SSB and VSB

Ref: Modern Digital and Analog Communication Systems, B. P. Lathi, 3rd Edition

Single Sideband (SSB) Modulation:

- Both sidebands of a standard AM signal or a DSB-SC signal carry the same information, so it is possible to remove one of them without losing any information.

- A Scheme in which only one sideband is transmitted is known as single sideband (SSB) transmission, which requires only one-half the bandwidth of the DSB signal.

SSB- SC AM

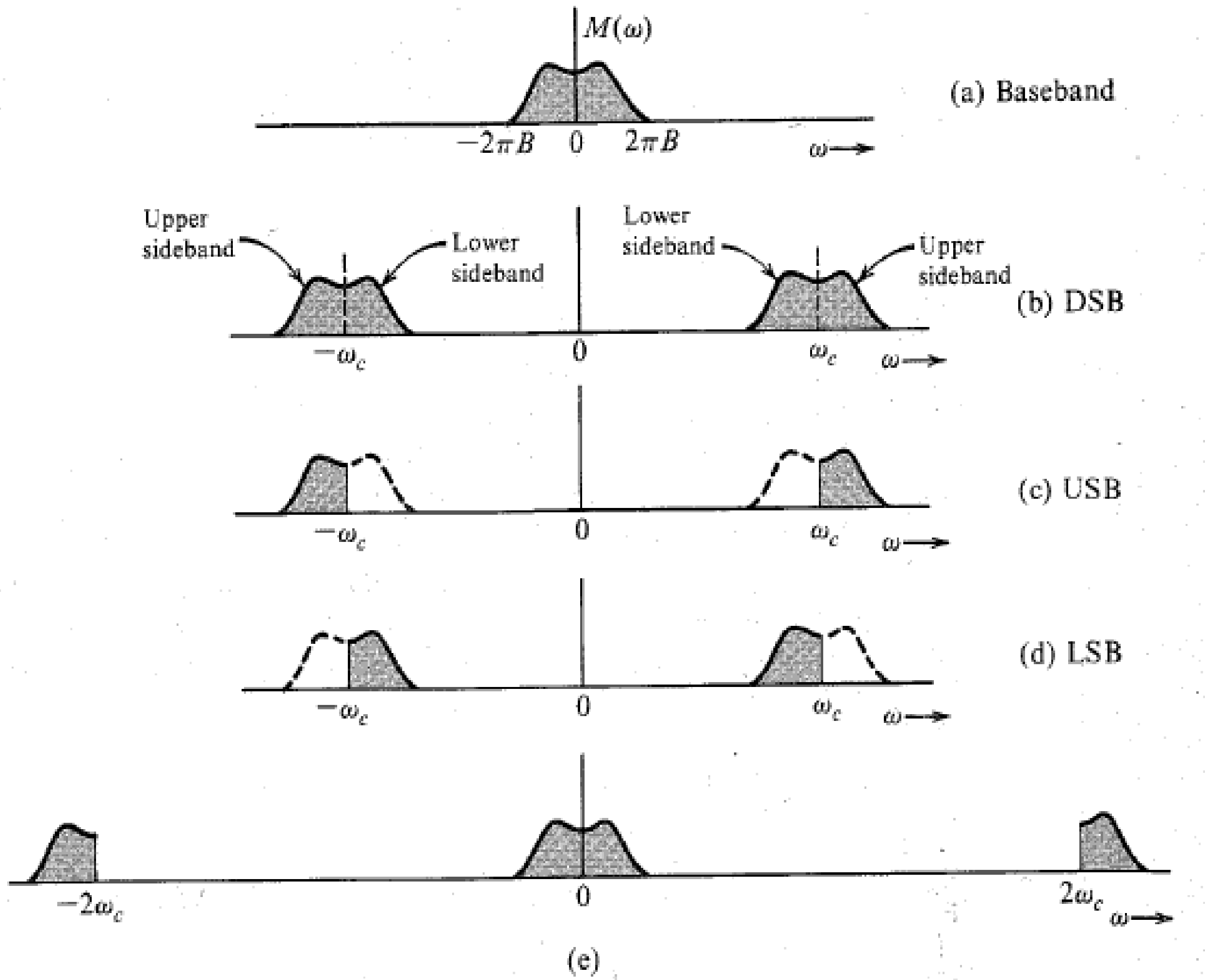


Fig: SSB Spectra

SSB

An SSB signal can be mathematically expressed as

$$\varphi_{SSB}(t) = m(t)cos\omega_c t \mp m_h(t)sin\omega_c t$$

Where the negative sign is used for upper sideband SSB and the positive sign is used for lower sideband SSB.

