CE 414: Prestressed Concrete
Lecture 15
Shear design and PC girder shape

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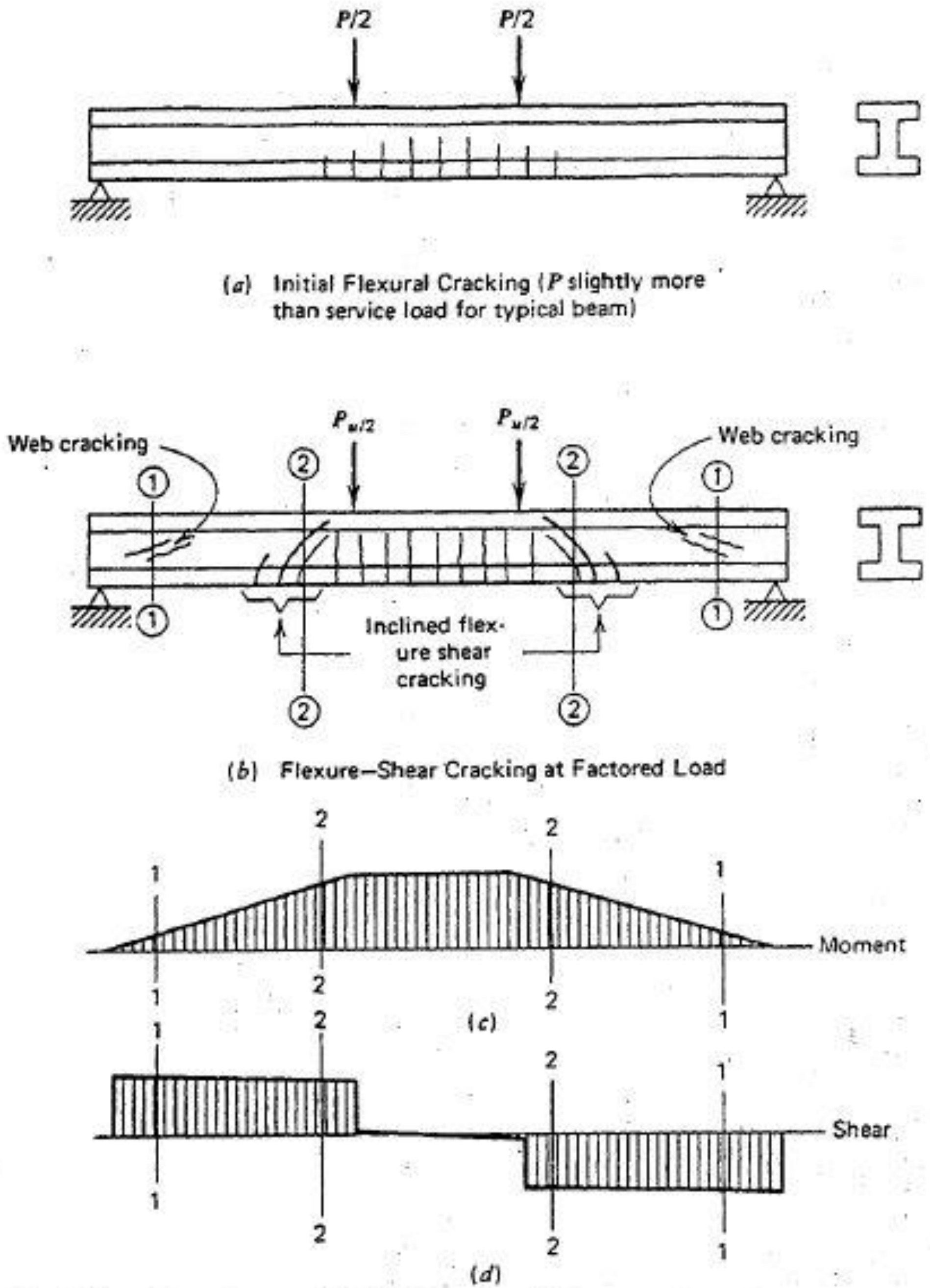


Fig. 7-3. Development of shear cracking.

6-8 Shapes of Concrete Sections

Having studied both the elastic and ultimate designs, we are now ready to discuss the selection of the best shapes for prestressed concrete sections under flexure. The simplest form is the rectangular shape possessed by all solid slabs and used for some short-span beams. As far as formwork is concerned, the rectangular section is the most economical. But the kern distances are small, and the available lever arm for the steel is limited. Concrete near the centroidal axis and on the tension side is not effective in resisting moment, especially at the ultimate stage. As observed in the previous section, the rectangular section is not as efficient in the use of the concrete section as is the I-shaped section.

Hence other shapes are frequently used for prestressed concrete, Fig. 6-20:

1. The symmetrical I-section.
2. The unsymmetrical I-section.
3. The T-section.
4. The inverted T-section.
5. The box section.

