CE 414: Prestressed Concrete
Lecture 17
Cambering and Deflection

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7-8 Transverse Tension at End Block

The portion of a prestressed member surrounding the anchorages of the tendons is often termed the end block. Throughout the length of the end block, prestress is transferred from more or less concentrated areas and distributed through the entire beam section. The theoretical length of the end block is the distance through which this change takes place and is sometimes called the lead length. It

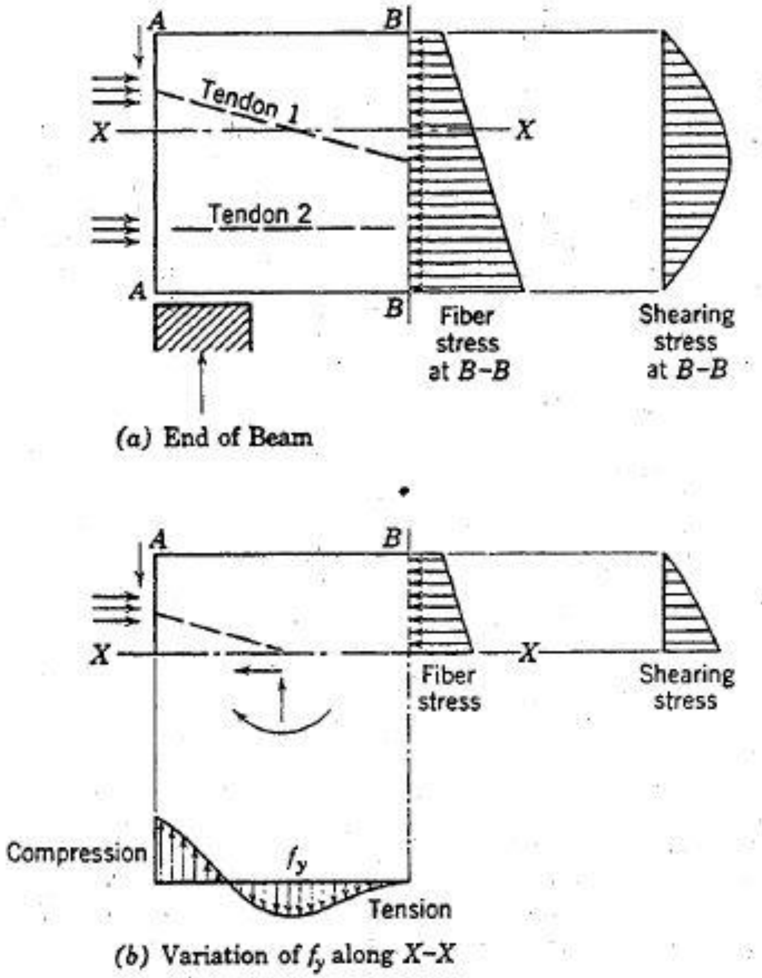

Fig. 7-23. Stresses at end block.

