CE 414: Prestressed Concrete
Lecture 12
Flexural Design (final design)

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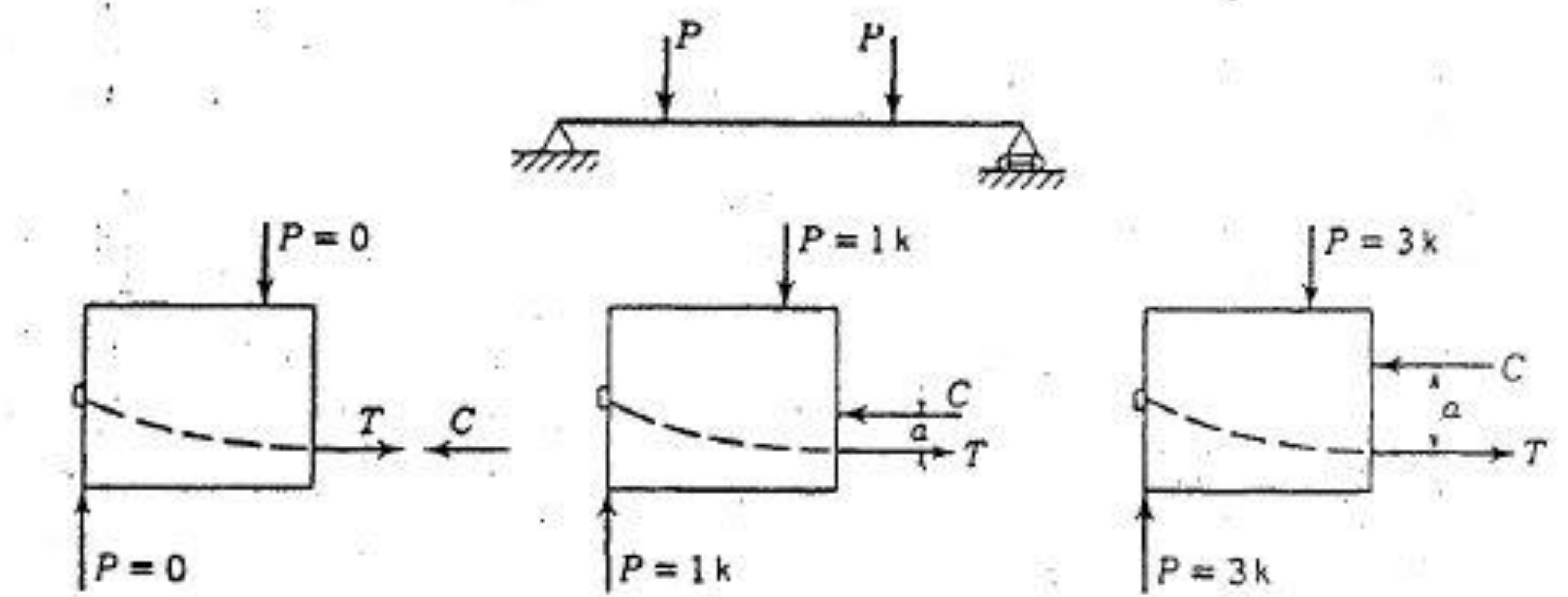
Contents

- Resisting moment in PC beam and RC beam
- Small M_G/M_T ratio
- Steps of final design



(a) External Moment = 0,
$a = 0$

(b) Small External Moment,
a is small;

(c) Large External Moment,
a is large.

Fig. 6-4. Variable *a* in a prestressed-concrete beam.

