CE 414: Prestressed Concrete
Lecture 14
Flexural Design (revised final design)

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- Stresses in transfer and service load
- Considering tension in concrete
- Revised design of concrete beam considering tension

1. *Stresses at transfer:*

Tension in members without auxiliary reinforcement—$3\sqrt{f'_{ci}}$ ($6\sqrt{f'_{ci}}$ at ends of precast simple beams).

Tension in members with properly designed auxiliary reinforcement—no limit.

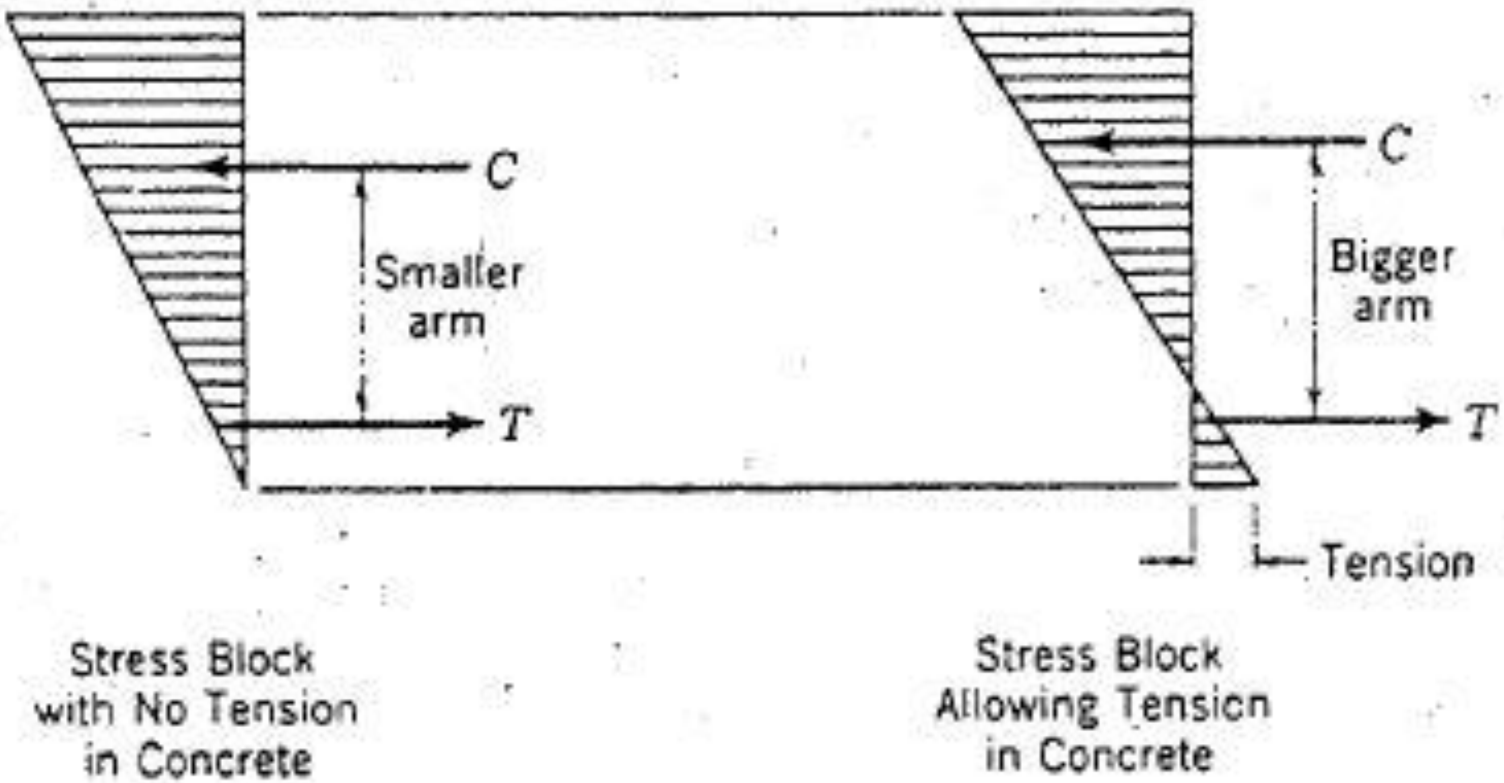

Fig. 6-11. Bigger arm for steel when allowing tension in concrete.