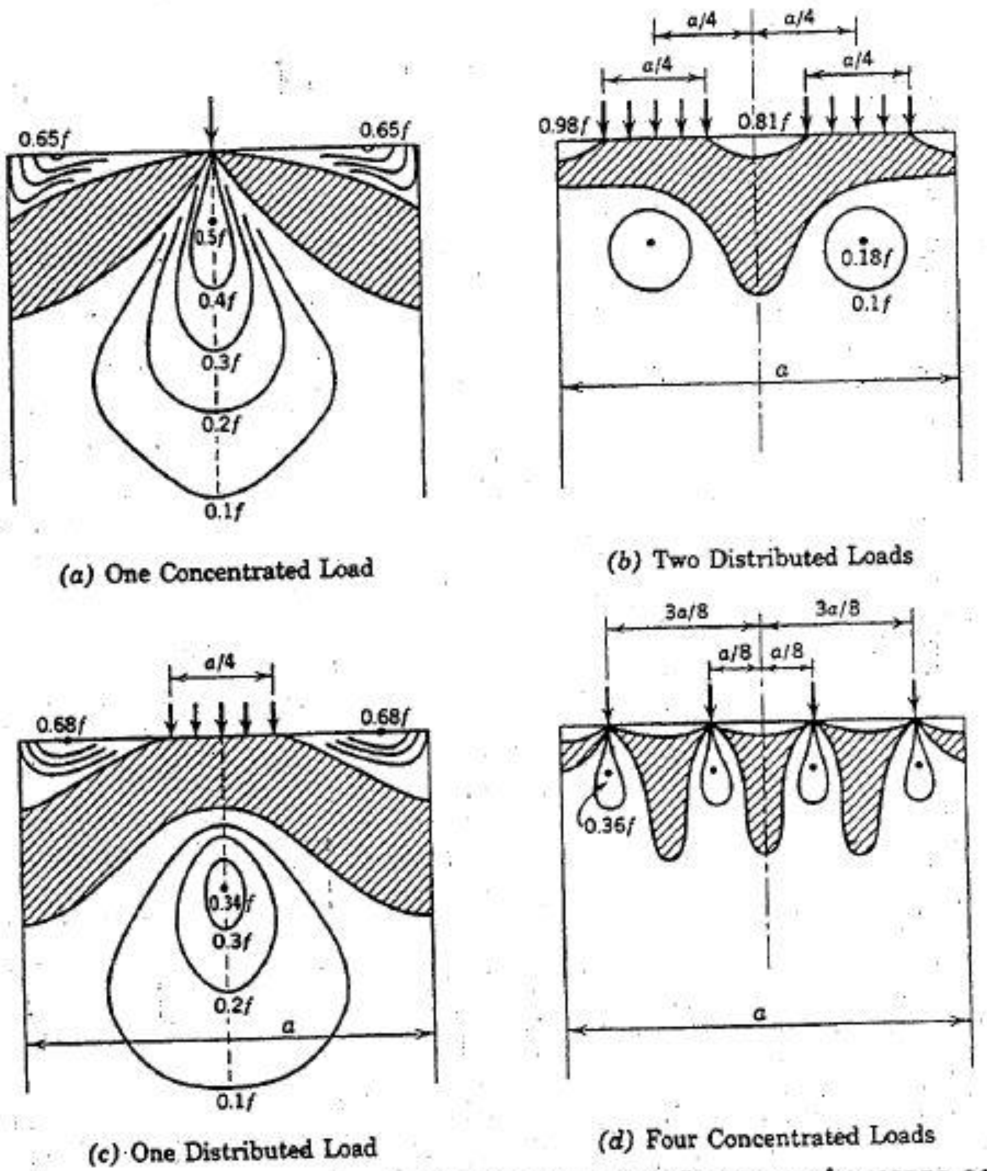

Fig. 7-24. Isobars for transverse tension in end block (in terms of average compression *f*). Shaded areas represent compressive zones. (From Guyon's *Prestressed Concrete.*)

8-1 Camber; Deflections

Before cracking, the deflections of prestressed-concrete beams can be predicted with greater precision than that of reinforced-concrete beams. Under working loads, prestressed-concrete beams do not crack; reinforced ones do. Since prestressed concrete is a more or less homogeneous elastic body which obeys quite closely the ordinary laws of flexure and shear, the deflections can be computed by methods available in elementary strength of materials.

As usually encountered for any concrete member, two difficulties still stand in the way when we wish to get an accurate prediction of the deflections. First, it is difficult to determine the value of E_c within an accuracy of 10% or even 20%. Tests on sample cylinders may not give the correct value of E_c, because E_c for beams may differ from that for cylinders. Besides, the value of E_c varies for different stress levels and changes with the age of concrete. The second difficulty lies in estimating the effect of creep on deflections.[1,2] The value of the creep coefficient as well as the duration and magnitude of the applied load cannot always be known in advance. However, for practical purposes, an accuracy of 10% or 20% is often sufficient, and that can be attained if all factors are carefully considered.

Deflections of prestressed beams differ from those of ordinary reinforced beams in the effect of prestress. While controlled deflections due to prestress can be advantageously utilized to produce desired cambers and to offset deflections due to loadings, there are also known cases where excessive cambers due to prestress have caused serious troubles.

EXAMPLE 8-1

A concrete beam of 32-ft simple span, Fig. 8-3, is posttensioned with 1.2 sq in. of high-tensile steel to an initial prestress of 140 ksi immediately after prestressing. Compute the initial deflection at midspan due to prestress and the beam's own weight, assuming $E_c = 4{,}000{,}000$ psi. Estimate the deflection after $1\frac{1}{2}$ months, assuming a creep coefficient of $C_c = 1.8$ and an effective prestress of 120 ksi at that time (span = 9.75 m, $A_{ps} = 774$ mm^2, $E_c = 27.58$ kN/mm^2, initial prestress = 965 N/mm^2, and effective prestress = 827 N/mm^2).

