Digital Modulation

2-2 INFORMATION CAPACITY, BITS, BIT RATE, BAUD, AND MARY ENCODING

- The Shannon limit for information capacity is

$$I = B \log_2\left(1 + \frac{S}{N}\right)$$

or

$$I = 3.32B \log_{10}\left(1 + \frac{S}{N}\right)$$

where I = information capacity (bps)

B = bandwidth (hertz)

$\frac{S}{N}$ = signal-to-noise power ratio (unitless)

Example:

For a standard telephone circuit with a signal-to-noise power ratio of 1000 (30 dB) and a bandwidth of 2.7 kHz, the Shannon limit for information capacity is

$$I = (3.32)(2700) \log_{10}(1 + 1000) = 26.9 \text{ kbps}$$

2-2-2 M-ary Encoding

M-ary is a term derived from the word *binary*.

M simply represents a digit that corresponds to the number of conditions, levels, or combinations possible for a given number of binary variables.

The number of bits necessary to produce a given number of conditions is expressed mathematically as

$$N = \log_2 M \tag{2.5}$$

where N = number of bits necessary

M = number of conditions, levels, or combinations possible with N bits

Equation 2-5 can be simplified and rearranged to express the number of conditions possible with N bits as

$$2^N = M \tag{2.6}$$

For example, with one bit, only $2^1 = 2$ conditions are possible. With two bits, $2^2 = 4$ conditions are possible, with three bits, $2^3 = 8$ conditions are possible, and so on.

2-2-3 Baud

Baud refers to the rate of change of a signal on the transmission medium after encoding and modulation have occurred.

Hence, baud is a unit of transmission rate, modulation rate, or symbol rate and, therefore, the terms symbols per second and baud are often used interchangeably.

Mathematically, baud is the reciprocal of the time of one output signaling element, and a signaling element may represent several information bits. Baud is expressed as

$$\text{baud} = \frac{1}{t_s} \qquad (2.7)$$

where baud = symbol rate (baud per second)

t_s = time of one signaling element (seconds)

2-2-3 Baud

In addition, since baud is the encoded rate of change, it also equals the bit rate divided by the number of bits encoded into one signaling element. Thus,

$$\text{Baud} = \left(\frac{f_b}{N}\right) \qquad (2.11)$$

where N is the number of bits encoded into each signaling element.

f_b = bit rate (bps)

2-3 AMPLITUDE-SHIFT KEYING

- Simplest digital modulation technique.
- In *amplitude-shift keying (ASK)* modulation technique, ***a binary information signal*** directly modulates the amplitude of an **analog carrier**.
- ASK is similar to standard amplitude modulation except there are only two output amplitudes possible.
- Amplitude shift keying is sometimes called ***digital amplitude modulation (DAM)***.

2-3 AMPLITUDE-SHIFT KEYING

Mathematically, amplitude-shift keying is

$$v_{(ask)}(t) = [1 + v_m(t)]\left[\frac{A}{2}\cos(\omega_c t)\right] \qquad (2.12)$$

where

$v_{ask}(t)$ = amplitude-shift keying wave

$v_m(t)$ = digital information (modulating) signal (volts)

$A/2$ = unmodulated carrier amplitude (volts)

ω_c = analog carrier radian frequency (radians per second, $2\pi f_c$)

2-3 AMPLITUDE-SHIFT KEYING

In Equation *2.12,* the modulating signal $[v_m(t)]$ *is* a normalized binary waveform, where + 1 V = logic 1 and -1 V = logic 0. Therefore, for a logic 1 input, $v_m(t)$ = + 1 V, Equation *2.12* reduces to

$$v_{(ask)}(t) = [1 + 1]\left[\frac{A}{2}\cos(\omega_c t)\right]$$

$$= A\cos(\omega_c t)$$

and for a logic 0 input, $v_m(t)$ = -1 V, Equation *2.12* reduces to

$$v_{(ask)}(t) = [1 - 1]\left[\frac{A}{2}\cos(\omega_c t)\right] = 0$$

Thus, the modulated wave $v_{ask}(t)$, is either A $\cos(\omega_c t)$ or 0. Hence, the carrier is either *"on"* or *"off,"* which is why amplitude-shift keying is sometimes referred to as *on-off keying* (OOK).