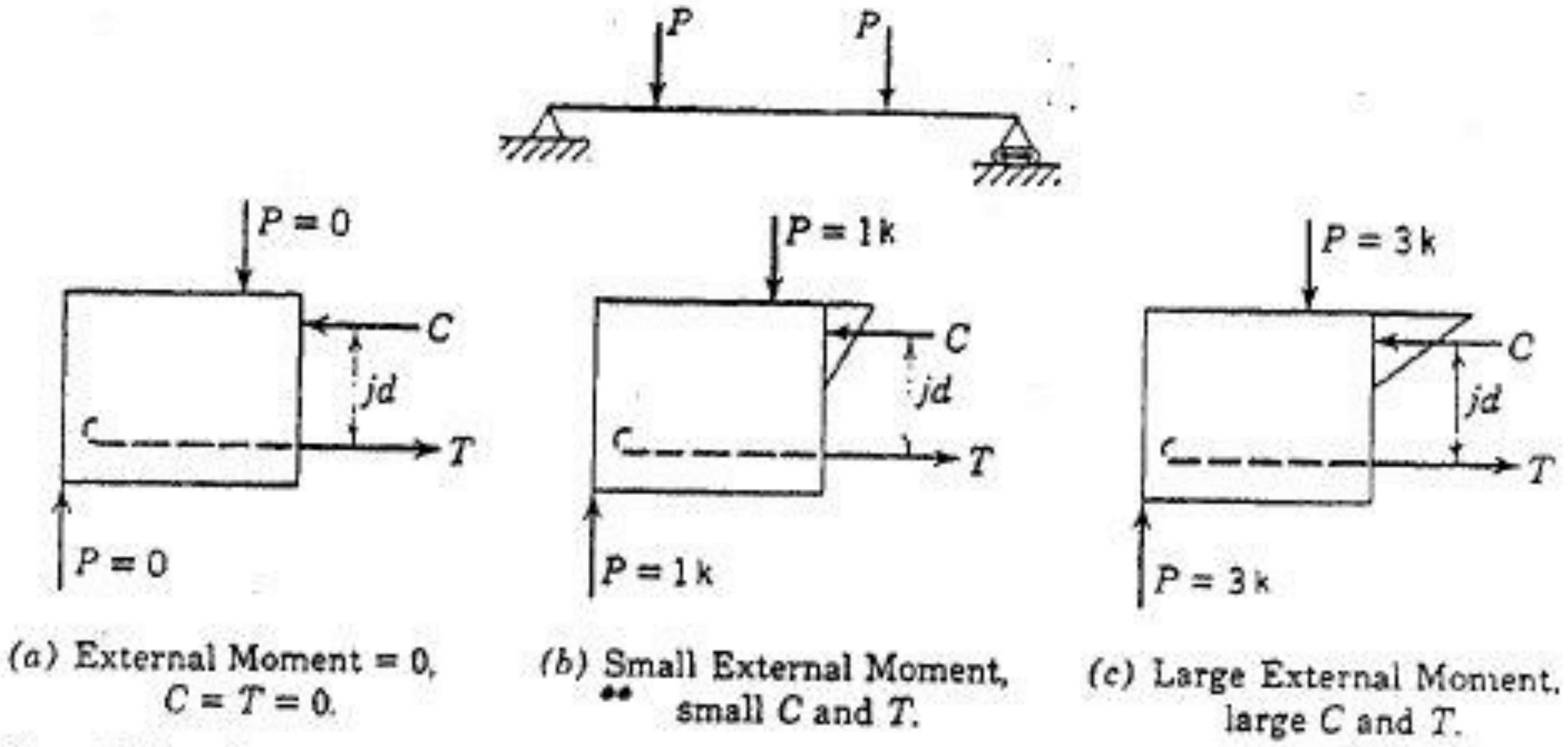


Fig. 6-5. Constant *jd* in a reinforced-concrete beam.

1. In a reinforced-concrete-beam section, as the external bending moment increases, the magnitude of the forces C and T is assumed to increase in direct proportion while the lever arm jd between the two forces remains practically unchanged, Fig. 6-5.
2. In a prestressed-concrete-beam section under working load, as the external bending moment increases, the magnitude of C and T remains practically constant while the lever arm a lengthens almost proportionately, Fig. 6-4.

Since the location of T remains fixed, we get a variable location of C in a prestressed section as the bending moment changes. For a given moment M, C can be easily located, since

$$Ca = Ta = M \tag{6-5}$$

$$a = M/C = M/T \tag{6-5a}$$

Thus, when $M=0$, $a=0$, and C must coincide with T, Fig. 6-4(a). When M is small, a is also small, Fig. 6-4(b). When M is large, a is also large, Fig. 6-4(c).