The term $m_h(t)$ denotes the **Hilbert transformation** of $m(t)$, which is given by

$$m_h(t) = m(t) * h_{HT}(t) = m(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\boldsymbol{\alpha})}{t-\boldsymbol{\alpha}} d\boldsymbol{\alpha}$$

$H_{HT}(\omega)$, the Fourier transform of $h_{HT}(t)$, corresponds to a **-90° phase** shift.

$$H_{HT}(\omega) = \begin{cases} -j, & \omega > 0 \\ j, & \omega < 0 \end{cases}$$

Example 4.7 Tone modulation: SSB

Find $\varphi_{SSB}(t)$ for a simple case of a tone modulation, that is, when the modulating signal is a sinusoid $m(t) = cos\omega_m t$.

Solution: $\varphi_{SSB}(t) = m(t)cos\omega_c t \mp m_h(t)sin\omega_c t$ ……(1)

$m_h(t)$ = Hilbert transformation of $m(t)$, Hilbert transform **delays** the phase of each spectral component by $\pi/2$.

Delaying the phase of $m(t)$ by $\pi/2$ yields

$$m_h(t) = cos(\omega_m t - \frac{\pi}{2}) = sin\omega_m t$$

From equation (1), $\varphi_{SSB}(t) = cos\omega_m t\ cos\omega_c t \mp sin\omega_m t\ sin\omega_c t$
$= cos(\omega_c \pm \omega_m)t$

Thus, $\varphi_{USB}(t) = cos(\omega_c + \omega_m)t$ and $\varphi_{LSB}(t) = cos(\omega_c - \omega_m)t$

