$$= \frac{29000+2900+290+29}{1000}$$

$$= \frac{32219}{1000}$$

$$= 32.219$$

03 Logarithm

Exponents & Logarithms

Exponents: If a is any number then the product of n numbers each of which is a, is defined as,

$$a^n$$

Where n **is called an exponent or index and** a **is called a base.**

Example: $2^5, 3^{-2}, x^6, p^{-7}$ *etc.*

Logarithms: If an expression is of the form

$$b^x = N \cdots (1) \quad where \;\; N > 0\,,\, b > 0 \;\&\; b \neq 1$$

then the logarithm of N to the base b is defined as

$$x = \log_b (N) \;\cdots (2) \quad where \;\; N > 0\,,\, b > 0 \;\&\; b \neq 1$$

The equations (1) & (2) are equivalent. The eq. (1) is in exponential form and the eq. (2) is in logarithmic form.

Example: Since $2^3 = 8$, then 3 is the logarithm of 8 to the base 2 i.e., $\log_2(8) = 3$.

Laws of Logarithms: The laws of logarithms are

6. $\log_b(MN) = \log_b(M) + \log_b(N)$
7. $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$
8. $\log_b(M^P) = P\log_b(M)$
9. $\log_b b = 1$

Problem-1: Using logarithmic laws write the followings:

b. $\log_2(3\cdot 5)$ b. $\log_3\left(\frac{17}{24}\right)$ c. $\log_3(5^7)$

Solution:

a. We have $\log_2(3\cdot 5) = \log_2 3 + \log_2 5$

b. We have $\log_3\left(\frac{17}{24}\right) = \log_3 17 - \log_3 24$

c. We have $\log_3(5^7) = 7\log_3 5$

Common Logarithms: The system of logarithms whose base is 10 is called the common logarithm system. When the base is omitted, it is understood that base 10 is to be used.

Thus, $\log 25 = \log_{10} 25$

Natural Logarithms: The system of logarithms whose base is the Eulerian constant e is called the natural logarithm system. When we want to indicate the base of a logarithm is e we write $\ln$.

Thus, $\ln 25 = \log_e 25$

NOTE: Since $10^{1.5377} = 34.49$ so $\log 34.49 = 1.5377$. Here the digit 1 before decimal point is called the characteristic and the digits .5377 after decimal point is called the mantissa of the log.

Problem-2: Express each of the following exponential form in logarithmic form:

a. $4^2 = 16$ b. $3^{-2} = \frac{1}{9}$ c. $8^{-\frac{2}{3}} = \frac{1}{4}$

Solution:

b. We have $4^2 = 16$

Using log of base 4 we get

$$\log_4 4^2 = \log_4 16$$

$$or,\ 2\log_4 4 = \log_4 16$$

$$or,\ 2 = \log_4 16$$

c. We have $8^{-\frac{2}{3}} = \frac{1}{4}$

Using log of base 8 we get

$$\log_8 8^{-\frac{2}{3}} = \log_8\left(\frac{1}{4}\right)$$

$$or,\ -\frac{2}{3}\log_8 8 = \log_8\left(\frac{1}{4}\right)$$

$$or,\ -\frac{2}{3} = \log_8\left(\frac{1}{4}\right)$$

a. We have $3^{-2} = \frac{1}{9}$

Using log of base 3 we get

$$\log_3 3^{-2} = \log_3\left(\frac{1}{9}\right)$$

$$or, -2\log_3 3 = \log_3\left(\frac{1}{9}\right)$$

$$or,\ -2 = \log_3\left(\frac{1}{9}\right)$$

Problem-3: Express each of the following logarithmic form in exponential form:

a. $\log_5 25 = 2$ b. $\log_2 64 = 6$ c. $\log_{1/4}\frac{1}{16} = 2$

Solution:

a. We have $\log_5 25 = 2$

By the definition of log we get

$$25 = 5^2$$

b. We have $\log_2 64 = 6$

By the definition of log we get

$$64 = 2^6$$

c. We have $\log_{1/4} \frac{1}{16} = 2$

By the definition of log we get

$$\frac{1}{16} = \left(\frac{1}{4}\right)^2$$

Problem-4: Find the logarithm of 1728 to the base $2\sqrt{3}$.

Solution: We have 1728

After factorization by prime number we get,

$$1728 = 2^6 \cdot 3^3$$

$$or,\ 2^6 \cdot \left(\sqrt{3}\right)^6 = 1728$$

$$or,\ \left(2\sqrt{3}\right)^6 = 1728$$

According to definition of logarithm we have,

$$6 = \log_{2\sqrt{3}} 1728$$

$$\therefore\ \log_{2\sqrt{3}} 1728 = 6$$

Problem-5: Find x if $\frac{1}{2}\log_{10}\left(11+4\sqrt{7}\right) = \log_{10}(2+x)$.

Solution: Given that, $\frac{1}{2}\log_{10}\left(11+4\sqrt{7}\right) = \log_{10}(2+x)$

$$or,\ \log_{10}\sqrt{11+4\sqrt{7}} = \log_{10}(2+x)$$

$$or,\ \sqrt{11+4\sqrt{7}} = 2+x$$

$$or,\ \left(\sqrt{11+4\sqrt{7}}\right)^2 = (2+x)^2$$

$$or,\ 11+4\sqrt{7} = x^2+4x+4$$

$$or,\ x^2+4x-7 = 4\sqrt{7}$$

$$or,\ x^2+4x-\left(7+4\sqrt{7}\right) = 0$$

$$\therefore\ x = \frac{-4 \pm \sqrt{4^2 - 4\cdot 1\cdot\left(7+4\sqrt{7}\right)}}{2\cdot 1}$$

$$= \frac{-4 \pm \sqrt{16-4\left(7+4\sqrt{7}\right)}}{2}$$

$$= \frac{-4 \pm \sqrt{16-28-16\sqrt{7}}}{2}$$

$$= \frac{-4 \pm \sqrt{-12-16\sqrt{7}}}{2}$$

$$= \frac{-4 \pm 2\sqrt{-3-4\sqrt{7}}}{2}$$

$$= -2 \pm \sqrt{-3-4\sqrt{7}}$$

Problem-6: Prove that $2\log x + 2\log x^2 + 2\log x^3 + \cdots + 2\log x^n = n(n+1)\log x$.

