6.4 POWER FLOW SOLUTION

Power flow studies, commonly known as *load flow*, form an important part of power system analysis. They are necessary for planning, economic scheduling, and control of an existing system as well as planning its future expansion. The problem consists of determining the magnitudes and phase angle of voltages at each bus and active and reactive power flow in each line.

In solving a power flow problem, the system is assumed to be operating under balanced conditions and a single-phase model is used. Four quantities are associated with each bus. These are voltage magnitude $|V|$, phase angle δ, real power P, and reactive power Q. The system buses are generally classified into three types.

Slack bus One bus, known as *slack* or *swing bus*, is taken as reference where the magnitude and phase angle of the voltage are specified. This bus makes up the difference between the scheduled loads and generated power that are caused by the losses in the network.

Load buses At these buses the active and reactive powers are specified. The magnitude and the phase angle of the bus voltages are unknown. These buses are called P-Q buses.

Regulated buses These buses are the *generator buses*. They are also known as *voltage-controlled buses*. At these buses, the real power and voltage magnitude are specified. The phase angles of the voltages and the reactive power are to be determined. The limits on the value of the reactive power are also specified. These buses are called P-V buses.

6.4.1 POWER FLOW EQUATION

Consider a typical bus of a power system network as shown in Figure 6.7. Transmission lines are represented by their equivalent π models where impedances have been converted to per unit admittances on a common MVA base.

Application of KCL to this bus results in

$$\begin{aligned} I_i &= y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \cdots + y_{in}(V_i - V_n) \\ &= (y_{i0} + y_{i1} + y_{i2} + \cdots + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - \cdots - y_{in}V_n \end{aligned} \quad (6.23)$$

or

$$I_i = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_j \qquad j \neq i \qquad (6.24)$$

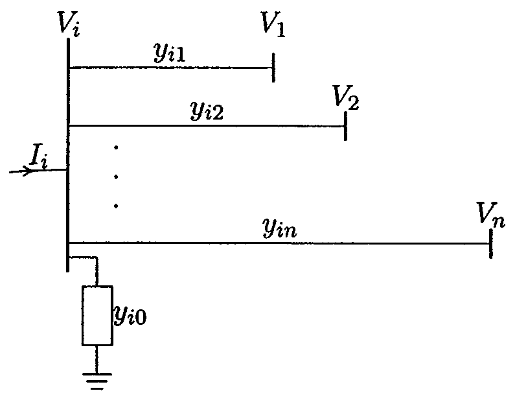

FIGURE 6.7
A typical bus of the power system.

The real and reactive power at bus i is

$$P_i + jQ_i = V_i I_i^* \tag{6.25}$$

or

$$I_i = \frac{P_i - jQ_i}{V_i^*} \tag{6.26}$$

Substituting for I_i in (6.24) yields

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j \qquad j \neq i \tag{6.27}$$

From the above relation, the mathematical formulation of the power flow problem results in a system of algebraic nonlinear equations which must be solved by iterative techniques.

6.5 GAUSS-SEIDEL POWER FLOW SOLUTION

In the power flow study, it is necessary to solve the set of nonlinear equations represented by (6.27) for two unknown variables at each node. In the Gauss-Seidel method (6.27) is solved for V_i, and the iterative sequence becomes

$$V_i^{(k+1)} = \frac{\frac{P_i^{sch} - jQ_i^{sch}}{V_i^{*(k)}} + \sum y_{ij} V_j^{(k)}}{\sum y_{ij}} \qquad j \neq i \tag{6.28}$$

where y_{ij} shown in lowercase letters is the actual admittance in per unit. P_i^{sch} and Q_i^{sch} are the net real and reactive powers expressed in per unit. In writing the KCL, current entering bus i was assumed positive. Thus, for buses where real and reactive powers are injected into the bus, such as generator buses, P_i^{sch} and Q_i^{sch} have positive values. For load buses where real and reactive powers are flowing away from the bus, P_i^{sch} and Q_i^{sch} have negative values. If (6.27) is solved for P_i and Q_i, we have

$$P_i^{(k+1)} = \Re\{V_i^{*(k)}[V_i^{(k)}\sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_j^{(k)}]\} \qquad j \neq i \qquad (6.29)$$

$$Q_i^{(k+1)} = -\Im\{V_i^{*(k)}[V_i^{(k)}\sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_j^{(k)}]\} \qquad j \neq i \qquad (6.30)$$

