18/02/2021
L3T2 - B(E)

Node/Bus elimination by Matrix Algebra.

Matrix partitioning

node/junction

Let, $\underset{3\times3}{A} = \left[\begin{array}{cc|c} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \hline a_{31} & a_{32} & a_{33} \end{array}\right] = \begin{bmatrix} E & F \\ G & H \end{bmatrix}$

where, $E = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $F = \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}$

$G = [a_{31} \; a_{32}]$, $H = [a_{33}]$

and, $\underset{3\times1}{B} = \left[\begin{array}{c} b_{11} \\ b_{21} \\ \hline b_{31} \end{array}\right] = \begin{bmatrix} Q \\ R \end{bmatrix}$

Then, $C = \underline{A \times B} = \underset{2\times2}{\begin{bmatrix} E & F \\ G & H \end{bmatrix}} \times \underset{2\times1}{\begin{bmatrix} Q \\ R \end{bmatrix}} = \underset{2\times1}{\begin{bmatrix} EQ + FR \\ GQ + HR \end{bmatrix}}$

Node/Bus current in matrix form.

$I_{BUS} = Y_{BUS} \cdot V_{BUS}$

I_{BUS}, V_{BUS} - column mat

Y_{BUS} - Sym. square mat

$\begin{bmatrix} I_A \\ I_X \end{bmatrix} = \begin{bmatrix} K & L \\ L^T & M \end{bmatrix} \begin{bmatrix} V_A \\ V_X \end{bmatrix}$

I_A, V_A - current/voltages need to retain ($I_A \neq 0$)

I_X, V_X - current/voltages need to eliminate ($I_X = 0$)

After multiplication,

$I_A = KV_A + LV_X$ — (1)

and $I_X = L^T V_A + MV_X$ — (2)

From eq^n ②, we get, (since all elements of I_X are equal to zero)

$$I_X = 0 = L^T V_A + M V_X$$

$$\Rightarrow V_X = -\frac{L^T V_A}{M} = -L^T V_A . M^{-1}$$

Substituting the value of V_X in eq^n ①

$$I_A = K V_A + L V_X$$

$$= K V_A + L.(-L^T V_A . M^{-1})$$

$$\Rightarrow \underline{I_A = (K - \underbrace{L.M^{-1}L^T}_{R}) V_A}$$

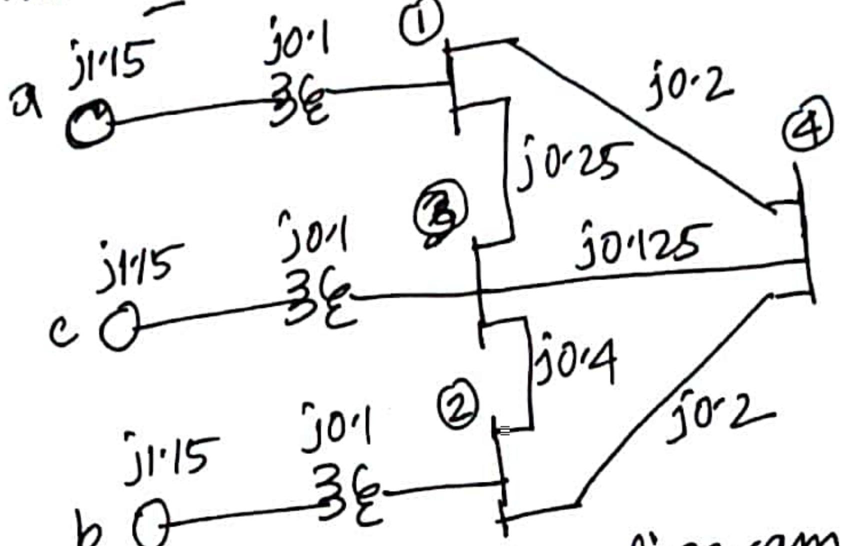
After eliminating bus(es)/node(s), we have

