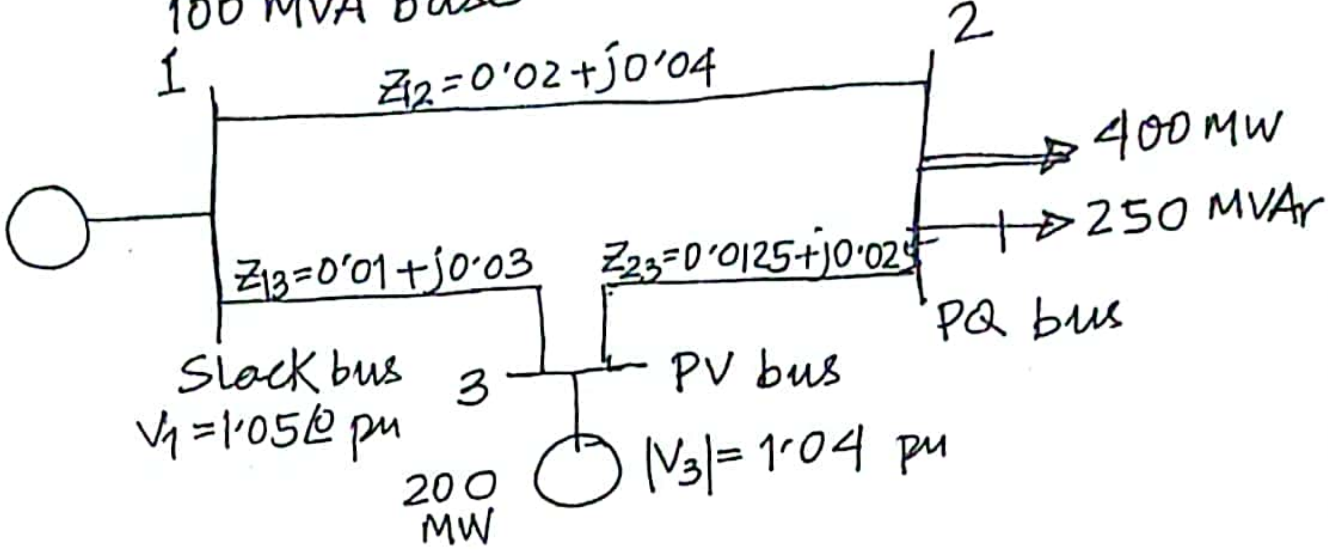
Example 6.8. PQ and PV Bus

100 MVA base



Obtain the power flow solution by the Gauss-Seidel method including line flows and line losses.

Solution: Line admittances are:

$$y_{12} = y_{21} = \frac{1}{Z_{12}} = 10 - j20$$

$$y_{13} = y_{31} = \frac{1}{Z_{13}} = 10 - j30$$

$$y_{23} = y_{32} = \frac{1}{Z_{23}} = 16 - j32$$

Y_{Bus} elements are

$$Y_{11} = y_{12} + y_{13} = 20 - j50 \text{ pu}$$

$$Y_{22} = y_{21} + y_{23} = 26 - j52 \text{ pu}$$

$$Y_{33} = y_{31} + y_{32} = 26 - j62 \text{ pu}$$

$$Y_{12} = Y_{21} = -y_{12} = -10 + j20 \text{ pu}$$

$$Y_{13} = Y_{31} = -y_{13} = -10 + j30 \text{ pu}$$

$$Y_{23} = Y_{32} = -y_{23} = -16 + j32 \text{ pu}.$$

Complex power in pu at bus 2 (PQ bus)

$$S_2^{sch} = -\frac{(P_2 + jQ_2)^{sch}}{\text{MVA base}} = -\frac{400 + j250}{100} = -4.0 - j2.5 \text{ pu}$$

Real power in pu at bus 3 (PV bus)

$$P_3^{sch} = \frac{P_3^{sch}}{\text{MVA base}} = \frac{200}{100} = 2.0 \text{ pu}.$$

Let,

Initial estimate; $V_2^0 = 1.0\angle 0$ pu, $V_3^0 = 1.04\angle 0$ pu

given, $V_1 = 1.05\angle 0$ pu

We Know, Gauss-Seidel power flow solution is

$$V_i^{K+1} = \frac{1}{Y_{ii}}\left\{\frac{(P_i - jQ_i)^{sch}}{V_i^{*K}} - \sum_{\substack{j=1\\ j\neq n}}^{n} Y_{ij}.V_j^K\right\}$$

