

Block Diagram fundamentals & reduction techniques

Lect# 4-5

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Introduction

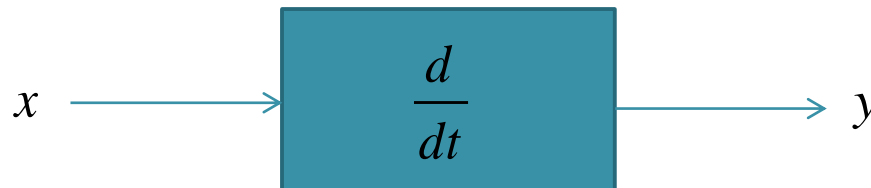
- Block diagram is a shorthand, graphical representation of a physical system, illustrating the functional relationships among its components.

OR

- A Block Diagram is a shorthand pictorial representation of the cause-and-effect relationship of a system.

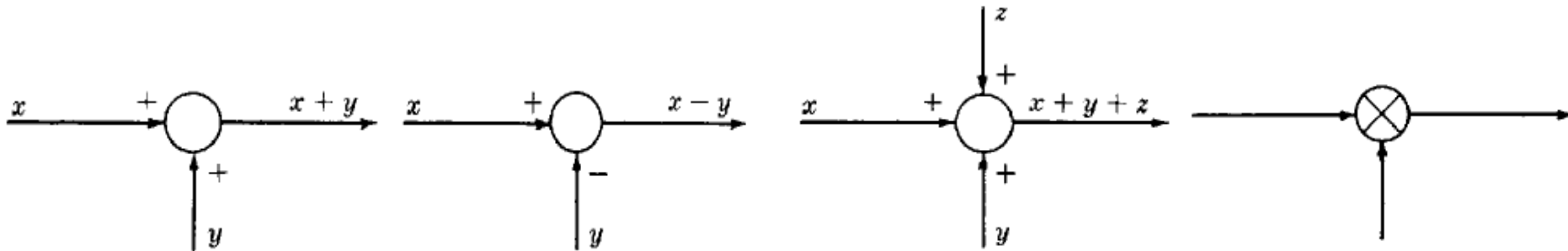
Introduction

- The simplest form of the block diagram is the single ***block***, ***with one input and one output***.
- The interior of the rectangle representing the block usually contains a description of or the name of the element, or the symbol for the mathematical operation to be performed on the input to yield the output.
- The arrows represent the direction of information or signal flow.



Introduction

- The operations of addition and subtraction have a special representation.
- The block becomes a small circle, called a summing point, with the appropriate plus or minus sign associated with the arrows entering the circle.
- Any number of inputs may enter a summing point.
- The output is the algebraic sum of the inputs.
- Some books put a cross in the circle.

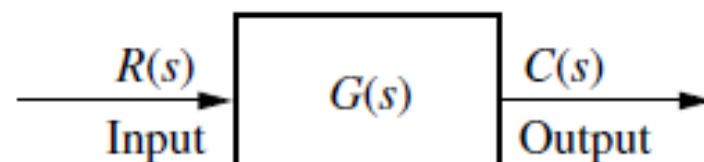


Components of a Block Diagram for a Linear Time Invariant System

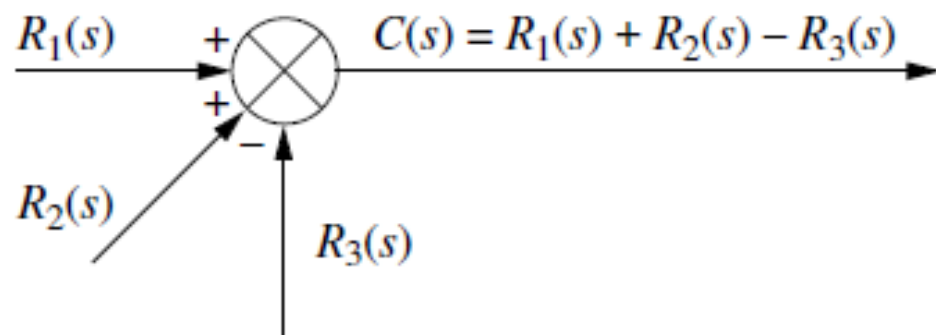
- System components are alternatively called elements of the system.
- Block diagram has four components:
 - *Signals*
 - *System/ block*
 - *Summing junction*
 - *Pick-off/ Take-off point*



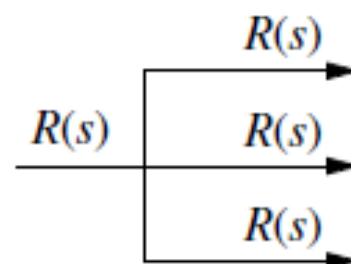
Signals
(a)



System
(b)

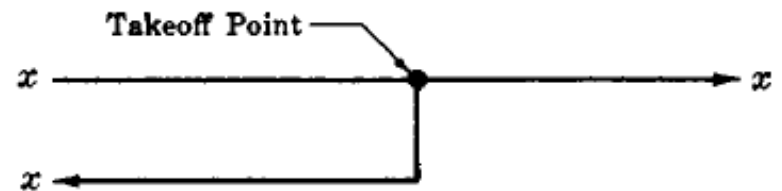
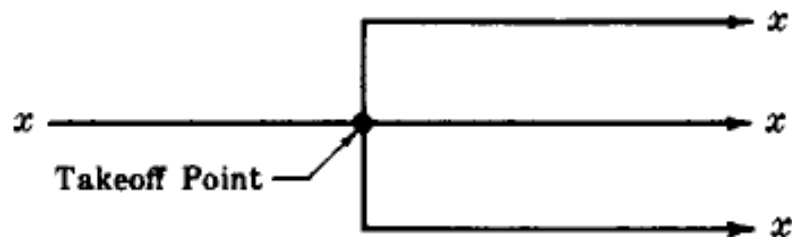


Summing junction
(c)



Pickoff point
(d)

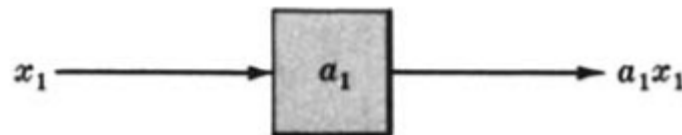
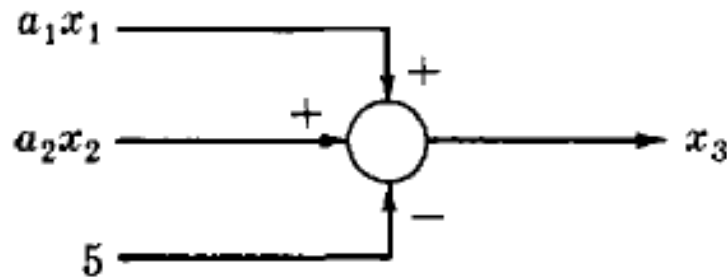
- In order to have the same signal or variable be an input to more than one block or summing point, a takeoff point is used.
- Distributes the input signal, undiminished, to several output points.
- This permits the signal to proceed unaltered along several different paths to several destinations.



Example-1

- Consider the following equations in which x_1 , x_2 , x_3 , are variables, and a_1 , a_2 are general coefficients or mathematical operators.

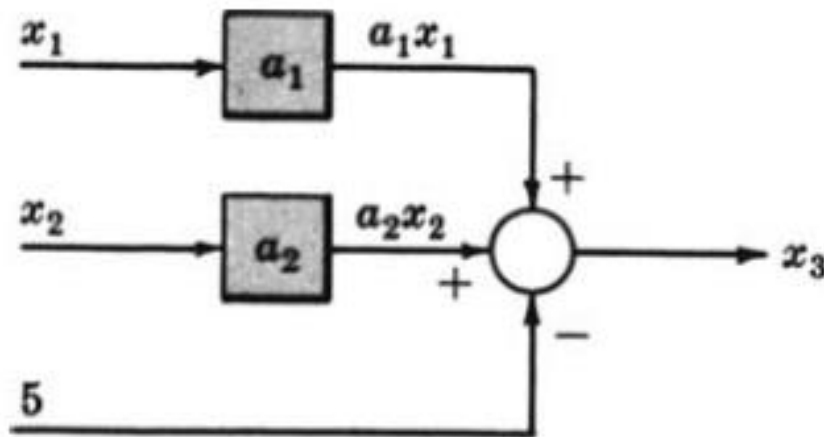
$$x_3 = a_1x_1 + a_2x_2 - 5$$



Example-1

- Consider the following equations in which x_1 , x_2 , x_3 , are variables, and a_1 , a_2 are general coefficients or mathematical operators.

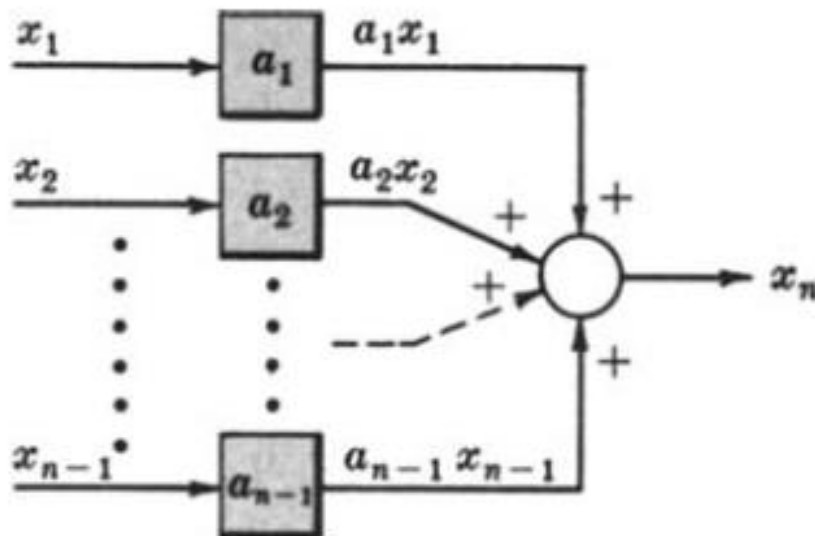
$$x_3 = a_1x_1 + a_2x_2 - 5$$



Example-2

- Consider the following equations in which x_1, x_2, \dots, x_n , are variables, and a_1, a_2, \dots, a_n , are general coefficients or mathematical operators.

$$x_n = a_1 x_1 + a_2 x_2 + a_{n-1} x_{n-1}$$



Example-3

- Draw the Block Diagrams of the following equations.

$$(1) \quad x_2 = a_1 \frac{dx_1}{dt} + \frac{1}{b} \int x_1 dt$$

$$(2) \quad x_3 = a_1 \frac{d^2 x_2}{dt^2} + 3 \frac{dx_1}{dt} - b x_1$$

Topologies

- We will now examine some common topologies for interconnecting subsystems and derive the single transfer function representation for each of them.
- These common topologies will form the basis for reducing more complicated systems to a single block.

CASCADE

- Any finite number of blocks in series may be algebraically combined by multiplication of transfer functions.
- That is, *n components or blocks with transfer functions G_1, G_2, \dots, G_n , connected in cascade* are equivalent to a single element G with a transfer function given by

$$G = G_1 \cdot G_2 \cdot G_3 \cdots G_n = \prod_{i=1}^n G_i$$

Example



- Multiplication of transfer functions is *commutative*; that is,

$$G_i G_j = G_j G_i$$

for any i or j .