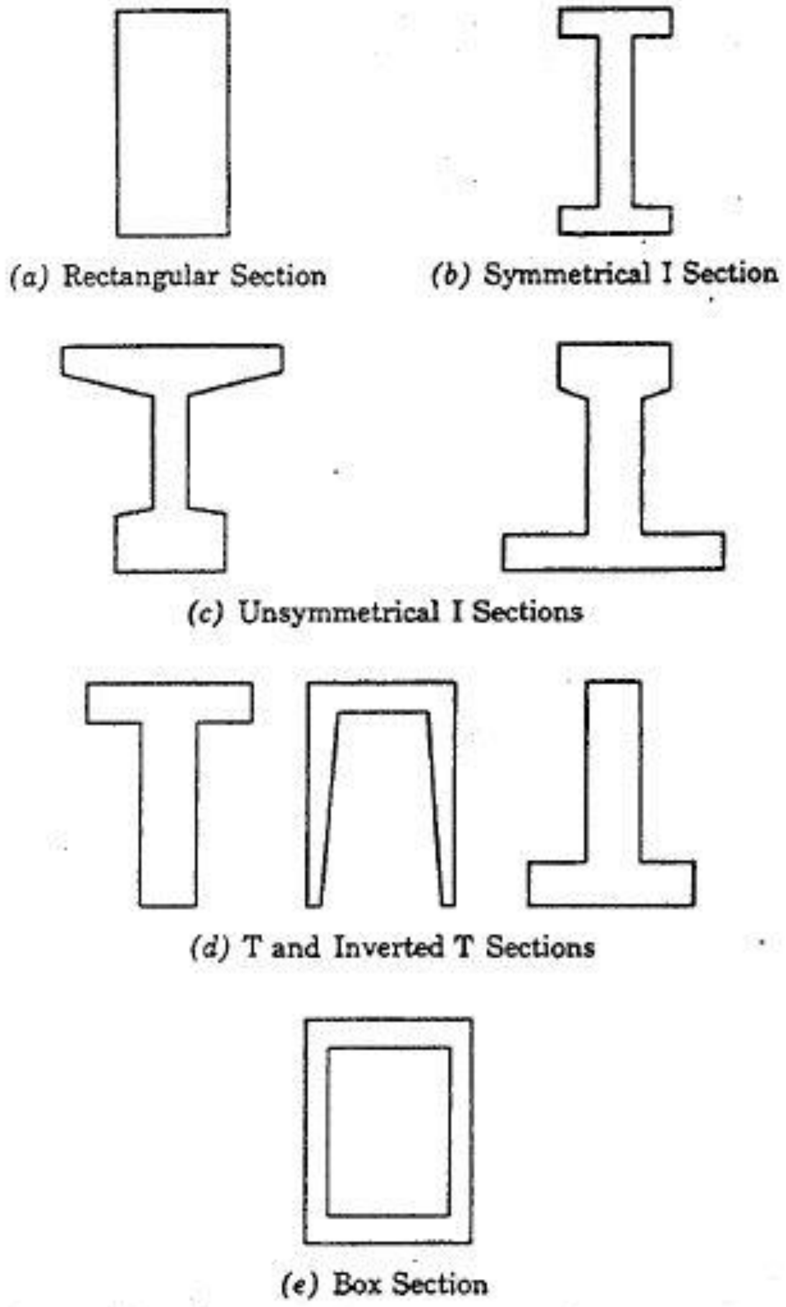


Fig. 6-20. Shapes of concrete sections.

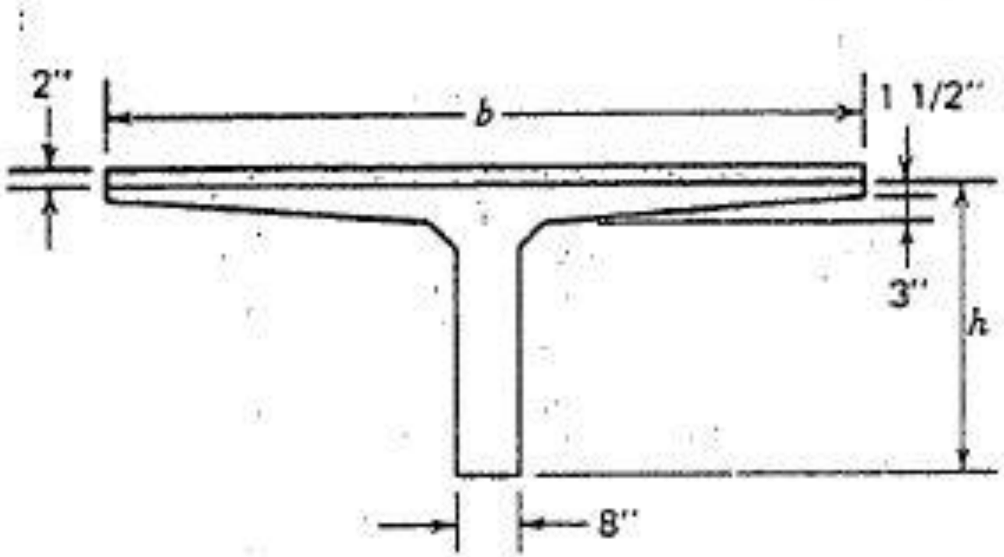


Fig. 6-21(a). Typical single-tee section (Lin Tee).

Table of Properties

b, ft	h, in.	A, in.^{2}	I, in.^{4}	y_t, in.	y_b, in.	V/S, in.	
8	36	570	68,917	9.99	26.01	2.16	without 2" topping
10	48	782	168,968	11.36	36.64	2.33	without 2" topping
8	36+2		88,260		29.09		with 2" topping

PCI Design Handbook gives tables of safe superimposed loads for these sections.

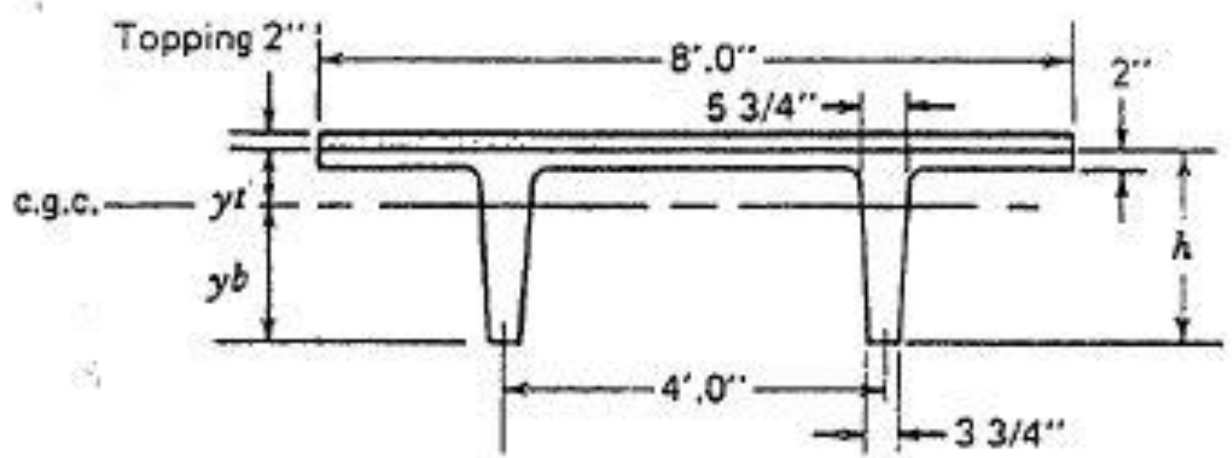


Fig. 6-21(*b*). Typical double-tee section.

Table of Properties

	h, in.	A, in.^{2}	I, in.^{4}	y_t, in.	y_b, in.	V/S, in.
Without topping	14	306	4,508	3.49	10.51	1.25
	18	344	9,300	4.73	13.27	1.32
	24	401	20,985	6.85	17.15	1.41
With topping 2" thick	14+2		7,173		12.40	
	18+2		13,799		15.51	
	24+2		29,853		19.94	

Other sections listed in *PCI Design Handbook* 12 in., 16 in., 20 in., and 32 in. deep with tables of safe superimposed load.