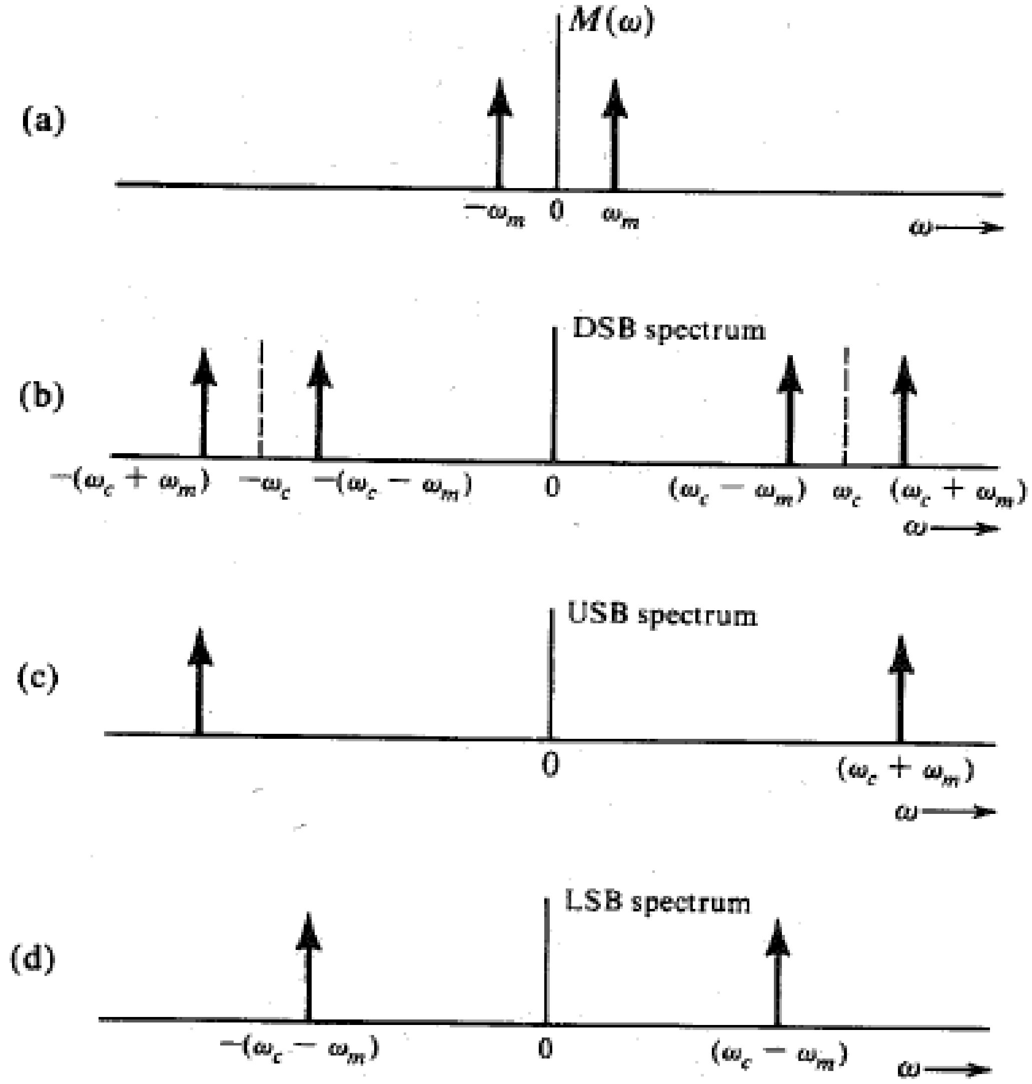


Figure 4.18 SSB spectra for tone modulation.

Generation of SSB Signal:

Two common techniques used for generating an SSB signal are:

- Selective-Filtering Method
- Phase-Shift Method

Selective-Filtering Method: This is the most commonly used method of generating SSB signals. In this method, a DSB-SC signal is passed through a sharp cutoff filter to eliminate the undesired sideband.

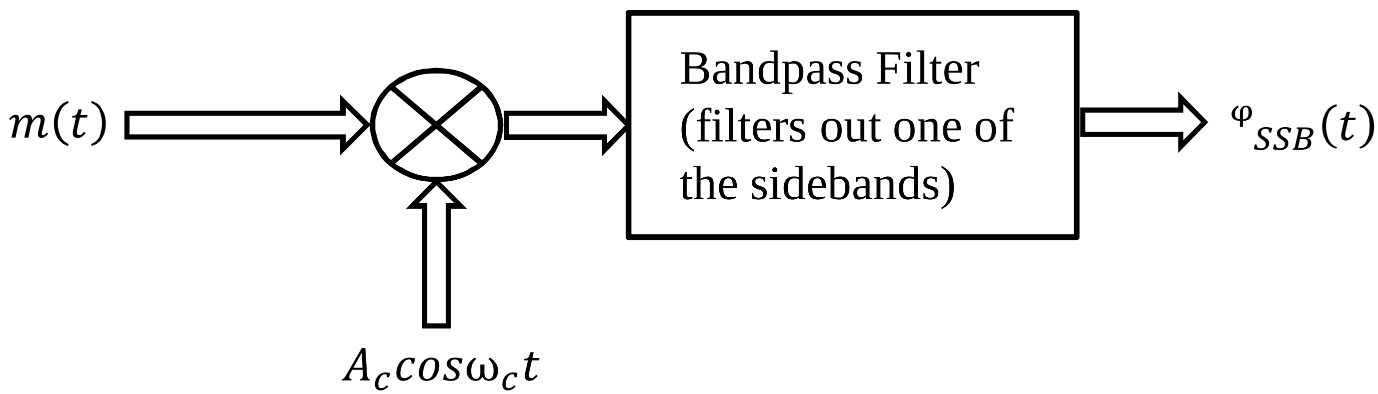


Fig: SSB generation by selective filtering method

Phase-Shift Method: Mathematical expression of SSB signal,

$$\varphi_{SSB}(t) = m(t)cos\omega_c t \mp m_h(t)sin\omega_c t$$

Phase shift method is the direct implementation of this equation. The modulating signal is split into two identical signals, one, which modulates the in-phase carrier, and the other which is passed through a -90° phase shifter before modulating a quadrature carrier. The sign used for the quadrature component determines whether USB or LSB is transmitted.