Cascade:

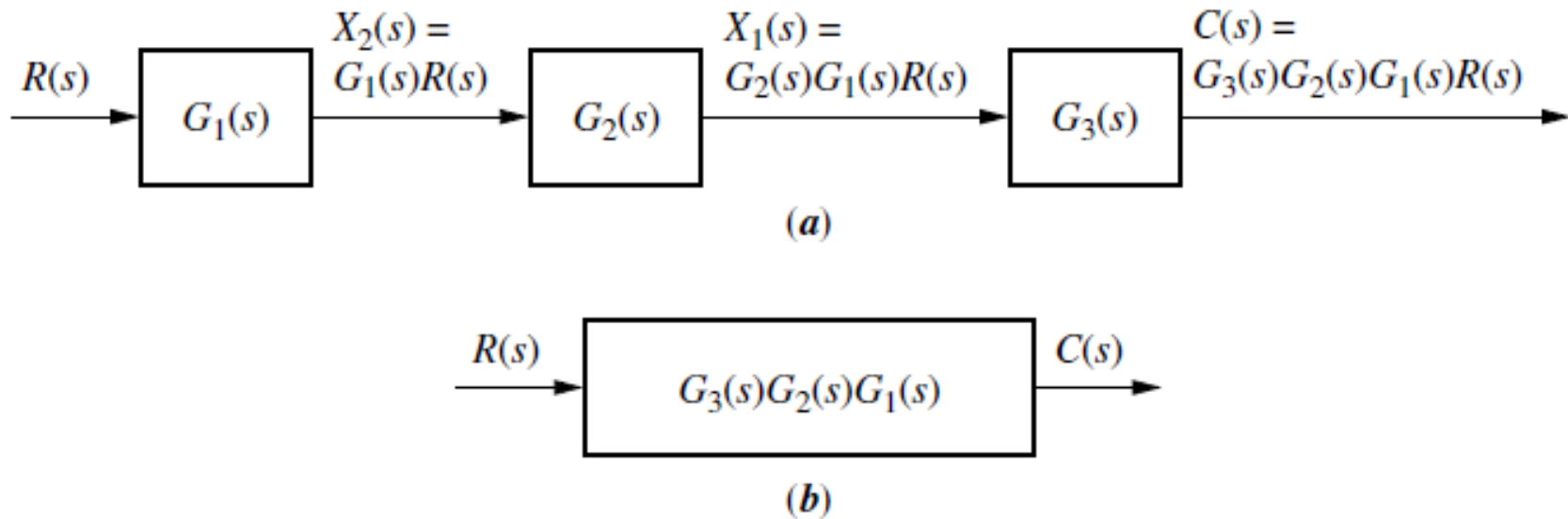


Figure:

a) Cascaded Subsystems.

b) Equivalent Transfer Function.

The equivalent transfer function is

$$G_e(s) = G_3(s)G_2(s)G_1(s)$$

Parallel Form:

- Parallel subsystems have a common input and an output formed by the algebraic sum of the outputs from all of the subsystems.

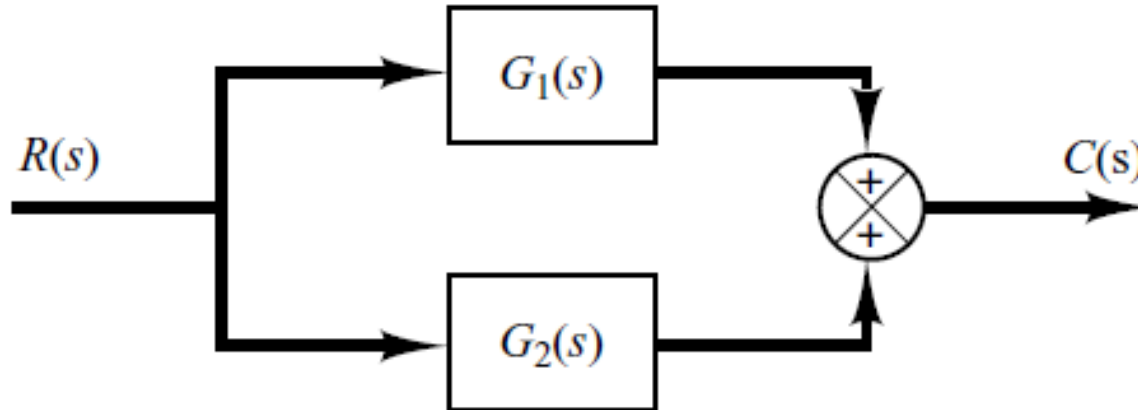
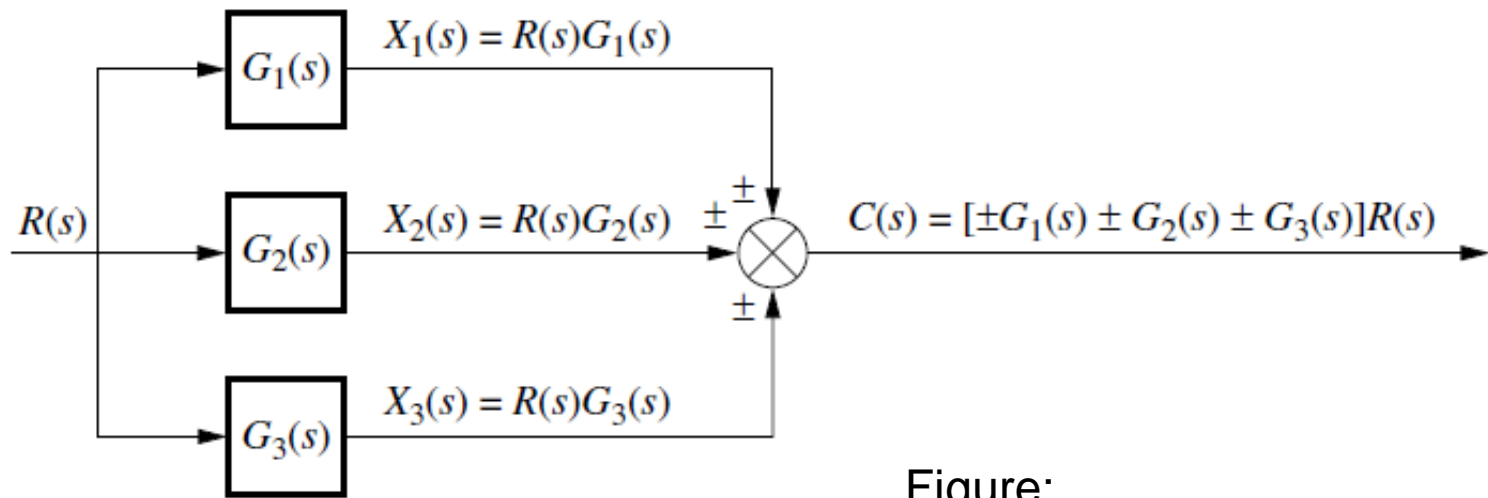


Figure: Parallel Subsystems.

Parallel Form:

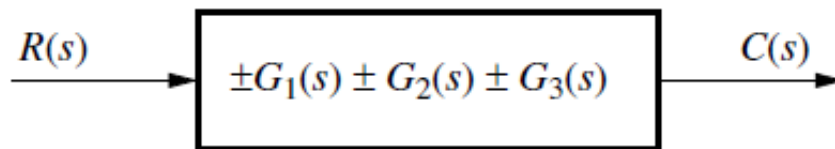


(a)

Figure:

a) Parallel Subsystems.

b) Equivalent Transfer Function.



(b)

The equivalent transfer function is

$$G_e(s) = \pm G_1(s) \pm G_2(s) \pm G_3(s)$$

Feedback Form:

- The third topology is the feedback form. Let us derive the transfer function that represents the system from its input to its output. The typical feedback system, shown in figure:

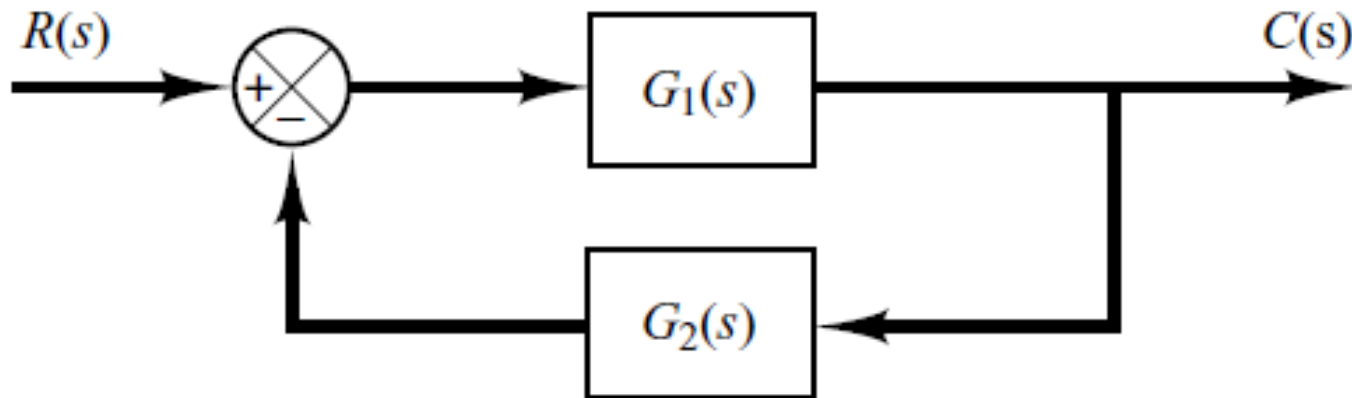


Figure: Feedback (Closed Loop) Control System.

The system is said to have negative feedback if the sign at the summing junction is negative and positive feedback if the sign is positive.

Feedback Form:

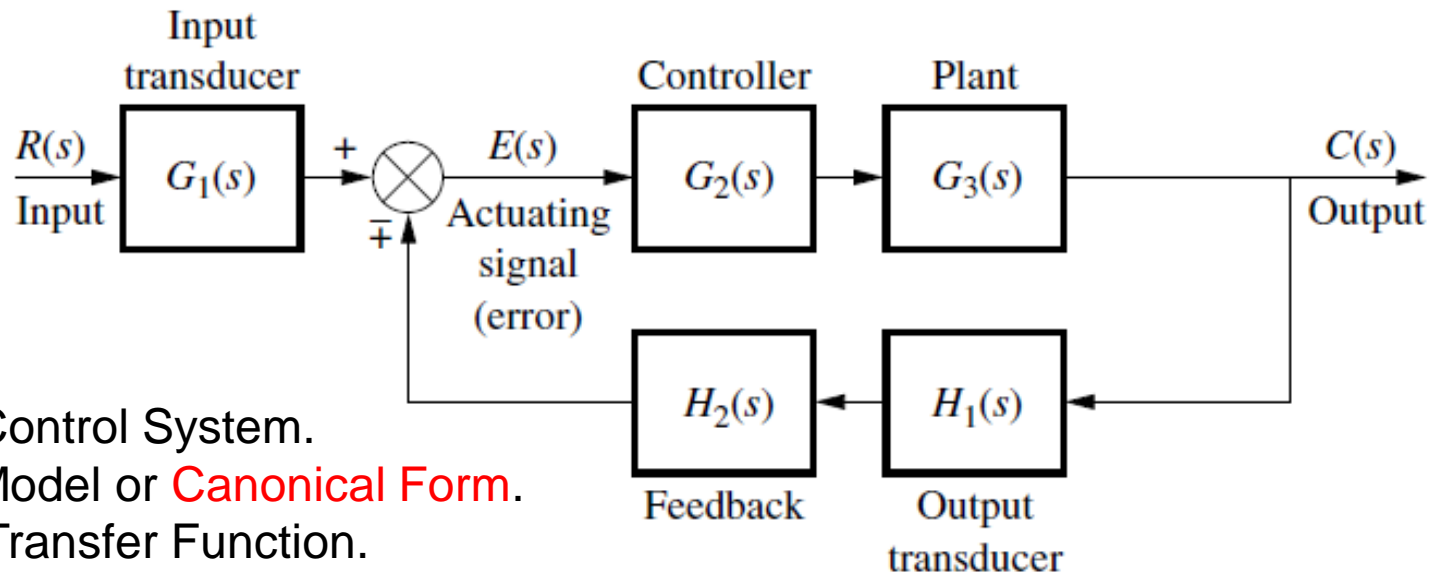
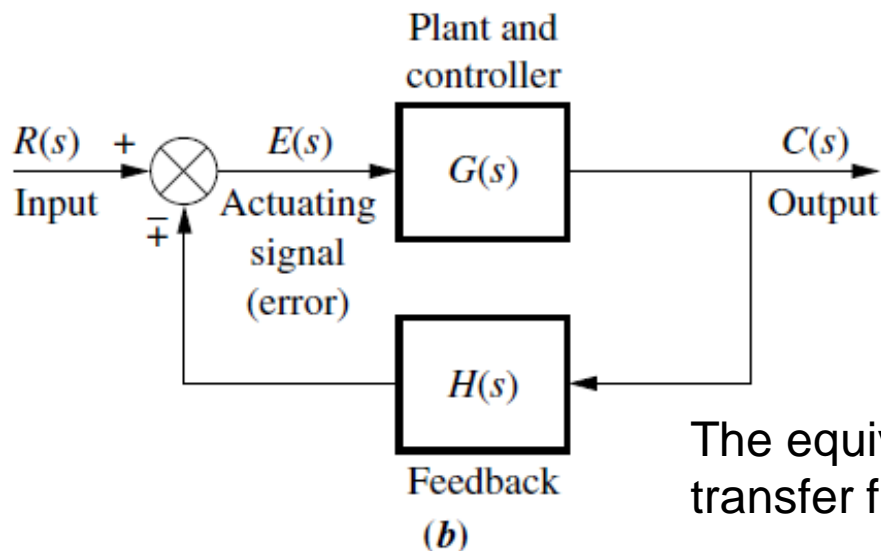
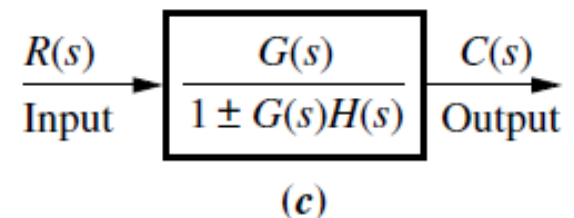


Figure:

- a) Feedback Control System.
- b) Simplified Model or **Canonical Form**.
- c) Equivalent Transfer Function.