Beam Properties

Type	Area, in.^{2}	y_B, in.	Moment of inertia, in.^{4}
40	253	15.16	31,000
60	332	18.63	70,100
80	476	22.53	154,900
100	546	27.90	249,000
120	626	35.60	456,000

Washington State Standard Bridge Beams

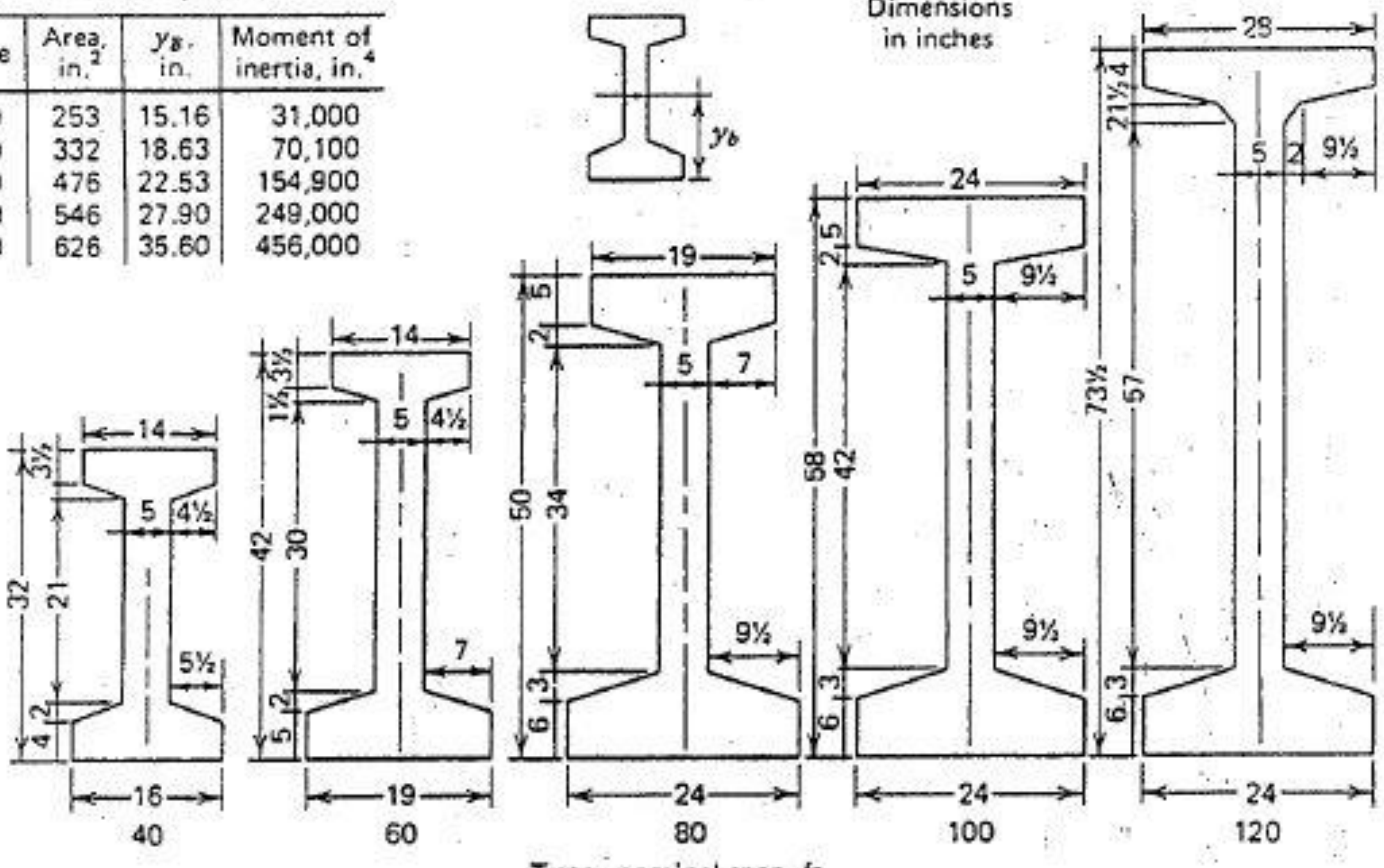


Fig. 6-21(c). Washington State standard beams.

