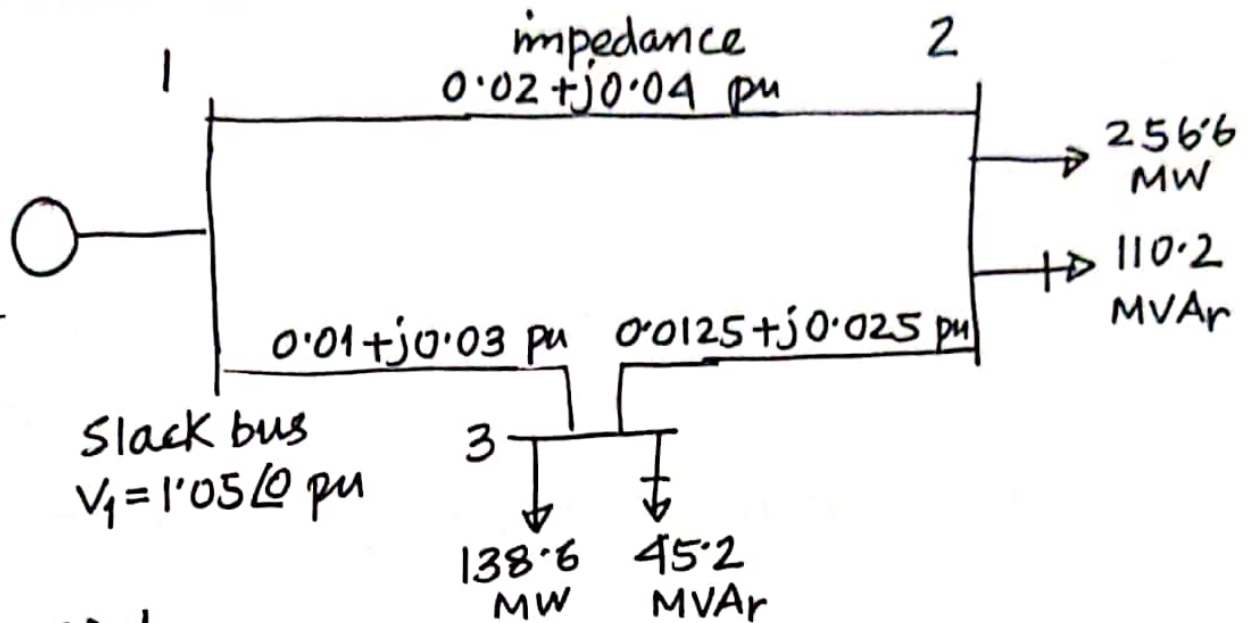


Example 6.7. 100 MVA base



Find

- (a) Phasor values of voltage at load bus 2 and 3 (P-Q buses) using Gauss-Seidel method (perform 2 iterations)
- (b) Slack bus real and reactive power. ← find
- (c) Determine line flow and line losses. Also construct a power flow diagram.

Solution: (a) Line admittances are

$$y_{12} = y_{21} = \frac{1}{0.02 + j0.04} = 10 - j20$$

$$y_{13} = y_{31} = \frac{1}{0.01 + j0.03} = 10 - j30$$

$$y_{23} = y_{32} = \frac{1}{0.0125 + j0.025} = 16 - j32$$

Y<sub>bus</sub> elements.

$$Y_{11} = y_{12} + y_{13} = 20 - j50$$

$$Y_{22} = y_{21} + y_{23} = 26 - j52$$

$$Y_{33} = y_{31} + y_{32} = 26 - j62$$

$$Y_{12} = Y_{21} = -y_{12} = -10 + j20$$

$$Y_{13} = Y_{31} = -y_{13} = -10 + j30$$

$$Y_{23} = Y_{32} = -y_{23} = -16 + j32$$

Bus 2 and 3 are load bus (P-Q bus)

At P-Q buses, The complex power in per-unit  
(on 100MVA base)

$$S_2^{sch} = -\frac{(P_2 + jQ_2)^{sch}}{100} = -\frac{256.6 + j110.2}{100} = -2.566 - j1.102 \text{ pu}$$

$$S_3^{sch} = -\frac{(P_3 + jQ_3)^{sch}}{100} = -\frac{138.6 + j45.2}{100} = -1.386 - j0.452 \text{ pu}$$

We know, the Gauss-Seidel power flow equation

$$V_i^{k+1} = \frac{1}{Y_{ii}} \left\{ \frac{(P_i - jQ_i)^{sch}}{V_i^{*k}} - \sum_{j=1, j \neq i}^n Y_{ij} V_j^k \right\}$$

Let initial estimate -  $V_2^0 = 1.0 \angle 0$  and  $V_3^0 = 1.0 \angle 0$   
given -  $V_1 = 1.05 \angle 0$  (slack bus)