(a)



The equivalent or closed-loop transfer function is

$$G_e(s) = \frac{G(s)}{1 \pm G(s)H(s)}$$

Characteristic Equation

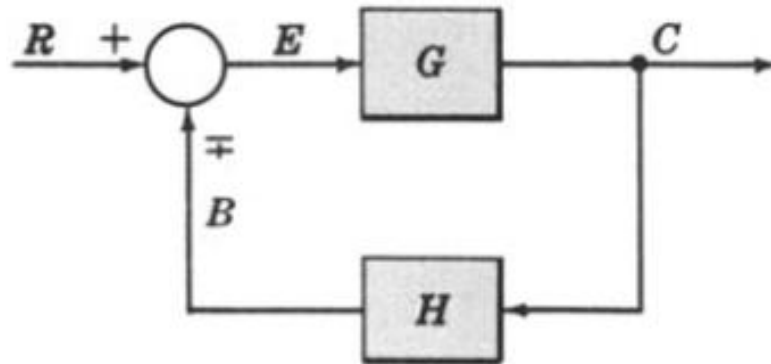
- The control ratio is the closed loop transfer function of the system.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

- The denominator of closed loop transfer function determines the characteristic equation of the system.
- Which is usually determined as:

$$1 \pm G(s)H(s) = 0$$

Canonical Form of a Feedback Control System



$G \equiv$ direct transfer function \equiv forward transfer function

$H \equiv$ feedback transfer function

$GH \equiv$ loop transfer function \equiv open-loop transfer function

$C/R \equiv$ closed-loop transfer function \equiv control ratio $\frac{C}{R} = \frac{G}{1 \pm GH}$

$E/R \equiv$ actuating signal ratio \equiv error ratio $\frac{E}{R} = \frac{1}{1 \pm GH}$

$B/R \equiv$ primary feedback ratio $\frac{B}{R} = \frac{GH}{1 \pm GH}$

The system is said to have negative feedback if the sign at the summing junction is negative and positive feedback if the sign is positive.

1. Open loop transfer function $\frac{B(s)}{E(s)} = G(s)H(s)$

2. Feed Forward Transfer function $\frac{C(s)}{E(s)} = G(s)$

3. control ratio $\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$

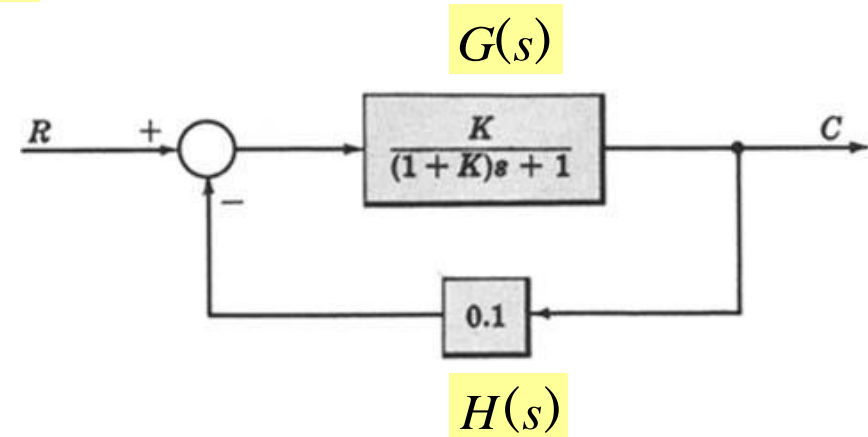
4. feedback ratio $\frac{B(s)}{R(s)} = \frac{G(s)H(s)}{1 + G(s)H(s)}$

5. error ratio $\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$

6. closed loop transfer function $\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$

7. characteristic equation $1 + G(s)H(s) = 0$

8. closed loop poles and zeros if $K=10$.



Characteristic Equation

$C/R \equiv$ closed-loop transfer function \equiv control ratio

$$\frac{C}{R} = \frac{G}{1 \pm GH}$$

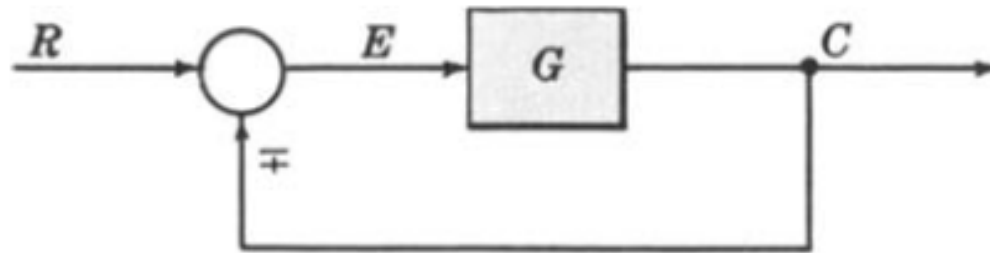
The denominator of C/R determines the *characteristic equation* of the system, which is usually determined from $1 \pm GH = 0$ or, equivalently,

$$D_{GH} \pm N_{GH} = 0$$

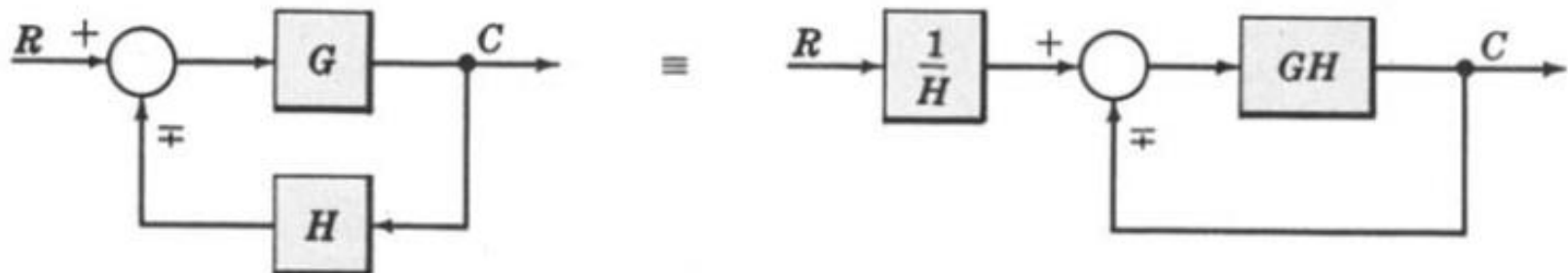
where D_{GH} is the denominator and N_{GH} is the numerator of GH

Unity Feedback System

A **unity feedback system** is one in which the primary feedback b is identically equal to the controlled output $H = 1$ for a linear, unity feedback system

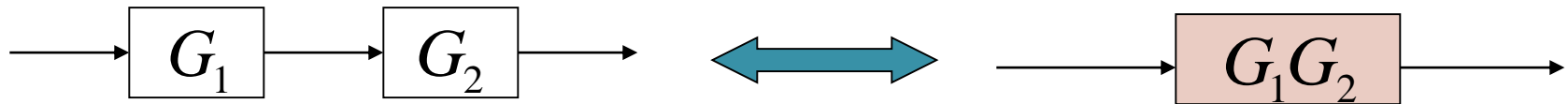


Any feedback system with only linear time-invariant elements can be put into the form of a unity feedback system by using Transformation 5.