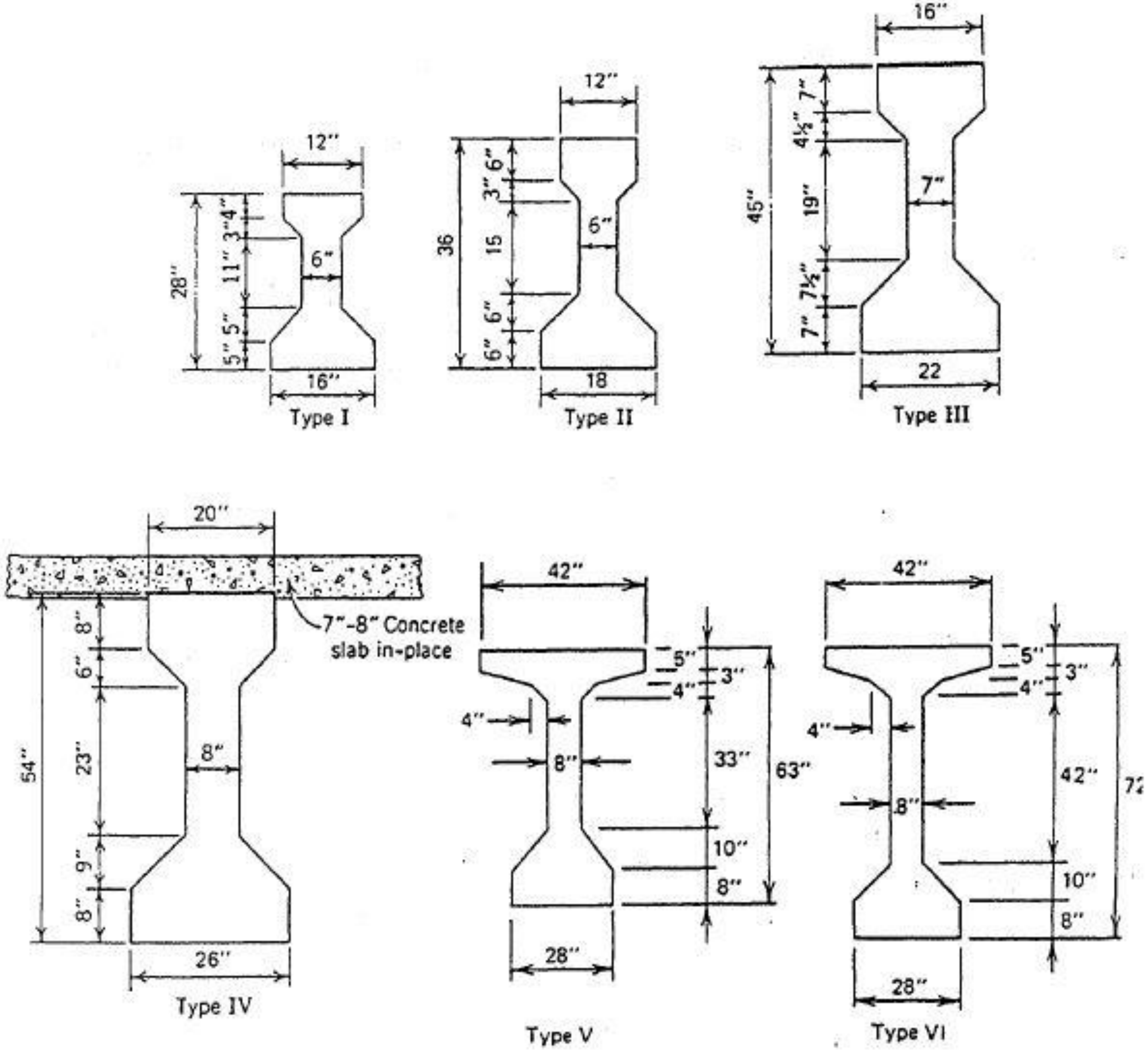
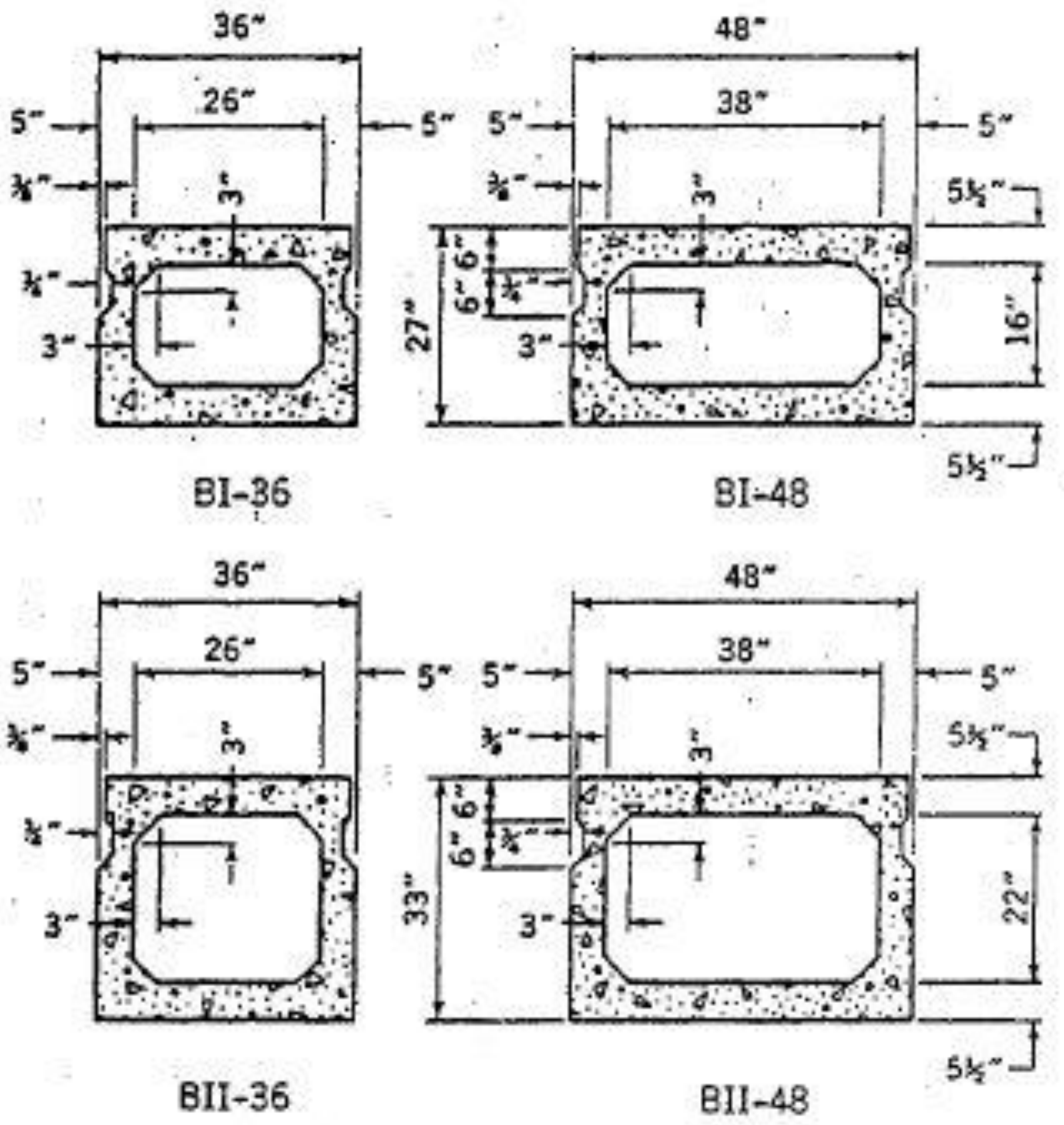


Fig. 6-21(*d*). Standard AASHTO-PCI prestressed concrete I-beams for highway bridges.[2]

Table of Properties
(Without in-place slab)

Beam Type	Area, in.^{2}	I, in.^{4}	c_b, in.	Recommended Span Limits, ft
I	276	22,750	12.59	30–45
II	369	50,980	15.83	40–60
III	560	125,390	20.27	55–80
IV	789	260,730	24.73	70–100
V	1013	521,180	31.96	90–120
VI	1085	733,320	36.38	110–140


