Angle Modulation

Ref: Modern Digital and Analog Communication Systems, B. P. Lathi, 3rd Edition, Chapter 5

Angle Modulation:

The techniques of modulation, where the angle of the carrier is varied in some manner with a modulating signal m(t), are known as angle modulation.

Two simple possibilities are:

- Phase Modulation (PM)
- Frequency Modulation (FM)

Phase Modulation (PM):

In phase modulation (PM), the angle θ(t) of the carrier is varied linearly with the modulating signal m(t).

Let us consider a carrier, $\varphi(t) = A\cos\theta(t)$

Angle of the carrier, $\theta(t) = \omega_c t + k_p m(t)$

Where k_p= constant, ω_c= carrier frequency

The resulting PM wave is: $\varphi_{\mathrm{PM}}(t) = A\cos[\omega_c t + k_p m(t)]$

The instantaneous frequency $\omega_i(t)$ is given by

$$\omega_i(t) = \frac{d\theta}{dt} = \omega_c + k_p \dot{m}(t)$$

So, in PM, the instantaneous frequency ω_i varies linearly with the **derivative of the modulating signal**.

Frequency Modulation:

In frequency modulation (FM), the **instantaneous frequency** $\omega_i(t)$ of the carrier is varied linearly with the modulating signal m(t).

Thus in FM, $\omega_i(t) = \omega_c + k_f m(t)$

The angle θ(t) is: $\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha) d\alpha$

Since $\omega_i(t) = \dfrac{d\theta}{dt}$

$$\Rightarrow \theta(t) = \int_{-\infty}^{t} [\omega_c + k_f m(\alpha)]\, d\alpha$$

$$= \omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha$$

So, the FM wave: $\varphi_{\mathrm{FM}}(t) = A \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]$

Generalized Concept of Angle Modulation:

PM wave: $\varphi_{PM}(t) = A\cos[\omega_c t + k_p m(t)] = A\cos[\omega_c t + k_p \int \dot{m}(t)dt]$

FM wave: $\varphi_{FM}(t) = A\cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right]$

A signal that is an FM wave corresponding to *m(t)* is also the PM wave corresponding to $\int m(\alpha)\,d\alpha$ (Fig 5.2a) .
Similarly, a PM wave corresponding to *m(t)* is the FM wave corresponding to $\dot{m}(t)$ (Fig 5.2 b).

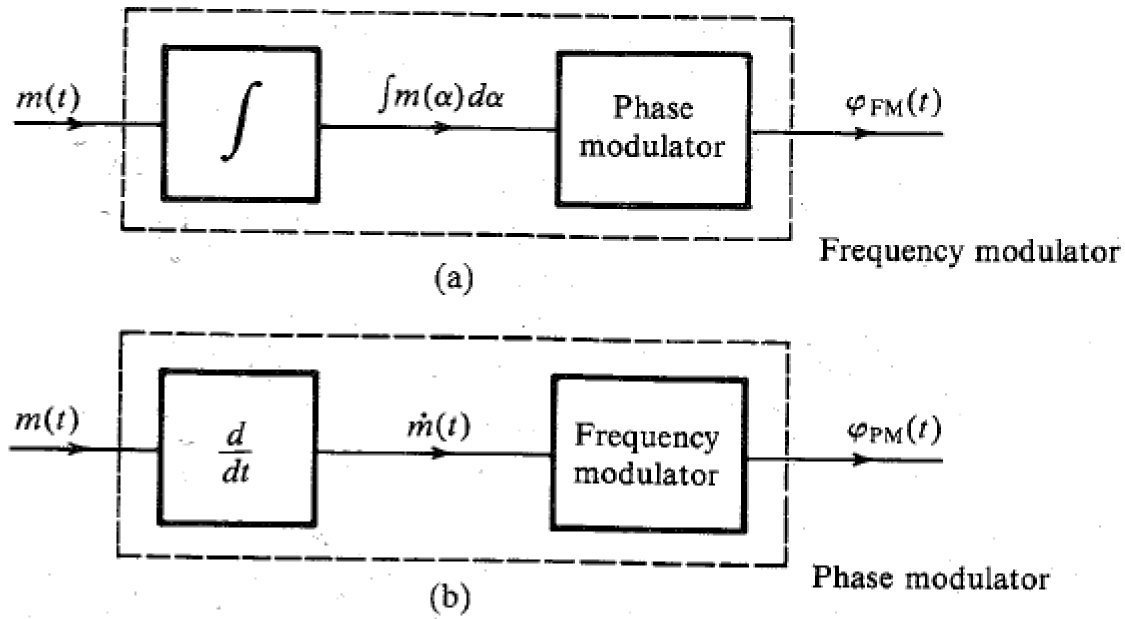
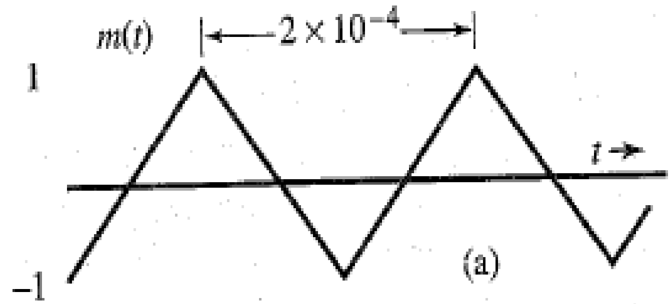


