CE 414: Prestressed Concrete
Lecture 16
Design for Shear and bearing stress

Course Instructor: Saurav Barua

Assistant Professor, Department of Civil Engineering, DIU

Email: saurav.ce@diu.edu.bd

Phone: 01715334075

Contents

- ACI code for stirrup design
- Nature of bond and transfer length
- Bearing stress
- Bearing plate area design

In composite construction, shearing stress v between precast and in-place portions computed on the basis of the ordinary elastic theory,

$$v = \frac{VQ}{Ib}$$

where V = the total shear in lb applied after the in-place portion has been cast

Q = statical moment of the cross-sectional area of in-place portion taken about the centroidal axis of the composite section

I = moment of inertia of the composite section

b = width of the contact area between the precast and the in-place portions

ACI Code Method for Stirrup Design. An ultimate strength method originally proposed for the 1963 ACI Code follows an empirical approach based primarily on the test results of the University of Illinois.[5] The method indicates that the yield strength in the stirrups extending over a distance d could be considered effective in transmitting the shear of $V_n - V_{ci}$; thus,

$$A_v f_y d/s = V_n - V_{ci} \tag{7-13}$$

where V_n is the design ultimate load shear, and V_{ci} is the shear at the section when the vertical flexural crack starts to develop into an inclined one. A similar criterion is recommended for the case of web cracking initiating by itself without flexural cracking; thus,

$$A_v f_y d/s = V_n - V_{cw} \tag{7-14}$$

where V_{cw} is the shear at the section when web cracking starts by itself, without any flexural cracking.

The ACI Code determines the shear strength V_{cw} and V_{ci} using the equations developed in section 7-3. We assume the nominal shear force as $V_n = V_u/\phi$ where $\phi = 0.85$ for shear strength design. This adds safety to the design in addition to that which is introduced by the load factors used for obtaining V_u. The shear force carried by stirrups, V_s, is expressed as follows:

$$V_s = \frac{V_u}{\phi} - V_c = \frac{A_v f_y d}{s} \tag{7-12}$$

The ACI Code has two expressions for the minimum A_v required, and the maximum spacing of stirrups is also limited. This maximum spacing is $0.75h$ in prestressed concrete, but not more than 24 in. Unless V_u is less than one-half of ϕV_c the minimum A_v requirement applies. If it is shown by test that the required ultimate flexural and shear capacity can be developed when shear reinforcement is omitted, the minimum A_v requirement can be waived under the ACI Code. For some standard precast concrete products this testing has been done, and the concrete alone carries the shear stress without stirrups.

The minimum A_v in square inches is given in the ACI Code by two expressions:

$$A_v = 50\frac{b_w s}{f_y} \tag{7-16}$$

or

$$A_v = \frac{A_{ps} f_{pu} s}{80 f_y d}\sqrt{\frac{d}{b_w}} \tag{7-17}$$

EXAMPLE 7-5

Design stirrups following ACI Code for the beam of examples 7-3 and 7-4. Use the plot of shear stress, Fig. 7-16, in this solution. Assume stirrups perpendicular to the longitudinal axis with $f_y = 50$ ksi (345 N/mm^2).

Solution Try #3U stirrups—$A_v = 0.22$ in^2 (142 mm^2) From Fig. 7-16, $V_s = \frac{V_u}{\phi} - V_c =$ 17.5 k (77.7 kN) at ends.

$$V_s = \frac{A_v f_y d}{s} \tag{7-12}$$

$$s = \frac{(0.22)(50)(30)}{17.5}$$

$s = 18.9$ in. (480 mm) (spacing required near ends) Use $s = 16$ in.

