$$27 = 3\times 9 = 3\times 3\times 3 = 3^3$$

HCF of Numerators is $= 3^1 = 3$

LCM of Numerators is $= 3^4 = 81$

Finally, the HCF of $\frac{2}{3}$, $\frac{8}{9}, \frac{16}{81}$ *and* $\frac{10}{27}$ is $= \frac{HCF(2,8,16,10)}{LCM(3,9,81,27)} = \frac{2}{81}$ & LCM $= \frac{LCM(2,8,16,10)}{HCF(3,9,81,27)} = \frac{80}{3}$ (Ans)

6. Evaluate $\sqrt{-16}\times\sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$.

Solution: We have $i^2 = -1$

Now, $\sqrt{-16}\times\sqrt{-4} = \sqrt{16i^2}\times\sqrt{4i^2} = \sqrt{4^2 i^2}\times\sqrt{2^2 i^2} = 4i\times 2i = 8i^2 = 8\times(-1) = -8$

Again,

$$\frac{\sqrt{-16}}{\sqrt{-4}} = \frac{\sqrt{16i^2}}{\sqrt{4i^2}} = \frac{\sqrt{4^2 i^2}}{\sqrt{2^2 i^2}} = \frac{4i}{2i} = 2$$

7. Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its polar, exponential form.

Solution: $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i} = \frac{\left(1+\sqrt{3}i\right)^2}{\left(1-\sqrt{3}i\right)\left(1+\sqrt{3}i\right)} = \frac{\left(1+\sqrt{3}i\right)^2}{1^2+\left(\sqrt{3}\right)^2} = \frac{1+2\sqrt{3}i+3i^2}{1^2+\left(\sqrt{3}\right)^2} = \frac{1+2\sqrt{3}i-3}{4} = \frac{-2+2\sqrt{3}i}{4} = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$= a+ib$ where $a = -\frac{1}{2}$ & $b = \frac{\sqrt{3}}{2}$.

Now $r = \sqrt{a^2+b^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{1} = 1$ and $\theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{\sqrt{3}/2}{-1/2}\right) = \tan^{-1}\left(-\sqrt{3}\right) = -\tan^{-1}\left(\sqrt{3}\right) = -\frac{\pi}{3} = \frac{2\pi}{3}$

So, the polar form is, $z = r(\cos\theta + i\sin\theta) = 1.\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ & Exponential form, $z = e^{i.\frac{2\pi}{3}}$

(Ans)

02 Radicals & Exponents

Radical: An expression containing the radical symbol $\left(\sqrt{\ }\right)$ is called a radical. The general form of a radical is $\sqrt[n]{a}$

Where n is the index and a is the radicand.

Note: 1. The index n is omitted if $n=2$.

2. Two or more radicals are called similar if the index and radicand are same.

Formulae for Radicals: The formulae for radicals are

1. $\left(\sqrt[n]{a}\right)^n = a$
2. $\sqrt[n]{ab} = \sqrt[n]{a}.\sqrt[n]{b}$
3. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad , b \neq 0$
4. $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
5. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Simplification of Radicals: The radicals can be simplified as,

1. By removing the perfect nth powers of the radicand.
2. By reducing the index of the radical
3. By rationalizing of the denominator of the radicand.

Problem-1: Find the simplest form the followings:

a. $\left(\sqrt[3]{6}\right)^3$ b. $\sqrt[3]{54}$ c. $\sqrt[5]{\frac{5}{32}}$ d. $\sqrt[3]{(27)^4}$ e. $\sqrt[3]{\sqrt{5}}$

<u>**Solution:**</u>

a. We have $\left(\sqrt[3]{6}\right)^3 = (6)^{3.\frac{1}{3}} = 6$

b. We have $\sqrt[3]{54} = \sqrt[3]{3^3 \cdot 2} = \sqrt[3]{3^3} \cdot \sqrt[3]{2} = 3\sqrt[3]{2}$

c. We have $\sqrt[5]{\frac{5}{32}} = \frac{\sqrt[5]{5}}{\sqrt[5]{32}} = \frac{\sqrt[5]{5}}{\sqrt[5]{2^5}} = \frac{1}{2}\sqrt[5]{5}$

d. We have $\sqrt[3]{(27)^4} = \left(\sqrt[3]{27}\right)^4 = \left(\sqrt[3]{3^3}\right)^4 = 3^4 = 81$

e. We have $\sqrt[3]{\sqrt{5}} = \sqrt[6]{5}$

Problem-2: Find the simplest form the followings:

a. $\sqrt{18}$ b. $\sqrt[4]{6480}$ c. $\sqrt[4]{\frac{16}{81}}$ d. $\sqrt[5]{(72)^4}$ e. $\sqrt[4]{\sqrt[3]{2}}$