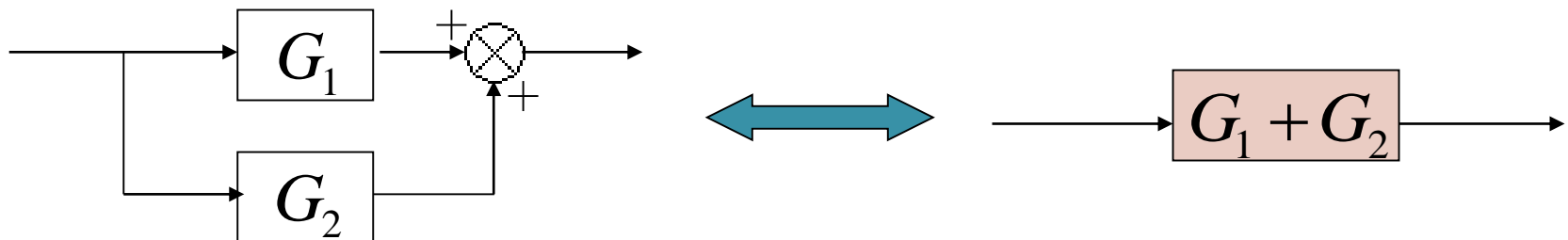


Reduction techniques

1. Combining blocks in cascade

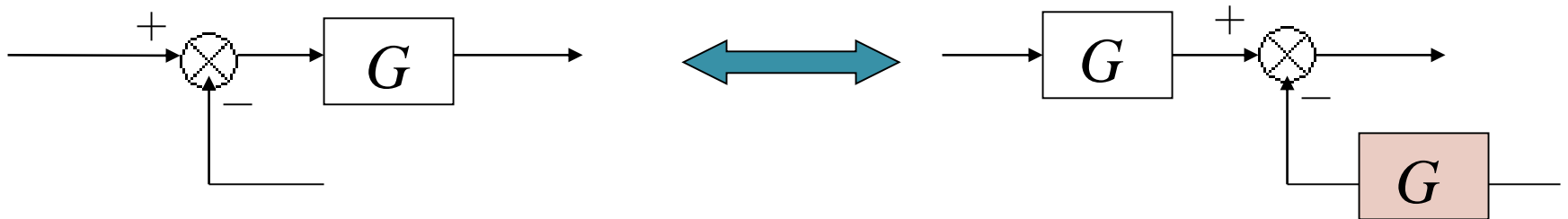


2. Combining blocks in parallel



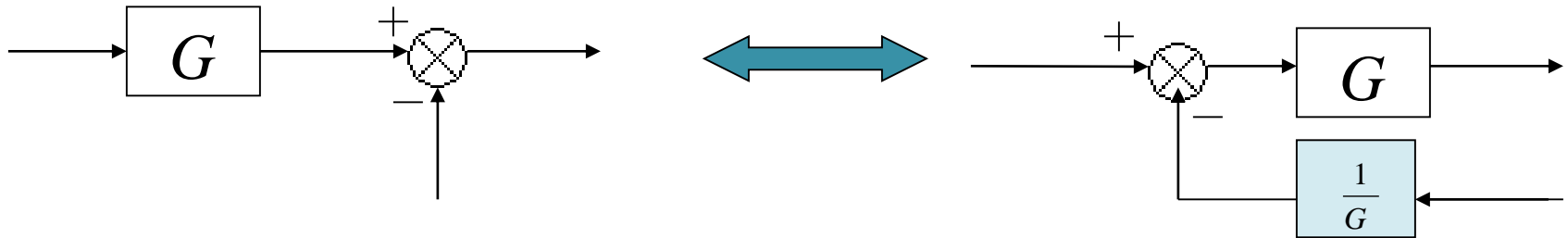
Reduction techniques

3. Moving a summing point behind a block

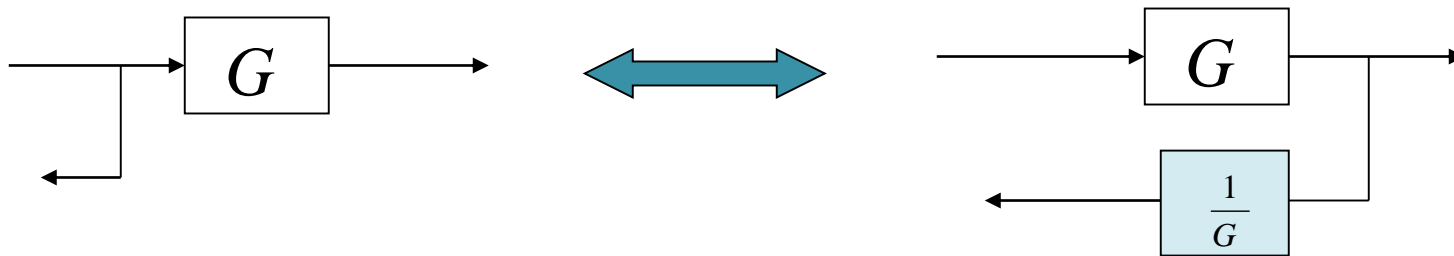


Reduction techniques

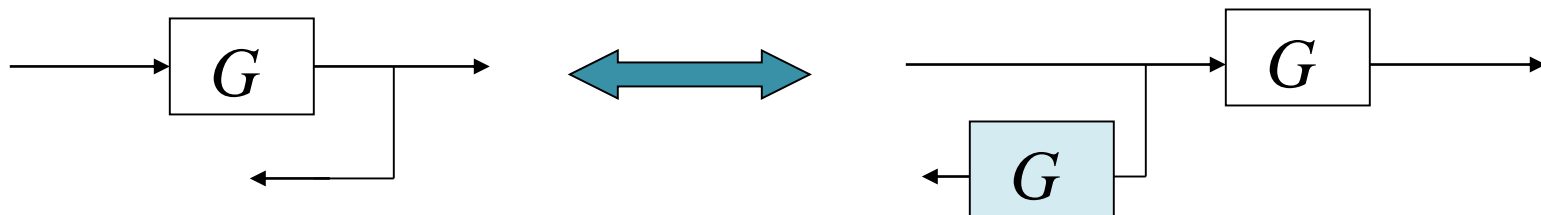
3. Moving a summing point ahead of a block



4. Moving a pickoff point behind a block



5. Moving a pickoff point ahead of a block



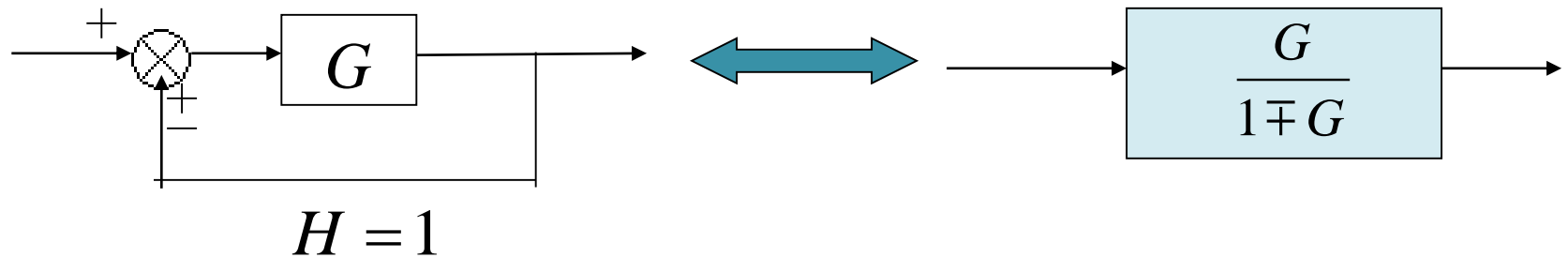
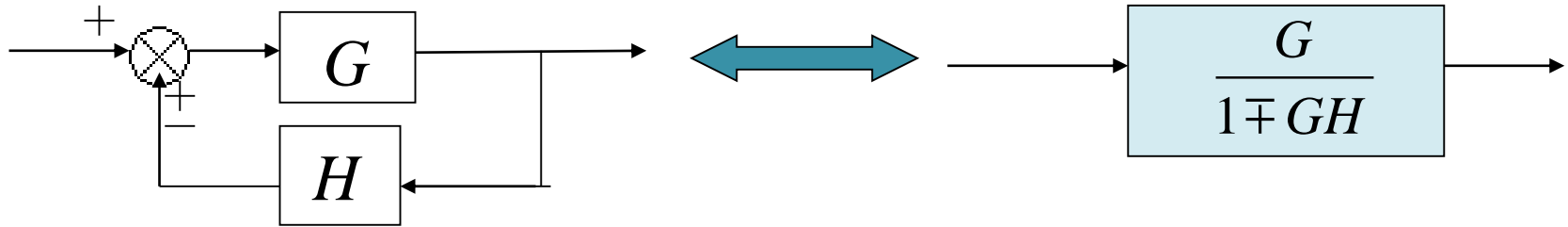
Block Diagram fundamentals & reduction techniques

Lect# 4-5

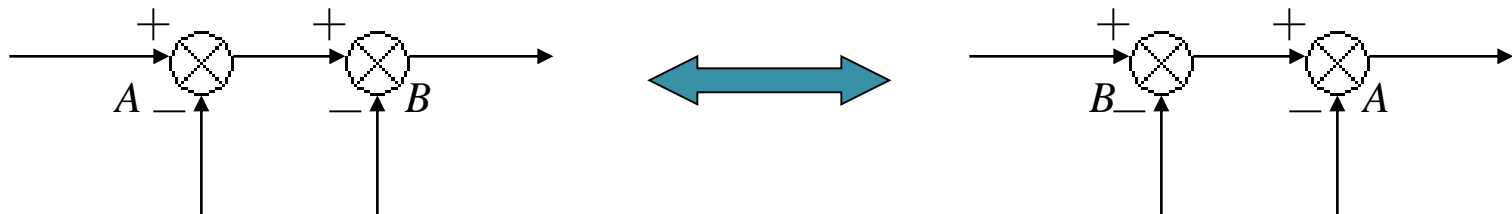
A series of horizontal bars in yellow and orange colors, some solid and some with multiple thin lines, extending across the width of the slide.

Reduction techniques

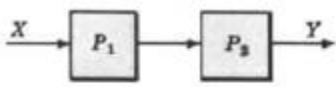
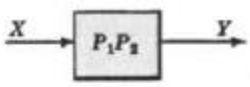
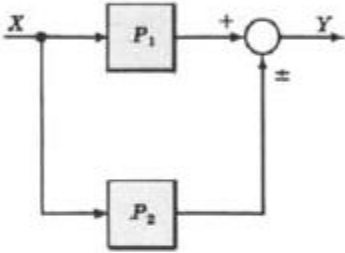
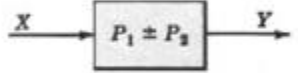
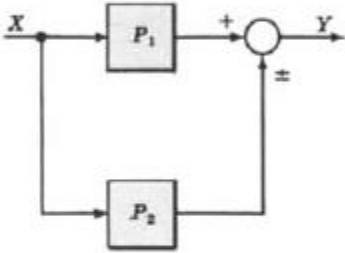
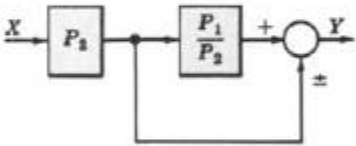
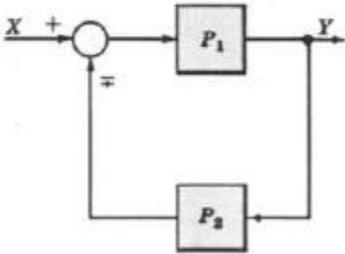
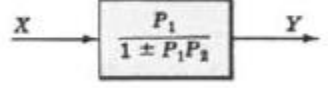
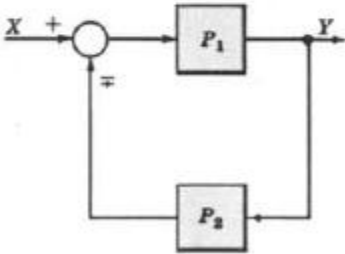
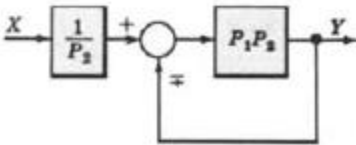
6. Eliminating a feedback loop



7. Swap with two neighboring summing points

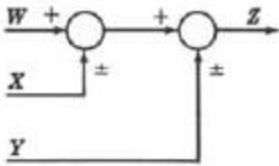
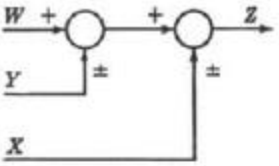
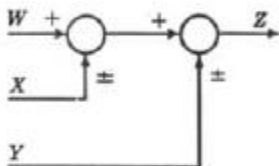
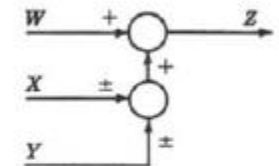
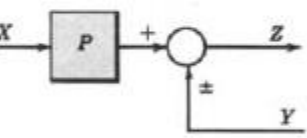
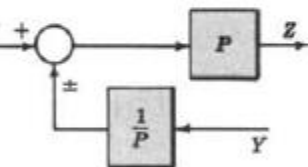
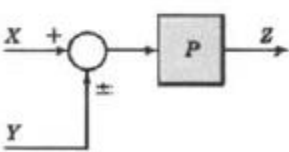
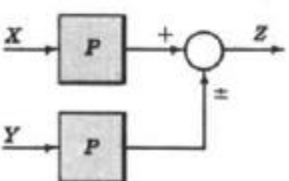


Block Diagram Transformation Theorems


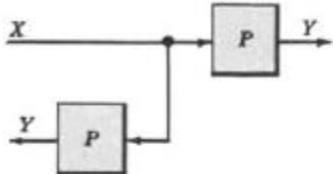
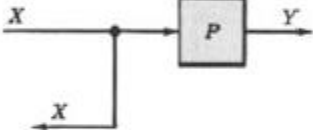
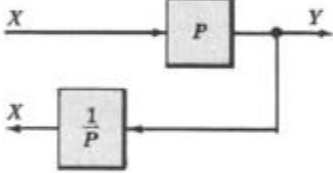
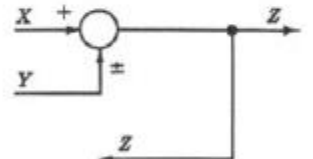
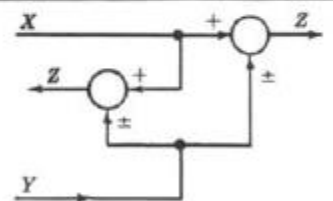
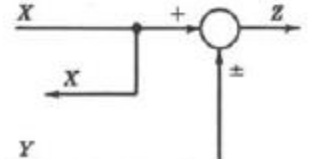
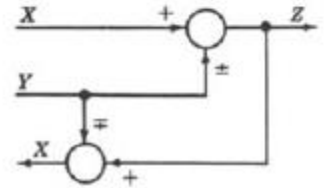
Transformation		Equation	Block Diagram	Equivalent Block Diagram
1	Combining Blocks in Cascade	$Y = (P_1 P_2)X$		
2	Combining Blocks in Parallel; or Eliminating a Forward Loop	$Y = P_1 X \pm P_2 X$		
3	Removing a Block from a Forward Path	$Y = P_1 X \pm P_2 X$		
4	Eliminating a Feedback Loop	$Y = P_1(X \mp P_2 Y)$		
5	Removing a Block from a Feedback Loop	$Y = P_1(X \mp P_2 Y)$		

The letter ***P*** is used to represent any transfer function, and ***W, X, Y, Z*** denote any transformed signals.