From Fig. 7-16, at 15 ft. from support $V_s = V_u/\phi - V_c = 9.8\ k_v(43.5\ kN)$.

$$s = \frac{(0.22)(50)(31.1)}{9.8}$$

$$s = 34.9\ \text{in.}\quad (886\ \text{mm}) > 24\ \text{in.}\ (588\ \text{mm})\ \text{max. spacing}$$

Maximum spacing controls except at ends.

$$\text{Check minimum } A_v = \frac{50\, b_w s}{f_y} = \frac{(50)(5.5)(24)}{50{,}000} \qquad (7\text{-}16)$$

$$\text{min. } A_v = 0.132\ \text{in}^2\ (85\ \text{mm}^2) < 0.22\ \text{in}^2\ (142\ \text{mm}^2)$$

Nature of Bond and Length of Transfer. When tendons are pretensioned, their stress is often transferred to the concrete solely by bond between the two materials. Thus there is a length of transfer at each end of the tendons to perform the function of anchorage, when mechanical end anchorages are not provided. The condition of bond stress existing at these ends is radically different from that along the intermediate length of a beam. At intermediate points, the bond stress is produced by the external shear or by the existence of cracks. Where there are no cracks and no shear, the bond stress is zero. At anchorage, bond stress exists immediately after transfer. The stress in the tendons varies from zero at the exposed end to a full prestress at some distance inside the concrete. That distance is known as the length of transfer; and such bond stress is termed prestress transfer bond.[12] A rather complete survey of the literature on transfer length and development bond is contained in reference 13.

The nature of prestress transfer bond is entirely different from the flexural bond stress produced by shear or cracks. At intermediate points along a beam, the bond stress is resisted by adhesion between steel and concrete, aided by mechanical resistance provided by corrugations in the steel when deformed bars are used. At end anchorages, the pre-tensioned tendons almost always slip and sink into the concrete at the moment of transfer. This slippage destroys most of the adhesion for the length of transfer and part of the mechanical resistance of the corrugations, leaving the bond stress to be carried largely by friction between steel and concrete.

Immediately after transfer, at end A, Fig. 7-19, the wire will have zero stress and its diameter will be restored to the unstressed diameter. At B, the inner end of the length of transfer, the wire will have almost full prestress, and, owing to Poisson's ratio effect, its diameter will be smaller than the unstressed diameter. Thus along the length of transfer, there is an expansion of the wire diameter which produces radial pressure against the surrounding concrete. Frictional force resulting from this pressure serves to transmit the stress between steel and concrete. In other words, a sort of wedging action takes place within that length of transfer.

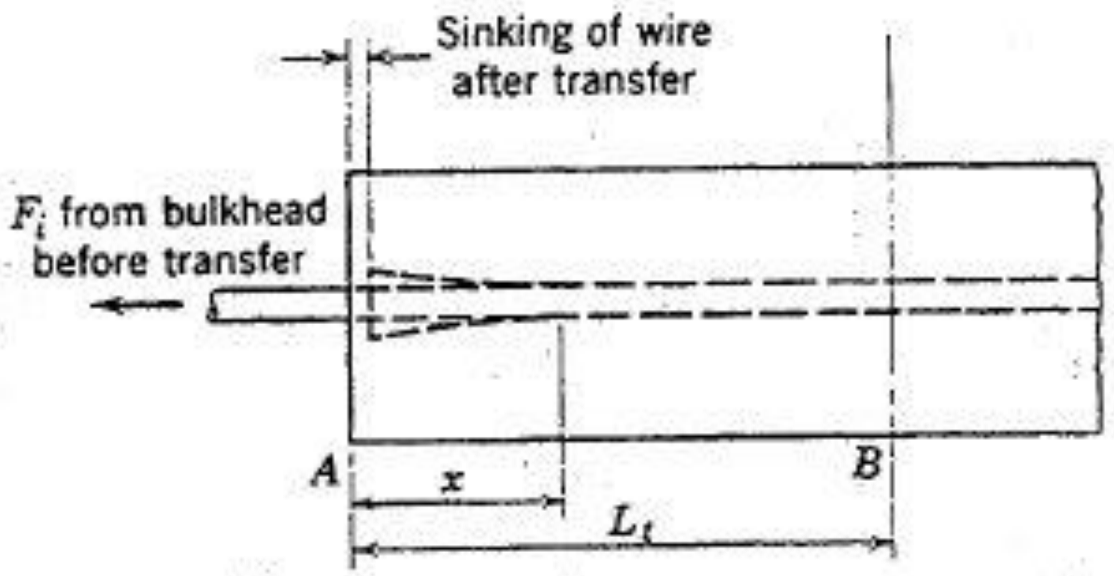

Fig. 7-19. Prestress transfer at end of pretensioned beam.