Iteration (K=0) 1

$i=2$, Bus 2

$$V_2^1 = \frac{1}{Y_{22}}\left\{\frac{(P_2 - jQ_2)^{sch}}{V_2^{*0}} - Y_{21}V_1 - Y_{23}V_3^0\right\}$$

$$\frac{\frac{A}{B} - CD - EF}{Y}$$

$$= \frac{1}{26-j52}\left\{\frac{-4.0+j2.5}{1.0\angle -0} - (-10+j20).1.05\angle 0 - (-16+j32)\,1.04\angle 0\right\}$$

$= 0.974615 - j0.042308$ pu

$= 0.975533\angle -2.485628$ pu.

$$P_i - jQ_i = V_i^{*K}\left(V_i^{K+1}Y_{ii} + \sum_{j=1;j\neq i}^{n} Y_{ij}V_j^K\right)$$

$$\Rightarrow Q_i^{K+1} = -Im\left[V_i^{*K}\left\{V_i^K Y_{ii} + \sum_{j=1;j\neq i}^{n} Y_{ij}V_j^K\right\}\right]$$

Reactive power at bus 3 ($i=3$)

$$Q_3^1 = -Im.\left[V_3^{*0}\left\{V_3^0.Y_{33} + Y_{31}V_1 + Y_{32}V_2^1\right\}\right]$$

$$= -Im.\left[1.04\angle 0\left\{1.04\angle 0 \times (26-j62) + (-10+j30)\times 1.05\angle 0 + (-16+j32)\times 0.975533\angle -2.485628\right\}\right]$$

$Q_3^1 = -Im[2.392003 - j1.160008]$

$= 1.160008$ pu

Then, complex power at bus 3 (PV bus)

$S_3^{sch} = 2.0 + j1.160008$ pu.

$i=3$ (Bus 3), complex bus voltage

$$V_{c3}^1 = \frac{1}{Y_{33}} \cdot \left\{ \frac{(P_3 - jQ_3)^{sch}}{V_3^{*0}} - Y_{31}V_1 - Y_{32}V_2^1 \right\}$$

$$= \frac{1}{26 - j62} \left\{ \frac{2 - j1.160008}{1.04\angle 0} - (-10 + j30) \cdot 1.05\angle 0 - (-16 + j32) \cdot 0.975533\angle -2.485628 \right\}$$

$= 1.037832 - j0.005170$ pu

We know,

$$|V_i|^2 = \sqrt{\text{real part}^2 + \text{imaginary part}^2}$$

$$= \sqrt{(e_i^{K+1})^2 + (f_i^{K+1})^2}$$

| $z = x + iy$
| $|z| = \sqrt{x^2 + y^2}$

For bus 3 = $i=3$

$$|V_3|^2 = \sqrt{(e_3^1)^2 + (f_3^1)^2}$$

$$\Rightarrow e_3^1 = \sqrt{|V_3|^2 - (f_3^1)^2}$$

$$= \sqrt{(1.04)^2 - (0.00517)^2}$$

$= 1.039987$ pu.

| $|V_3| = 1.04$
| $f_3^1 = 0.005170$
| $e_3^1 = ?$

Then, $V_3^1 = 1.039987 - j0.005170$ pu

$= 1.04\angle -0.284827$ pu.

Iteration (K=1) 2:

i=3; Bus 2

$$V_2^2 = \frac{1}{Y_{22}}\left\{\frac{(P_2 - jQ_2)^{sch}}{V_2^{*1}} - Y_{21}V_1 - Y_{23}V_3^1\right\}$$

$$= \frac{1}{26-j52}\left\{\frac{-4+j2.5}{0.975533\angle 2.485628} - (-10+j20).\,1.05\angle 0 - (-16+j32).\,1.04\angle -0.284827\right\}$$

$$= 0.971057 - j0.043432 \text{ pu}$$

$$= 0.972028\angle -2.560920 \text{ pu}$$

Then, $Q_3^2 = -\text{Im}\left[V_3^{*1}\left\{V_3^1 Y_{33} + Y_{31}V_1 + Y_{32}V_2^2\right\}\right]$

$$= -\text{Im}\left[1.04\angle 0.284827\left\{1.04\angle -0.284827 \times (26-j62) + (-10+j30).\,1.05\angle 0 + (-16+j32).\,0.972028\angle -2.56092\right\}\right]$$