Transformation Theorems Continue:

	Transformation	Equation	Block Diagram	Equivalent Block Diagram
6a	Rearranging Summing Points	$Z = W \pm X \pm Y$		
6b	Rearranging Summing Points	$Z = W \pm X \pm Y$		
7	Moving a Summing Point Ahead of a Block	$Z = PX \pm Y$		
8	Moving a Summing Point Beyond a Block	$Z = P[X \pm Y]$		

Transformation Theorems Continue:

Transformation		Equation	Block Diagram	Equivalent Block Diagram
9	Moving a Takeoff Point Ahead of a Block	$Y = PX$		
10	Moving a Takeoff Point Beyond a Block	$Y = PX$		
11	Moving a Takeoff Point Ahead of a Summing Point	$Z = X \pm Y$		
12	Moving a Takeoff Point Beyond a Summing Point	$Z = X \pm Y$		

Reduction of Complicated Block Diagrams:

The block diagram of a practical feedback control system is often quite complicated. It may include several feedback or feedforward loops, and multiple inputs. By means of systematic block diagram reduction, every multiple loop linear feedback system may be reduced to canonical form.

The following general steps may be used as a basic approach in the reduction of complicated block diagrams.

Step 1: Combine all cascade blocks using Transformation 1.

Step 2: Combine all parallel blocks using Transformation 2.

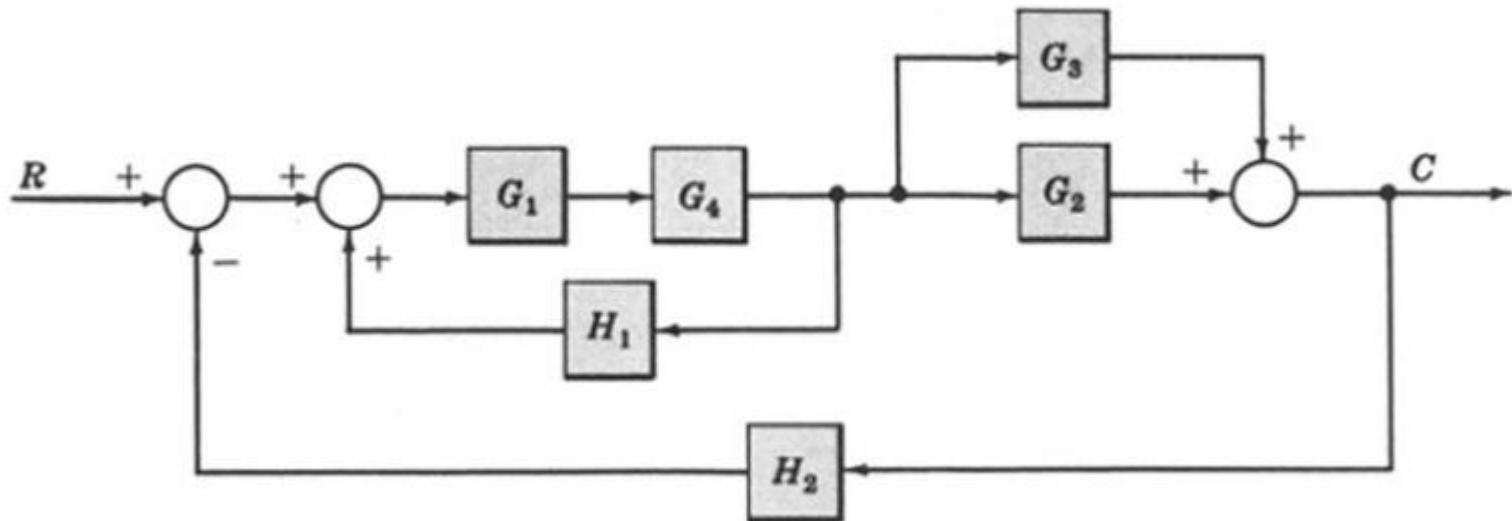
Step 3: Eliminate all minor feedback loops using Transformation 4.

Step 4: Shift summing points to the left and takeoff points to the right of the major loop, using Transformations 7, 10, and 12.

Step 5: Repeat Steps 1 to 4 until the canonical form has been achieved for a particular input.

Step 6: Repeat Steps 1 to 5 for each input, as required.

Example-4: Reduce the Block Diagram to Canonical Form.



Step 1: Combine all cascade blocks using Transformation 1.



Step 2: Combine all parallel blocks using Transformation 2.



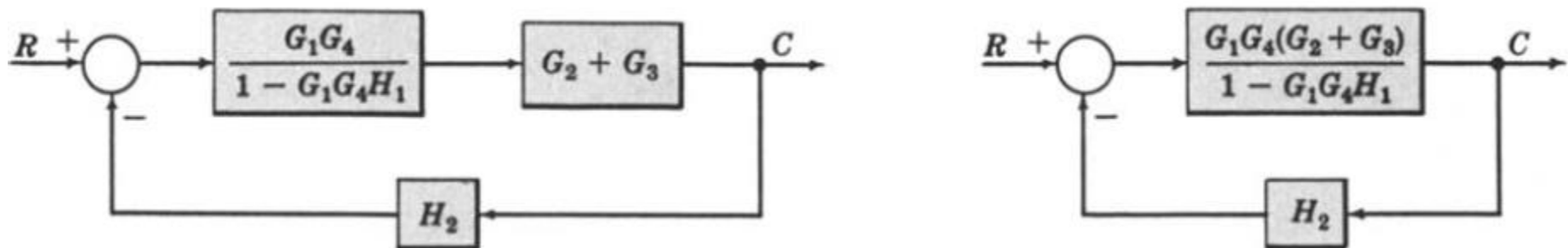
Example-4: Continue.

Step 3: Eliminate all minor feedback loops using Transformation 4.



Step 4: Shift summing points to the left and takeoff points to the right of the major loop, using Transformations 7, 10, and 12. However in this example step-4 does not apply.

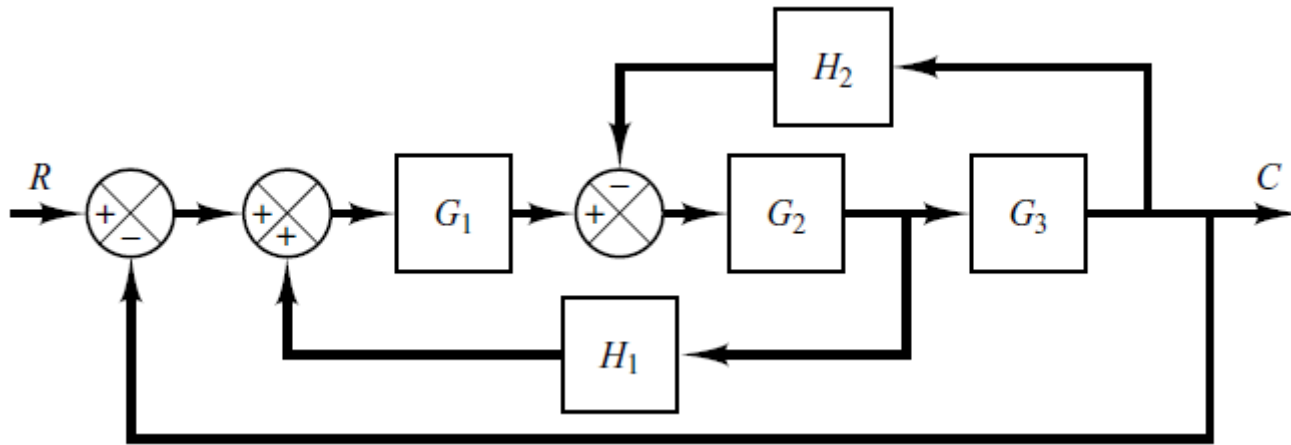
Step 5: Repeat Steps 1 to 4 until the canonical form has been achieved for a particular input.



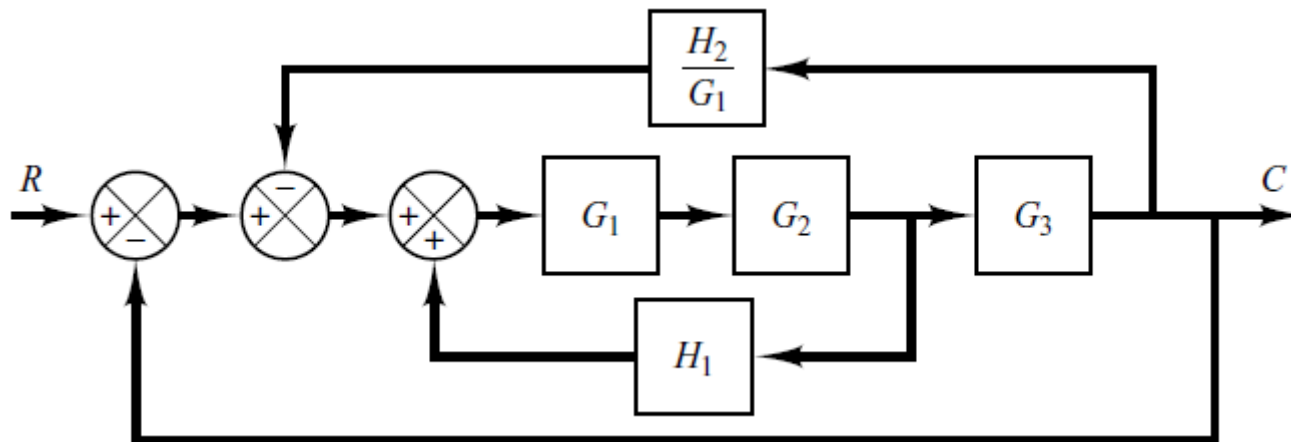
Step 6: Repeat Steps 1 to 5 for each input, as required.

However in this example step-6 does not apply.

Example-5: Simplify the Block Diagram.

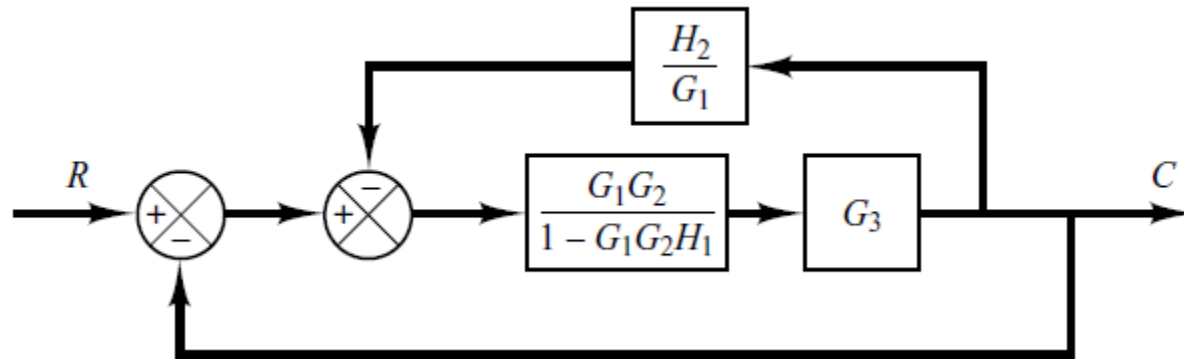


By moving the summing point of the negative feedback loop containing H_2 outside the positive feedback loop containing H_1 , we obtain Figure

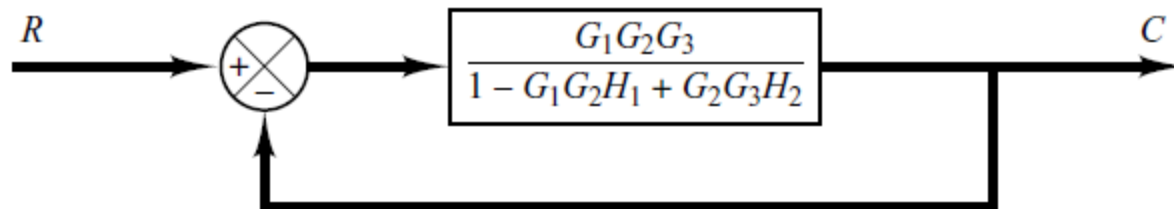


Example-5: Continue.