$$Y_{Bus,new} = \underline{K - L M^{-1} L^T}$$

$$Y_{Bus} = \left[\begin{array}{c|c} K & \\ \hline L^T & \end{array}\right]$$

Stevension Jr

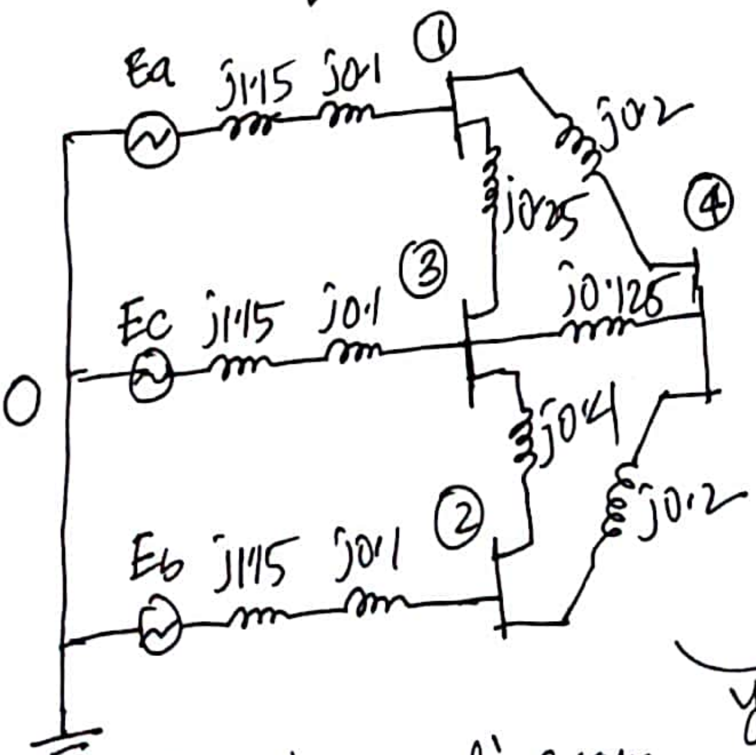
7.3 If the generator and transformer at bus ③ are removed from the circuit, eliminate node ③ and ④ by matrix algebra procedure.



$E_a = E_c = 1.5\angle 0$

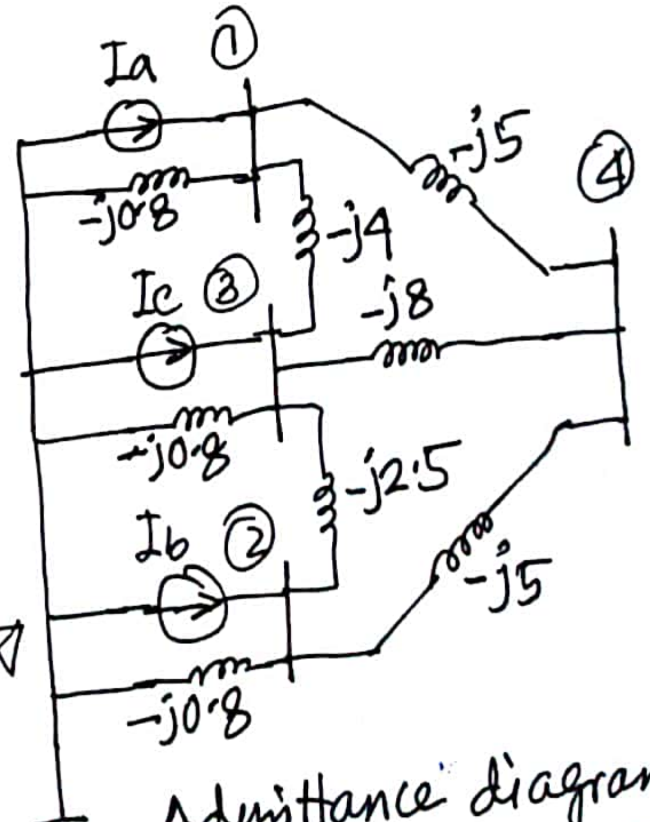
$E_b = 1.5\angle -36.87°$

Single line diagram



Impedance diagram

$y = \frac{1}{Z}$



Admittance diagram

23/02/2021
L3T2 - B(E)

Without
~~After~~ removing generator and transformer from bus ③ we get,

$$Y_{Bus} = \begin{bmatrix} -j9.8 & 0 & j4 & j5 \\ 0 & -j8.3 & j2.5 & j5 \\ j4 & j2.5 & -j15.3 & j8 \\ j5 & j5 & j8 & -j18 \end{bmatrix}$$