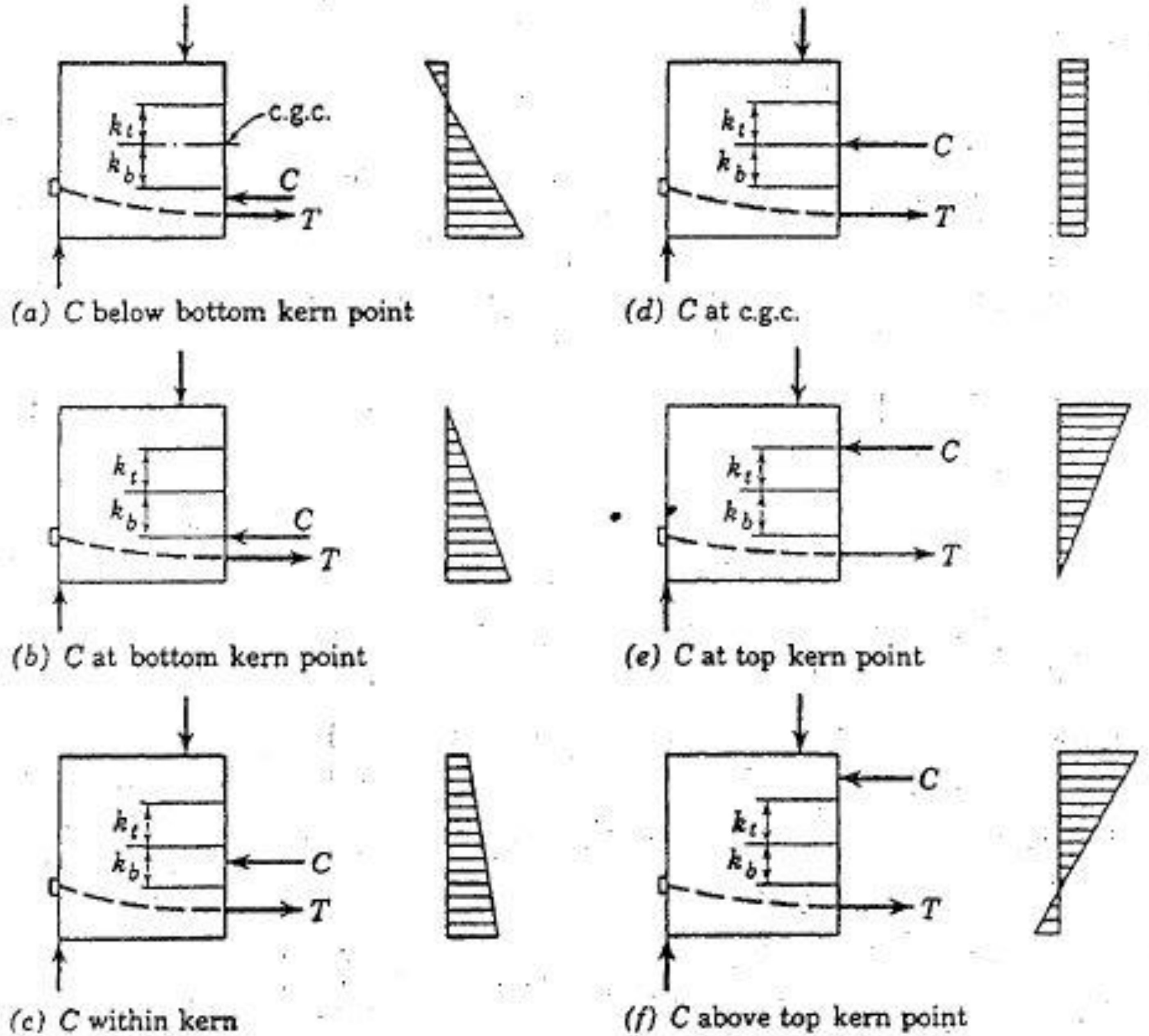


Fig. 6-6. Stress distribution in concrete by the elastic theory.