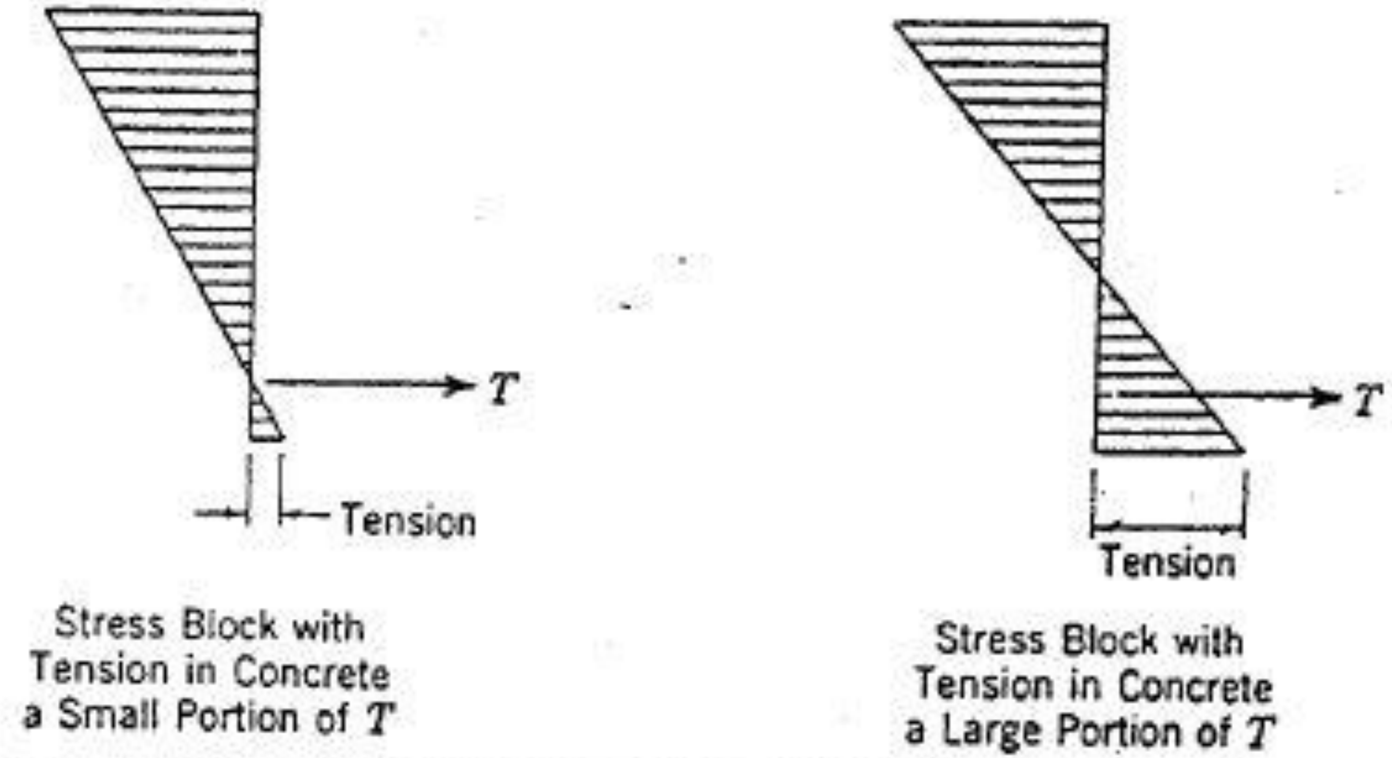

Fig. 6-12. Relative significance of tension in concrete.

2. *Stresses at service loads:*
   Tension in precompressed tensile zone.
   Tension in excess of above limiting values may be permitted when shown to be not detrimental to proper structural behavior ($12\sqrt{f'_c}$).