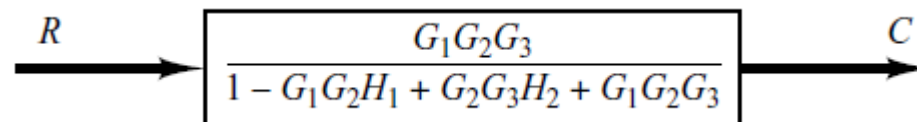
Eliminating the positive feedback loop, we have



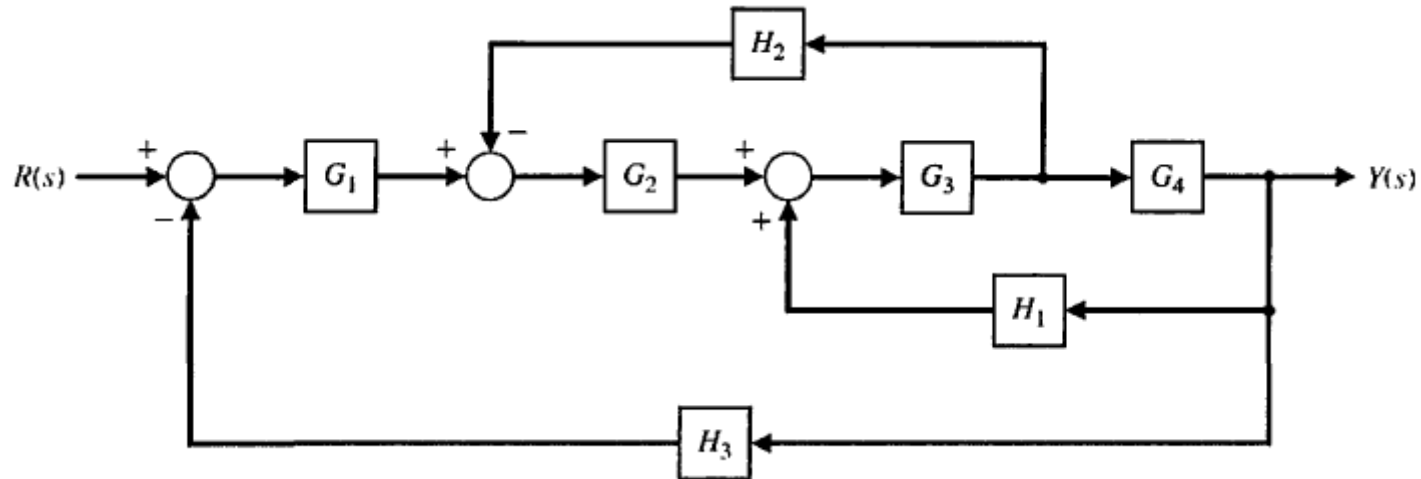
The elimination of the loop containing H_2/G_1 gives



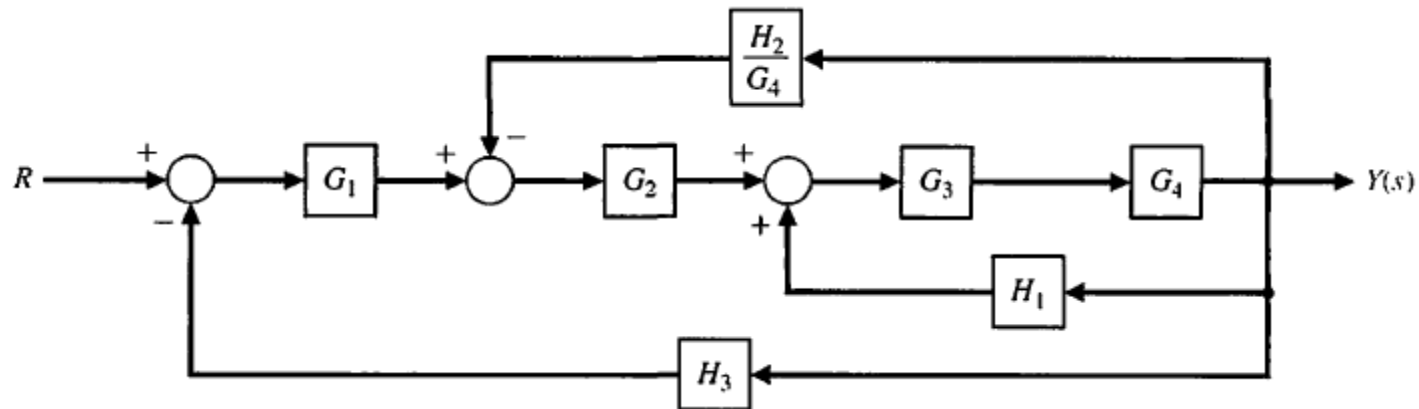
Finally, eliminating the feedback loop results in



Example-6: Reduce the Block Diagram.

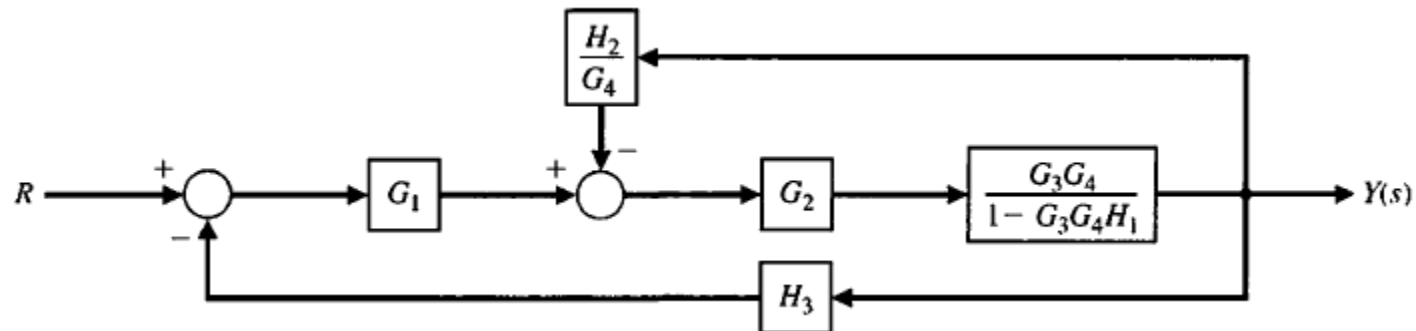


First, to eliminate the loop $G_3G_4H_1$, we move H_2 behind block G_4

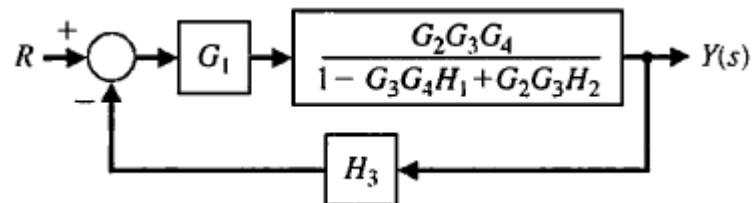


Eliminating the loop $G_3G_4H_1$ we obtain

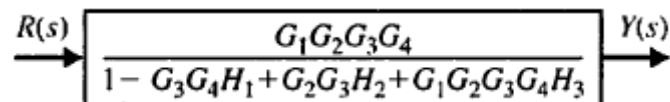
Example-6: Continue.



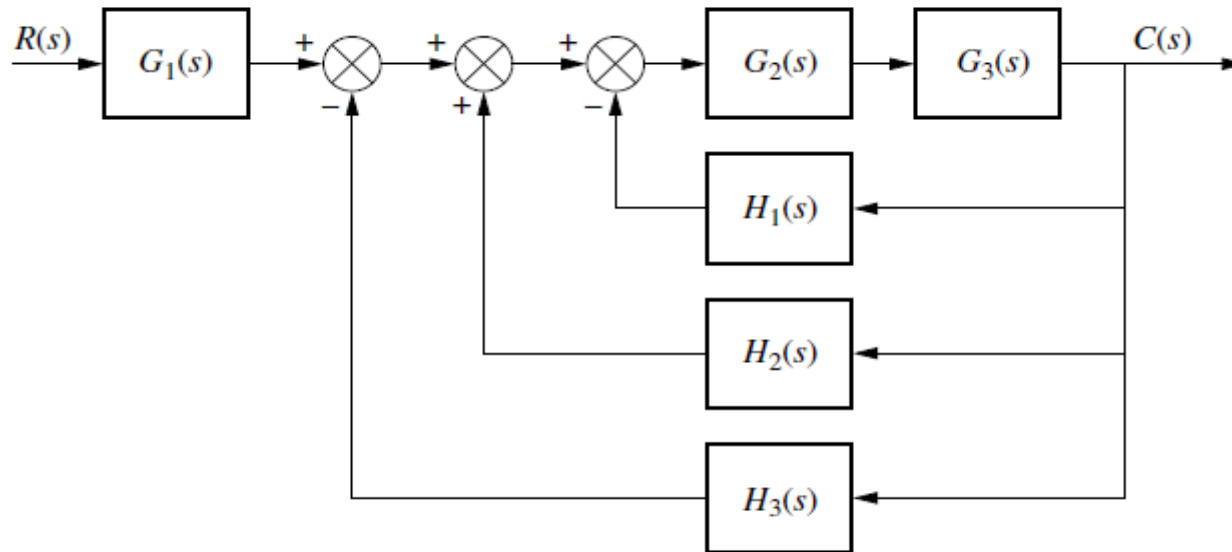
Then, eliminating the inner loop containing H_2/G_4 , we obtain



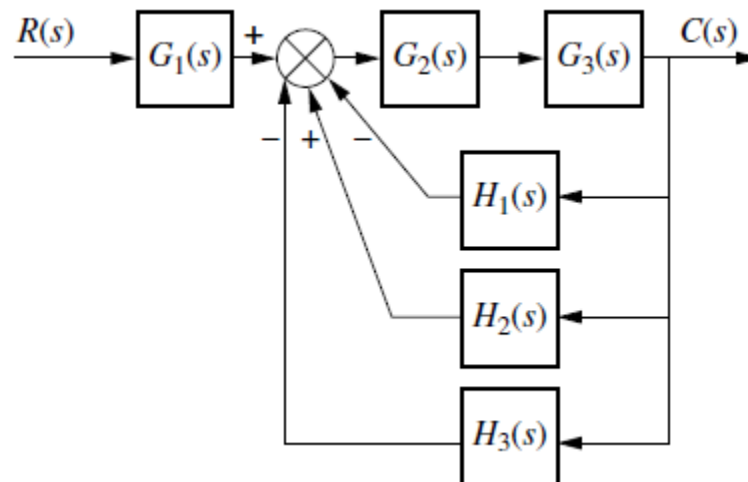
Finally, by reducing the loop containing H_3 , we obtain



Example-7: Reduce the Block Diagram. (from Nise: page-242)

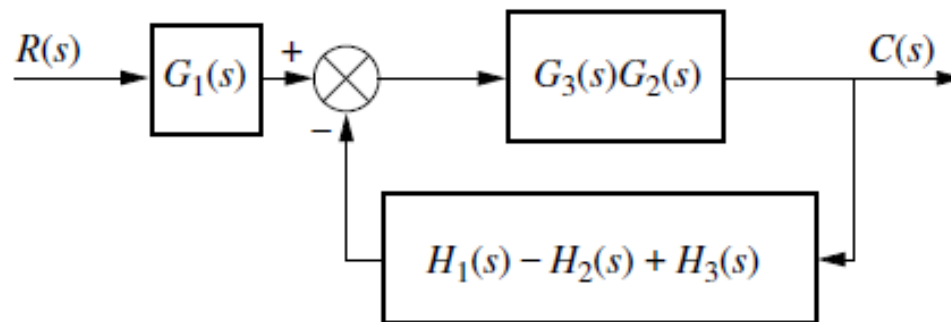


First, the three summing junctions can be collapsed into a single summing junction,

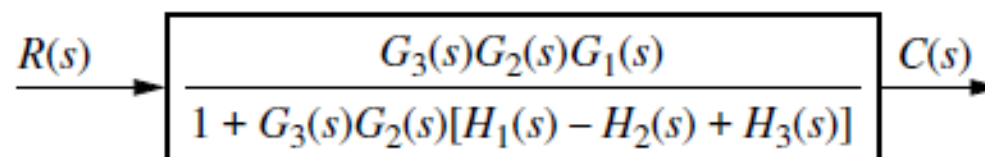


Example-7: Continue.

Second, recognize that the three feedback functions, $H_1(s)$, $H_2(s)$, and $H_3(s)$, are connected in parallel. They are fed from a common signal source, and their outputs are summed. Also recognize that $G_2(s)$ and $G_3(s)$ are connected in cascade.