Figure 5.2 Phase and frequency modulation are inseparable.

Example 5.1: Sketch FM and PM waves for the modulating signal $m(t)$ shown in the figure. The constants k_f and k_p are $2\pi \times 10^5$ and 10π, respectively, and the carrier frequency f_c is 100 MHz.



<u>For FM:</u> $\omega_i = \omega_c + k_f m(t)$

$$\Rightarrow f_i = f_c + \frac{k_f}{2\pi} m(t) \qquad \text{[dividing by } 2\pi\text{]}$$

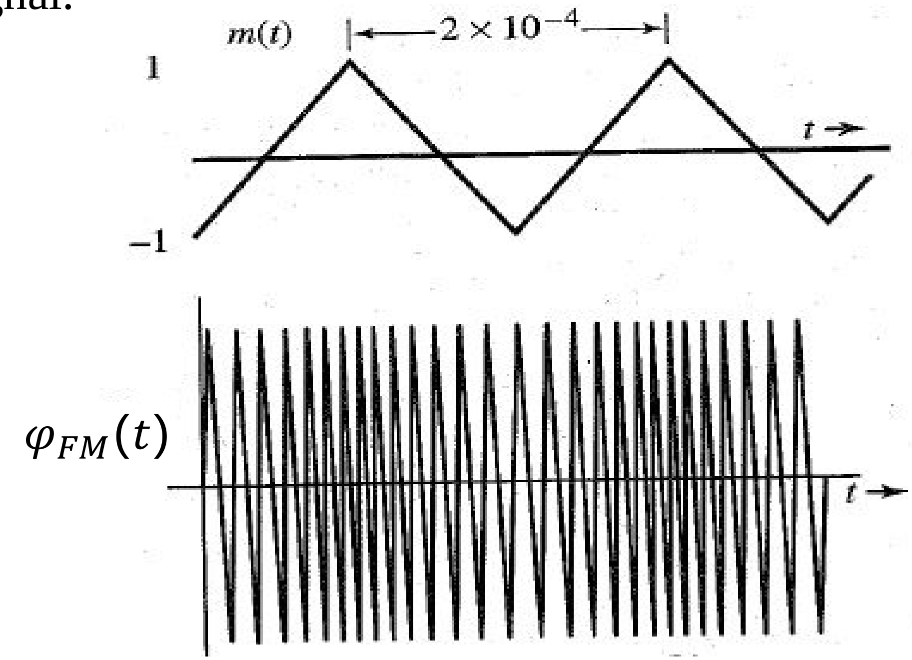
$$= 100 \times 10^6 + \frac{2\pi \times 10^5}{2\pi} m(t)$$

$$\Rightarrow f_i = 10^8 + 10^5 m(t)$$

So, $\quad (f_i)_{min} = 10^8 + 10^5[m(t)]_{min} = 10^8 + 10^5(-1) = 99.9 \text{ MHz}$