<u>**Solution:**</u>

a. We have $\sqrt{18} = \sqrt{3^2 \cdot 2} = \sqrt{3^2} \cdot \sqrt{2} = 3\sqrt{2}$

b. We have $\sqrt[4]{6480} = \sqrt[4]{3^4 \cdot 2^4 \cdot 5} = \sqrt[4]{3^4} \cdot \sqrt[4]{2^4} \cdot \sqrt[4]{5}$
$= 3 \cdot 2\sqrt[4]{5} = 6\sqrt[4]{5}$

c. We have $\sqrt[4]{\frac{16}{81}}=\frac{\sqrt[4]{16}}{\sqrt[4]{81}}=\frac{\sqrt[4]{2^4}}{\sqrt[4]{3^4}}=\frac{2}{3}$

e. We have $\sqrt[4]{\sqrt[3]{2}}=\sqrt[12]{2}$

d. We have $\sqrt[5]{(72)^4}=\sqrt[5]{\left(2^3\cdot 3^2\right)^4}=\sqrt[5]{2^{12}\cdot 3^8}$

$$=\sqrt[5]{2^{12}}\cdot\sqrt[5]{3^8}=2^{\frac{12}{5}}\cdot 3^{\frac{8}{5}}=2^{2+\frac{2}{5}}\cdot 3^{1+\frac{3}{5}}$$

$$=\left(2^2\cdot 2^{\frac{2}{5}}\right)\cdot\left(3\cdot 3^{\frac{3}{5}}\right)=\left(4\sqrt[5]{2^2}\right)\cdot\left(3\cdot\sqrt[5]{3^3}\right)$$

$$=12\sqrt[5]{2^2\cdot 3^3}=12\sqrt[5]{108}$$

Problem-3: Find the simplest form the followings:

a. $\sqrt[6]{81a^2}$ b. $\left(7\sqrt[3]{4ab}\right)^2$ c. $\sqrt[3]{64x^7y^{-6}}$ d. $\sqrt[3]{\frac{(x+1)^3}{(y-2)^6}}$

Solution:

a. We have $\sqrt[6]{81a^2}=\sqrt[6]{3^4a^2}=\sqrt[6]{3^4}.\sqrt[6]{a^2}$

$$=3^{\frac{4}{6}}.a^{\frac{2}{6}}=3^{\frac{2}{3}}.a^{\frac{1}{3}}=\sqrt[3]{3^2}.\sqrt[3]{a}=\sqrt[3]{9a}$$

b. We have $\left(7\sqrt[3]{4ab}\right)^2=49\left(\sqrt[3]{2^2ab}\right)^2$

$$=49\sqrt[3]{2^3\cdot 2a^2b^2}=98\sqrt[3]{2a^2b^2}$$

c. We have $\sqrt[3]{64x^7y^{-6}}=\sqrt[3]{2^6\cdot x^7y^{-6}}=\sqrt[3]{2^6}\cdot\sqrt[3]{x^7}\cdot\sqrt[3]{y^{-6}}$

$$=2^{\frac{6}{3}}\cdot x^{\frac{7}{3}}\cdot y^{-\frac{6}{3}}=2^2\cdot x^{2+\frac{1}{3}}\cdot y^{-2}=4x^2\cdot x^{\frac{1}{3}}\cdot y^{-2}$$

$$=\frac{4x^2}{y^2}\sqrt[3]{x}$$

d. We have $\sqrt[3]{\frac{(x+1)^3}{(y-2)^6}}=\frac{\sqrt[3]{(x+1)^3}}{\sqrt[3]{(y-2)^6}}=\frac{x+1}{(y-2)^2}$

Exercise: Find the simplest form the followings:

a. $\sqrt{40}$ b. $\sqrt[3]{648}$ c. $\sqrt[6]{343}$ d. $\frac{x-25}{\sqrt{x}+5}$ e. $\sqrt[3]{\sqrt{246}}$ f. $\sqrt[4]{\sqrt[3]{6ab^2}}$

Solutions: $a.\ 2\sqrt{10}$ $b.\ 6\sqrt[3]{3}$ $c.\ \sqrt{7}$ $d.\sqrt{x}-5$ $e.2\sqrt[3]{2}$ $f.\sqrt[12]{6ab^2}$

Problem-4: Calculate the followings:

a. $\sqrt{18}+\sqrt{50}-\sqrt{72}$ b. $2\sqrt{27}-4\sqrt{12}$ c. $\sqrt{248+\sqrt{52+\sqrt{144}}}$ d. $\frac{112}{\sqrt{196}}\times\frac{\sqrt{576}}{12}\times\frac{\sqrt{256}}{8}$