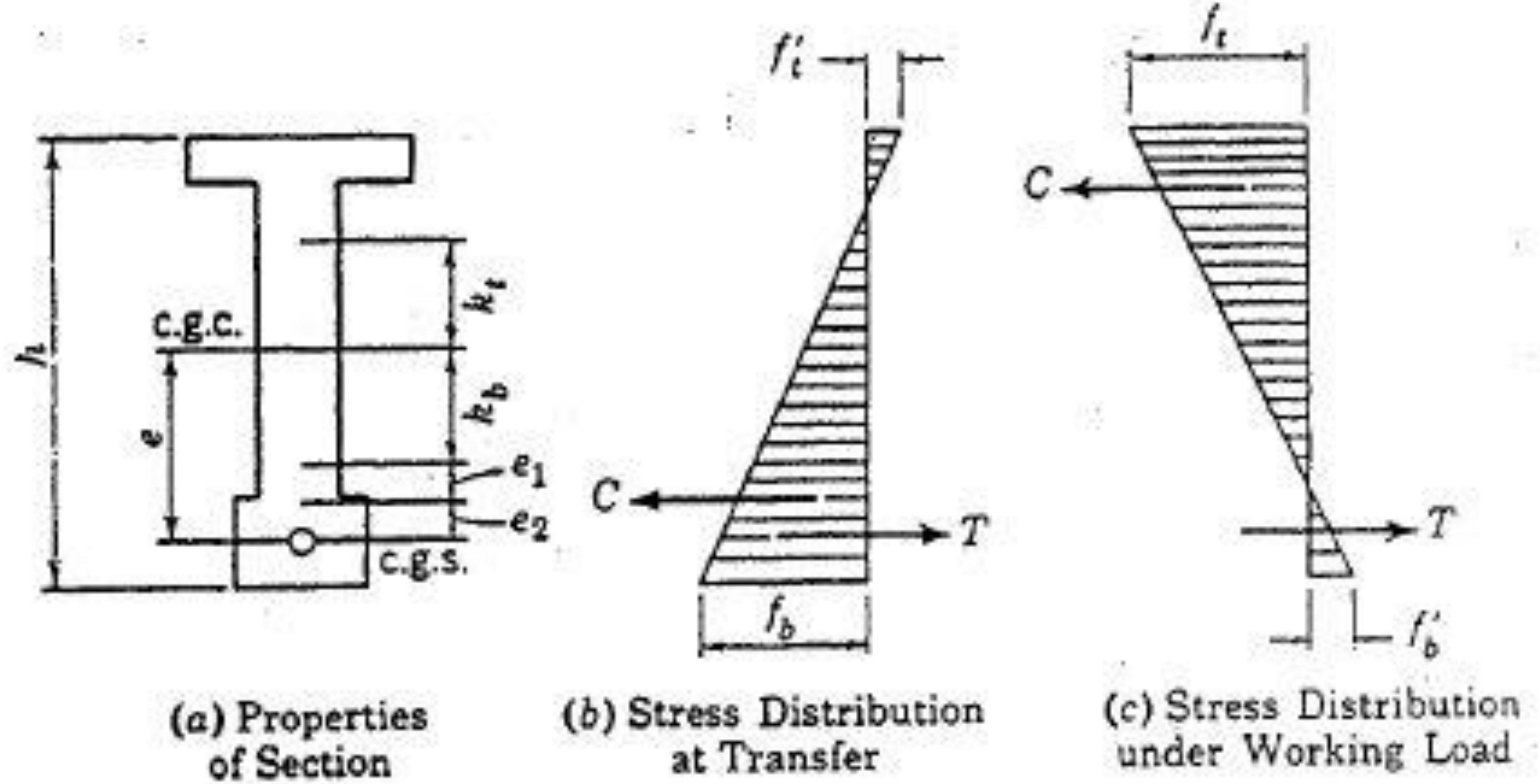

Fig. 6-13. Allowing and considering tension in concrete.

Small Ratios of M_G/M_T. If tensile stress f_t' is permitted in the top fibers, the center of compression C can be located below the bottom kern by the amount of

$$e_1 = f_t' I / F_0 c_t = f_t' A k_b / F_0 \tag{6-11}$$

For a given moment M_G, the c.g.s. can be further located below C by the amount of

$$e_2 = M_G / F_0 \tag{6-12}$$

Hence the maximum total amount that the c.g.s. can be located below the kern is given by

$$e_1 + e_2 = \frac{M_G + f_t' A k_b}{F_0} \tag{6-13}$$

The c.g.s. having been located at some value e below c.g.c., the lever arm a under working load is known. For an allowable tension in the bottom fiber, the moment carried by the concrete is

$$f_b' I / c_b = f_b' A k_t$$

The net moment $M_T - f_b' A k_t$ is to be carried by the prestress F with a lever arm acting up to the top kern point; hence the total arm is (Fig. 6-13).