Solution: $L.H.S = 2\log x + 2\log x^2 + 2\log x^3 + \cdots + 2\log x^n$

$$= 2\log x + 2\log x^2 + 2\log x^3 + \cdots + 2\log x^n$$

$$= 2\log x + 4\log x + 6\log x + \cdots + 2n\log x$$

$$= (1+2+3+\cdots+n)2\log x$$

$$= \frac{n(n+1)}{2}\cdot 2\log x$$

$$= n(n+1)\log x$$

$$= R.H.S \quad (Proved)$$

Problem-7 : Express the logarithm of $\frac{\sqrt{a^3}}{c^5b^2}$ in terms of $\log a, \log b \,\&\, \log c$.

Solution: We have $\frac{\sqrt{a^3}}{c^5b^2}$

The logarithm of this part is,

$$\log\left(\frac{\sqrt{a^3}}{c^5b^2}\right)$$

$$\log\left(\frac{\sqrt{a^3}}{c^5b^2}\right)$$

$$or,\ x\ln a-2x\ln c=(3x+1)\ln b$$

$$or,\ x\ln a-2x\ln c=(3x+1)\ln b$$

$$or,\ x\ln a-2x\ln c=3x\ln b+\ln b$$

$$or,\ x\ln a-2x\ln c-3x\ln b=\ln b$$

$$or,\ x(\ln a-2\ln c-3\ln b)=\ln b$$

$$or,\ x\left(\ln a-\ln c^2-\ln b^3\right)=\ln b$$

$$or,\ x=\frac{\ln b}{\ln a-\ln c^2-\ln b^3}$$

$$=\frac{\ln b}{\ln\left(\frac{a}{c^2b^3}\right)}$$

Problem-8: Find x from the equation $a^x.c^{-2x}=b^{3x+1}$

Solution: We have $a^x.c^{-2x}=b^{3x+1}$

$$or,\ln\left(a^x.c^{-2x}\right)=\ln b^{3x+1}$$

$$or,\ln a^x+\ln c^{-2x}=\ln b^{3x+1}$$

$$or,\ x\ln a-2x\ln c=(3x+1)\ln b$$

$$or,\ x\ln a-2x\ln c=(3x+1)\ln b$$

$$or,\ x\ln a-2x\ln c=3x\ln b+\ln b$$

$$or,\ x\ln a-2x\ln c-3x\ln b=\ln b$$

$$or,\ x(\ln a-2\ln c-3\ln b)=\ln b$$

$$or,\ x\left(\ln a-\ln c^2-\ln b^3\right)=\ln b$$

$$or,\ x=\frac{\ln b}{\ln a-\ln c^2-\ln b^3}$$

$$=\frac{\ln b}{\ln\left(\frac{a}{c^2b^3}\right)}$$

Problem-09: Solve $\log_{10}(3x+2)+\log_{10}(x-1)=1$.

Solution: We have $\log_{10}(3x+2)+\log_{10}(x-1)=1$

$$or,\ \log_{10}(3x+2)(x-1)=1$$

$$or,\ \log_{10}\left(3x^2-3x+2x-2\right)=1$$

$$or,\ \log_{10}\left(3x^2-x-2\right)=1$$

$$or,\ 3x^2-x-2=10^1$$

$$or,\ 3x^2-x-2=10$$

$$or,\ 3x^2-x-12=0$$

$$\therefore\ x=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot 3\cdot(-12)}}{2\cdot 1}$$

$$=\frac{1\pm\sqrt{1+24}}{2}$$

$$=\frac{1\pm\sqrt{25}}{2}$$

$$=\frac{1\pm 5}{2}$$

$$=-2,\ 3$$

Problem-10: Solve the equation $\frac{e^x-1}{e^{-x}-1}=-3$

Solution: We have $\frac{e^x-1}{e^{-x}-1}=-3$

$$or,\ \frac{e^x-1}{\frac{1}{e^x}-1}=-3$$

$$or,\ \frac{e^x-1}{\frac{1-e^x}{e^x}}=-3$$

$$or,\ \frac{e^{2x}-e^x}{1-e^x}=-3$$

$$or,\ e^{2x}-e^x=-3+3e^x$$

$$or,\ \left(e^x\right)^2-4e^x+3=0$$

$$\therefore\ e^x=\frac{-(-4)\pm\sqrt{(-4)^2-4\cdot 1\cdot 3}}{2\cdot 1}$$

$$=\frac{4\pm\sqrt{16-12}}{2}$$

$$= \frac{4 \pm \sqrt{4}}{2}$$

$$= \frac{4 \pm 2}{2}$$

$$= 1, 3$$

Problem-11: Calculate the value of p from $\log_{10} 4 + 2\log_{10} p = 2$

Solution: We have $\log_{10} 4 + 2\log_{10} p = 2$

$$or, \log_{10} 4 + \log_{10} p^2 = 2$$

$$or, \log_{10} 4p^2 = 2$$

$$or, 4p^2 = 10^2$$

$$or, 4p^2 = 100$$

$$or, p^2 = 25$$

$$or, p = \pm 5$$

04 Inequality

Number Line: A straight line whose each point indicates a single number is called a number line. Graphically it is denoted by