36"
26"
5"
3"
BI-36
48"
38"
27"
16"
5½"
BI-48
BII-36
33"
22"
BII-48

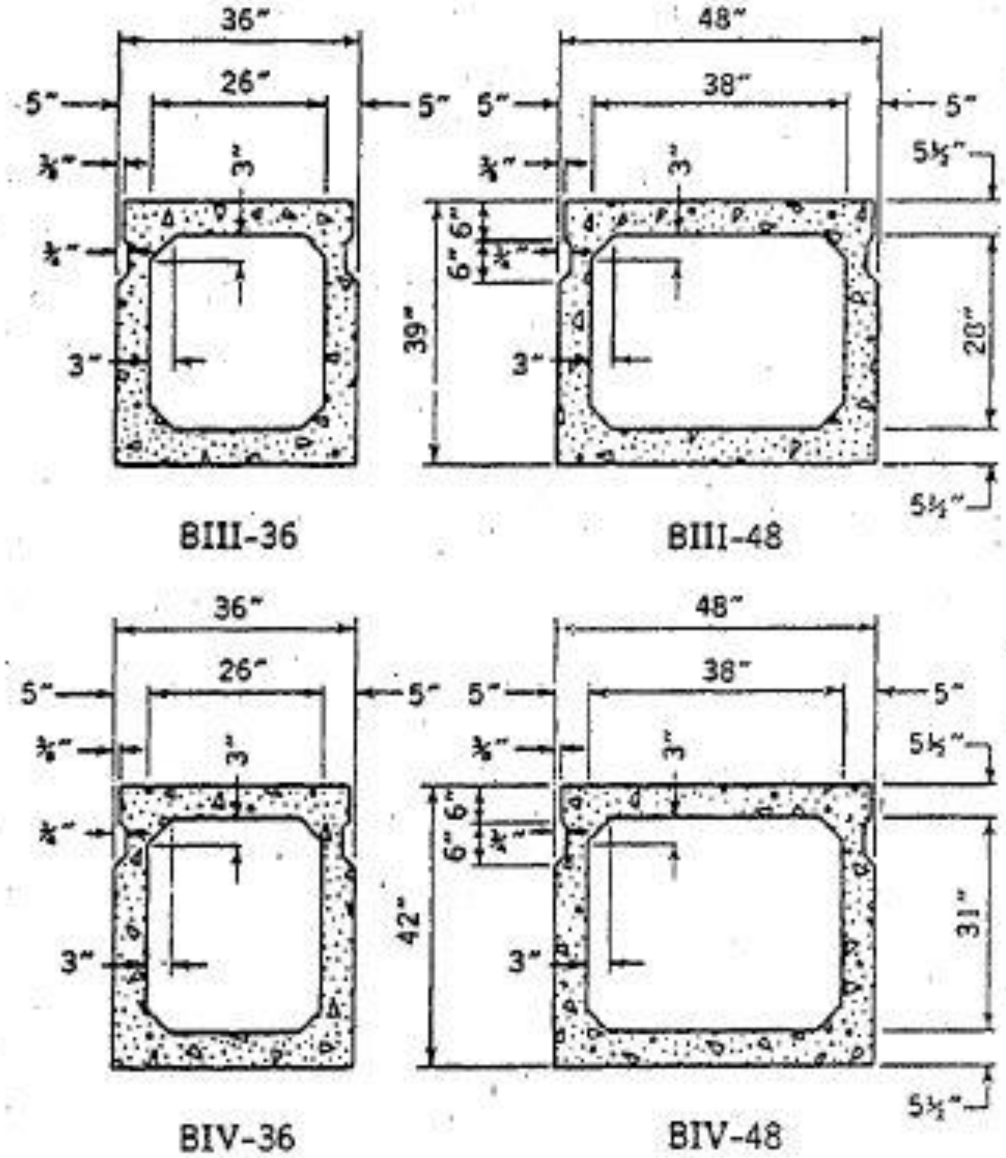


Fig. 6-21(*e*). Standard AASHTO-PCI prestressed concrete box beams for highway bridges.

Table of Properties

Beam Type	Area, in.^{2}	I, in.^{4}	c_b, in.	Recommended Span Limits, ft Draped Strand	Recommended Span Limits, ft Straight Strand
BI-36	560.5	50,334	13.35	74	62
BI-48	692.5	65,941	13.37	73	63
BII-36	620.5	85,153	16.29	86	73
BII-48	752.5	110,499	16.33	86	74
BIII-36	680.5	131,145	19.25	97	83
BIII-48	812.5	168,367	19.29	96	83
BIV-36	710.5	158,644	20.73	103	87
BIV-48	842.5	203,088	20.78	103	88