$(f_i)_{max} = 10^8 + 10^5[m(t)]_{max} = 10^8 + 10^5(1) = 100.1 \text{ MHz}$

Because $m(t)$ increases and decreases linearly with time, the instantaneous frequency increases linearly from 99.9 MHz to 100.1 MHz over a half-cycle and decreases linearly from 100.1 to 99.9 MHz over the remaining half-cycle of the modulating signal.



FM waveform

For PM: $\omega_i = \omega_c + k_p \dot{m}(t)$

$$\Rightarrow f_i = f_c + \frac{k_p}{2\pi}\dot{m}(t) \qquad \text{[dividing by } 2\pi\text{]}$$

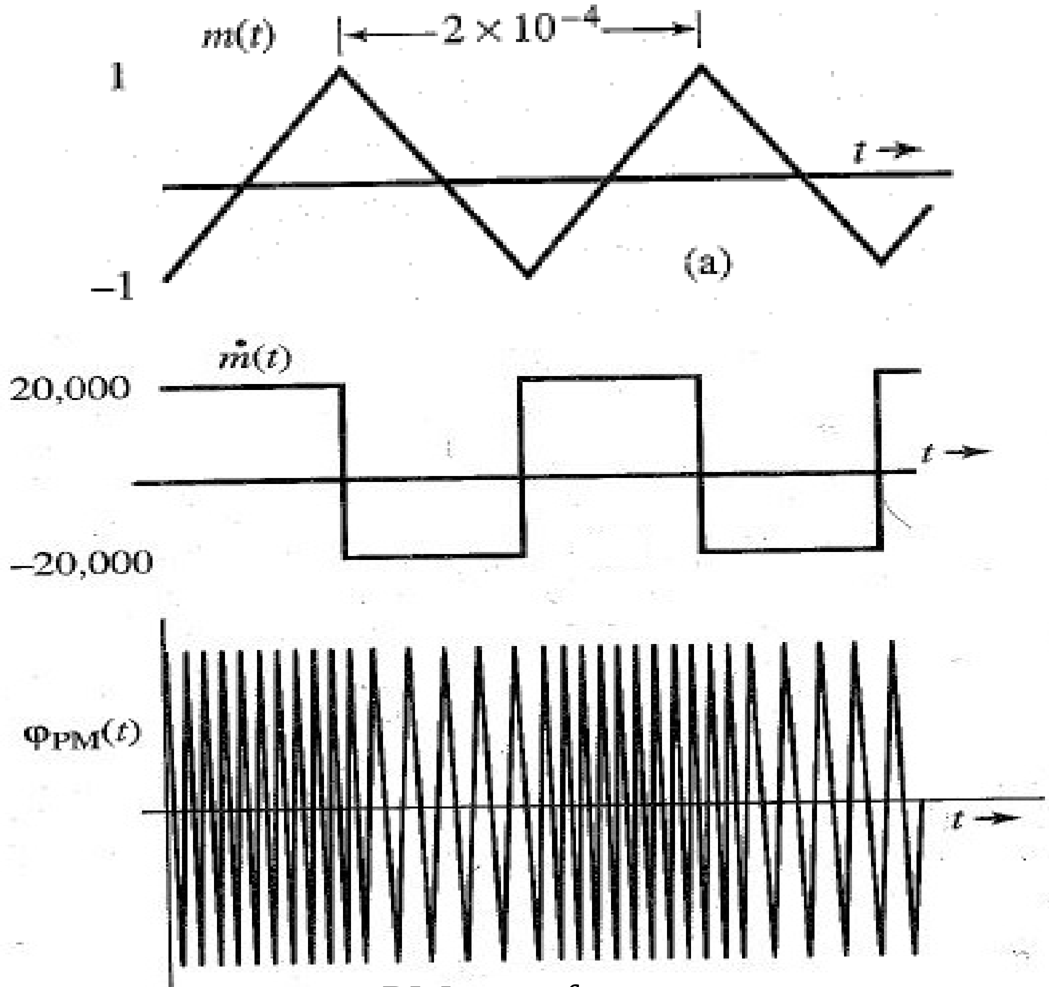
$$= 100 \times 10^6 + \frac{10\pi}{2\pi}\dot{m}(t)$$