Solution Using the first method, take the concrete as a freebody and replace the tendon with forces acting on the concrete. The parabolic tendon with 6-in. (152.4 mm) midordinate is replaced by a uniform load acting along the beam with intensity

$$w = \frac{8Fh}{L^2} = \frac{8 \times 140{,}000 \times 1.2 \times 6}{32^2 \times 12} = 655 \text{ plf } (9.56 \text{ kN/m})$$

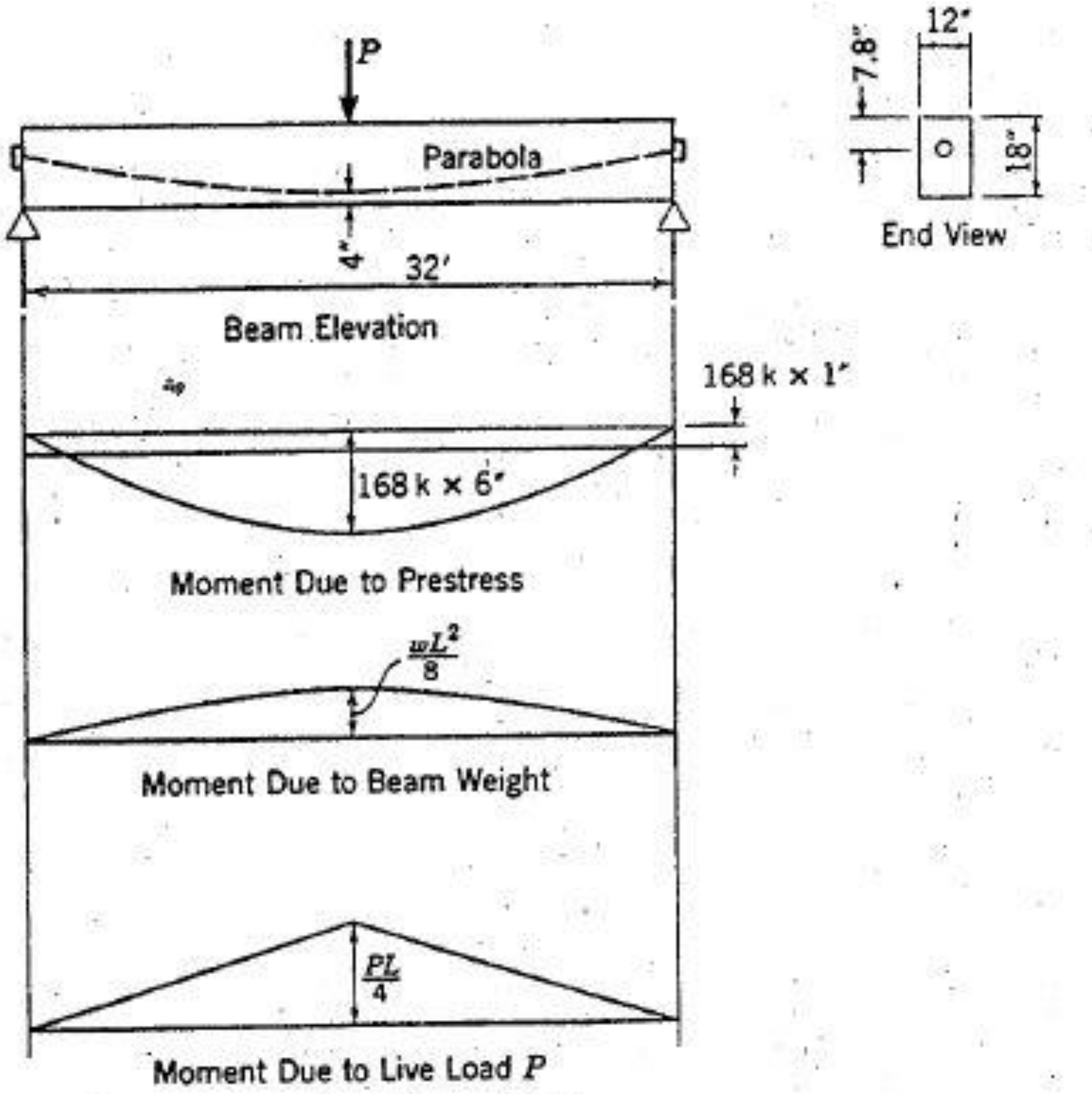

Fig. 8-3. Examples 8-1 and 8-2.