2-3 AMPLITUDE-SHIFT KEYING

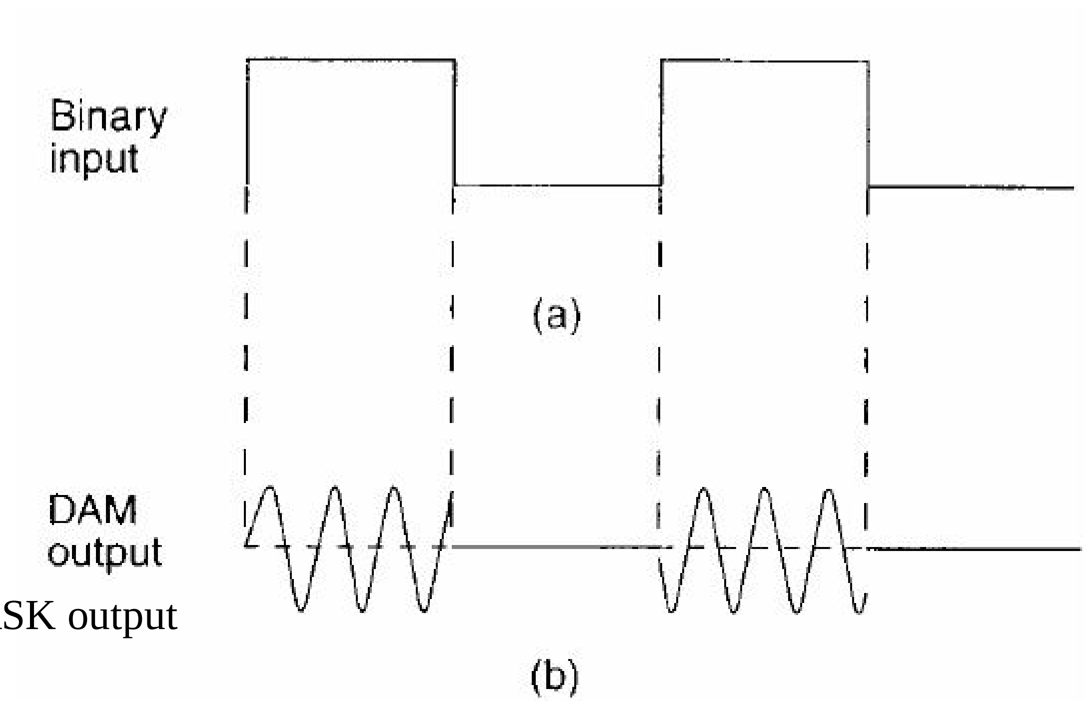


FIGURE 2-2 Digital amplitude modulation: (a) input binary; (b) output DAM waveform

2-3 AMPLITUDE-SHIFT KEYING

Example: Determine the baud necessary to pass a 10 kbps binary signal using amplitude shift keying.

Solution: For ASK, N=1.

Here, $f_b = 10 \text{ kbps}$

So, $$\text{Baud} = \frac{f_b}{N} = \frac{10{,}000}{1} = 10{,}000$$

The use of amplitude-modulated analog carriers to transport digital information is a relatively low-quality, low-cost type of digital modulation and, therefore, is seldom used except for very low-speed telemetry circuits.

2-4 FREQUENCY-SHIFT KEYING

- FSK is a form of **constant-amplitude** angle modulation similar to standard frequency modulation (FM) except the modulating signal is a **binary signal** that varies between two discrete voltage levels rather than a continuously changing analog waveform.
- Consequently, FSK is sometimes called *binary FSK (BFSK).* The general expression for FSK is

$$v_{fsk}(t) = V_c \cos\{2\pi[f_c + v_m(t)\,\Delta f]t\} \qquad (2.13)$$

$v_{fsk}(t)$ = binary FSK waveform
V_c = peak analog carrier amplitude (volts)
f_c = analog carrier center frequency (hertz)
Δf = peak change (shift) in the analog carrier frequency (hertz)
$v_m(t)$ = binary input (modulating) signal (volts)

2-4 FREQUENCY-SHIFT KEYING

The modulating signal is a normalized binary waveform where a logic 1 = + 1 V and a logic 0 = -1 V. Thus, for a logic l input, $v_m(t) = +1$, Equation 2.13 can be rewritten as

$$v_{fsk}(t) = V_c \cos[2\pi(f_c + \Delta f)t]$$

For a logic 0 input, $v_m(t) = -1$, Equation 2.13 becomes

$$v_{fsk}(t) = V_c \cos[2\pi(f_c - \Delta f)t]$$

2-4 FREQUENCY-SHIFT KEYING

With binary FSK, the carrier center frequency (f_c) is shifted (deviated) up and down in the frequency domain by the binary input signal as shown in Figure 2-3.