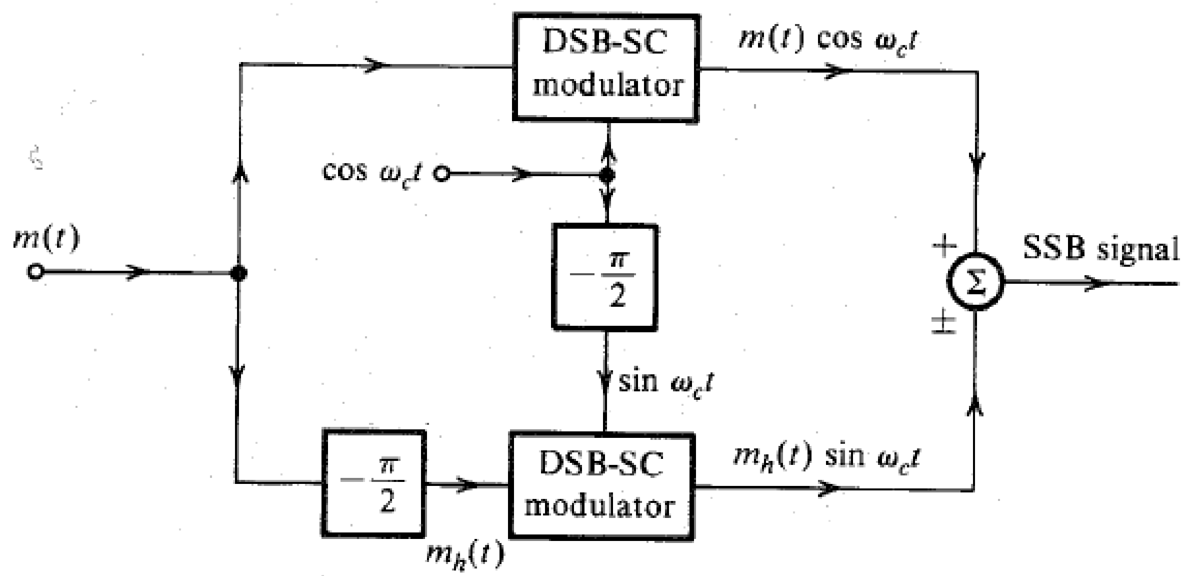


Figure 4.20 SSB generation by phase-shift method.

Demodulation of SSB-SC Signals:

Coherent method: SSB-SC signals can be coherently demodulated.

$$\varphi_{SSB}(t) = m(t)cos\omega_c t \mp m_h(t)sin\omega_c t$$

Hence, $\varphi_{SSB}(t)\ cos\omega_c t = [m(t)cos\omega_c t \mp m_h(t)sin\omega_c t]\ cos\omega_c t$

$$= m(t)cos^2\omega_c t \mp m_h(t)sin\omega_c t\ cos\omega_c t$$

$$= \frac{1}{2}m(t)[1 + cos2\omega_c t] \mp \frac{1}{2}m_h(t)sin2\omega_c t$$

$$= \frac{1}{2}m(t) + \frac{1}{2}[m(t)cos2\omega_c t \mp m_h(t)sin2\omega_c t]$$

The product $\varphi_{SSB}(t)\ cos\omega_c t$ yields the baseband signal and another SSB signal with a carrier frequency $2\omega_c$. A low pass filter will suppress the unwanted SSB terms, giving the desired baseband signal $\frac{1}{2}m(t)$.

Envelope Detection of SSB Signals with a Carrier (SSB+C):

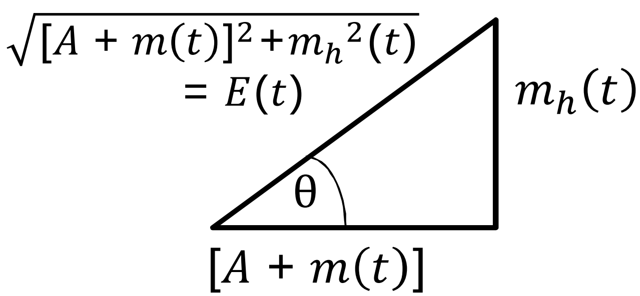
SSB signal with an additional carrier (SSB+C) can be expressed as

$$\varphi_{SSB+C}(t) = A\cos\omega_c t + [m(t)\cos\omega_c t + m_h(t)\sin\omega_c t]$$

If the carrier amplitude, A is large enough, $m(t)$ can also be recovered from $\varphi_{SSB+C}(t)$ by envelope or rectifier detection.