The power flow equation is usually expressed in terms of the elements of the bus admittance matrix. Since the off-diagonal elements of the bus admittance matrix Y_{bus}, shown by uppercase letters, are $Y_{ij} = -y_{ij}$, and the diagonal elements are $Y_{ii} = \sum y_{ij}$, (6.28) becomes

$$V_i^{(k+1)} = \frac{\frac{P_i^{sch} - jQ_i^{sch}}{V_i^{*(k)}} - \sum_{j \neq i} Y_{ij}V_j^{(k)}}{Y_{ii}} \qquad (6.31)$$

and

$$P_i^{(k+1)} = \Re\{V_i^{*(k)}[V_i^{(k)}Y_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij}V_j^{(k)}]\} \quad j \neq i \qquad (6.32)$$

$$Q_i^{(k+1)} = -\Im\{V_i^{*(k)}[V_i^{(k)}Y_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij}V_j^{(k)}]\} \quad j \neq i \qquad (6.33)$$

Y_{ii} includes the admittance to ground of line charging susceptance and any other fixed admittance to ground. In Section 6.7, a model is presented for transformers containing off-nominal ratio, which includes the effect of transformer tap setting.

Since both components of voltage are specified for the slack bus, there are $2(n-1)$ equations which must be solved by an iterative method. Under normal operating conditions, the voltage magnitude of buses are in the neighborhood of 1.0 per unit or close to the voltage magnitude of the slack bus. Voltage magnitude at load buses are somewhat lower than the slack bus value, depending on the reactive power demand, whereas the scheduled voltage at the generator buses are somewhat higher. Also, the phase angle of the load buses are below the reference angle in accordance to the real power demand, whereas the phase angle of the generator buses may be above the reference value depending on the amount of real power flowing into the bus. Thus, for the Gauss-Seidel method, an initial voltage estimate of $1.0 + j0.0$ for unknown voltages is satisfactory, and the converged solution correlates with the actual operating states.

For P-Q buses, the real and reactive powers P_i^{sch} and Q_i^{sch} are known. Starting with an initial estimate, (6.31) is solved for the real and imaginary components of voltage. For the voltage-controlled buses (P-V buses) where P_i^{sch} and $|V_i|$ are specified, first (6.33) is solved for $Q_i^{(k+1)}$, and then is used in (6.31) to solve for $V_i^{(k+1)}$. However, since $|V_i|$ is specified, only the imaginary part of $V_i^{(k+1)}$ is retained, and its real part is selected in order to satisfy

$$(e_i^{(k+1)})^2 + (f_i^{(k+1)})^2 = |V_i|^2 \tag{6.34}$$

or

$$e_i^{(k+1)} = \sqrt{|V_i|^2 - (f_i^{(k+1)})^2} \tag{6.35}$$

where $e_i^{(k+1)}$ and $f_i^{(k+1)}$ are the real and imaginary components of the voltage $V_i^{(k+1)}$ in the iterative sequence.

The rate of convergence is increased by applying an acceleration factor to the approximate solution obtained from each iteration.

$$V_i^{(k+1)} = V_i^{(k)} + \alpha(V_{i\,cal}^{(k)} - V_i^{(k)}) \tag{6.36}$$

where α is the acceleration factor. Its value depends upon the system. The range of 1.3 to 1.7 is found to be satisfactory for typical systems.