<u>Solution</u>:

a. We have $\sqrt{18}+\sqrt{50}-\sqrt{72}$

$$=\sqrt{2\cdot 3^2}+\sqrt{2.5^2}-\sqrt{2^3\cdot 3^2}$$

$$=3\sqrt{2}+5\sqrt{2}-3\sqrt{2^2\cdot 2}$$

$$=3\sqrt{2}+5\sqrt{2}-6\sqrt{2}$$

$$=2\sqrt{2}$$

b. We have $2\sqrt{27}-4\sqrt{12}$

$$=2\sqrt{3^2\cdot 3}-4\sqrt{2^2\cdot 3}$$

$$=2\cdot 3\sqrt{3}-4\cdot 2\sqrt{3}$$

$$=6\sqrt{3}-8\sqrt{3}$$

$$=-2\sqrt{3}$$

c. We have $\sqrt{248+\sqrt{52+\sqrt{144}}}$

$$=\sqrt{248+\sqrt{52+\sqrt{2^4\cdot 3^2}}}$$

$$=\sqrt{248+\sqrt{52+2^2\cdot 3}}$$

$$=\sqrt{248+\sqrt{52+12}}$$

$$=\sqrt{248+\sqrt{64}}=\sqrt{248+\sqrt{2^6}}$$

$$=\sqrt{248+2^3}=\sqrt{248+8}$$

$$=\sqrt{2^8}=2^4=16$$

b. We have $\frac{112}{\sqrt{196}}\times\frac{\sqrt{576}}{12}\times\frac{\sqrt{256}}{8}$

$$=\frac{112}{\sqrt{2^2\cdot 7^2}}\times\frac{\sqrt{2^6\cdot 3^2}}{12}\times\frac{\sqrt{2^8}}{8}$$

$$=\frac{112}{2\cdot 7}\times\frac{2^3\cdot 3}{12}\times\frac{2^4}{8}$$

$$=\frac{112}{2\cdot 7}\times\frac{8\cdot 3}{12}\times\frac{16}{8}=32$$

Problem-5: Show that $\sqrt{5+2\sqrt{6}}=\sqrt{3}+\sqrt{2}$.

Solution: L.H.S $=\sqrt{5+2\sqrt{6}}$

$$=\sqrt{5+2\sqrt{3\cdot 2}}=\sqrt{5+2\sqrt{3}\cdot\sqrt{2}}=\sqrt{3+2\sqrt{3}\cdot\sqrt{2}+2}$$

$$=\sqrt{\left(\sqrt{3}\right)^2+2\sqrt{3}\cdot\sqrt{2}+\left(\sqrt{2}\right)^2}$$

$$=\sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2}$$

$$=\sqrt{3}+\sqrt{2}$$

= R.H.S

Problem-5: Find the cube root of 2744.

Solution: The cube root is $=\sqrt[3]{2744}$

$$=\sqrt[3]{2^3\cdot 7^3}$$

$$=2\cdot 7$$

$$=14$$

Problem-6: If $a*b*c=\sqrt{\frac{(a+2)(b+3)}{c+1}}$ then find the value of $6*15*3$.

Solution: We have $a*b*c=\sqrt{\frac{(a+2)(b+3)}{c+1}}$

Using $a=6, b=15 \ \& \ c=3$ we get

$$6*15*3=\sqrt{\frac{(6+2)(15+3)}{3+1}}$$

$$=\sqrt{\frac{8\times 45}{4}}$$

$$=\sqrt{90}$$

$$=\sqrt{2\cdot 3^2\cdot 5}$$

$$=3\sqrt{10}$$

Problem-7: Show that $3+\frac{1}{\sqrt{3}}+\frac{1}{3+\sqrt{3}}-\frac{1}{3-\sqrt{3}}=3$.

Solution: L.H.S = $3+\frac{1}{\sqrt{3}}+\frac{1}{3+\sqrt{3}}-\frac{1}{3-\sqrt{3}}$

$$=3+\frac{\sqrt{3}}{\left(\sqrt{3}\right)\left(\sqrt{3}\right)}+\frac{\left(3-\sqrt{3}\right)}{\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)}-\frac{\left(3+\sqrt{3}\right)}{\left(3-\sqrt{3}\right)\left(3+\sqrt{3}\right)}$$

$$=3+\frac{\sqrt{3}}{3}+\frac{\left(3-\sqrt{3}\right)}{9-3}-\frac{\left(3+\sqrt{3}\right)}{9-3}$$

$$=3+\frac{\sqrt{3}}{3}+\frac{\left(3-\sqrt{3}\right)}{6}-\frac{\left(3+\sqrt{3}\right)}{6}$$

$$=\frac{18+2\sqrt{3}+3-\sqrt{3}-3-\sqrt{3}}{6}$$

$$=\frac{18}{6}=3$$

= R.H.S

Problem-8: If $x=1+\sqrt{2} \ \& \ y=1-\sqrt{2}$, then find the value of $\left(x^2+y^2\right)$.