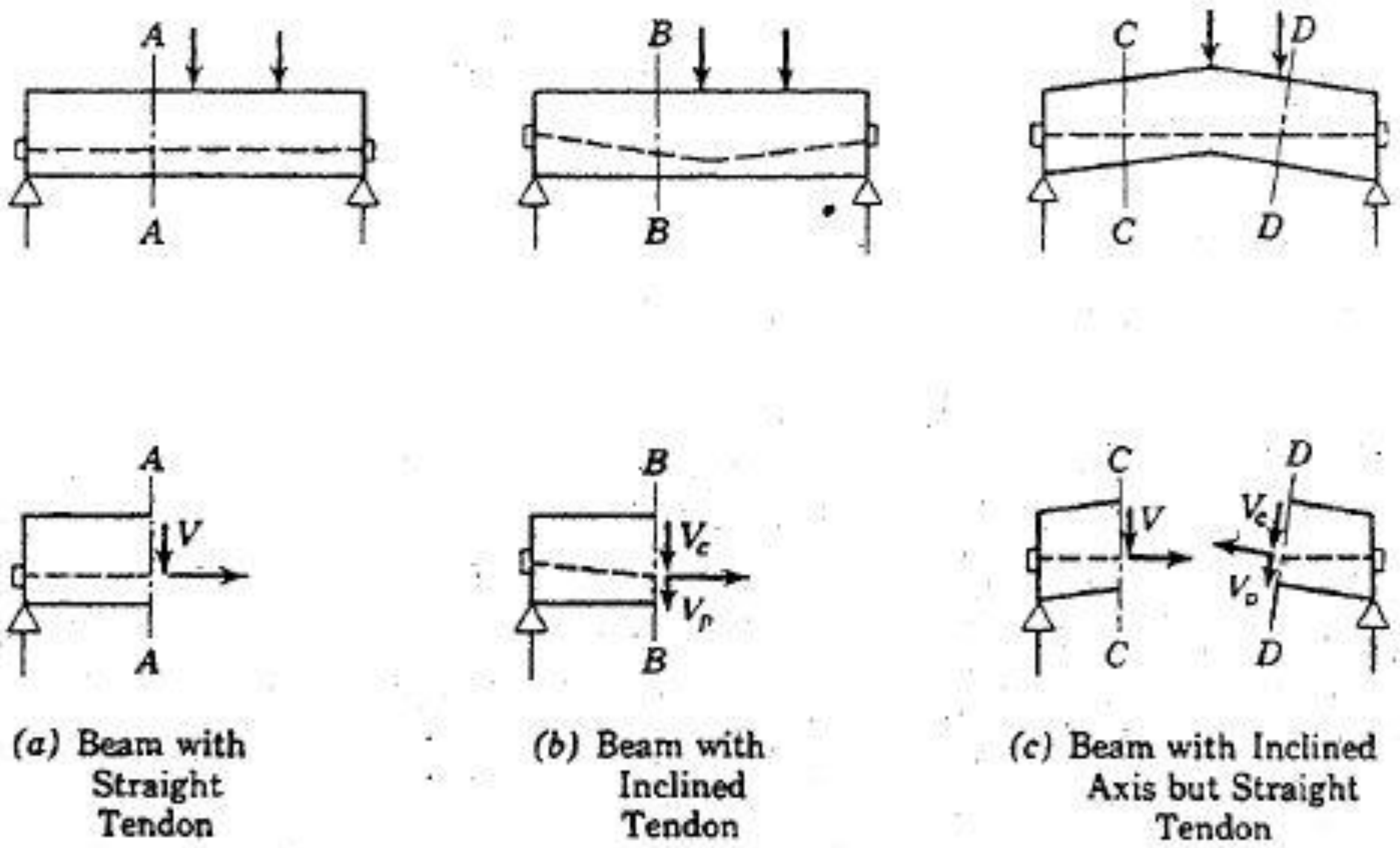
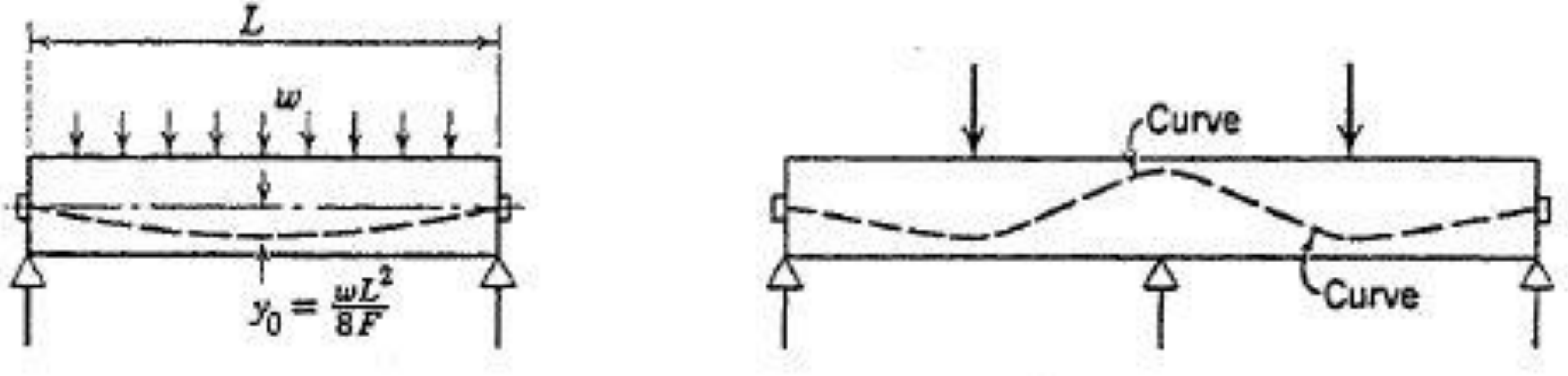


Fig. 7-1. Shear carried by concrete and tendons.



(a) Beam with No Shear in Concrete

(b) Continuous Beam with Concentrated Loads

Fig. 7-2. Varying inclination of tendons to carry shear.

1. From the total external shear V across the section, deduct the shear V_p carried by the tendon to obtain the shear V_c carried by the concrete, thus,

$$V_c = V - V_p \tag{7-1}$$

Note again that occasionally, though rarely, $V_c = V + V_p$; this happens when the cable inclination is such that it adds to the shear on the concrete.

2. Compute the distribution of V_c across the concrete section by the usual formula, Fig 7-6,

$$v = V_c Q / Ib$$

where v = shearing unit stress at any given level
Q = statical moment of the cross-sectional area above (or below) that level about the centroidal axis.
b = width of section at that level.

3. Compute the fiber stress distribution for that section due to external moment M, the prestress F, and its eccentricity e by the formula