In addition, there will be two eccentric loads acting at the ends of the beam, each producing a moment of $140{,}000 \times 1.2 \times \frac{1}{12} = 14{,}000$ ft-lb. (19.0 kN-m)

Since the weight of the beam is 225 plf (3.28 kN/m), the net uniform load on concrete is $655 - 225 = 430$ plf (6.28 kN/m), which produces an upward deflection at midspan given by the usual deflection formula

$$\Delta = \frac{5wL^4}{384EI}$$

$$= \frac{5 \times 430 \times 32^4 \times 12^3}{384 \times 4{,}000{,}000 \times (12 \times 18^3)/12}$$

$$= 0.434 \text{ in. } (11.02 \text{ mm})$$

The end moments produce a downward deflection given by the formula

$$\Delta = \frac{ML^2}{8EI}$$

$$= \frac{140 \times 1.2 \times 1 \times 32^2 \times 12^2}{8 \times 4{,}000{,}000 \times (12 \times 18^3)/12}$$

$$= 0.133 \text{ in. } (3.38 \text{ mm})$$

Thus the net deflection due to prestress and beam weight is

$$0.434 - 0.133 = 0.301 \text{ in. } (7.64 \text{ mm}) \text{ upward}$$

If we follow the second method, it will not be necessary to compute the forces between the tendon and the concrete. Instead, the moment diagram is drawn from the eccentricity curve of the tendon, and the deflection computed therefrom. For convenience is computation, the moment diagram can be divided into two parts, a parabola and a rectangle (Fig. 8-3). By area-moment principles or any similar method, the upward deflection due to prestress can be computed to be

$$\Delta = \frac{5FhL^2}{48EI} - \frac{ML^2}{8EI}$$

$$= \frac{5 \times 140 \times 1.2 \times 6 \times 32^2 \times 12^2}{48 \times 4{,}000{,}000 \times (12 \times 18^3)/12} - \frac{140 \times 1.2 \times 1 \times 32^2 \times 12^2}{8 \times 4{,}000{,}000 \times (12 \times 18^3)/12}$$

$$= 0.661 - 0.133$$

$$= 0.528 \text{ in. } (13.41 \text{ mm})$$

Downward deflection due to beam weight of 225 plf is given by

$$\Delta = \frac{5wL^4}{384EI} = \frac{5\times 225\times 32^4\times 12^3}{384\times 4{,}000{,}000\times (12\times 18^3)/12} = 0.227 \text{ in. } (5.77 \text{ mm})$$

The resultant deflection is $0.528 - 0.227 = 0.301$ in. (7.64 mm) upward, the same answer as by the first method.

While the above gives the initial deflection, the eventual deflection should be modified by two factors: first, the loss of prestress, which tends to decrease the deflection; and second, the creep effect, which tends to increase the deflection. Since the prestress is reduced from 140 to 120 ksi (965–827 N/mm^2), the deflection due to prestress can be modified by the factor 120/140. Then, for the creep effect, the net deflection should be increased by the coefficient 1.8. Thus, if the beam is not subject to external loads the eventual deflection after $1\frac{1}{2}$ months can be estimated as

$$\left(0.528\times \frac{120}{140} - 0.227\right)1.8 = 0.407 \text{ in. } (10.34 \text{ mm}) \text{ upward}$$

The calculation for deflections due to external loads is similar to that for nonprestressed beams. So long as the concrete has not cracked, the beam can be treated as a homogeneous body and the usual elastic theory applied to it for deflection computations.