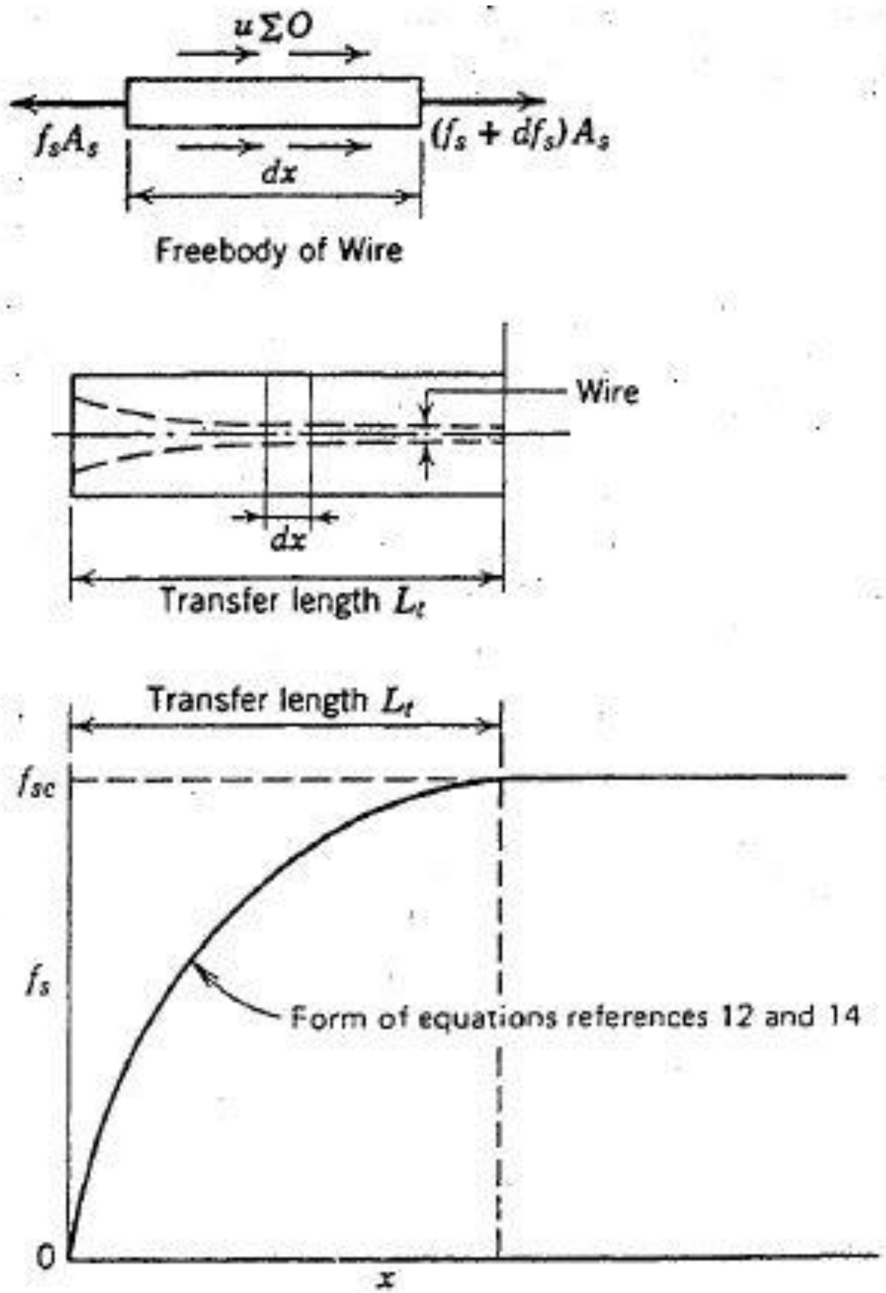

Fig. 7-20. Transfer length.

The transfer length for prestressing steel is affected by many parameters[13], the most important of them being:

1. Type of steel (e.g., wire, strand).
2. Steel size (diameter).
3. Steel stress level.
4. Surface condition of steel—clean, oiled, rusted.
5. Concrete strength.
6. Type of loading (e.g., static, repeated, impact).
7. Type of release [e.g., gradual, sudden (flame cutting, sawing)].
8. Confining reinforcement around steel (e.g., helix or stirrups).
9. Time-dependent effect.
10. Consolidation and consistency of concrete around steel.
11. Amount of concrete coverage around steel.

