located at 638 mm from the bottom fiber. The area and moment of inertia of the rectangular and the composite sections are computed and listed below:

	Rectangular Section	Composite Section
Area, mm²	2.76×10^5	4.14×10^5
I, mm⁴	1.95×10^{10}	4.62×10^{10}

(*a*) Immediately after prestressing, the stresses in the rectangular section will be

$$f = \frac{F}{A} \pm \frac{(M - Fe)c}{I}$$

$$= \frac{-2450 \times 10^3}{2.76 \times 10^5} \pm \frac{(270 \times 10^6 - 2450 \times 10^3 \times 260)460}{1.95 \times 10^{10}}$$

$$= -8.88 \pm 8.66$$

$= -0.22$ MPa top fiber

$= -17.54$ MPa bottom fiber

(*b*) After loss of prestress, the stresses will be

$$f = \frac{-2150 \times 10^3}{2.76 \times 10^5} \pm \frac{(270 \times 10^6 - 2150 \times 10^3 \times 260)460}{1.95 \times 10^{10}}$$

$$= -7.79 \pm 6.82$$

$= -0.97$ MPa top fiber

$= -14.61$ MPa bottom fiber

$= -8.88 \pm 8.66$

$= -0.22$ MPa top fiber

$= -17.54$ MPa bottom fiber

(*b*) After loss of prestress, the stresses will be

$$f = \frac{-2150\times10^3}{2.76\times10^5} \pm \frac{(270\times10^6 - 2150\times10^3\times260)460}{1.95\times10^{10}}$$

$= -7.79 \pm 6.82$

$= -0.97$ MPa top fiber

$= -14.61$ MPa bottom fiber

(*c*) After pouring top slab, the stresses will be 270+135=405 kN·m

$$f = \frac{-2150\times10^3}{2.76\times10^5} \pm \frac{(405\times10^6 - 2150\times10^3\times260)460}{1.95\times10^{10}}$$

$= -7.79 \pm 3.63$

$= -4.16$ MPa top fiber

$= -11.42$ MPa bottom fiber

(*d*) The live load acts on the composite section, producing stresses,

$$f = \frac{-750\times10^6\times432}{4.62\times10^{10}} = -7.01 \text{ MPa top fiber of composite section}$$

$\frac{Mc}{I}$

$$f = \frac{750\times10^6\times638}{4.62\times10^{10}} = +10.36 \text{ MPa bottom fiber}$$

By proportioning, the stress at top fiber of the rectangular portion is found to be 4.57 MPa due to this live load.

(*e*) The combined stresses due to prestress and dead and live loads are given in Fig. 5-37(*e*), which yields −1.06 MPa for bottom fiber and −8.73 MPa for top fiber of the rectangular section.

(*f*) The ultimate moment capacity of the section can be estimated as follows.

$$\rho_p = \frac{2400}{(920)(870)} = 0.003 \qquad \rho_p = \frac{A_{ps}}{b_t d}$$

$$f_{ps} = f_{pu}\left(1 - 0.5\rho_p \frac{f_{pu}}{f'_c}\right) = 1650\left[1-(0.5)(0.003)\left(\frac{1650}{34}\right)\right]$$

$$f_{ps} = 1530 \text{ MPa}$$

Total tensile force at ultimate $= f_{ps}A_{ps}$.

$$T' = 1530 \times 2400 = 3672 \text{ kN} \quad \nearrow 39$$

Area of compression concrete, for an average stress of $0.85f'_c = 29$ MPa, is

$$\frac{3672 \times 10^3}{29} = 126 \times 10^3 \text{ mm}^2$$

or a depth of $126 \times 10^3/920 = 137$ mm. The center of compressive force is about $137/2 = 69$ mm from top; hence the lever arm for the resisting moment is $1070 - 69 - 200 = 801$ mm, and the ultimate moment capacity is

$$3672 \times 0.801 = 2941 \text{ kN-m} \quad 750 + 270 + 135$$

The total applied dead and live load moment is only 1155 kN-m, indicating a load factor (factor of safety) of $2941/1155 = 2.5$.

The ACI code introduces the strength reduction factor $\phi = 0.9$, which would change the indicated load factor to $0.9 \times 2941/1155 = 2.3$. This is more than adequate for factored loads of $1.4D + 1.7L$ required by the ACI.