Solution: We have $x=1+\sqrt{2} \ \& \ y=1-\sqrt{2}$

Now $\left(x^2+y^2\right)$

$$=\left(1+\sqrt{2}\right)^2+\left(1-\sqrt{2}\right)^2$$

$$=1+2\sqrt{2}+\left(\sqrt{2}\right)^2+1-2\sqrt{2}+\left(\sqrt{2}\right)^2$$

$$=1+2\sqrt{2}+2+1-2\sqrt{2}+2$$

$$=6$$

Problem-9: If $\sqrt{1+\frac{x}{144}}=\frac{13}{12}$ then find the value of x.

Solution: We have $\sqrt{1+\frac{x}{144}}=\frac{13}{12}$

$$or,\ 1+\frac{x}{144}=\left(\frac{13}{12}\right)^2$$

$$or,\ \frac{x}{144}=\frac{169}{144}-1$$

$$or,\ \frac{x}{144}=\frac{169-144}{144}$$

$$or,\ \frac{x}{144}=\frac{25}{144}$$

$$or,\ x=25$$

Problem-10: What will come in the place of question mark $(?)^{\frac{1}{4}}=\frac{48}{(?)^{\frac{3}{4}}}$

Solution: Let the required value is *x*
According to question we can write,

$$(x)^{\frac{1}{4}}=\frac{48}{(x)^{\frac{3}{4}}}$$

$$or,(x)^{\frac{3}{4}}(x)^{\frac{1}{4}}=48$$

$$or,(x)^{\frac{3}{4}+\frac{1}{4}}=48$$

$$or,(x)^{\frac{4}{4}}=48$$

$$or,\ x=48$$

Problem-11: Find the value of $\frac{(243)^{\frac{n}{5}}\times 3^{2n+1}}{9^n\times 3^{n-1}}$.

Solution: We have, $\frac{(243)^{\frac{n}{5}}\times 3^{2n+1}}{9^n\times 3^{n-1}}$

$$=\frac{(3)^{5\cdot\frac{n}{5}}\times 3^{2n+1}}{3^{2n}\times 3^{n-1}}$$

$$=\frac{3^n \times 3^{2n+1}}{3^{2n+n-1}}$$

$$=\frac{3^{n+2n+1}}{3^{2n+n-1}}$$

$$=\frac{3^{3n+1}}{3^{3n-1}}$$

$$=\frac{3^{3n}\times 3}{3^{3n}\times 3^{-1}}$$

$$=3\times\frac{3}{1}=9$$

Problem-12: What will come in the place of question mark $\sqrt{86.49}+\sqrt{5+(?)}=12.3$

Solution: Let the required value is *x*

According to question we can write,

$$\sqrt{86.49}+\sqrt{5+(x)}=12.3$$

$$or, \sqrt{5+(x)}=12.3-\sqrt{86.49}$$

$$or, 5+x=\left(\frac{123}{10}-\sqrt{\frac{8649}{100}}\right)^2$$

$$or,\ x=\left(\frac{123}{10}-\frac{\sqrt{8649}}{\sqrt{100}}\right)^2-5$$

$$or,\ x=\left(\frac{123}{10}-\frac{93}{10}\right)^2-5$$

$$or,\ x=\left(\frac{30}{10}\right)^2-5$$

$$or,\ x=(3)^2-5$$

$$or,\ x=9-5$$

$$or,\ x=4$$

Problem-13: If $\sqrt{841}=29$, then find the value of $\sqrt{841}+\sqrt{8.41}+\sqrt{0.0841}+\sqrt{0.000841}$.

Solution: Since $\sqrt{841}=29$

Now $\sqrt{841}+\sqrt{8.41}+\sqrt{0.0841}+\sqrt{0.000841}$

$$=\sqrt{841}+\sqrt{\frac{841}{100}}+\sqrt{\frac{841}{10000}}+\sqrt{\frac{841}{1000000}}$$

$$=\sqrt{841}+\frac{\sqrt{841}}{\sqrt{100}}+\frac{\sqrt{841}}{\sqrt{10000}}+\frac{\sqrt{841}}{\sqrt{1000000}}$$

$$=29+\frac{29}{10}+\frac{29}{100}+\frac{29}{1000}$$

$$= \frac{29000 + 2900 + 290 + 29}{1000}$$

$$= \frac{32219}{1000}$$

$$= 32.219$$

03 Logarithm

Exponents & Logarithms

Exponents: If a is any number then the product of n numbers each of which is a, is defined as,