Sometimes it is necessary to design or to check the bearing areas for end anchorage, as governed by the allowable bearing in concrete. Since the cost of anchorage increases greatly if the allowable bearing stress is low, it has been the practice to use as high a bearing stress as is consistent with safety, much higher than permitted in reinforced concrete. This is true for practically all systems of prestressing. Besides reasons of economy, such high bearing stress can be justified on the following grounds.

1. The highest bearing stress that will ever exist at the anchorage occurs at transfer. As loss of prestress takes place, the bearing stress gradually diminishes.
2. The strength of concrete increases with time. Hence, if failure does not take place immediately at transfer, there is little possibility that it will happen later.
3. For bonded tendons with anchorages at the end of members, externally applied load will not increase the force on the anchorage. For unbonded tendons, the force on the anchorage will increase with load; but the increase is limited, hence a high factor of safety is not required.

The allowable bearing stress depends on several factors, such as the amount of reinforcement at the anchorage, the ratio of bearing to total area, and the method of stress computation.

Previously used equations were found to be too conservative[25] for design. Both the Post-Tensioning Institute Guide Specifications[26] and the 1977 ACI Commentary use the following equations for average bearing stress on the concrete, f_{cp}:

At service load—

$$f_{cp} = 0.6 f'_c \sqrt{A'_b / A_b} \qquad (7\text{-}25)$$

but not greater than f'_c

At transfer load—

$$f_{cp} = 0.8 f'_{ci} \sqrt{(A'_b / A_b) - 0.2} \qquad (7\text{-}26)$$

but not greater than $1.25 f'_{ci}$

where
f_{cp} = permissible compressive concrete stress
f'_c = compressive strength of concrete
f'_{ci} = compressive strength of concrete at time of initial prestress
A'_b = maximum area of the portion of the concrete anchorage surface that is geometrically similar to and concentric with the area of the anchorage
A_b = bearing area of the anchorage

EXAMPLE 7-6

Determine the bearing plate area required for a tendon consisting of $12 - \frac{1}{2}$ in. diameter, 7-wire strands, Fig. 7-22. At time of posttensioning assume that f'_{ci} is approximately 4000 psi (28 N/mm^2) and at service load after losses $f'_c = 5500$ psi (38 N/mm^2). The tendon forces for design are: 397 k (1,766 kN) due to maximum jacking force and 297 k (1321 kN) at service load after losses. Follow the Guide Specification of the Post-Tensioning Institute (PTI) for allowable bearing stresses on the concrete.