The updated voltages immediately replace the previous values in the solution of the subsequent equations. The process is continued until changes in the real and imaginary components of bus voltages between successive iterations are within a specified accuracy, i.e.,

$$\begin{aligned} |e_i^{(k+1)} - e_i^{(k)}| &\leq \epsilon \\ |f_i^{(k+1)} - f_i^{(k)}| &\leq \epsilon \end{aligned} \tag{6.37}$$

For the power mismatch to be reasonably small and acceptable, a very tight tolerance must be specified on both components of the voltage. A voltage accuracy in the range of 0.00001 to 0.00005 pu is satisfactory. In practice, the method for determining the completion of a solution is based on an accuracy index set up on the power mismatch. The iteration continues until the magnitude of the largest element in the ΔP and ΔQ columns is less than the specified value. A typical power mismatch accuracy is 0.001 pu

Once a solution is converged, the net real and reactive powers at the slack bus are computed from (6.32) and (6.33).

6.6 LINE FLOWS AND LOSSES

After the iterative solution of bus voltages, the next step is the computation of line flows and line losses. Consider the line connecting the two buses i and j in Figure 6.8. The line current I_{ij}, measured at bus i and defined positive in the direction

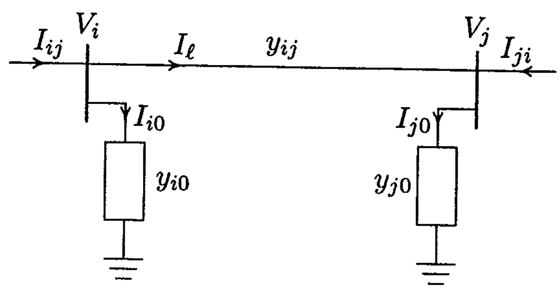

FIGURE 6.8
Transmission line model for calculating line flows.

$i \rightarrow j$ is given by

$$I_{ij} = I_\ell + I_{i0} = y_{ij}(V_i - V_j) + y_{i0}V_i \tag{6.38}$$

Similarly, the line current I_{ji} measured at bus j and defined positive in the direction $j \rightarrow i$ is given by

$$I_{ji} = -I_\ell + I_{j0} = y_{ij}(V_j - V_i) + y_{j0}V_j \tag{6.39}$$

The complex powers S_{ij} from bus i to j and S_{ji} from bus j to i are

$$S_{ij} = V_i I_{ij}^* \tag{6.40}$$
$$S_{ji} = V_j I_{ji}^* \tag{6.41}$$

The power loss in line $i - j$ is the algebraic sum of the power flows determined from (6.40) and (6.41), i.e.,

$$S_{L\,ij} = S_{ij} + S_{ji} \tag{6.42}$$

The power flow solution by the Gauss-Seidel method is demonstrated in the following two examples.

Example 6.7

Figure 6.9 shows the one-line diagram of a simple three-bus power system with generation at bus 1. The magnitude of voltage at bus 1 is adjusted to 1.05 per unit. The scheduled loads at buses 2 and 3 are as marked on the diagram. Line impedances are marked in per unit on a 100-MVA base and the line charging susceptances are neglected.

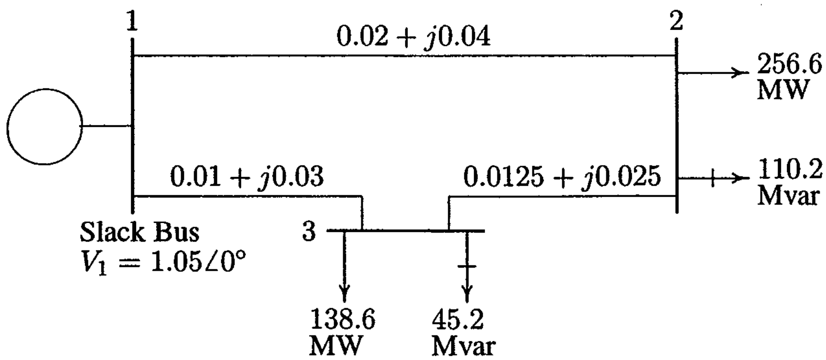

FIGURE 6.9
One-line diagram of Example 6.7 (impedances in pu on 100-MVA base).