After removing generator and transformer at bus ③

$$Y_{Bus} = \begin{array}{c} \\ \\ r_3 \\ r_4 \\ \\ \end{array} \begin{array}{c} \left[\begin{array}{cc|cc} -j9.8 & 0 & j4 & j5 \\ 0 & -j8.3 & j2.5 & j5 \\ \hline j4 & j2.5 & \boxed{-j14.5} & j8 \\ j5 & j5 & j8 & -j18 \end{array}\right] \\ \begin{array}{cccc} \quad & \quad & c_3 & c_4 \end{array} \end{array} = \begin{bmatrix} K & L \\ L^T & M \end{bmatrix}$$

After eliminating node ③ and ④

$$Y_{Bus(new)} = K - LM^{-1}L^T.$$

$$A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adj(A)$$

$$= \frac{1}{ad-bc} \times \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Here, $M = \begin{bmatrix} -j14.5 & j8 \\ j8 & -j18 \end{bmatrix}$

Then, $M^{-1} = \frac{1}{|M|} \times Adj(M)$

$$= \frac{1}{(-j14.5 \times -j18) - j8 \times j8} \times \begin{bmatrix} -j18 & -j8 \\ -j8 & -j14.5 \end{bmatrix}$$

$$= \frac{1}{-261 - (-64)} \times \begin{bmatrix} -j18 & -j8 \\ -j8 & -j14.5 \end{bmatrix}$$

$$\Rightarrow M^{-1} = \frac{1}{-197}\begin{bmatrix} -j18 & -j8 \\ -j8 & -j14.5 \end{bmatrix} = \begin{bmatrix} j0.0914 & j0.0406 \\ j0.0406 & j0.0736 \end{bmatrix}$$

$$M^{\#} = \frac{1}{M}$$

$$= \frac{1}{j}\begin{bmatrix} -14.5 & 8 \\ 8 & -18 \end{bmatrix}$$

$$= -j\begin{bmatrix} -0.091 & -0.04 \\ -0.04 & -0.073 \end{bmatrix}$$

Calculator

Again,

$LM^{-1}L^T$

$$= \begin{bmatrix} j4 & j5 \\ j2.5 & j5 \end{bmatrix} \times \begin{bmatrix} j0.0914 & j0.0406 \\ j0.0406 & j0.0736 \end{bmatrix} \times \begin{bmatrix} j4 & j2.5 \\ j5 & j5 \end{bmatrix}$$

$$= (j \times j \times j) \times \underset{\text{MATA}}{\begin{bmatrix} 4 & 5 \\ 2.5 & 5 \end{bmatrix}} \underset{\text{MATB}}{\begin{bmatrix} 0.0914 & 0.0406 \\ 0.0406 & 0.0736 \end{bmatrix}} \underset{\text{MATC}}{\begin{bmatrix} 4 & 2.5 \\ 5 & 5 \end{bmatrix}}$$

$$= -j \times \begin{bmatrix} 4.9264 & 4.0735 \\ 4.0735 & 3.4262 \end{bmatrix}$$

$$= \begin{bmatrix} -j4.9264 & -j4.0735 \\ -j4.0735 & -j3.4262 \end{bmatrix}$$

After eliminating node 3 and 4, we get

$$Y_{Bus(new)} = K - LM^{-1}L^T$$

$$= \begin{bmatrix} -j9.8 & 0 \\ 0 & -j8.3 \end{bmatrix} - \begin{bmatrix} -j4.9264 & -j4.0735 \\ -j4.0735 & -j3.4262 \end{bmatrix}$$