$$f_c = \frac{F}{A} \pm \frac{Fec}{I} \pm \frac{Mc}{I}$$

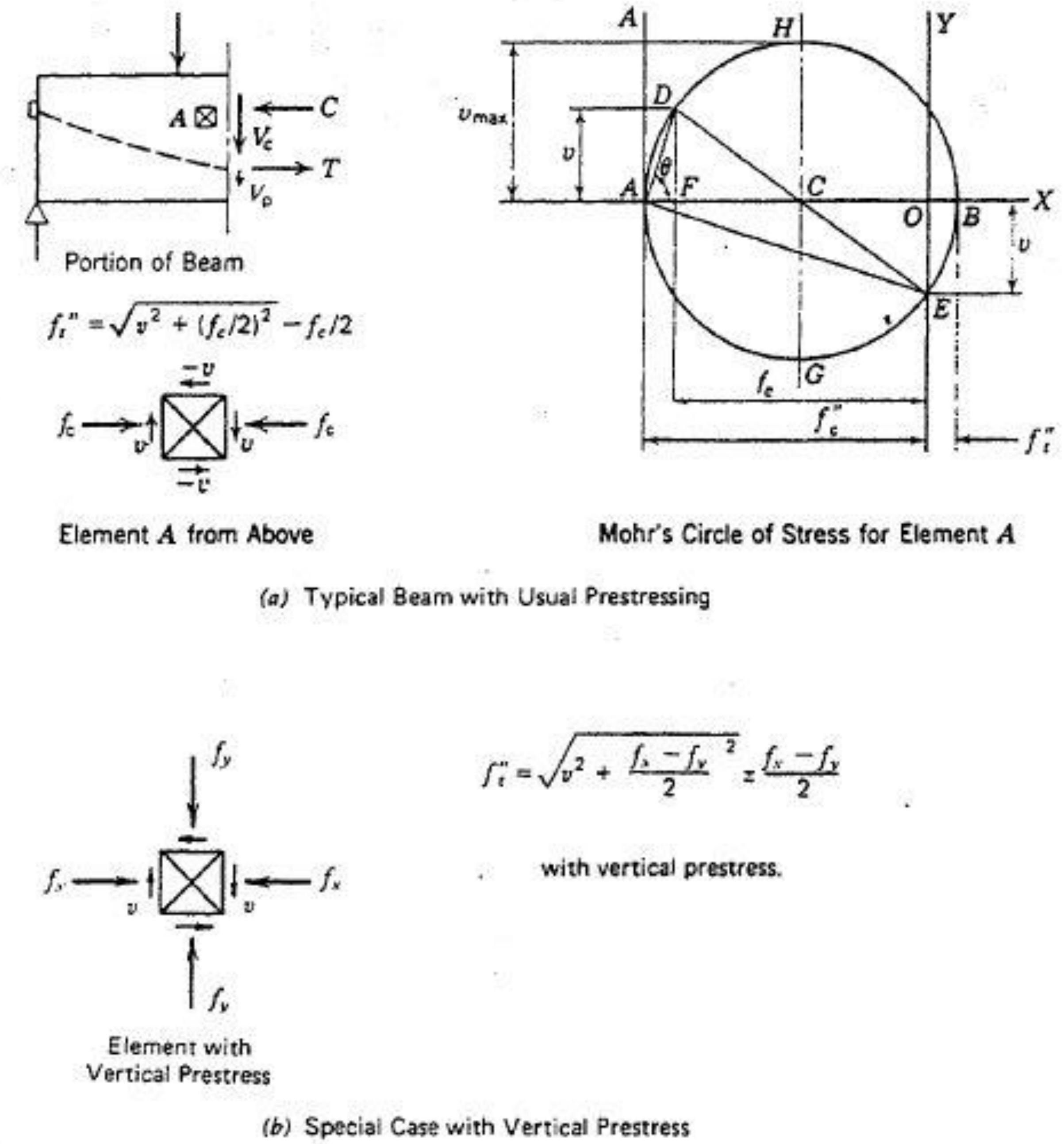


Fig. 7-6. State of stress in concrete.

4. The maximum principal tensile stress f_t'' corresponding to the above v and f_c is then given by the formula

$$f_t'' = \sqrt{v^2 + (f_c/2)^2} - (f_c/2) \tag{7-2}$$

principal tension as shown in Fig. 7-6 and listed in the table. (Note: AA = plane perpendicular to AB.)

Plane	Shearing Stress	Normal Stress
AD = vertical plane	v	f_c
AE = horizontal plane	$-v$	0
AB = principal tensile plane	0	f_t''
AA = principal compressive plane	0	f_c''

EXAMPLE 7-1

A prestressed-concrete beam section under the action of a given moment has a fiber stress distribution as shown in Fig. 7-7. The total vertical shear in the concrete at the section is 520 k (2313 kN). Compute and compare the principal tensile stresses at the centroidal axis *N–N* and the junction of the web with the lower flange *M–M*.

Solution I of the section about its centroidal axis is computed as 3,820,000 in.^{4} Other values are listed separately for the two levels *M–M* and *N–N* as tabulated.

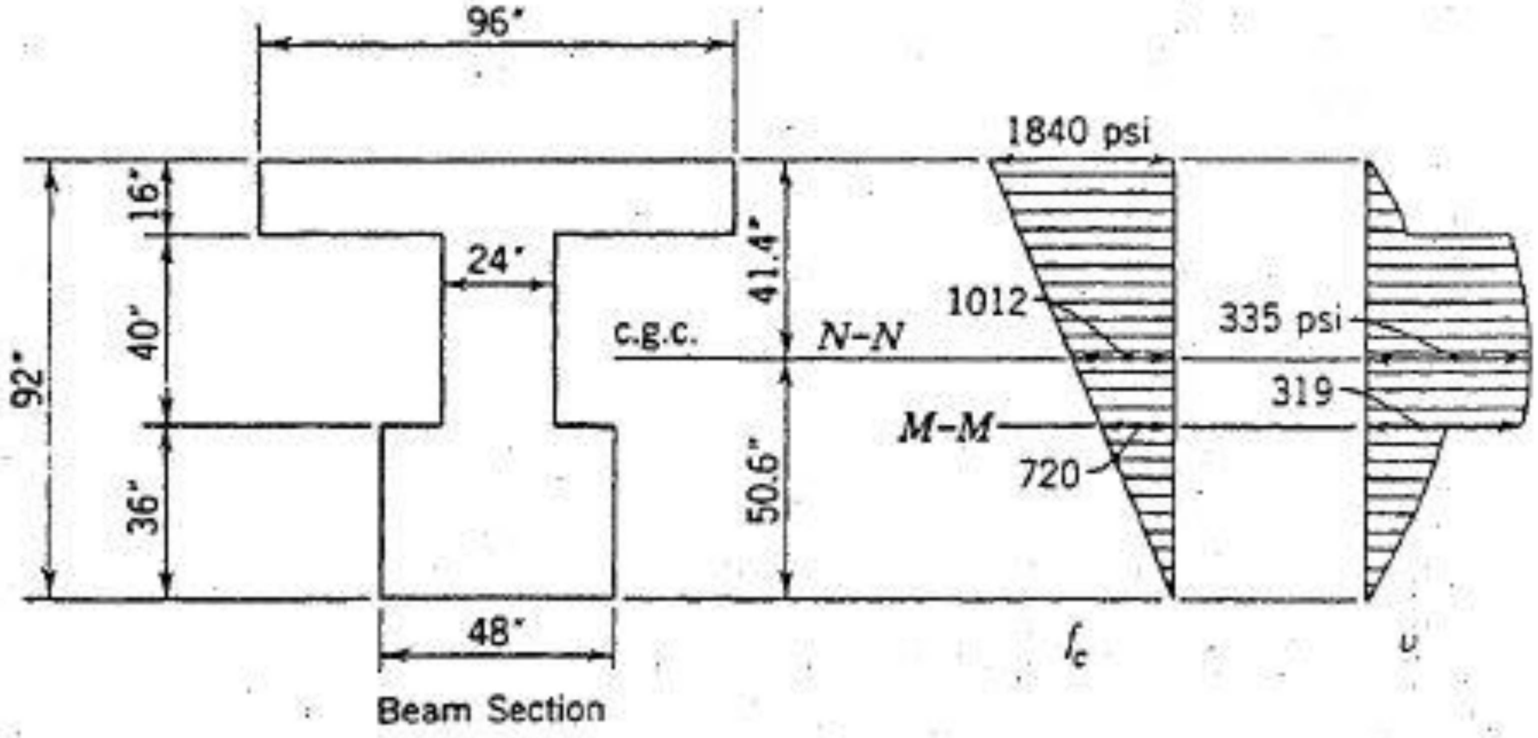


Fig. 7-7. Example 7-1.