$$\varphi_{SSB+C}(t) = A\cos\omega_c t + [m(t)\cos\omega_c t + m_h(t)\sin\omega_c t]$$

$$= [A + m(t)]\cos\omega_c t + m_h(t)\sin\omega_c t$$



$$= E(t)\frac{[A+m(t)]}{E(t)}\cos\omega_c t + E(t)\frac{m_h(t)}{E(t)}\sin\omega_c t$$

$$= E(t)[\cos\omega_c t\cos\theta + \sin\omega_c t\sin\theta]$$

$$\Rightarrow \varphi_{SSB+C}(t) = E(t)\cos(\omega_c t - \theta)$$

where $E(t)$ is the envelope of $\varphi_{SSB+C}(t)$ and

$$\theta = \tan^{-1}\frac{m_h(t)}{[A + m(t)]}$$

Now, $$E(t) = \{[A + m(t)]^2 + {m_h}^2(t)\}^{\frac{1}{2}}$$

$$= [A^2 + 2Am(t) + m^2(t) + {m_h}^2(t)]^{1/2}$$

$$= A[1 + \frac{2m(t)}{A} + \frac{m^2(t)}{A^2} + \frac{m_h^2(t)}{A^2}]^{1/2}$$

If $A \gg |m(t)|$, then in general $A \gg |m_h(t)|$ and the terms $\frac{m^2(t)}{A^2}$ and $\frac{m_h^2(t)}{A^2}$ can be ignored. Thus,

$$E(t) \simeq A\,[1 + \frac{2m(t)}{A}]^{1/2}$$

Using binomial expansion and discarding higher order terms [because $m(t)/A \ll 1$], we get

$$E(t) \simeq A[1 + \frac{m(t)}{A}]$$

$$= A + m(t)$$

So, $$\varphi_{SSB+C}(t) = [A + m(t)]\cos(\omega_c t - \theta)$$

It is evident that for a large carrier, SSB+C can be demodulated by an envelope detector.

In AM, envelope detection requires the condition $\mathsf{A} \geq |m(t)|$, whereas for SSB+C, the condition is $\mathsf{A} \gg |m(t)|$. Hence, in SSB case, the required carrier amplitude is much larger than that in AM, and consequently, the efficiency of SSB+C is pathetically low.

Amplitude Modulation: Vestigial Sideband (VSB)

- Generation of SSB signals is difficult (Limitation of SSB).

- Generation of DSB signals is simpler but requires twice the bandwidth (Limitation of DSB).

- A vestigial sideband (VSB), also called asymmetric sideband system is a compromise between DSB and SSB.

- VSB signals are relatively easy to generate, and, at the same time, their bandwidth is only (typically 25%) greater than that of SSB signals.

VSB

In VSB, instead of rejecting one sideband completely, a gradual cutoff of one sideband is accepted.