Iteration 1:  $k=0$

$$\begin{aligned} i=2 \Rightarrow \text{Bus 2} \Rightarrow V_2^1 &= \frac{1}{Y_{22}} \left\{ \frac{(P_2 - jQ_2)^{sch}}{V_2^{*0}} - (Y_{21} V_1 + Y_{23} V_3^0) \right\} \\ &= \frac{1}{26 - j52} \left[ \frac{-2.566 + j1.102}{1.0 \angle 0} - \left\{ (-10 + j20) \times 1.05 \angle 0 \right. \right. \\ &\quad \left. \left. + (-16 + j32) \times 1.0 \angle 0 \right\} \right] \\ &= 0.9825 - j0.031 = 0.983 \angle -1.8071 \end{aligned}$$

$$\begin{aligned} i=3 \Rightarrow \text{Bus 3} \Rightarrow V_3^1 &= \frac{1}{Y_{33}} \left\{ \frac{(P_3 - jQ_3)^{sch}}{V_3^{*0}} - (Y_{31} V_1 + Y_{32} V_2^1) \right\} \\ &= \frac{1}{26 - j62} \left[ \frac{-1.386 + j0.452}{1.0 \angle 0} - \left\{ (-10 + j30) \times 1.05 \angle 0 \right. \right. \\ &\quad \left. \left. + (-16 + j32) \times (0.983 \angle -1.8071) \right\} \right] \\ &= 1.0011 - j0.0353 = 1.0017 \angle -2.0172 \end{aligned}$$

Iteration 2: K=1

$$V_2^2 = \frac{1}{Y_{22}} \left\{ \frac{(P_2 - jQ_2)^{sch}}{V_2^{*1}} - (Y_{21}V_1 + Y_{23}V_3^1) \right\}$$

$$= \frac{1}{26 - j52} \left[ \frac{-2.566 + j1.102}{0.983 / 1.8071} - \left\{ (-10 + j20) \times 1.05 \angle 0 + (-16 + j32) \times 1.0017 / -2.0172 \right\} \right]$$

$$= 0.9816 - j0.0520 = 0.983 / -3.0348$$

$$V_3^2 = \frac{1}{Y_{33}} \left\{ \frac{(P_3 - jQ_3)^{sch}}{V_3^{*1}} - (Y_{31}V_1 + Y_{32}V_2^2) \right\}$$

$$= \frac{1}{26 - j62} \left[ \frac{-1.386 + j0.452}{1.0017 / 2.0172} - \left\{ (-10 + j30) \times 1.05 \angle 0 + (-16 + j32) \times 0.983 / -3.0348 \right\} \right]$$

$$= 1.0008 - j0.0459 = 1.0019 / -2.6275$$

The approximate values of phasor voltages are

$$V_2 = 0.983 / -3.0348 \text{ pu}$$

$$V_3 = 1.0019 / -2.6275 \text{ pu} \checkmark$$

Actual after 7 iter..
$V_2 = 0.98183 / -3.5035$
$V_3 = 1.00125 / -2.8624$

(b) The complex power at slack bus

$$P_1 - jQ_1 = V_1^* \{ Y_{11}V_1 + (Y_{12}V_2 + Y_{13}V_3) \}$$

$$= (1.05 \angle 0) \times \left\{ (20 - j50) \times 1.05 \angle 0 + \left\{ (-10 + j20) \times 0.983 / -3.0348 + (-10 + j30) \times 1.0019 / -2.6275 \right\} \right\}$$

$$P_1 - jQ_1 = 3.7738 - j1.9556 \text{ pu}$$

$$\text{Real power, } P_1 = 3.7738 \text{ pu} = 3.7738 \times 100 = 377.38 \text{ MW}$$

$$\text{Reactive Power, } Q_1 = 1.9556 \text{ pu} = 1.9556 \times 100 = 195.56 \text{ MVAR}$$

(c) The line currents are -

$$I_{12} = Y_{12}(V_1 - V_2) = (10 - j20)(1.05 \angle 0 - 0.983 \angle -3.0348)$$

$$= 1.7246 - j0.8471$$

$$I_{21} = -I_{12} = -1.7246 + j0.8471$$

$$I_{13} = Y_{13}(V_1 - V_3) = (10 - j30)(1.05 \angle 0 - 1.0019 \angle -2.6275)$$

$$= 1.8694 - j1.0153$$

$$I_{31} = -I_{13} = -1.8694 + j1.0153$$

$$I_{23} = Y_{23}(V_2 - V_3) = (16 - j32)(0.983 \angle -3.0348 - 1.0019 \angle -2.6275)$$

$$= -0.5032 + j0.5174$$

$$I_{32} = -I_{23} = 0.5032 - j0.5174$$

Line flows are

$$S_{12} = V_1 I_{12}^* = 1.05 \angle 0 \times (1.7246 + j0.8471) = 1.8108 + j0.8895 \text{ pu}$$