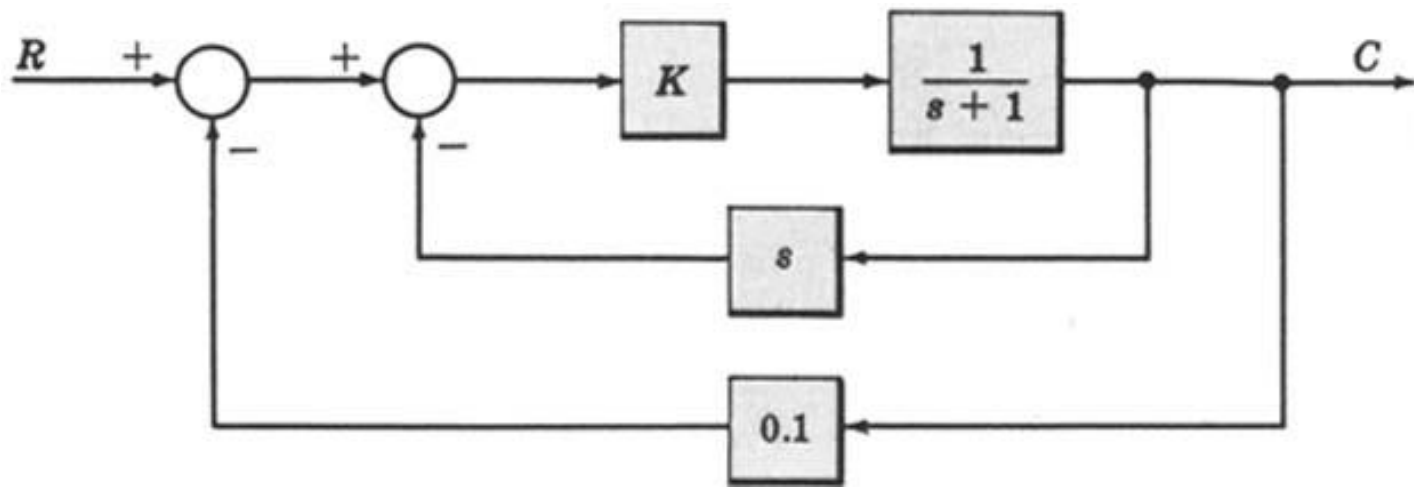


Finally, the feedback system is reduced and multiplied by $G_1(s)$ to yield the equivalent transfer function shown in Figure



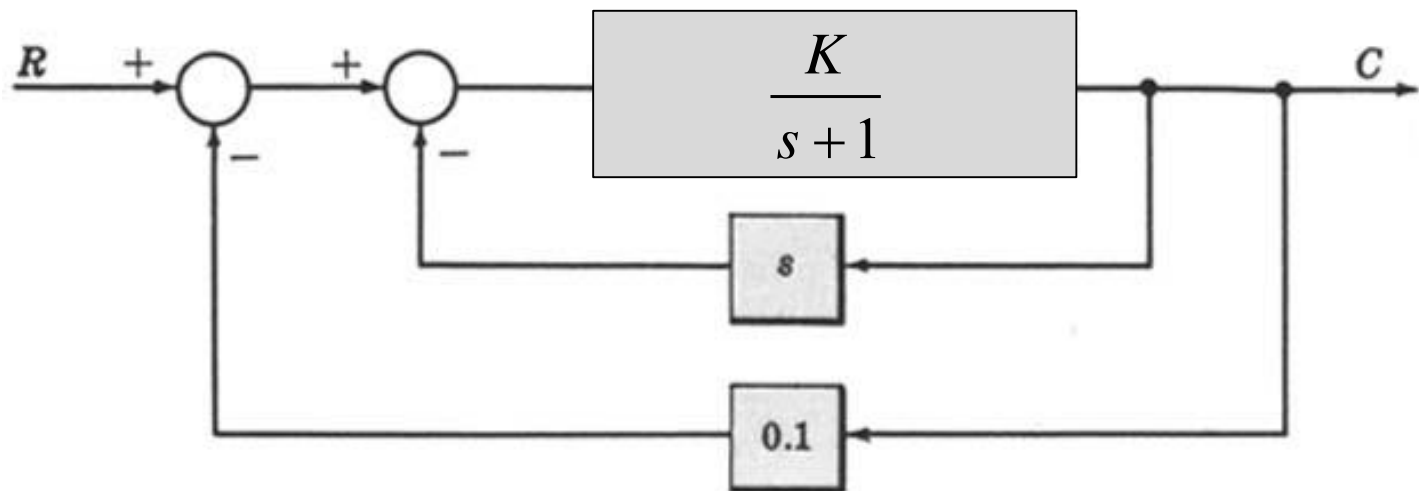
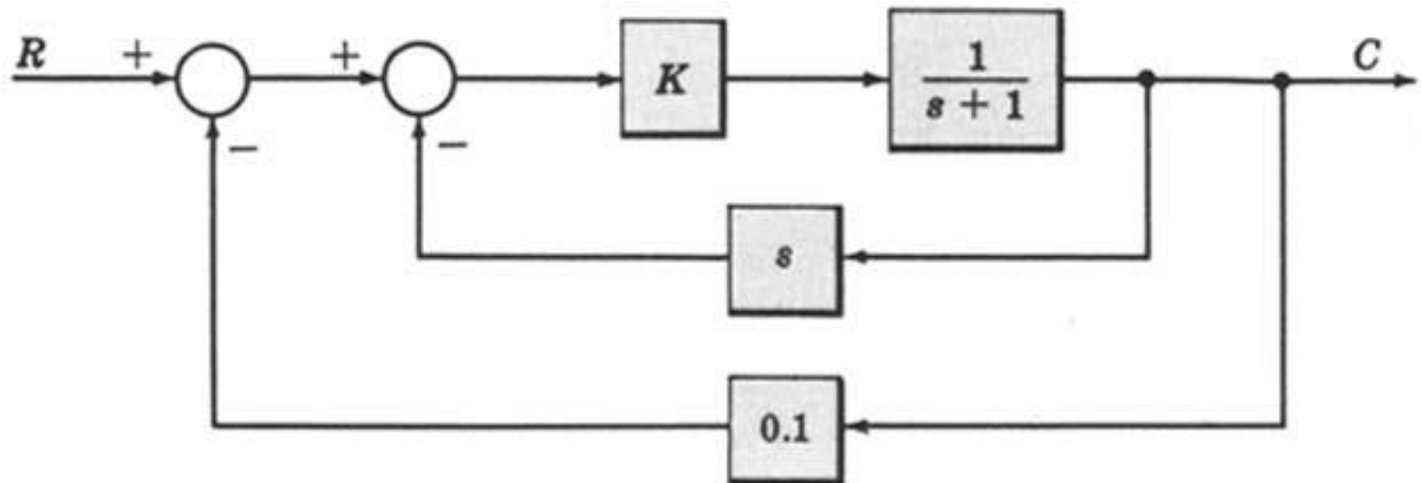
Example-8: For the system represented by the following block diagram determine:

1. Open loop transfer function
2. Feed Forward Transfer function
3. control ratio
4. feedback ratio
5. error ratio
6. closed loop transfer function
7. characteristic equation
8. closed loop poles and zeros if $K=10$.

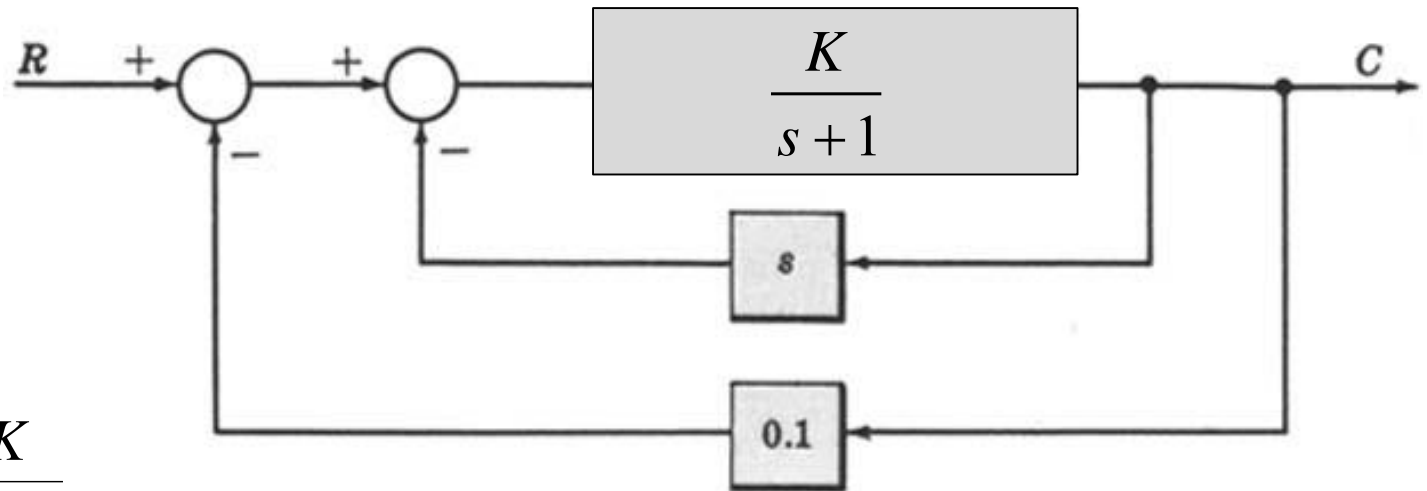


Example-8: Continue

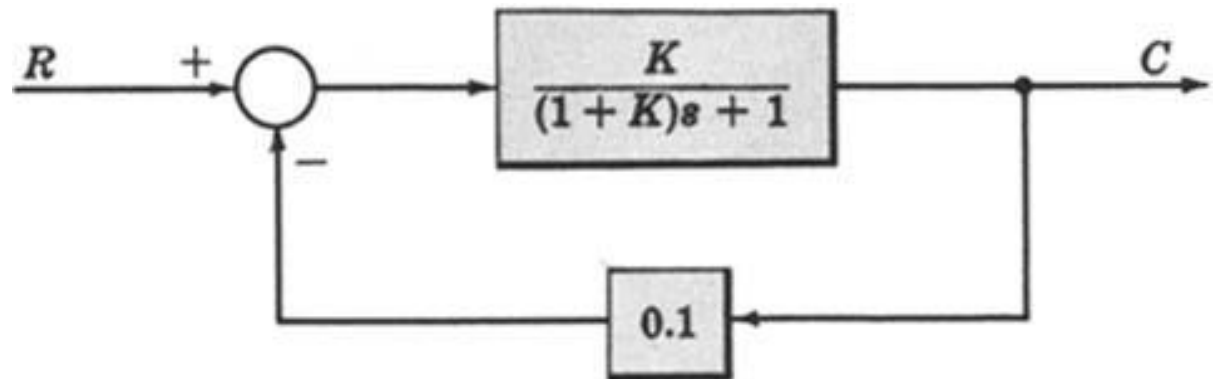
- First we will reduce the given block diagram to canonical form



Example-8: Continue



$$\frac{G}{1+GH} = \frac{\frac{K}{s+1}}{1 + \frac{K}{s+1}s}$$



Example-8: Continue

1. Open loop transfer function $\frac{B(s)}{E(s)} = G(s)H(s)$

2. Feed Forward Transfer function $\frac{C(s)}{E(s)} = G(s)$

3. control ratio $\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$

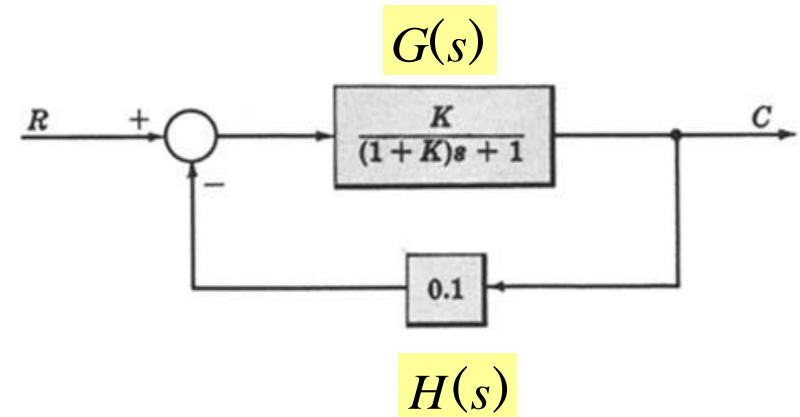
4. feedback ratio $\frac{B(s)}{R(s)} = \frac{G(s)H(s)}{1 + G(s)H(s)}$