$$\Rightarrow f_i = 10^8 + 5\,\dot{m}(t)$$

So, $(f_i)_{min} = 10^8 + 5\,[\dot{m}(t)]_{min} = 10^8 + 5(-20000) = 99.9$ MHz

$(f_i)_{max} = 10^8 + 5\,[\dot{m}(t)]_{max} = 10^8 + 5(20000) = 100.1$ MHz

Because $\dot{m}(t)$ switches back and forth from a value of -20,000 to 20,000, the carrier frequency switches back and forth from 99.9 MHz to 100.1 MHz every half-cycle of $\dot{m}(t)$.



PM waveform

Bandwidth of angle modulated wave:

<u>FM:</u> $\omega_i = \omega_c + k_f m(t)$

Now, $\omega_{i(max)} = \omega_c + k_f m_p$ where m_p= peak value of m(t)

and $\omega_{i(min)} = \omega_c - k_f m_p$

So, Carrier frequency deviation, $\Delta\omega = k_f m_p$

Carrier frequency deviation in hertz, $\Delta f = \frac{k_f m_p}{2\pi}$

FM bandwidth (in hertz), $B_{FM} = 2(\Delta f + B)$

Where, B =Modulating signal bandwidth

Deviation ratio, $\beta = \frac{\Delta f}{B}$

Another form of FM bandwidth, $B_{FM} = 2B(\beta + 1)$

PM: $\omega_i = \omega_c + k_p \dot{m}(t)$

The frequency deviation, $\Delta\omega = k_p m'_p$ where $m'_p = [\dot{m}(t)]_{max}$

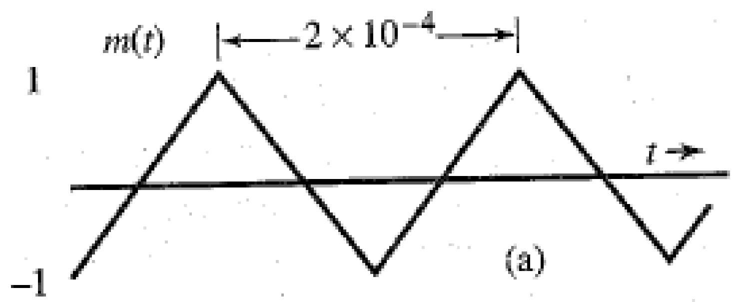
in hertz, $\Delta f = \frac{k_f m'_p}{2\pi}$

PM bandwidth (in hertz), $B_{PM} = 2(\Delta f + B)$

Deviation ratio, $\beta = \frac{\Delta f}{B}$

Another form of PM bandwidth, $B_{PM} = 2B(\beta + 1)$

Example 5.3: (a) Estimate B_{FM} and B_{PM} for the modulating signal *m(t)* in the figure for $k_f = 2\pi \times 10^5$, $k_p = 5\pi$ and B=15 KHz.
(b) Repeat the problem if the amplitude of *m(t)* is doubled.