$$= 181.08 \text{ MW} + j88.95 \text{ MVAR}$$

$$S_{21} = V_2 I_{21}^* = 0.983 \angle -3.0348 \times (-1.7246 - j0.8471)$$

$$= -1.737 - j0.7418 \text{ pu} = -173.7 \text{ MW} - j74.18 \text{ MVAR}$$

$$S_{13} = V_1 I_{13}^* = 1.05 \angle 0 (1.8694 + j1.0153) = 1.9629 + j1.1606 \text{ pu}$$

$$= 196.29 \text{ MW} + 116.06 \text{ MVAR}$$

$$S_{31} = V_3 I_{31}^* = 1.0019 \angle -2.6275 \times (-1.8694 - j1.0153)$$

$$= -1.9176 - j0.9303 \text{ pu} = 191.76 \text{ MW} - 93.03 \text{ MVAR}$$

$$S_{23} = V_2 I_{23}^* = 0.983 \angle -3.0348 \times (-0.5023 - j0.5174)$$

$$= -0.52 - j0.4817 \text{ pu} = -52 \text{ MW} - j48.17 \text{ MVAR}$$

$$S_{32} = V_3 I_{32}^* = 1.0019 \angle -2.6275 \times (0.5023 + j0.5174)$$

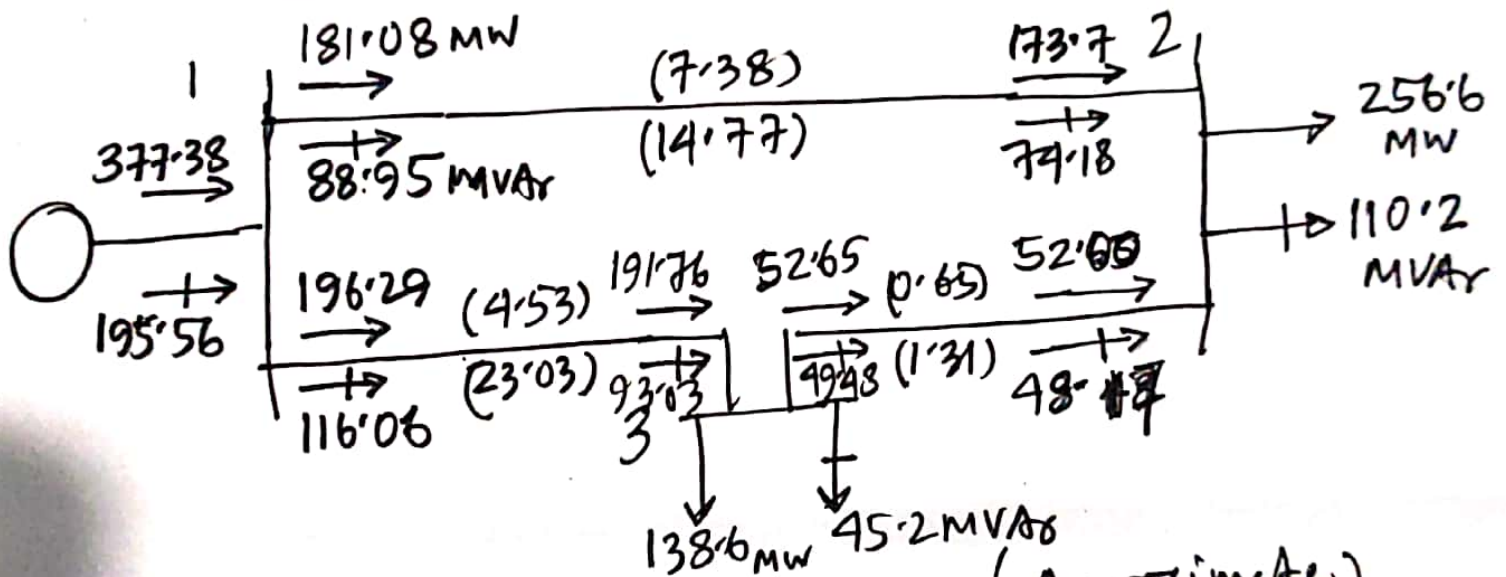
$$= 0.5265 + j0.4948 \text{ pu} = 52.65 \text{ MW} + j49.48 \text{ MVAR}$$

Line losses are

$$S_{L,12} = S_{12} + S_{21} = 181.08 \text{ MW} + j88.95 \text{ MVAR} - 173.7 \text{ MW} - j74.18 \text{ MVAR} = 7.38 \text{ MW} + j14.77 \text{ MVAR}$$

$$S_{L,13} = S_{13} + S_{31} = 4.53 \text{ MW} + j23.03 \text{ MVAR}$$

$$S_{L,23} = S_{23} + S_{32} = 0.65 \text{ MW} + j1.31 \text{ MVAR}$$



Power Flow diagram (Approximate)