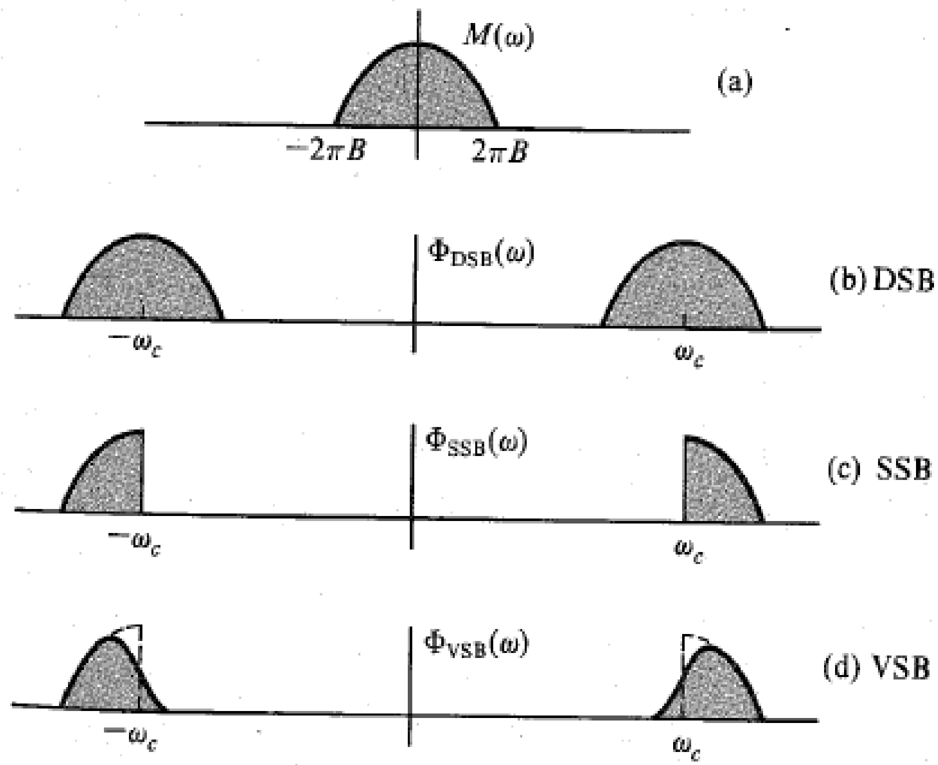


Figure 4.21 Spectra of the modulating signal and corresponding DSB, SSB, and VSB signals.

VSB Modulator:

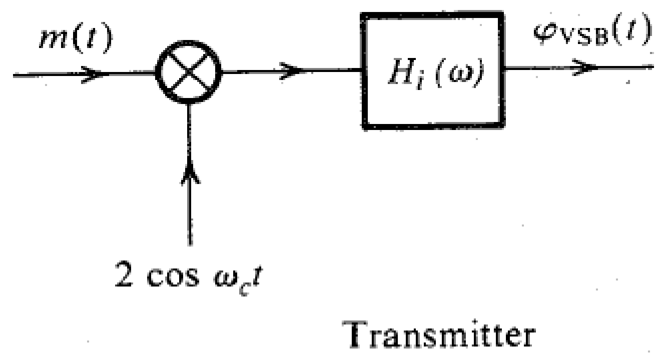


Fig: VSB Modulator

$H_i(\omega)$= Vestigial shaping filter that produces VSB from DSB

VSB signal spectrum: $\Phi_{VSB}(\omega) = [M(\omega + \omega_c) + M(\omega - \omega_c)]H_i(\omega)$

The VSB shaping filter $H_i(\omega)$ allows the transmission of one sideband, but suppresses the other sideband, **not completely, but gradually**. This makes it easy to realize such a filter, but the transmission bandwidth is now somewhat **higher** than that of the SSB. The bandwidth of the VSB signal is typically 25 to 33% higher than that of the SSB signals.

VSB Demodulator:

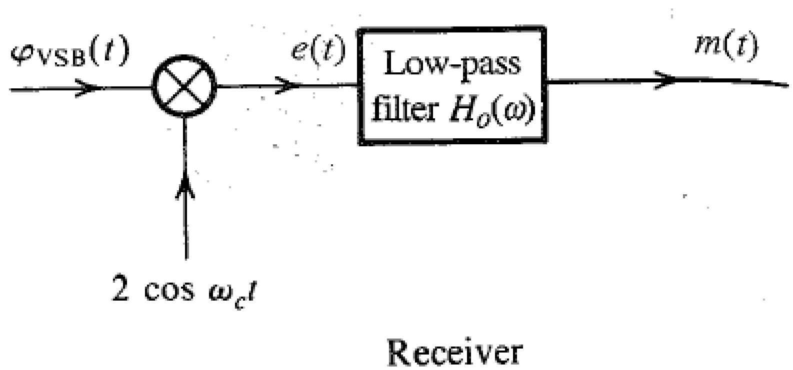


Fig: VSB Demodulator

To recover $m(t)$ from $\varphi_{VSB}(t)$, synchronous demodulation can be done. This is done by multiplying the incoming VSB signal $\varphi_{VSB}(t)$ by $2cos\omega_c t$. The product $e(t)$ is given by, $e(t) = 2\,\varphi_{VSB}(t)cos\omega_c t$

$$\Rightarrow E(\omega) = \varphi_{VSB}(\omega + \omega_c) + \varphi_{VSB}(\omega - \omega_c)$$

The signal $e(t)$ is further passed through the low-pass equalizer filter of transfer function $H_o(\omega)$. The output of the equalizer filter is $m(t)$. Hence, the output signal spectrum is given by

$$M(\omega) = [\varphi_{VSB}(\omega + \omega_c) + \varphi_{VSB}(\omega - \omega_c)]\, H_o(\omega)$$