$$a = k_t + e \qquad (6\text{-}14)$$

and the prestress F required is

$$F = \frac{M_T - f_b' A k_t}{a} \tag{6-15}$$

The bottom fiber stress at transfer is given by

$$f_b = \frac{F_0 h}{A_c c_t} + f_t' \frac{c_b}{c_t} \tag{6-16}$$

from which we have

$$A_c = \frac{F_0 h}{f_b c_t - f_t' c_b} \tag{6-16a}$$

Similarly, the top fiber stress under working load is given by

$$f_t = \frac{Fh}{A_c c_b} + f_b' \frac{c_t}{c_b} \qquad (6\text{-}17)$$

from which

$$A_c = \frac{Fh}{f_t c_b - f_b' c_t} \qquad (6\text{-}17a)$$

EXAMPLE 6-5

Redesign the beam section in example 6-3, allowing and considering tension in concrete. $f_t'=0.30$ ksi, $f_b'=0.24$ ksi. Other given values were: $M_T=320$ k-ft; $M_G=40$ k-ft; $f_t=-1.60$ ksi; $f_b=-1.80$ ksi; $F=184$ k; $F_0=221$ k ($f_t'=2.07$ N/mm², $f_b'=1.65$ N/mm², $M_T=434$ kN-m, $M_G=54$ kN-m, $f_t=-11.3$ N/mm², $f_b=-12.41$ N/mm², $F=818$ kN, and $F_0=983$ kN).

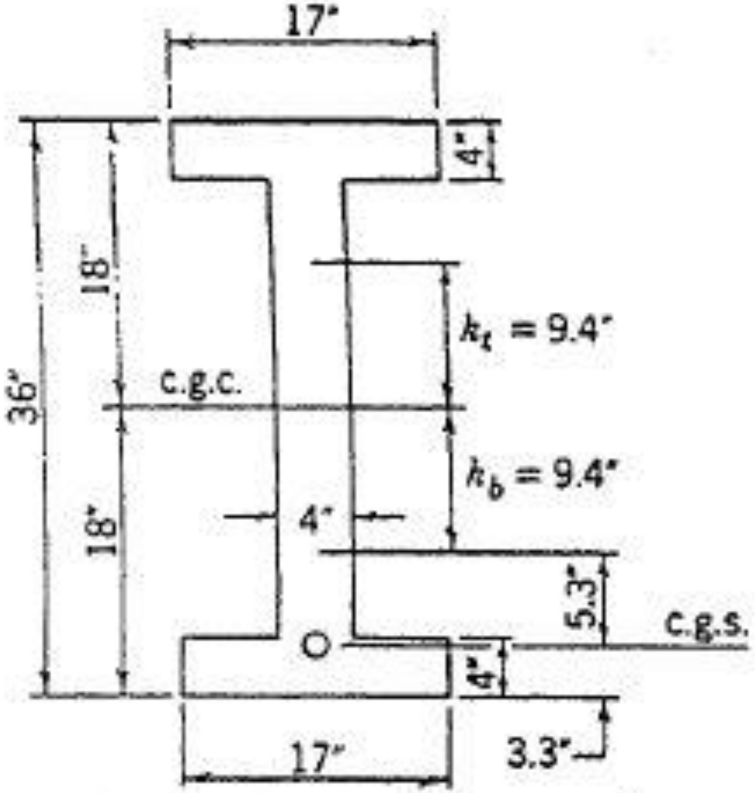

Fig. 6-14. Example 6-5.

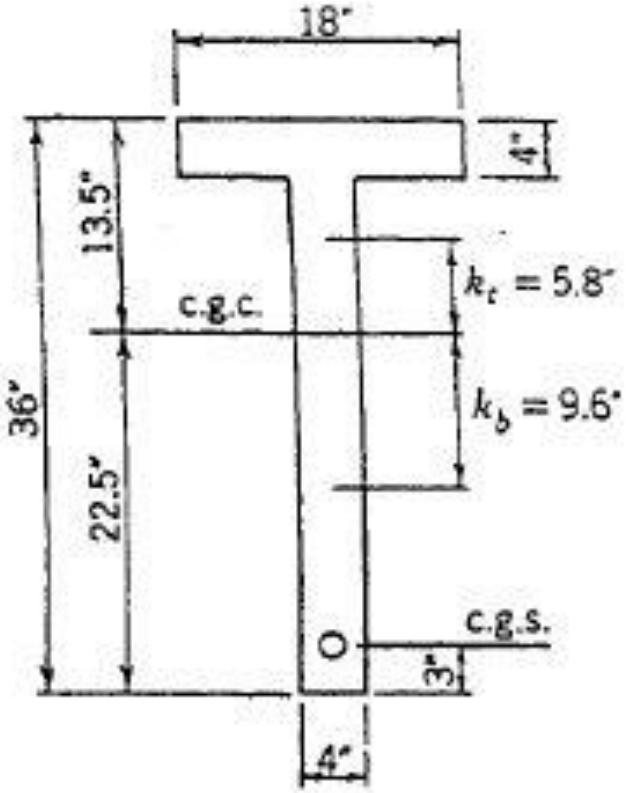

Fig. 6-15. Example 6-6.