(a) Using the Gauss-Seidel method, determine the phasor values of the voltage at the load buses 2 and 3 (P-Q buses) accurate to four decimal places.
(b) Find the slack bus real and reactive power.
(c) Determine the line flows and line losses. Construct a power flow diagram showing the direction of line flow.

(a) Line impedances are converted to admittances

$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j20$$

Similarly, $y_{13} = 10 - j30$ and $y_{23} = 16 - j32$. The admittances are marked on the network shown in Figure 6.10.
At the P-Q buses, the complex loads expressed in per units are

$$S_2^{sch} = -\frac{(256.6 + j110.2)}{100} = -2.566 - j1.102 \quad \text{pu}$$

$$S_3^{sch} = -\frac{(138.6 + j45.2)}{100} = -1.386 - j0.452 \quad \text{pu}$$

Since the actual admittances are readily available in Figure 6.10, for hand calculation, we use (6.28). Bus 1 is taken as reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.0 + j0.0$, V_2 and V_3 are computed from (6.28) as follows

$$V_2^{(1)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}}$$

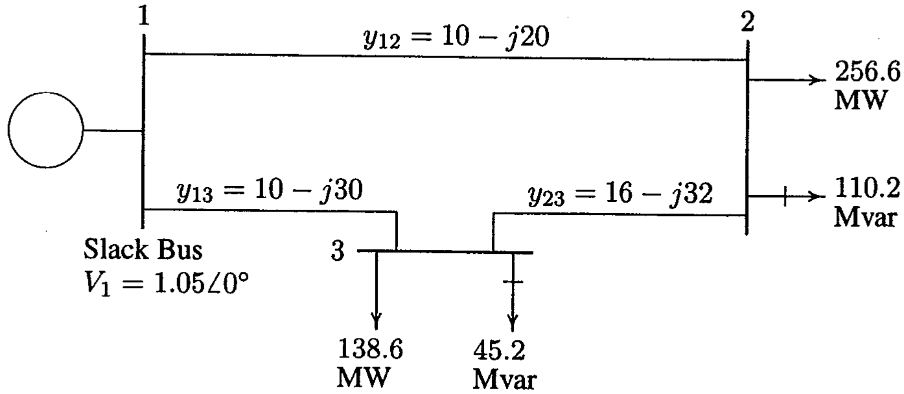

FIGURE 6.10
One-line diagram of Example 6.7 (admittances in pu on 100-MVA base).

$$= \frac{\frac{-2.566+j1.102}{1.0-j0} + (10-j20)(1.05+j0) + (16-j32)(1.0+j0)}{(26-j52)}$$

$$= 0.9825 - j0.0310$$

and

$$V_3^{(1)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$

$$= \frac{\frac{-1.386+j0.452}{1-j0} + (10-j30)(1.05+j0) + (16-j32)(0.9825-j0.0310)}{(26-j62)}$$

$$= 1.0011 - j0.0353$$

For the second iteration we have

$$V_2^{(2)} = \frac{\frac{-2.566+j1.102}{0.9825+j0.0310} + (10-j20)(1.05+j0) + (16-j32)(1.0011-j0.0353)}{(26-j52)}$$

$$= 0.9816 - j0.0520$$

and

$$V_3^{(2)} = \frac{\frac{-1.386+j0.452}{1.0011+j0.0353} + (10-j30)(1.05+j0) + (16-j32)(0.9816-j0.052)}{(26-j62)}$$

$$= 1.0008 - j0.0459$$

The process is continued and a solution is converged with an accuracy of 5×10^{-5} per unit in seven iterations as given below.

$$V_2^{(3)} = 0.9808 - j0.0578 \qquad V_3^{(3)} = 1.0004 - j0.0488$$

$$V_2^{(4)} = 0.9803 - j0.0594 \qquad V_3^{(4)} = 1.0002 - j0.0497$$
$$V_2^{(5)} = 0.9801 - j0.0598 \qquad V_3^{(5)} = 1.0001 - j0.0499$$
$$V_2^{(6)} = 0.9801 - j0.0599 \qquad V_3^{(6)} = 1.0000 - j0.0500$$
$$V_2^{(7)} = 0.9800 - j0.0600 \qquad V_3^{(7)} = 1.0000 - j0.0500$$