Small Ratios of M_G/M_T. For the section obtained from the preliminary design, the values of M_G, k_t, k_b, A_c are computed. When the ratio of M_G/M_T is small, c.g.s. is located outside the kern just as much as the M_G will allow. Since no tension is permitted in the concrete, c.g.s. will be located below the kern by the amount of Fig. 6-7(b).

$$e-k_b=M_G/F_0 \tag{6-6}$$

If c.g.s. is so located, C will be exactly at the bottom kern point for the given M_G, and the stresses at the top and bottom fibers will be

$$f_t=0$$

$$f_b=\frac{F_0}{A_c}\frac{h}{c_t}\leqslant 0.6f'_{ci} \tag{6-7}$$

Hence,

$$A_c=\frac{F_0 h}{f_b c_t} \tag{6-7a}$$

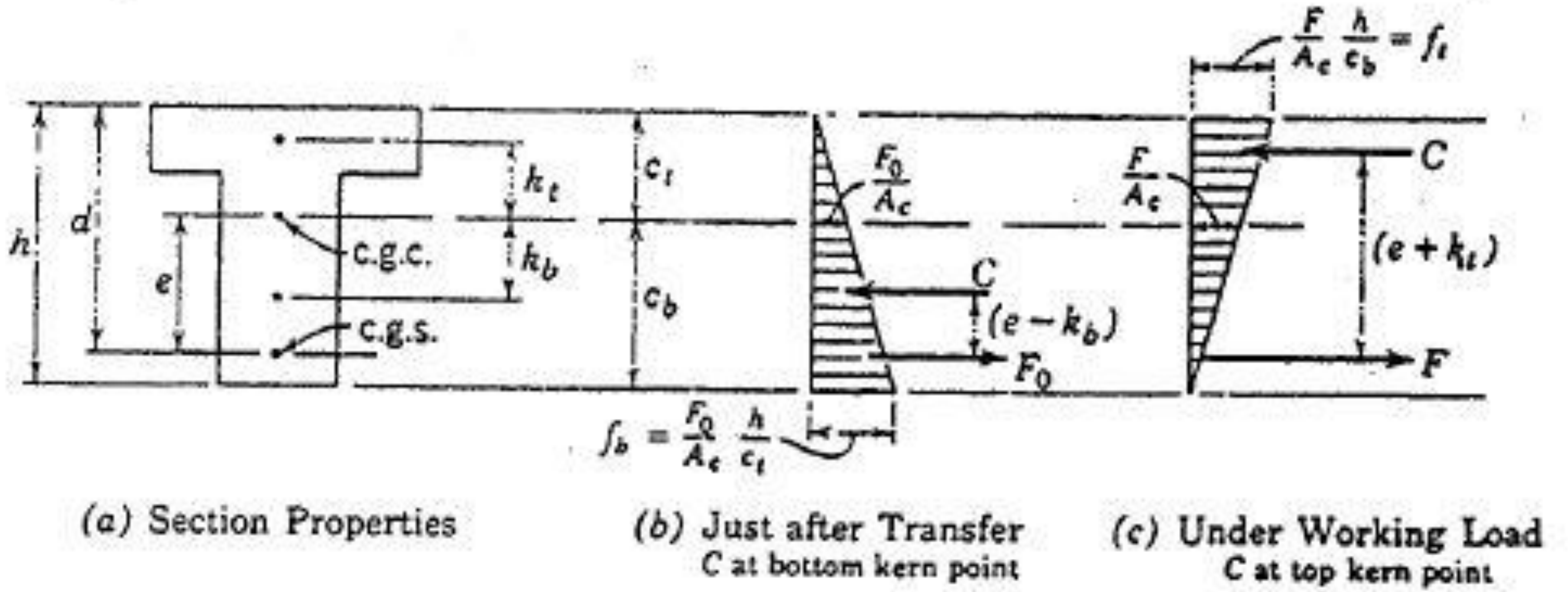


Fig. 6-7. Stress distribution, no tension in concrete (small ratios of M_G/M_T).

be stressed higher than given by equation 6-7. The bottom fiber allowable stress is 0.60 f'_{ci} following the ACI Code and F_0 is the prestress force acting at transfer.