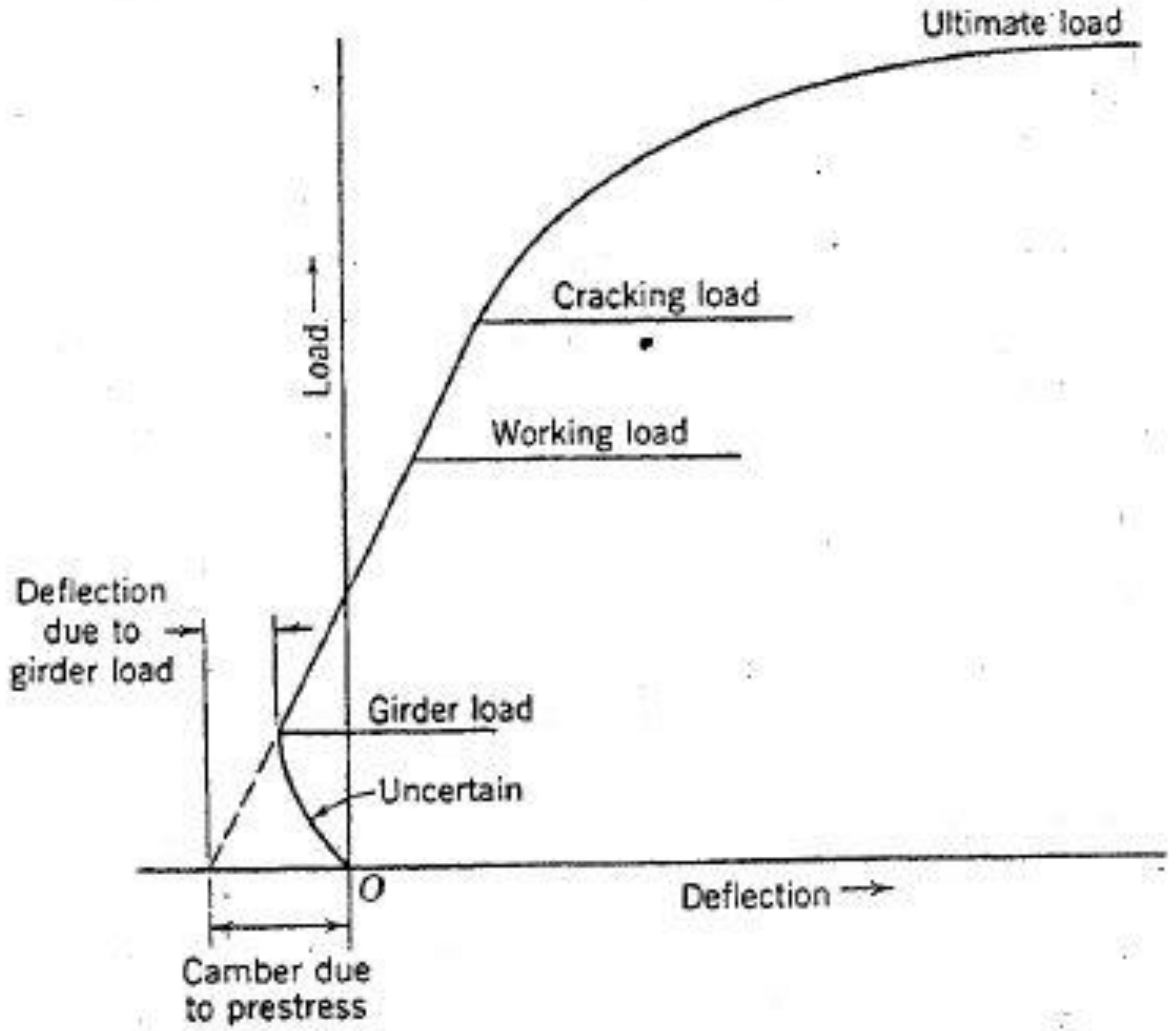

Fig. 8-4. Load deflection curve of a prestressed beam.

EXAMPLE 8-2

For the beam in example 8-1, compute the center deflection due to a 10-k (44.48 kN) concentrated load applied at midspan, when the beam is $1\frac{1}{2}$ months old after prestressing. Assume camber is 0.407 in. (10.34 mm) at this time prior to application of the 10-k (44.48 kN) load as computed in example 8-1.

Solution If the beam is bonded, the moment of inertia for the section should be computed on the basis of the transformed section including steel, but it can be approximated by using the gross concrete section. Also note that the modulus of elasticity E_c may be greater at the time of application of load than at transfer, but will be assumed to be 4,000,000 psi (27.58 kN/mm^2) for simplicity. Using the usual formula for deflection, we have

$$\Delta = \frac{PL^3}{48EI}$$

$$= \frac{10{,}000 \times 32^3 \times 12^3}{48 \times 4{,}000{,}000 \times (12 \times 18^3)/12}$$

$$= 0.505 \text{ in. } (12.83 \text{ mm})$$

which is the instantaneous downward deflection due to a load of 10 k (44.48 kN). Since the deflection before the application of load was 0.407 in. (10.34 mm) upward, the resultant deflection is 0.505 − 0.407 = 0.098 in. (2.49 mm) downward. If the load is kept on for a time, the creep effect due to that load must be considered. Also, if the load is heavy enough to produce cracking, then the elastic theory for computing deflection can be used only as guidance for an approximation.