$$= \begin{bmatrix} -j4.8736 & j4.0735 \\ j4.0735 & -j4.8738 \end{bmatrix}$$

$$= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

Admittance between bus ① and ② -

$y_{12} = -Y_{12} = -Y_{21} = -j4.0735$ pu

$Y_{12} = -y_{12} = Y_{21}$

Admittance between bus ① and ground

$y_{10} = Y_{11} - y_{12} = -j4.8736 - (-j4.0735)$

$= -j0.8001$ pu

$Y_{11} = y_{10} + y_{12}$

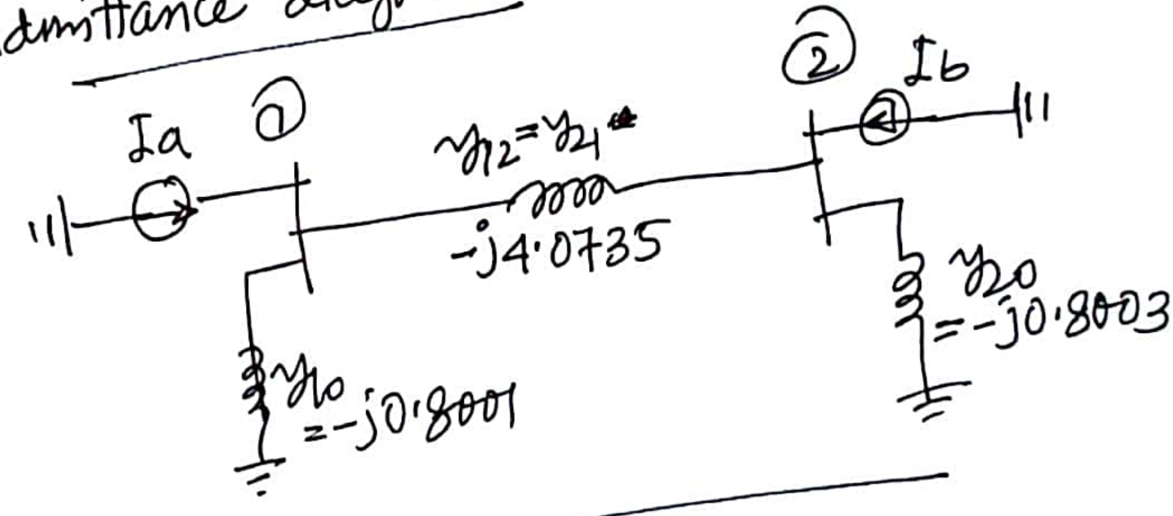
Admittance between bus ② and ground

$y_{20} = Y_{22} - y_{21} = -j4.8738 - (-j4.0735)$

$= -j0.8003$ pu.

$y_{12} = y_{21}$

Admittance diagram



25/02/2021

L3T2 - B (E)

KRON method of node elimination

Sofiul

4877

~~4864~~

$$Y_{ij(new)} = Y_{ij(old)} - \left(\frac{Y_{in}\,Y_{nj}}{Y_{nn}}\right)_{(old)}$$

where, i, j — row, column

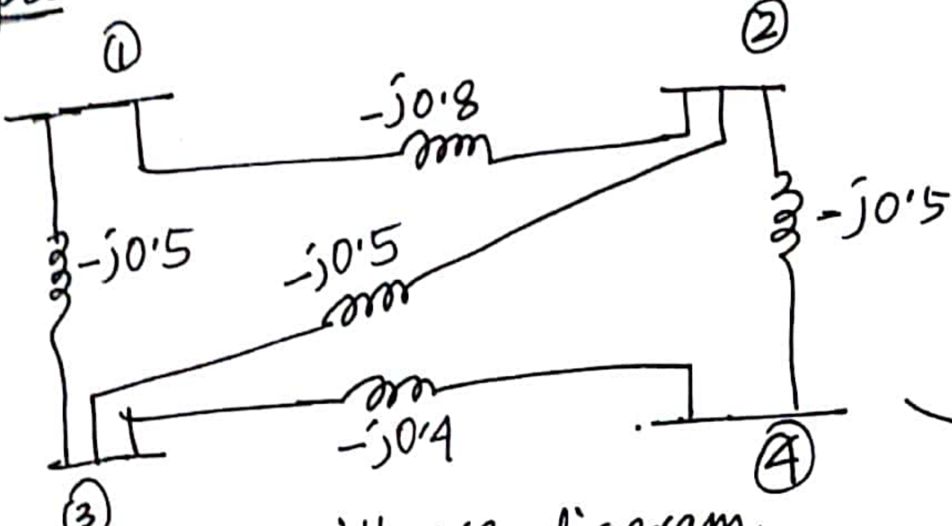
n - node to be eliminated

Example

4891

4871

~~4860~~



Admittance diagram.

$\rightarrow$ Y_{BUS} 4×4 $\rightarrow$ $3\times$

Eliminate node ④ from the network and find new bus admittance matrix.