(a)

For FM:

$$\Delta f = \frac{1}{2\pi} k_f m_p = \frac{1}{2\pi}(2\pi \times 10^5)(1) = 100\,\text{kHz}$$

and

$$B_{\text{FM}} = 2(\Delta f + B) = 230\,\text{kHz}$$

Alternately, the deviation ratio β is given by

$$\beta = \frac{\Delta f}{B} = \frac{100}{15}$$

and

$$B_{\text{FM}} = 2B(\beta + 1) = 30\left(\frac{100}{15} + 1\right) = 230\,\text{kHz}$$

For PM: The peak amplitude of $\dot{m}(t)$, is 20,000, and

$$\Delta f = \frac{1}{2\pi} k_p m'_p = 50 \text{ kHz}$$

Hence,

$$B_{\text{PM}} = 2(\Delta f + B) = 130\,\text{kHz}$$

Alternately, the deviation ratio β is given by

$$\beta = \frac{\Delta f}{B} = \frac{50}{15}$$

and

$$B_{\text{PM}} = 2B(\beta + 1) = 30\left(\frac{50}{15} + 1\right) = 130\,\text{kHz}$$

(b) Doubling $m(t)$ doubles its peak value. Hence, $m_p = 2$. But its bandwidth is unchanged so that $B = 15$ kHz.

For FM:

$$\Delta f = \frac{1}{2\pi} k_f m_p = \frac{1}{2\pi} (2\pi \times 10^5)(2) = 200\,\text{kHz}$$

and

$$B_{\text{FM}} = 2(\Delta f + B) = 430\,\text{kHz}$$

Alternately, the deviation ratio β is given by

$$\beta = \frac{\Delta f}{B} = \frac{200}{15}$$

and

$$B_{\text{FM}} = 2B(\beta + 1) = 30\left(\frac{200}{15} + 1\right) = 430\,\text{kHz}$$

For PM: Doubling $m(t)$ doubles its derivative so that now $m'_p = 40,000$, and

$$\Delta f = \frac{1}{2\pi} k_p m'_p = 100\text{kHz}$$

and

$$B_{\text{PM}} = 2(\Delta f + B) = 230\,\text{kHz}$$

Alternately, the deviation ratio β is given by

$$\beta = \frac{\Delta f}{B} = \frac{100}{15}$$

and

$$B_{\text{PM}} = 2B(\beta + 1) = 30\left(\frac{100}{15} + 1\right) = 230\,\text{kHz}$$

Observe that doubling the signal amplitude [doubling $m(t)$] roughly doubles the bandwidth of both FM and PM waveforms.

Example 5.5:

An angle-modulated signal with carrier frequency $\omega_c = 2\pi \times 10^5$ is described by the equation

$$\varphi_{EM}(t) = 10\cos(\omega_c t + 5\sin 3000t + 10\sin 2000\pi t)$$

(a) Find the power of the modulated signal.
(b) Find the frequency deviation Δf.
(c) Find the deviation ratio β.
(d) Find the phase deviation $\Delta\phi$.
(e) Estimate the bandwidth of $\varphi_{EM}(t)$.

The signal bandwidth is the highest frequency in $m(t)$ (or its derivative). In this case $B = 2000\pi/2\pi = 1000$ Hz.

(a) The carrier amplitude is 10, and the power is

$$P = 10^2/2 = 50$$

(b) To find the frequency deviation Δf, we find the instantaneous frequency ω_i, given by

$$\omega_i = \frac{d}{dt}\theta(t) = \omega_c + 15,000 \cos 3000t + 20,000\pi \cos 2000\pi t$$

The carrier deviation is $15,000 \cos 3000t + 20,000\pi \cos 2000\pi t$. The two sinusoids will add in phase at some point, and the maximum value of this expression is $15,000 + 20,000\pi$. This is the maximum carrier deviation $\Delta\omega$. Hence,

$$\Delta f = \frac{\Delta\omega}{2\pi} = 12,387.32 \text{ Hz}$$

(c)

$$\beta = \frac{\Delta f}{B} = \frac{12,387.32}{1000} = 12.387$$

(d) The angle $\theta(t) = \omega t + (5 \sin 3000t + 10 \sin 2000\pi t)$. The phase deviation is the maximum value of the angle inside the parentheses, and is given by $\Delta\phi = 15$ rad.

(e)

$$B_{EM} = 2(\Delta f + B) = 26,774.65 \text{ Hz}$$

Observe the generality of this method of estimating the bandwidth of an angle-modulated waveform. We need not know whether it is FM, PM, or some other kind of angle modulation. It is applicable to any angle-modulated signal.