The final solution is

$$V_2 = 0.9800 - j0.0600 = 0.98183\angle{-3.5035°} \quad \text{pu}$$
$$V_3 = 1.0000 - j0.0500 = 1.00125\angle{-2.8624°} \quad \text{pu}$$

(b) With the knowledge of all bus voltages, the slack bus power is obtained from (6.27)

$$\begin{aligned} P_1 - jQ_1 &= V_1^*[V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)] \\ &= 1.05[1.05(20 - j50) - (10 - j20)(0.98 - j.06) - \\ &\quad (10 - j30)(1.0 - j0.05)] \\ &= 4.095 - j1.890 \end{aligned}$$

or the slack bus real and reactive powers are $P_1 = 4.095$ pu $= 409.5$ MW and $Q_1 = 1.890$ pu $= 189$ Mvar.

(c) To find the line flows, first the line currents are computed. With line charging capacitors neglected, the line currents are

$$I_{12} = y_{12}(V_1 - V_2) = (10 - j20)[(1.05 + j0) - (0.98 - j0.06)] = 1.9 - j0.8$$
$$I_{21} = -I_{12} = -1.9 + j0.8$$
$$I_{13} = y_{13}(V_1 - V_3) = (10 - j30)[(1.05 + j0) - (1.0 - j0.05)] = 2.0 - j1.0$$
$$I_{31} = -I_{13} = -2.0 + j1.0$$
$$I_{23} = y_{23}(V_2 - V_3) = (16 - j32)[(0.98 - j0.06) - (1 - j0.05)] = -.64 + j.48$$
$$I_{32} = -I_{23} = 0.64 - j0.48$$

The line flows are

$$\begin{aligned} S_{12} = V_1 I_{12}^* &= (1.05 + j0.0)(1.9 + j0.8) = 1.995 + j0.84 \ \text{pu} \\ &= 199.5 \ \text{MW} + j84.0 \ \text{Mvar} \\ S_{21} = V_2 I_{21}^* &= (0.98 - j0.06)(-1.9 - j0.8) = -1.91 - j0.67 \ \text{pu} \\ &= -191.0 \ \text{MW} - j67.0 \ \text{Mvar} \\ S_{13} = V_1 I_{13}^* &= (1.05 + j0.0)(2.0 + j1.0) = 2.1 + j1.05 \ \text{pu} \\ &= 210.0 \ \text{MW} + j105.0 \ \text{Mvar} \end{aligned}$$

$$S_{31} = V_3 I_{31}^* = (1.0 - j0.05)(-2.0 - j1.0) = -2.05 - j0.90 \text{ pu}$$
$$= -205.0 \text{ MW} - j90.0 \text{ Mvar}$$
$$S_{23} = V_2 I_{23}^* = (0.98 - j0.06)(-0.656 + j0.48) = -0.656 - j0.432 \text{ pu}$$
$$= -65.6 \text{ MW} - j43.2 \text{ Mvar}$$
$$S_{32} = V_3 I_{32}^* = (1.0 - j0.05)(0.64 + j0.48) = 0.664 + j0.448 \text{ pu}$$
$$= 66.4 \text{ MW} + j44.8 \text{ Mvar}$$

and the line losses are

$$S_{L\,12} = S_{12} + S_{21} = 8.5 \text{ MW} + j17.0 \text{ Mvar}$$
$$S_{L\,13} = S_{13} + S_{31} = 5.0 \text{ MW} + j15.0 \text{ Mvar}$$
$$S_{L\,23} = S_{23} + S_{32} = 0.8 \text{ MW} + j1.60 \text{ Mvar}$$

The power flow diagram is shown in Figure 6.11, where real power direction is indicated by $\rightarrow$ and the reactive power direction is indicated by $\mapsto$. The values within parentheses are the real and reactive losses in the line.