With c.g.s. located as above, the available lever arm for the resisting moment is given by $e+k_t$, and the effective prestress F is given by

$$F=\frac{M_T}{e+k_t} \tag{6-8}$$

Under the action of this effective prestress F and the total moment M_T, C will be located at the top kern point, and the top and bottom fiber stresses are Fig. 6-7 (c),

$$f_b=0$$

$$f_t=\frac{F}{A_c}\frac{h}{c_b}<0.45f'_c \tag{6-9}$$

Hence,

$$A_c=\frac{Fh}{f_t c_b} \tag{6-9a}$$

To summarize the procedure of design, we have:

Step 1. From the preliminary design section, locate c.g.s. by

$$e-k_b=M_G/F_0$$

Step 2. With the above location of c.g.s., compute the effective prestress F (and then the initial prestress F_0) by

$$F=\frac{M_T}{e+k_t}$$

Step 3. Compute the required A_c by

$$A_c=F_0h/f_bc_t$$

and

$$A_c = Fh/f_t c_b$$

Step **4.** Revise the preliminary section to meet the above requirements for F and A_c. Repeat steps 1 through 4 if necessary.

From the above discussion, the following observations regarding the properties of a section can be made.

1. $e+k_t$ is a measure of the total moment-resisting capacity of the beam section. Hence the greater this value, the more desirable is the section.
2. $e-k_b$ locates the c.g.s. for the section, and is determined by the value of M_G. Thus, within certain limits, the amount of M_G does not seriously affect the capacity of the section for carrying M_L.
3. h/c_b is the ratio of the maximum top fiber stress to the average stress on the section under working load. Thus, the smaller this ratio, the lower will be the maximum top fiber stress.
4. h/c_t is the ratio of the maximum bottom fiber stress to the average stress on the section at transfer. Hence, the smaller this ratio, the lower will be the maximum bottom fiber stress.

EXAMPLE 6-3

For the preliminary section obtained in example 6-2, make a final design, allowing $f_b = -1.80$ ksi, $f_0 = 150$ ksi. Other given values were: $M_T = 320$ k-ft; $M_G = 40$ k-ft; $f_t = -1.60$ ksi; $f_{se} = 125$ ksi; $F = 187$ k. And the preliminary section is the same as in Fig. 6-3 ($f_b = -12.41$ N/mm^2, $f_0 = 1034$ N/mm^2, $M_T = 434$ kN-m, $M_G = 54$ kN-m, $f_t = -11.03$ N/mm^2, $f_{se} = 862$ N/mm^2, and $F = 832$ kN).

Solution For the trial preliminary section, compute the properties as follows

$$A_c = 2 \times 4 \times 15 + 4 \times 28 = 232 \text{ sq in. } (150 \times 10^3 \text{ mm}^2)$$

$$I = \frac{15 \times 36^3}{12} - \frac{11 \times 28^3}{12}$$

$$= 58{,}200 - 20{,}100$$

$$= 38{,}100 \text{ in.}^4 \; (15.86 \times 10^9 \text{ mm}^4)$$

$$r^2 = 38{,}100/232$$

$$= 164 \text{ in.}^2 \; (106 \times 10^3 \text{ mm}^2)$$

$$k_t = k_b = 164/18 = 9.1 \text{ in. } (231 \text{ mm})$$

Step 1. For an assumed

$$F = 187 \text{ k } (832 \text{ kN})$$

$$F_0 = \frac{150}{125} 187 = 225 \text{ k } (1001 \text{ kN})$$

c.g.s. should be located at $e - k_b$ below the bottom kern, where

$$e - k_b = \frac{M_G}{F_0} = \frac{40 \times 12}{225} = 2.1 \text{ in. } (53 \text{ mm})$$