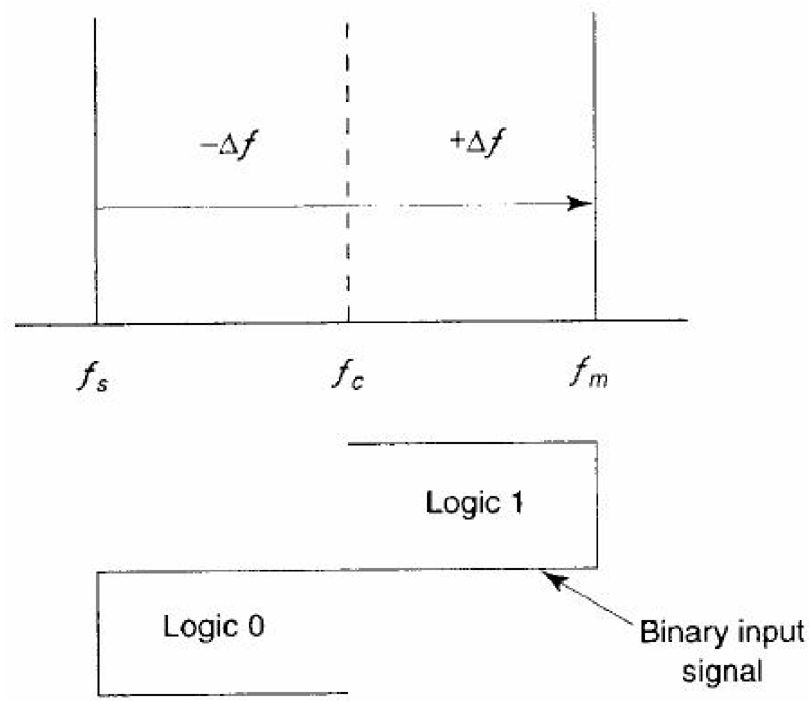


FIGURE 2-3 FSK in the frequency domain

2-4 FREQUENCY-SHIFT KEYING

- As the binary input signal changes from a logic 0 to a logic 1 and vice versa, the output frequency shifts between two frequencies: a mark, or logic 1 frequency (f_m), and a space, or logic 0 frequency (f_s).

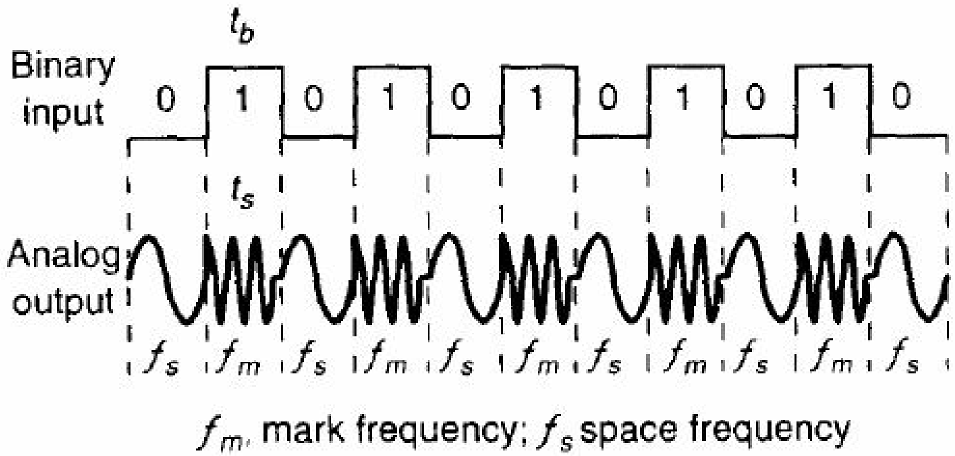
Frequency deviation is illustrated in Figure 2-3 and expressed mathematically as

$$\Delta f = |f_m - f_s| / 2 \tag{2.14}$$

where Δf = frequency deviation (hertz)

$|f_m - f_s|$ = absolute difference between the mark and space frequencies (hertz)

2-4 FREQUENCY-SHIFT KEYING



(a)

binary input	frequency output
0	space (f_s)
1	mark (f_m)

(b)

FIGURE 2-4 FSK in the time domain: (a) waveform: (b) truth table

2-4 FREQUENCY-SHIFT KEYING

Example 2-2

Determine (a) the peak frequency deviation, ~~(b) minimum bandwidth, and (c) baud~~ for a binary FSK signal with a mark frequency of 49 kHz, a space frequency of 51 kHz, ~~and an input bit rate of 2 kbps.~~

Solution

a. The peak frequency deviation is determined from Equation 2.14:

$$\Delta f = |49\text{kHz} - 51\text{ kHz}| / 2 = 1\text{ kHz}$$

2-5 PHASE-SHIFT KEYING

- Phase-shift keying (PSK) is another form of angle-modulated, constant-amplitude digital modulation.

2-5-1 Binary Phase-Shift Keying

The simplest form of PSK is *binary phase-shift keying* (BPSK), where N = 1 and $M = 2$.