Bus admittance matrix without eliminating node ④, we get

$$\begin{array}{cc} & \begin{array}{cccc} \phantom{-j1{\cdot}3} & \phantom{j0{\cdot}8} & \phantom{j0{\cdot}5} & c4 \end{array} \\ Y_{BUS} = & \left[\begin{array}{ccc|c} -j1{\cdot}3 & j0{\cdot}8 & j0{\cdot}5 & 0 \\ j0{\cdot}8 & -j1{\cdot}8 & j0{\cdot}5 & j0{\cdot}5 \\ j0{\cdot}5 & j0{\cdot}5 & -j1{\cdot}4 & j0{\cdot}4 \\ \hline 0 & j0{\cdot}5 & j0{\cdot}4 & -j0{\cdot}9 \end{array}\right] \begin{array}{c} \\ \\ \\ -r4 \end{array} \end{array}$$

4890
4862
4890
4873

Here, $n=4$:

$i=1, j=1;$ $Y_{11}(new) = Y_{11}(old) - \left(\frac{Y_{14}Y_{41}}{Y_{44}}\right)_{old}$

$= -j1.3 - \frac{0\times0}{-j0.9} = -j1.3$ pu

$i=2, j=2;$ $Y_{22}(new) = Y_{22}(old) - \left(\frac{Y_{24}Y_{42}}{Y_{44}}\right)_{old}$

$= -j1.8 - \frac{j0.5\times j0.5}{-j0.9} = -j1.522$ pu

$i=3, j=3;$ $Y_{33}(new) = Y_{33}(old) - \left(\frac{Y_{34}Y_{43}}{Y_{44}}\right)_{old}$

$= -j1.4 - \frac{j0.4\times j0.4}{-j0.9} = -j1.222$ pu

$i=1, j=2;$ $Y_{12}(new) = Y_{21}(new) = Y_{12}(old) - \left(\frac{Y_{14}Y_{42}}{Y_{44}}\right)_{old}$

$= j0.8 - \frac{0\times j0.5}{-j0.9} = j0.8$ pu

$i=1, j=3;$ $Y_{13}(new) = Y_{31}(new) = Y_{13}(old) - \left(\frac{Y_{14}Y_{43}}{Y_{44}}\right)_{old}$

$= j0.5 - \frac{0\times j0.4}{-j0.9} = j0.5$ pu

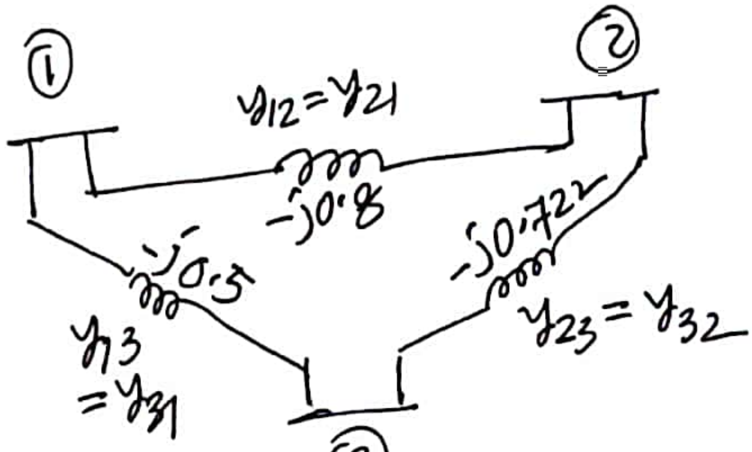
$i=2, j=3;$ $Y_{23}(new) = Y_{32}(new) = Y_{23}(old) - \left(\frac{Y_{24}Y_{43}}{Y_{44}}\right)_{o}$

$= j0.5 - \frac{j0.5\times j0.4}{-j0.9}$

$= j0.722$ pu

Then new bus admittance matrix after eliminating bus 4, we have—

$$Y_{BUS(new)} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} = \begin{bmatrix} -j1.3 & j0.8 & j0.5 \\ j0.8 & -j1.522 & j0.722 \\ j0.5 & j0.722 & -j1.22 \end{bmatrix}$$



$y_{12} = y_{21} = -Y_{12} = -j0.8$

$y_{13} = y_{31} = -Y_{13} = -j0.5$

$y_{23} = y_{32} = -Y_{23} = -j0.722$

new admittance diagram

$Z = \frac{1}{y}$

Quiz-2 : 4 March

11:30 – 12:30 pm

contact me