$$e = 9.1 + 2.1 = 11.2 \text{ in. } (285 \text{ mm})$$

Step 2. Effective prestress required is recomputed as

$$F = \frac{M_T}{e + k_t} = \frac{320 \times 12}{11.2 + 9.1}$$

$$= 189 \text{ k } (841 \text{ kN})$$

$$F_0 = \frac{150}{125} 189 = 227 \text{ k } (1010 \text{ kN})$$

Step 3. A_c required is

$$A_c = \frac{F_0 h}{f_b c_t}$$

$$= \frac{227 \times 36}{1.80 \times 18}$$

$$= 252 \text{ sq in. } (163 \times 10^3 \text{ mm}^2) \text{ controlling}$$

$$A_c = \frac{Fh}{f_t c_b}$$

$$= \frac{189 \times 36}{1.60 \times 18}$$

$$= 236 \text{ sq in. } (152 \times 10^3 \text{ mm}^2)$$

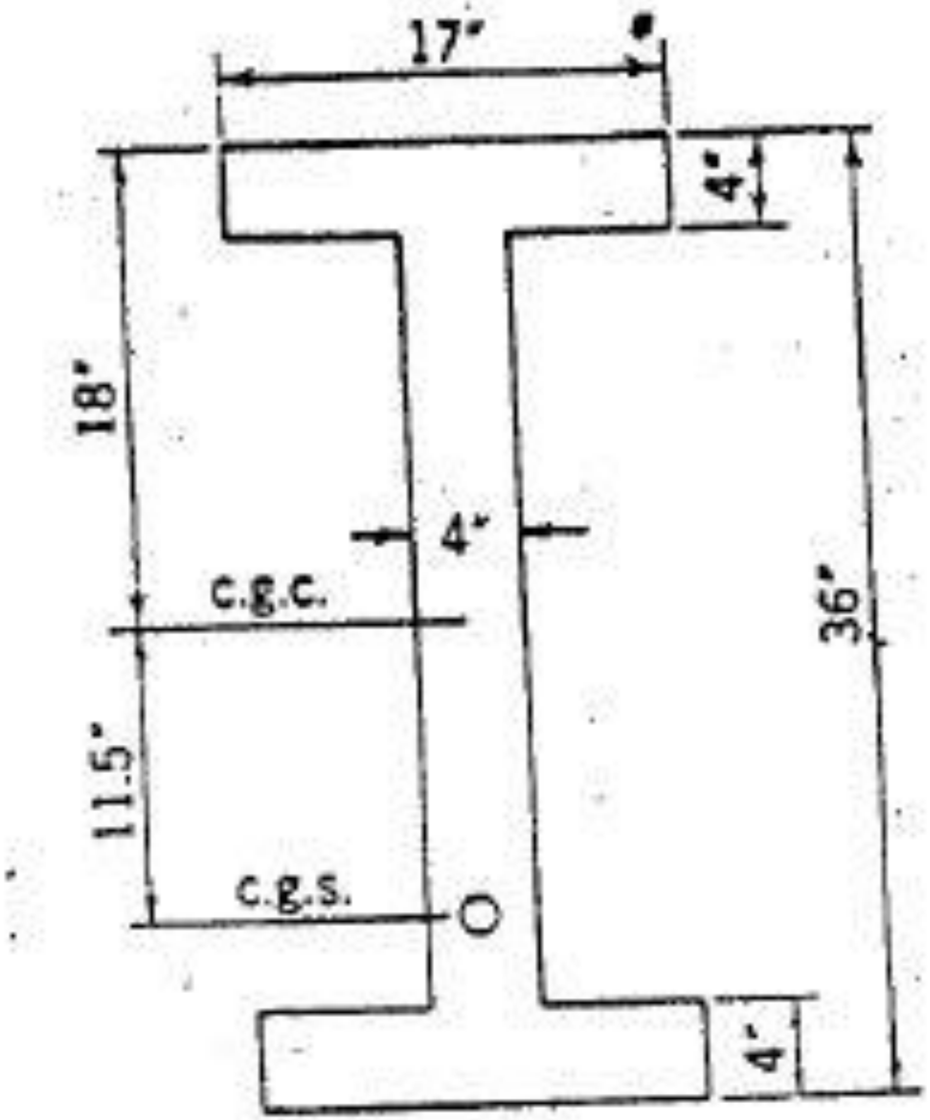


Fig. 6-8. Example 6-3.

Step 4. Try a new section as shown in Fig. 6-8, with $A_c = 248$ sq in. (160×10^3 mm^2). For this new section, $I = 42{,}200$ in.^{4} (17.57×10^9 mm^4); $k_t = k_b = 9.4$ in. (239 mm $e - k_b = 2.1$ in (53 mm); $F = 320 \times 12/(11.5 + 9.4) = 184$ k (818 kN); $F_0 = 221$ k (983 kN); A_c required for bottom fiber $= 246$ sq in. (159×10^3 mm^2), for top fiber $= 230$ sq in. (148×10^3 mm^2). Hence the section seems to be quite satisfactory. And no further revision is needed.