Therefore, with BPSK, two phases ($2^1 = 2$) are possible for the carrier.

One phase represents a logic 1, and the other phase represents a logic 0. As the input digital signal changes state (i.e., from a 1 to a 0 or from a 0 to a 1), the phase of the output carrier shifts between two angles that are separated by 180°.

2-5 PHASE-SHIFT KEYING

As the binary input shifts between a logic 1 and a logic 0 condition and vice versa, the phase of the BPSK waveform shifts between 0° and 180°, respectively.

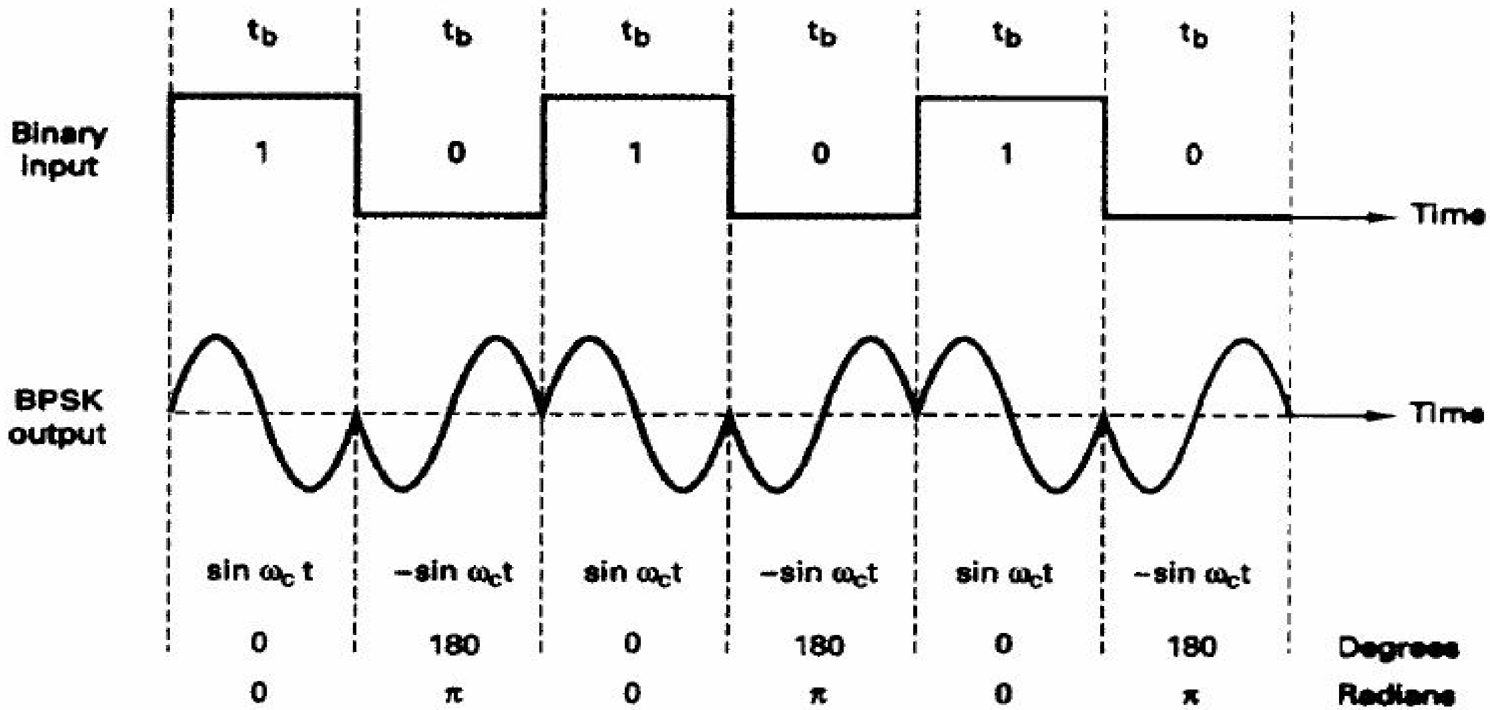


FIGURE 2-15 Output phase-versus-time relationship for a BPSK modulator

2-5-2 Quaternary Phase-Shift Keying

QPSK is an M-ary encoding scheme where N = 2 and $M = 4$ (hence, the name "quaternary" meaning "4"). A QPSK modulator is a binary (base 2) signal, to produce four different input combinations,: 00, 01, 10, and 11.

Therefore, with QPSK, the binary input data are combined into groups of two bits, called *dibits*. In the modulator, each dibit code generates one of the four possible output phases (+45°, +135°, -45°, and -135°).