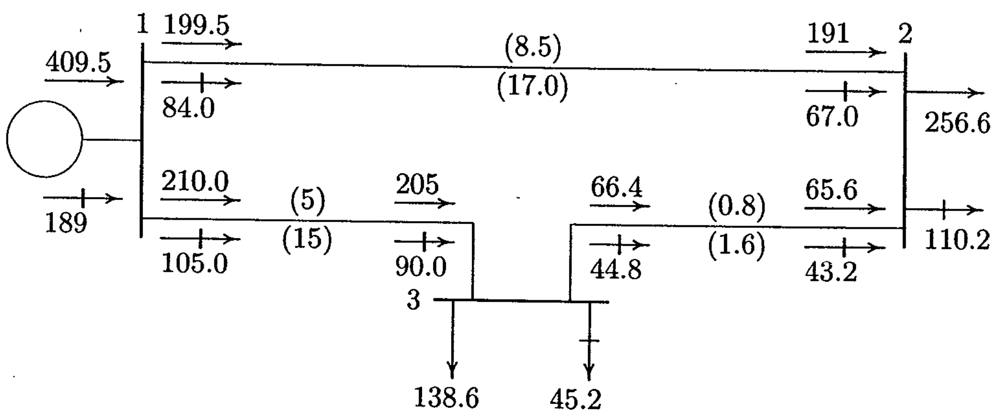

FIGURE 6.11
Power flow diagram of Example 6.7 (powers in MW and Mvar).

Example 6.8

Figure 6.12 shows the one-line diagram of a simple three-bus power system with generators at buses 1 and 3. The magnitude of voltage at bus 1 is adjusted to 1.05 pu. Voltage magnitude at bus 3 is fixed at 1.04 pu with a real power generation of 200 MW. A load consisting of 400 MW and 250 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100 MVA base, and the line charging susceptances are neglected. Obtain the power flow solution by the Gauss-Seidel method including line flows and line losses.

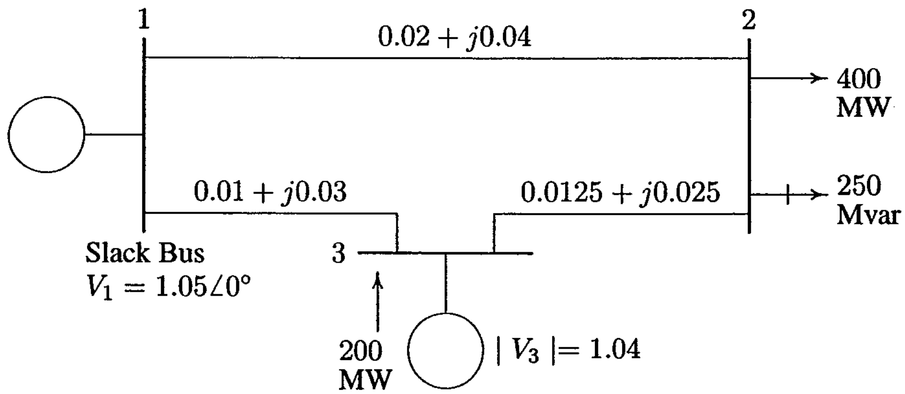

FIGURE 6.12
One-line diagram of Example 6.8 (impedances in pu on 100-MVA base).

Line impedances converted to admittances are $y_{12} = 10 - j20$, $y_{13} = 10 - j30$ and $y_{23} = 16 - j32$. The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5 \quad \text{pu}$$

$$P_3^{sch} = \frac{200}{100} = 2.0 \quad \text{pu}$$

Bus 1 is taken as the reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.04 + j0.0$, V_2 and V_3 are computed from (6.28).

$$\begin{aligned} V_2^{(1)} &= \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}} \\ &= \frac{\frac{-4.0 + j2.5}{1.0 - j0} + (10 - j20)(1.05 + j0) + (16 - j32)(1.04 + j0)}{(26 - j52)} \\ &= 0.97462 - j0.042307 \end{aligned}$$