$$= -\text{Im}\left[2.161835 - j1.387950\right]$$

$$= 1.387950 \text{ pu}$$

Then, complex power in pu at bus 3 (PV bus)

$$S_3^{sch} = 2.0 + j1.387950 \text{ pu.}$$ ✓

i=3 (Bus 3), complex bus voltage

$$V_{c3}^2 = \frac{1}{Y_{33}}\left\{\frac{(P_3 - jQ_3)^{sch}}{V_3^{*1}} - Y_{31}V_1 - Y_{32}V_2^2\right\}$$

$$= \frac{1}{26-j62}\left\{\frac{2-j1.387950}{1.04\angle 0.284827} - (-10+j30).\,1.05\angle 0 - (-16+j32).\,0.972028\angle -2.56092\right\}$$

$$= 1.039081 - j0.007300 \text{ pu}$$

Then, real part, $e_3^{2} = \sqrt{|V_3|^2 - (f_3)^2}$

$|V_3| = 1.04$

$f_3^{2} = 0.0073$

$e_3^{2} = \sqrt{1.04^2 - 0.0073^2} = 1.039974$

Then, $V_3^{2} = 1.039974 - j0.007300$ pu

$= 1.04 \angle -0.402176$ pu.

After 2 iteration—

$V_2^{2} = 0.972028 \angle -2.56092$ pu, $Q_3^{2} = 1.387950$ pu

$V_3^{2} = 1.04 \angle -0.402176$ pu ——— Approximate Vale

In Book – 7 iteration..

$V_2^{7} = 0.97061 - j0.04569 = 0.971685 \angle -2.695123$ pu

$Q_3^{7} = 1.46082$ pu $= 146.08$ MVAr (100 MVA base)

$V_3^{7} = 1.039960 - j0.00903 = 1.04 \angle -0.497488$ pu.

Bus 2: $V_2 = 0.971685 \angle -2.695123$ pu

$S_2 = -4.0 - j2.5$ pu

Bus 3: $V_3 = 1.04 \angle -0.497488$ pu

$S_3 = 2.0 + j1.46082$ pu.

Bus 1: $V_1 = 1.05 \angle 0$ pu.

$S_1 = 2.182940 + j1.408861$ pu

$P_1 - jQ_1 = V_1^* (Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3)$

~~$= 1.05\{(20 - j50).1.05 + (-10 + j20).$~~

$P_1 - jQ_1 = V_1^* (Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3)$

$= 1.05\{(20 - j50).1.05 + (-10 + j20).0.971685 \angle -2.69512$

$+ (-10 + j30).1.04 \angle -0.497488\}$

$P_1 - jQ_1 = 2.182940 - j1.408861$ pu
$= 218.29$ MW $+ j140.89$ MVAr.

Line flows and line losses:

Line currents:

$I_{12} = y_{12}(V_1 - V_2) = (10 - j20)(1.05\angle 0 - 0.971685\angle -2.695123)$
$= 1.707698 - j1.130896$ pu

$I_{21} = -I_{12} = -1.707698 + j1.130896$ pu

$I_{13} = y_{13}(V_1 - V_3) = (10 - j30)(1.05\angle 0 - 1.04\angle -0.497488)$
$= 0.371292 - j0.210876$ pu

$I_{31} = -I_{13} = -0.371292 + j0.210876$ pu

$I_{23} = y_{23}(V_2 - V_3) = (16 - j32)(0.971685\angle -2.695123 - 1.04\angle -0.497488)$
$= -2.282730 + j1.632659$ pu

$I_{32} = -I_{23} = 2.282730 - j1.632659$ pu

Line flows (complex power)

$S_{12} = V_1 \cdot I_{12}^* = 1.05\angle 0 \times (1.707698 + j1.130896)$
$= 1.793083 + j1.187441$ pu
$= 179.31$ MW $+ j118.74$ MVAr (100 MVA base)

$S_{21} = V_2 I_{21}^* = 0.971685\angle -2.695123 \times (-1.707698 - j1.130896)$
$= -1.709180 - j1.019634$ pu
$= -170.92$ MW $- j101.96$ MVAr

$S_{13} = V_1 \cdot I_{13}^* = 1.05\angle 0 \times (0.371292 + j0.210876)$
$= 0.389857 + j0.221420$ pu
$= 38.99$ MW $+ j22.14$ MVAr

$S_{31} = V_3 \cdot I_{31}^* = 1.04\angle{-0.497488} \times (-0.371292 - j0.210876)$
$= -0.388033 - j0.215950$ pu
$= -38.80$ MW $- j21.595$ MVAr