5. error ratio $\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$

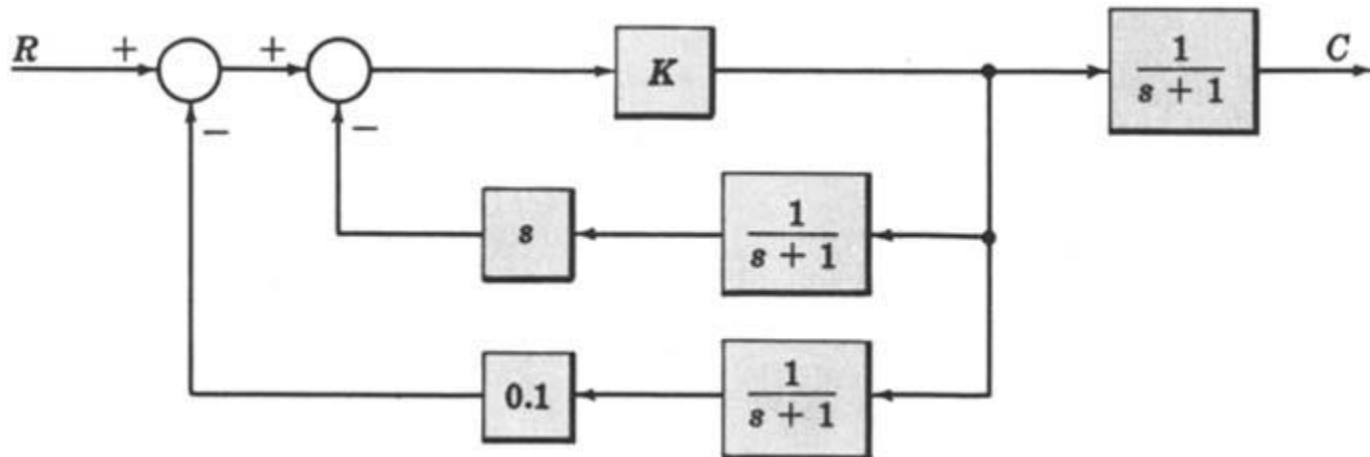
6. closed loop transfer function $\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$

7. characteristic equation $1 + G(s)H(s) = 0$

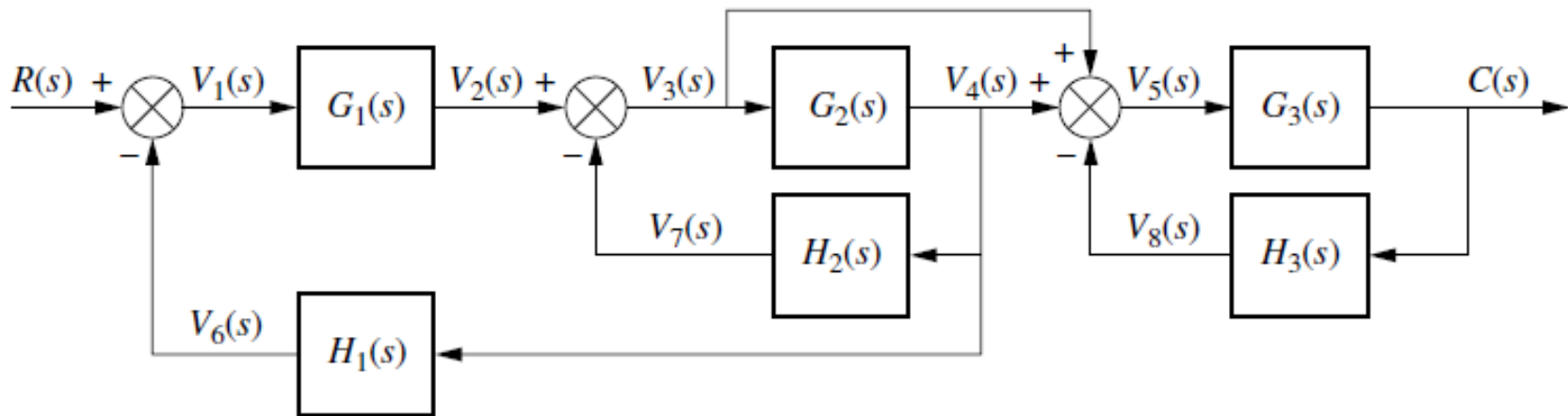
8. closed loop poles and zeros if $K=10$.



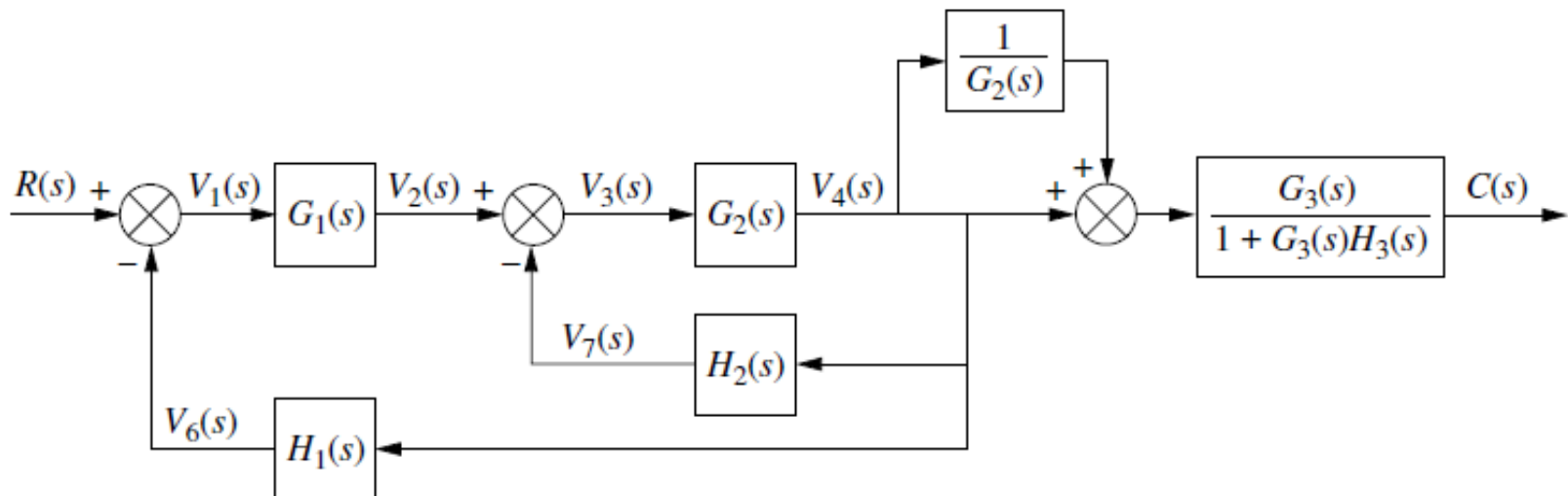
- Example-9: For the system represented by the following block diagram determine:
 1. Open loop transfer function
 2. Feed Forward Transfer function
 3. control ratio
 4. feedback ratio
 5. error ratio
 6. closed loop transfer function
 7. characteristic equation
 8. closed loop poles and zeros if $K=100$.



Example-10: Reduce the system to a single transfer function. (from Nise:page-243).

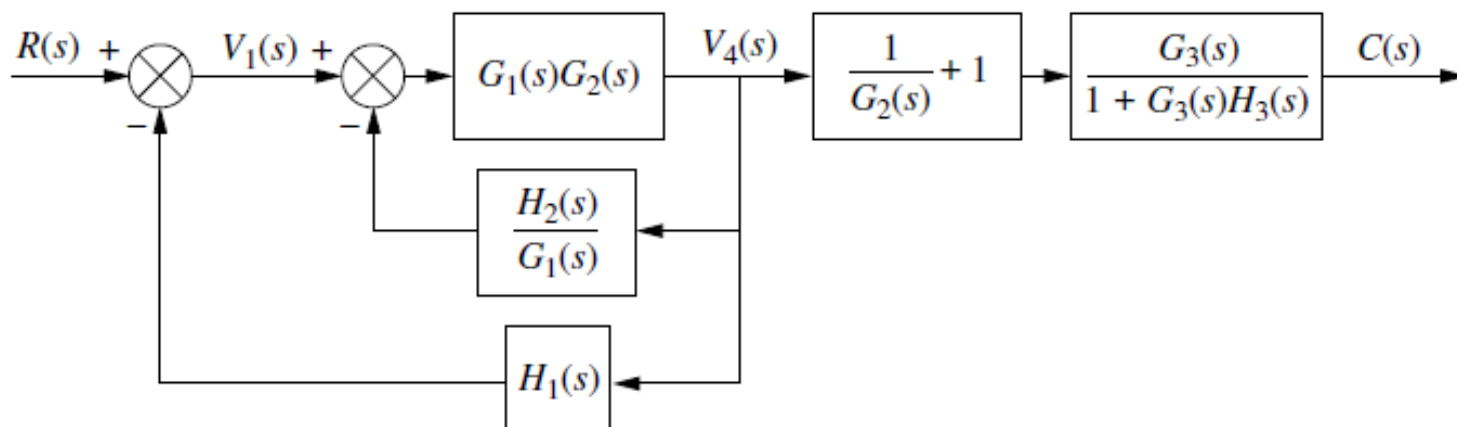


First, move $G_2(s)$ to the left past the pickoff point to create parallel subsystems, and reduce the feedback system consisting of $G_3(s)$ and $H_3(s)$.

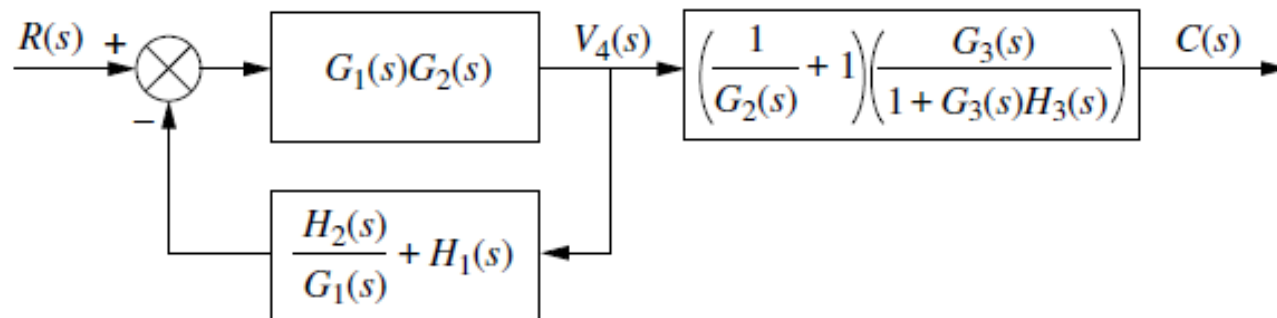


Example-10: Continue.

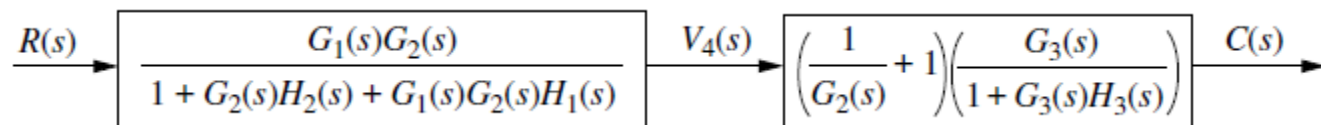
Second, reduce the parallel pair consisting of $1/G_2(s)$ and unity, and push $G_1(s)$ to the right past the summing junction, creating parallel subsystems in the feedback.



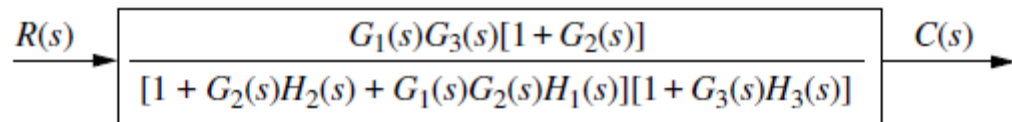
Third, collapse the summing junctions, add the two feedback elements together, and combine the last two cascaded blocks.



Fourth, use the feedback formula to obtain Figure 10. Continue.



Finally, multiply the two cascaded blocks and obtain the final result,



Answer of Skill Assessment Exercise:

$$\text{ANSWER: } T(s) = \frac{s^3 + 1}{2s^4 + s^2 + 2s}$$