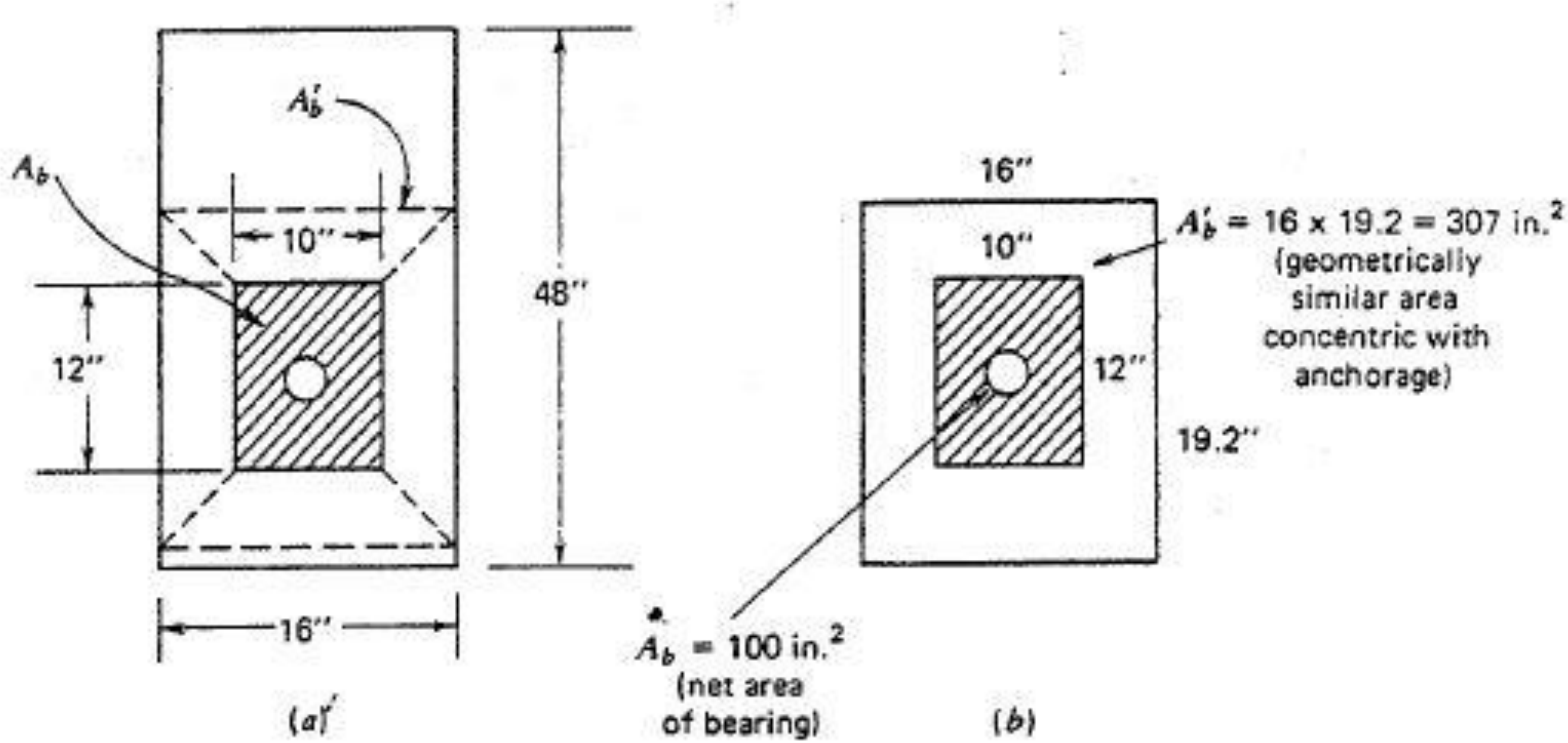

Fig. 7-22. Example 7-6.

Solution Note that we may assume $A'_b/A_b > 1.0$ as illustrated in Fig. 7-22 and estimate size of plate for service load requirements:

$$f_{cp} = 0.6 f'_c \sqrt{\frac{A'_b}{A_b}} = (0.6)(5500)(1) = 3300 \text{ psi} = 3.3 \text{ ksi } (22.75 \text{ N/mm}^2) \qquad (7\text{-}25)$$

Bearing area required $= \dfrac{297}{3.3} = 90$ in.^{2} (58×10^3 mm^2)

Assume an area of approximately 20 in.^{2} (13×10^3 mm^2) (5 in. diameter circular area) is lost as a bearing area for the tendon to pass through the plate.

Gross plate area $\cong 90 + 20 = 110$ in.^{2} (71×10^3 mm^2) Try plate 10 in. $\times$ 12 in. ($A_b = 120 - 20 = 100$ in.$^2 > 90$ in.^{2}) Check bearing pressure at transfer load. Assume maximum jacking load is 397 k (1766 kN) and $f'_{ci} = 4{,}000$ psi. (28 N/mm^2)

$$f_{cp} = 0.8 f'_{ci} \sqrt{\left(\frac{A'_b}{A_b}\right) - 0.2} \qquad (7\text{-}26)$$

$$f_{cp} = (0.8)(4{,}000)\sqrt{\frac{307}{100} - 0.2} = 5420 \text{ psi} = 5.42 \text{ ksi } (37.4 \text{ N/mm}^2)$$

$$A_b = \frac{397}{5.42} = 73.2 \text{ in.}^2 \; (47 \times 10^3 \text{ mm}^2) < 100 \text{ in.}^2 \; (65 \times 10^3 \text{ mm}^2) \text{ provided}$$

Use 10 in. × 12 in. Plate (O.K. at both stages)