Generation of FM waves:

Basically, there are two ways of generating FM waves:

- Indirect generation
- Direct generation

Indirect Method of Armstrong:

In this method, NBFM (Narrow-Band FM) is generated by integrating *m(t)* and using it to phase modulate a carrier. The NBFM is then converted to WBFM (Wide-Band FM) by using frequency multipliers.

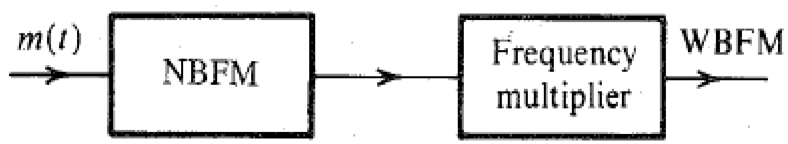


Figure 5.9 Simplified block diagram of Armstrong indirect FM wave generator.

For NBFM, $\varphi_{\text{FM}}(t) \simeq A[\cos \omega_c t - k_f a(t) \sin \omega_c t]$

Where, $a(t) = \int_{-\infty}^{t} m(\alpha)\, d\alpha$

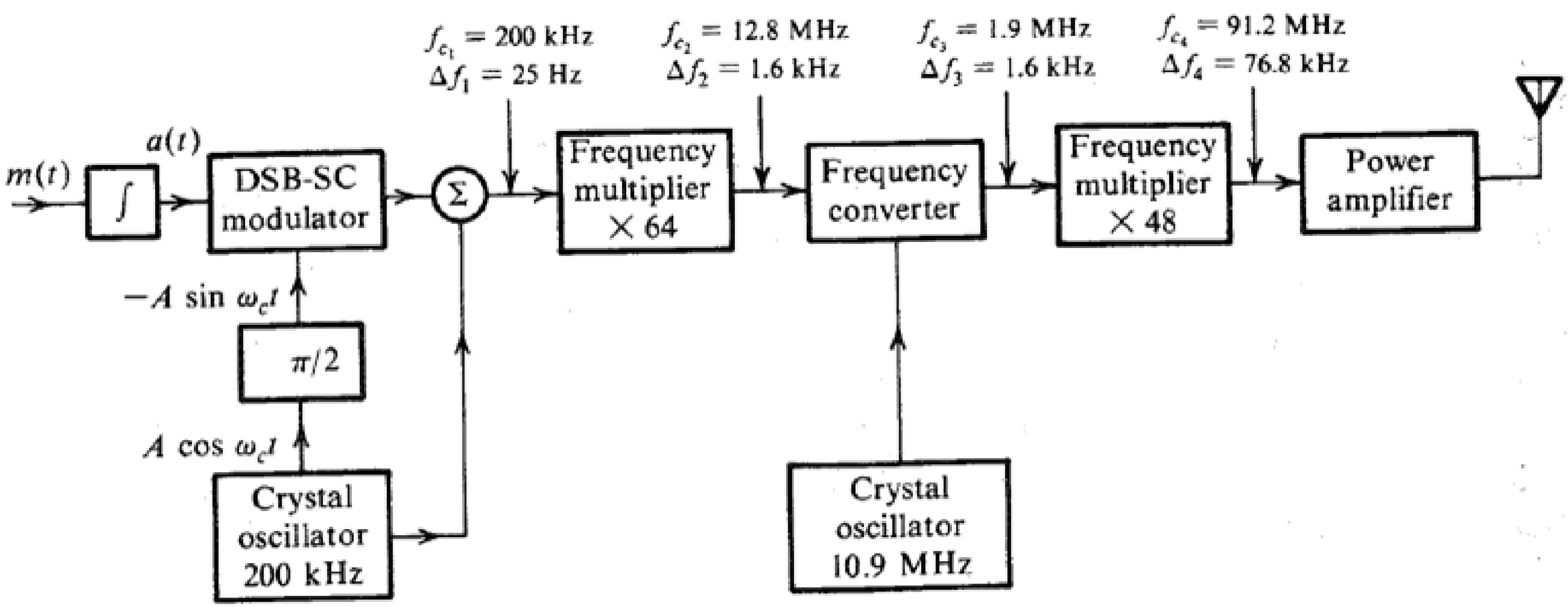


Figure 5.10 Armstrong indirect FM transmitter.