Section	$M-M$	$N-N$
Q, in.^{3}	$36\times48\times32.6$ $=56{,}200$	$14.6\times24\times7.3$ $+56{,}200=58{,}800$
$v=\dfrac{V_cQ}{Ib}$, psi	$\dfrac{520{,}000\times56{,}200}{3{,}820{,}000\times24}$ $=319$	$\dfrac{520{,}000\times58{,}800}{3{,}820{,}000\times24}$ $=335$
f_c, psi	720	1012
$f_t''=\sqrt{v^2+\left(\dfrac{f_c}{2}\right)^2}-\dfrac{f_c}{2}$	$\sqrt{319^2+\left(\dfrac{720}{2}\right)^2}-\dfrac{720}{2}$ $=121$	$\sqrt{334^2+\left(\dfrac{1012}{2}\right)^2}-\dfrac{1012}{2}$ 100

EXAMPLE 7-2

For the beam section in example 7-1, suppose that the external load is increased by 25% so that the fiber stress distribution is shown in Fig. 7-9 and the total vertical shear in the concrete is $520 \times 1.25 = 650$ k (2,891 kN). Compute the principal tensile stress at *M-M*.

Solution The compressive fiber stress at *M-M* is computed to be 560 psi (3.86 N/mm^2), assuming that cracks have not occurred at the bottom fiber for the tensile stress of 561 psi. THe unit vertical shearing stress is

$$v = \frac{V_c Q}{Ib} = \frac{650{,}000 \times 56{,}200}{3{,}820{,}000 \times 24} = 399 \text{ psi } (2.75 \text{ N/mm}^2)$$

Hence the principal tensile stress at *M-M* is

$$f_t'' = \sqrt{v^2 + \left(\frac{f_c}{2}\right)^2} - \frac{f_c}{2}$$

$$= \sqrt{399^2 + \left(\frac{560}{2}\right)^2} - \frac{560}{2} = 208 \text{ psi } (1.43 \text{ N/mm}^2)$$

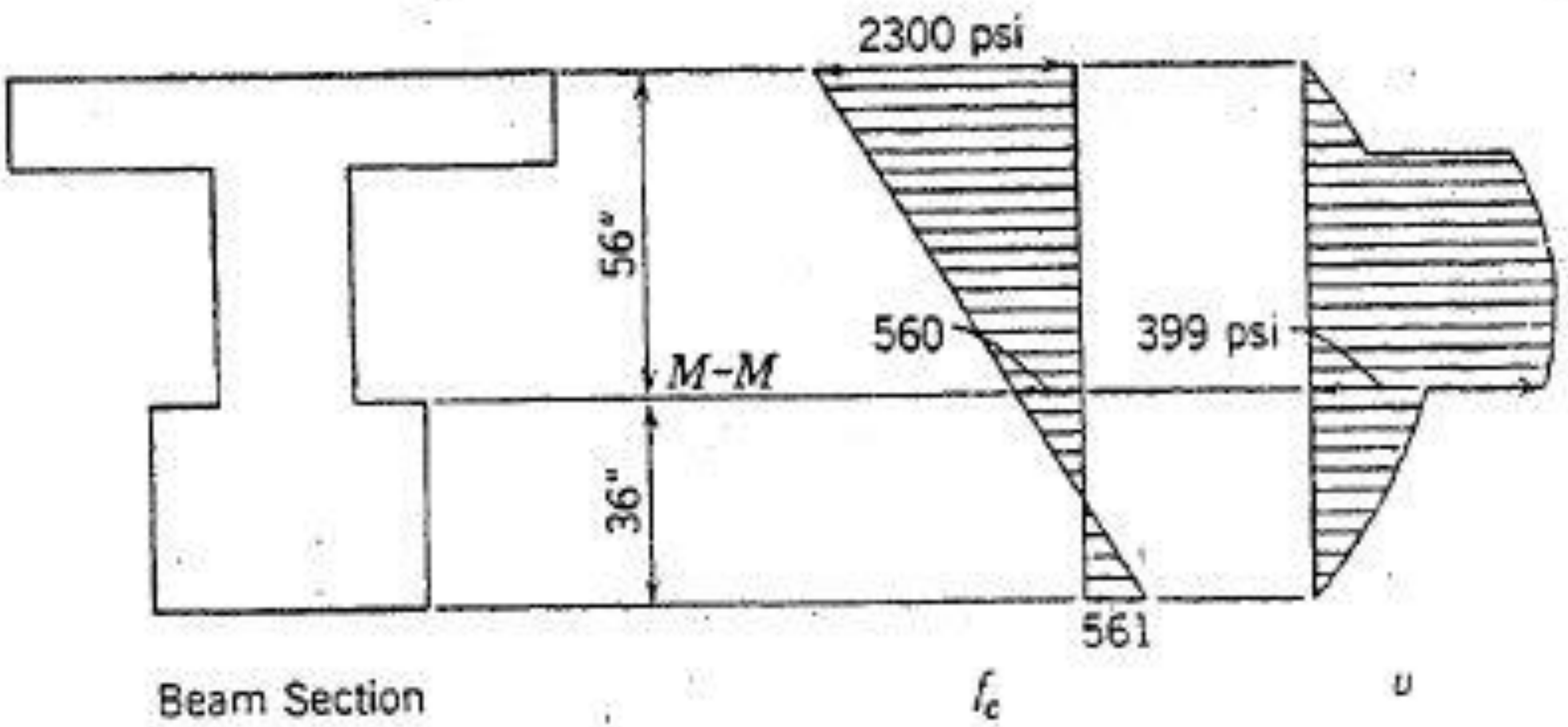


Fig. 7-9. Example 7-2.