Solution *Step* 1. From example 6-3, we have $k_t = k_b = 9.4$ in. (239 mm); $A_c = 248$ sq in. (160×10^3 mm^2). Using equation 6-13, we have

$$e_1 + e_2 = \frac{40 \times 12 + 0.3 \times 248 \times 9.4}{221} = 5.3 \text{ in. (135 mm)}$$

Hence c.g.s. can be located 5.3 in. (135 mm) below the bottom kern, or 3.3 in. (84 mm) above the bottom fiber, Fig. 6-14.

Step 2. The net moment to be carried by the prestress is

$$M_T - f_b' A k_t = 320 \times 12 - 0.240 \times 248 \times 9.4$$
$$= 3840 - 560 = 3280 \text{ k-in. (371 kN-m)}$$

For a resisting lever arm of $9.4 + 9.4 + 5.3 = 24.1$ in. (612 mm), the prestress required is

$$F = 3280/24.1 = 136 \text{ k (605 kN)}$$
$$F_0 = 136 \times 150/125 = 163 \text{ k (725 kN)}$$

Step 3. To limit the bottom fibers to -1.80 ksi (-12.41 N/mm^2), we need

$$A_c = \frac{163 \times 36}{1.80 \times 18 - 0.30 \times 18}$$

$$= 218 \text{ sq in. } (141 \times 10^3 \text{ mm}^2)$$

To keep the top fibers to -1.60 ksi (-11.03 N/mm^2), we need

$$A_c = \frac{136 \times 36}{1.60 \times 18 - 0.24 \times 18}$$

$$= 200 \text{ sq in. } (129 \times 10^3 \text{ mm}^2)$$

which indicates that the trial section can be appreciably reduced and a new section tried over again.

over again.

Large Ratios of M_G/M_T. When M_G/M_T is large, C will be within the kern at transfer, and the allowing of tension on top fiber will have no effect on the design. The c.g.s. has to be located within practical limits. Otherwise, the design is made as for the first case. This is illustrated in the next example.

EXAMPLE 6-6

Revise the design for the section in example 6-4 allowing and considering tension in concrete. Other values given were: $M_T = 320$ k-ft; $M_G = 210$ k-ft; $F = 152$ k; $F_0 = 182$ k; $A_c = 200$; $c_t = 13.5$ in.; $c_b = 22.5$ in.; $k_t = 5.8$ in.; $k_b = 9.6$ in. (Fig. 6-15) ($M_T = 434$ kN-m, $M_G = 285$ kN-m, $F = 676$ kN, $F_0 = 810$ kN, $A_c = 129 \times 10^3$ mm^2, $c_t = 343$ mm, $c_b = 572$ mm, $k_t = 147$ mm, and $k_b = 244$ mm).

Solution *Step* 1. Referring to example 6-6, since the possible theortical location for c.g.s. is 13.8 in. (351 mm) below the bottom kern (0.9 in. (23 mm) below bottom fiber) without producing tension in top fiber, whereas the practical location of c.g.s. has to be 3 in. (76.2 mm) above bottom fiber, no tension will exist in top fiber.

Step 2. Net amount to be carried by prestress is

$$M_T - f_b' A k_t = 320 \times 12 - 0.240 \times 200 \times 5.8$$
$$= 3840 - 280 = 3560 \text{ k-in. } (402 \text{ kN-m})$$

The resisting lever arm is

$$36 - 3 - 13.5 + 5.8 = 25.3 \text{ in. } (643 \text{ mm})$$

The required prestress is

$$F = 3560/25.3 = 141 \text{ k } (627 \text{ kN})$$
$$F_0 = 141(150/125) = 169 \text{ k } (752 \text{ kN})$$

To keep the bottom fiber stress within limits, we can apply equation 6-10,

$$A_c = \frac{F_0}{f_b}\left(1 + \frac{e-(M_G/F_0)}{k_t}\right)$$

$$= \frac{169}{1.80}\left(1 + \frac{19.5-(210\times 12/169)}{5.8}\right)$$

$$= 168 \text{ sq in. } (108\times 10^3 \text{ mm}^2)$$

To keep the top fiber stress within limit, we have, from equation 6-17*a*,

$$A_c = \frac{141\times 36}{1.60\times 22.5 - 0.24\times 13.5}$$

$$= 155 \text{ sq in. } (100\times 10^3 \text{ mm}^2)$$

The area furnished is 200 sq in. (129×10^3 mm^2), which can be reduced if desired.