$S_{23} = V_2 \cdot I_{23}^* = 0.971685\angle{-2.695123} \times (-2.282730 - j1.632659)$
$= -2.290237 - j1.480378$ pu
$= -229.02$ MW $- j148.04$ MVAr

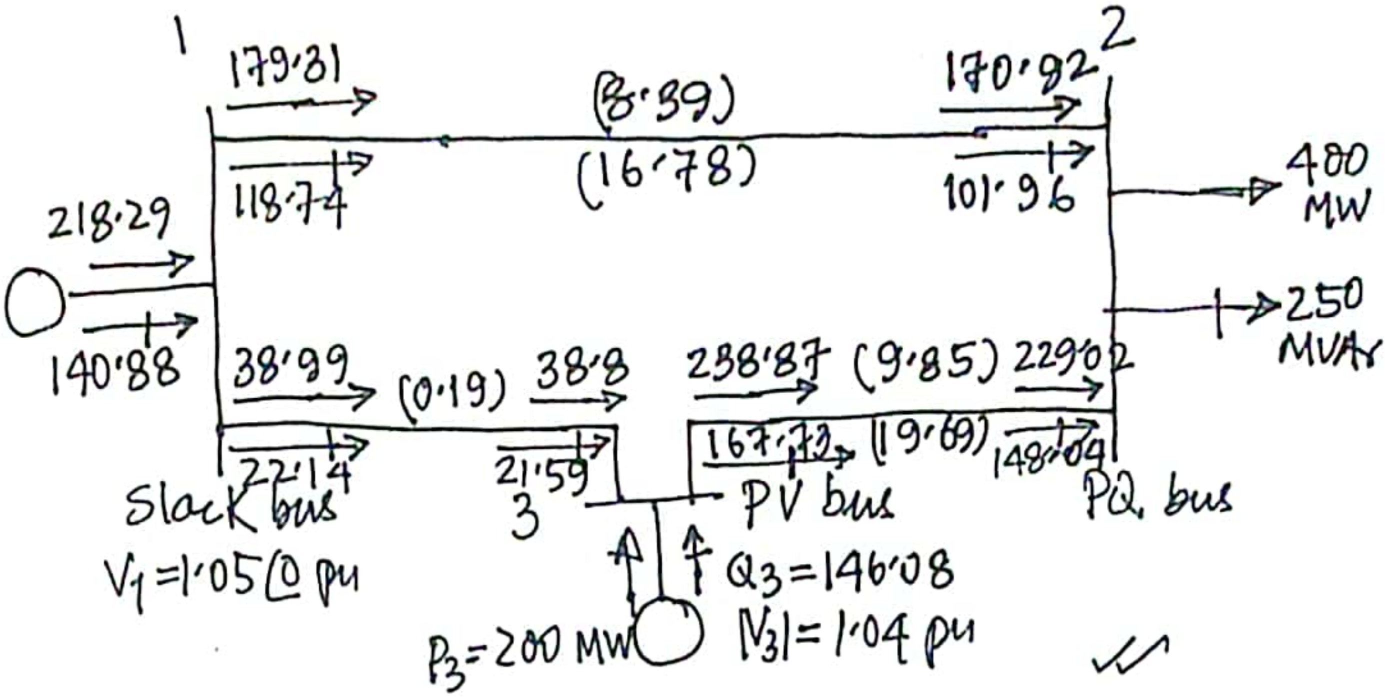
$S_{32} = V_3 \cdot I_{32}^* = 1.04\angle{-0.497488} \times (2.282730 + j1.632659)$
$= 2.388693 + j1.677288$ pu
$= 238.87$ MW $+ j167.73$ MVAr

Line losses are:

$S_{L,12} = S_{12} + S_{21} = 179.31$ MW $+ j118.74$ MVAr $- 170.92$ MW $- j101.96$ MVAr
$= 8.39$ MW $+ j16.78$ MVAr

$S_{L,13} = S_{13} + S_{31} = 38.99$ MW $+ j22.14$ MVAr $- 38.8$ MW $- j21.595$ MVAr
$= 0.19$ MW $+ j0.545$ MVAr

$S_{L,23} = S_{23} + S_{32} = -229.02$ MW $- j148.04$ MVAr $+ 238.87$ MW $+ j167.73$ MVAr
$= 9.85$ MW $+ j19.69$ MVAr.



Power flow diagram