Bus 3 is a regulated bus where voltage magnitude and real power are specified. For the voltage-controlled bus, first the reactive power is computed from (6.30)

$$\begin{aligned} Q_3^{(1)} &= -\Im\{V_3^{*(0)}[V_3^{(0)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(1)}]\} \\ &= -\Im\{(1.04 - j0)[(1.04 + j0)(26 - j62) - (10 - j30)(1.05 + j0) - \\ &\quad (16 - j32)(0.97462 - j0.042307)]\} \\ &= 1.16 \end{aligned}$$

The value of $Q_3^{(1)}$ is used as Q_3^{sch} for the computation of voltage at bus 3. The complex voltage at bus 3, denoted by $V_{c3}^{(1)}$, is calculated

$$V_{c3}^{(1)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$
$$= \frac{\frac{2.0 - j1.16}{1.04 - j0} + (10 - j30)(1.05 + j0) + (16 - j32)(0.97462 - j0.042307)}{(26 - j62)}$$
$$= 1.03783 - j0.005170$$

Since $|V_3|$ is held constant at 1.04 pu, only the imaginary part of $V_{c3}^{(1)}$ is retained, i.e, $f_3^{(1)} = -0.005170$, and its real part is obtained from

$$e_3^{(1)} = \sqrt{(1.04)^2 - (0.005170)^2} = 1.039987$$

Thus

$$V_3^{(1)} = 1.039987 - j0.005170$$

For the second iteration, we have

$$V_2^{(2)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(1)}} + y_{12}V_1 + y_{23}V_3^{(1)}}{y_{12} + y_{23}}$$
$$= \frac{\frac{-4.0 + j2.5}{.97462 + j.042307} + (10 - j20)(1.05) + (16 - j32)(1.039987 + j0.005170)}{(26 - j52)}$$
$$= 0.971057 - j0.043432$$

$$Q_3^{(2)} = -\Im\{V_3^{*(1)}[V_3^{(1)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(2)}]\}$$
$$= -\Im\{(1.039987 + j0.005170)[(1.039987 - j0.005170)(26 - j62) -$$
$$(10 - j30)(1.05 + j0) - (16 - j32)(0.971057 - j0.043432)]\}$$
$$= 1.38796$$

$$V_{c3}^{(2)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(1)}} + y_{13}V_1 + y_{23}V_2^{(2)}}{y_{13} + y_{23}}$$
$$= \frac{\frac{2.0 - j1.38796}{1.039987 + j0.00517} + (10 - j30)(1.05) + (16 - j32)(.971057 - j.043432)}{(26 - j62)}$$
$$= 1.03908 - j0.00730$$

Since $|V_3|$ is held constant at 1.04 pu, only the imaginary part of $V_{c3}^{(2)}$ is retained, i.e, $f_3^{(2)} = -0.00730$, and its real part is obtained from

$$e_3^{(2)} = \sqrt{(1.04)^2 - (0.00730)^2} = 1.039974$$

or

$$V_3^{(2)} = 1.039974 - j0.00730$$

The process is continued and a solution is converged with an accuracy of 5×10^{-5} pu in seven iterations as given below.

$$\begin{array}{lll}
V_2^{(3)} = 0.97073 - j0.04479 & Q_3^{(3)} = 1.42904 & V_3^{(3)} = 1.03996 - j0.00833 \\
V_2^{(4)} = 0.97065 - j0.04533 & Q_3^{(4)} = 1.44833 & V_3^{(4)} = 1.03996 - j0.00873 \\
V_2^{(5)} = 0.97062 - j0.04555 & Q_3^{(5)} = 1.45621 & V_3^{(5)} = 1.03996 - j0.00893 \\
V_2^{(6)} = 0.97061 - j0.04565 & Q_3^{(6)} = 1.45947 & V_3^{(6)} = 1.03996 - j0.00900 \\
V_2^{(7)} = 0.97061 - j0.04569 & Q_3^{(7)} = 1.46082 & V_3^{(7)} = 1.03996 - j0.00903
\end{array}$$