Deflection response to a transient live load would be made using the elastic modulus, E_c, for concrete. A realistic estimate of the actual strength for concrete at the time of loading must be made, and the ACI equation for E_c will give a reasonable estimate of the modulus of elasticity to use in deflection calculations.

$$E_c = w^{1.5} 33 \sqrt{f_c'} \tag{2-1}$$

where w = unit weight of concrete
f_c' = concrete strength
E_c = elastic modulus

Note that this equation is valid for lightweight concrete as well as normal weight concrete. For typical lightweight concrete the value of E_c will be about 75% of the value for normal weight concrete.

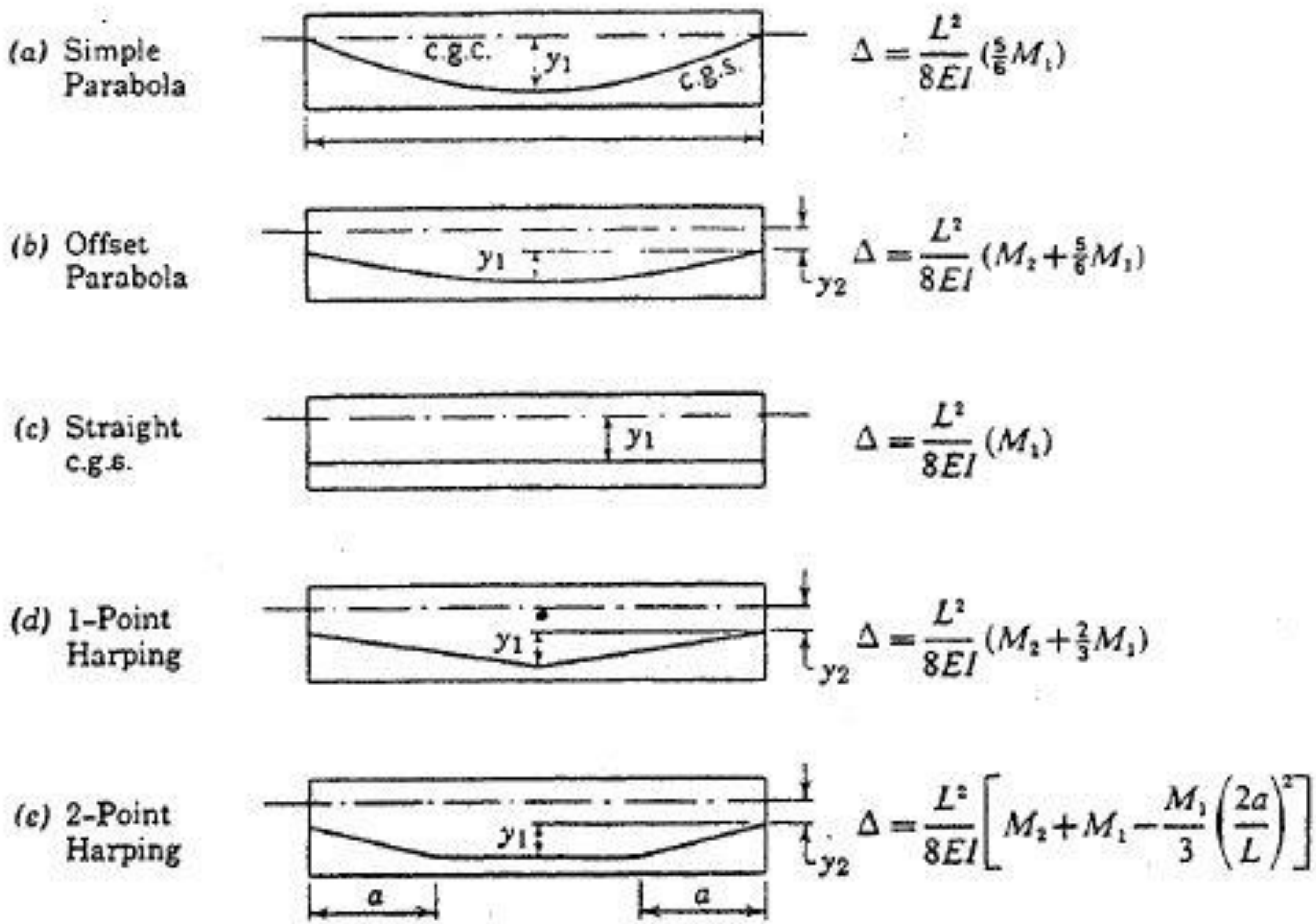

Fig. 8-6. Formulas for computing midspan camber due to prestress (simple beams).