The final output is required to have a carrier frequency of 91.2 MHz and $\Delta f = 75$ KHz. We begin with NBFM with a carrier frequency f_{c1}=200 kHz generated by a crystal oscillator.
The deviation Δf is chosen to be 25 Hz to maintain $\beta \ll 1$ as required in NBFM. For tone modulation $\beta = \Delta f / f_m$. The baseband spectrum ranges from 50Hz to15kHz. We chose $\Delta f = 25$ Hz so that $\beta = 0.5$ for the worst possible case (f_m= 50 Hz).

In order to achieve $\Delta f = 75$ KHz , we need a multiplication of 75,000/25=3000. This can be done by two multiplier stages of 64 and 48 giving a total multiplication of 64 $\times$ 48 = 3072 and $\Delta f = 25 \times 3072 = 76.8$ kHz.

Now, $f_{c2} = 200 \text{ kHz} \times 64 = 12.8 \text{ MHz}$
and $\Delta f_2 = 25 \text{ Hz} \times 64 = 1.6 \text{ kHz}$

Then, the entire spectrum is shifted using a frequency converter with carrier frequency 10.9 MHz.

So, $f_{c3} = 12.8 - 10.9 = 1.9$ MHz

The frequency converter shifts the entire spectrum without altering Δf. Hence, Δf_3=1.6 kHz.

Further multiplication by 48, yields

$$f_{c4} = 1.9\ \text{MHz} \times 48 = 91.2\ \text{MHz}$$
$$\Delta f_4 = 1.6\ \text{kHz} \times 48 = 76.8\ \text{kHz}$$

Direct Generation:

In a voltage–controlled oscillator (VCO), the frequency is controlled by an external voltage. The oscillation frequency varies linearly with the control voltage. We can generate an FM wave by using the modulating signal m(t) as a control signal.

This gives, $\omega_i(t) = \omega_c + k_f m(t)$

One way of accomplishing this goal is to vary one of the reactive parameters (C or L) of the resonant circuit of an oscillator. A reverse-biased semiconductor diode acts as a capacitor whose capacitance varies with the bias voltage. The capacitance of these diodes can be approximated as a linear function of the bias voltage *m(t)* over a limited range.

In Hartley or Colpitt oscillators, the frequency of oscillation is given by,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

If the capacitance C is varied by the modulating signal *m(t)*,
Then,

$$C = C_o - k\, m(t) = C_o[1 - \frac{km(t)}{C_o}]$$

Now, $$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{LC_o\left[1 - \frac{km(t)}{C_o}\right]}}$$

$$= \frac{1}{\sqrt{LC_o}\left[1 - \frac{km(t)}{C_o}\right]^{1/2}}$$

$$= \frac{1}{\sqrt{LC_o}}\left[1 - \frac{km(t)}{C_o}\right]^{-1/2}$$

$$\approx \frac{1}{\sqrt{LC_o}}\left[1 + \frac{km(t)}{2C_o}\right] \quad ; \quad \frac{km(t)}{C_o} \ll 1$$

Here, we have used the binomial approximation $(1+x)^n \approx 1 + nx \; for \; |x| \ll 1$.

So, $$\omega_0 = \omega_c\left[1 + \frac{km(t)}{2C_o}\right] \qquad \omega_c = \frac{1}{\sqrt{LC_o}}$$

$$= \omega_c + \frac{k\omega_c}{2C_o}m(t) \qquad k_f = \frac{k\omega_c}{2C_o}$$

$$\Rightarrow \omega_0 = \omega_c + k_f m(t)$$