The final solution is

$$V_2 = 0.97168\angle -2.6948^\circ \quad \text{pu}$$

$$\begin{aligned}
S_3 &= 2.0 + j1.4617 \quad \text{pu} \\
V_3 &= 1.04\angle -.498^\circ \quad \text{pu} \\
S_1 &= 2.1842 + j1.4085 \quad \text{pu}
\end{aligned}$$

Line flows and line losses are computed as in Example 6.7, and the results expressed in MW and Mvar are

$$\begin{array}{lll}
S_{12} = 179.36 + j118.734 & S_{21} = -170.97 - j101.947 & S_{L12} = 8.39 + j16.79 \\
S_{13} = 39.06 + j22.118 & S_{31} = -38.88 - j21.569 & S_{L13} = 0.18 + j0.548 \\
S_{23} = -229.03 - j148.05 & S_{32} = 238.88 + j167.746 & S_{L23} = 9.85 + j19.69
\end{array}$$

The power flow diagram is shown in Figure 6.13, where real power direction is indicated by $\rightarrow$ and the reactive power direction is indicated by $\mapsto$. The values within parentheses are the real and reactive losses in the line.

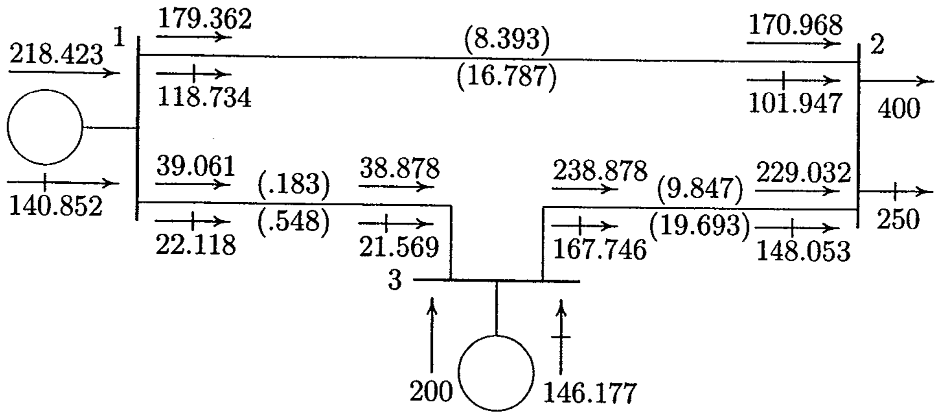

FIGURE 6.13
Power flow diagram of Example 6.8 (powers in MW and Mvar).

6.7 TAP CHANGING TRANSFORMERS

In Section 2.6 it was shown that the flow of real power along a transmission line is determined by the angle difference of the terminal voltages, and the flow of reactive power is determined mainly by the magnitude difference of terminal voltages. Real and reactive powers can be controlled by use of tap changing transformers and regulating transformers.

In a tap changing transformer, when the ratio is at the nominal value, the transformer is represented by a series admittance y_t in per unit. With off-nominal ratio, the per unit admittance is different from both sides of the transformer, and the admittance must be modified to include the effect of the off-nominal ratio. Consider a transformer with admittance y_t in series with an ideal transformer representing the off-nominal tap ratio 1:a as shown in Figure 6.14. y_t is the admittance in per unit based on the nominal turn ratio and a is the per unit off-nominal tap position allowing for small adjustment in voltage of usually ± 10 percent. In the case of phase shifting transformers, a is a complex number. Consider a fictitious bus x between the turn ratio and admittance of the transformer. Since the complex power on either side of the ideal transformer is the same, it follows that if the voltage goes through a positive phase angle shift, the current will go through a negative phase angle shift. Thus, for the assumed direction of currents, we have

$$V_x = \frac{1}{a} V_j \tag{6.43}$$

$$I_i = -a^* I_j \tag{6.44}$$

The current I_i is given by

$$I_i = y_t(V_i - V_x)$$