CHAPTER

10

SYMMETRICAL COMPONENTS AND UNBALANCED FAULT

10.1 INTRODUCTION

Different types of unbalanced faults are the *single line-to-ground fault*, *line-to-line fault*, and *double line-to-ground fault*.

The fault study that was presented in Chapter 9 has considered only three-phase balanced faults, which lends itself to a simple per phase approach. Various methods have been devised for the solution of unbalanced faults. However, since the one-line diagram simplifies the solution of the balanced three-phase problems, the method of symmetrical components that resolves the solution of unbalanced circuit into a solution of a number of balanced circuits is used. In this chapter, the symmetrical components method is discussed. It is then applied to the unbalanced faults, which allows once again the treatment of the problem on a simple per phase basis. Two functions are developed for the symmetrical components transformations. These are **abc2sc**, which provides transformation from phase quantities to symmetrical components, and **sc2abc** for the inverse transformation. In addition, these functions produce plots of unbalanced phasors and their symmetrical components. Finally, unbalanced faults are computed using the concept of symmetrical components. Three functions named **lgfault(zdata0, zbus0, zdata1, zbus1, zdata2, zbus2, V)**, **llfault(zdata1, zbus1, zdata2, zbus2, V)**, and **dlgfault(zdata0, zbus0, zdata1, zbus1, zdata2, zbus2, V)** are developed for the line-to-ground, line-to-line, and the double line-to-ground fault studies.

10.2 FUNDAMENTALS OF SYMMETRICAL COMPONENTS

Symmetrical components allow unbalanced phase quantities such as currents and voltages to be replaced by three separate balanced symmetrical components.

In three-phase system the phase sequence is defined as the order in which they pass through a positive maximum. Consider the phasor representation of a three-phase balanced current shown in Figure 10.1(a).

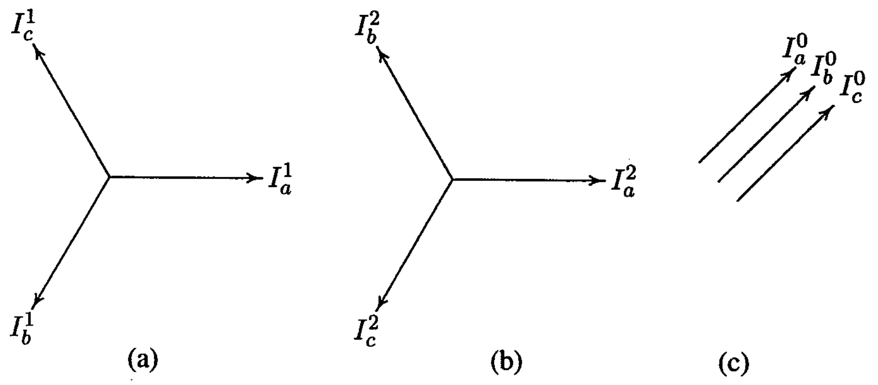

FIGURE 10.1
Representation of symmetrical components.

By convention, the direction of rotation of the phasors is taken to be counterclockwise. The three phasors are written as

$$\begin{aligned} I_a^1 &= I_a^1 \angle 0^\circ &&= I_a^1 \\ I_b^1 &= I_a^1 \angle 240^\circ &&= a^2 I_a^1 \\ I_c^1 &= I_a^1 \angle 120^\circ &&= a I_a^1 \end{aligned} \tag{10.1}$$

where we have defined an operator a that causes a counterclockwise rotation of 120°, such that

$$\begin{aligned} a &= 1\angle 120^\circ = -0.5 + j0.866 \\ a^2 &= 1\angle 240^\circ = -0.5 - j0.866 \\ a^3 &= 1\angle 360^\circ = 1 + j0 \end{aligned} \tag{10.2}$$

From above, it is clear that

$$1 + a + a^2 = 0 \tag{10.3}$$

The order of the phasors is *abc*. This is designated the *positive phase sequence.* When the order is *acb* as in Figure 10.1(b), it is designated the *negative phase* *sequence*. The negative phase sequence quantities are represented as

$$\begin{aligned} I_a^2 &= I_a^2\angle 0^\circ &&= I_a^2 \\ I_b^2 &= I_a^2\angle 120^\circ &&= aI_a^2 \\ I_c^2 &= I_a^2\angle 240^\circ &&= a^2I_a^2 \end{aligned} \tag{10.4}$$

When analyzing certain types of unbalanced faults, it will be found that a third set of balanced phasors must be introduced. These phasors, known as the *zero phase sequence*, are found to be in phase with each other. Zero phase sequence currents, as in Figure 10.1(c), would be designated

$$I_a^0 = I_b^0 = I_c^0 \tag{10.5}$$

The superscripts 1, 2, and 0 are being used to represent positive, negative, and zero-sequence quantities, respectively. In some texts the notation 0, +, − is used instead of 0, 1, 2. The symmetrical components method was introduced by Dr. C. L. Fortescue in 1918. Based on his theory, three-phase unbalanced phasors of a three-phase system can be resolved into three balanced systems of phasors as follows.

1. Positive-sequence components consisting of a set of balanced three-phase components with a phase sequence *abc*.
2. Negative-sequence components consisting of a set of balanced three-phase components with a phase sequence *acb*.
3. Zero-sequence components consisting of three single-phase components, all equal in magnitude but with the same phase angles.

Consider the three-phase unbalanced currents I_a, I_b, and I_c shown in Figure 10.2 (page 405). We are seeking to find the three symmetrical components of the current such that

$$\begin{aligned} I_a &= I_a^0 + I_a^1 + I_a^2 \\ I_b &= I_b^0 + I_b^1 + I_b^2 \\ I_c &= I_c^0 + I_c^1 + I_c^2 \end{aligned} \tag{10.6}$$

According to the definition of the symmetrical components as given by (10.1), (10.4), and (10.5), we can rewrite (10.6) all in terms of phase a components.

$$\begin{aligned} I_a &= I_a^0 + I_a^1 + I_a^2 \\ I_b &= I_a^0 + a^2I_a^1 + aI_a^2 \\ I_c &= I_a^0 + aI_a^1 + a^2I_a^2 \end{aligned} \tag{10.7}$$

or

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} \tag{10.8}$$

We rewrite the above equation in matrix notation as

$$\mathbf{I}^{abc} = \mathbf{A}\ \mathbf{I}_a^{012} \tag{10.9}$$

where $\mathbf{A}$ is known as *symmetrical components transformation matrix* (SCTM) which transforms phasor currents $\mathbf{I}^{abc}$ into component currents $\mathbf{I}_a^{012}$, and is

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{10.10}$$

Solving (10.9) for the symmetrical components of currents, we have

$$\mathbf{I}_a^{012} = \mathbf{A}^{-1}\ \mathbf{I}^{abc} \tag{10.11}$$

The inverse of $\mathbf{A}$ is given by

$$\mathbf{A}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{10.12}$$

From (10.10) and (10.12), we conclude that

$$\mathbf{A}^{-1} = \frac{1}{3}\mathbf{A}^* \tag{10.13}$$

Substituting for $\mathbf{A}^{-1}$ in (10.11), we have

$$\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{10.14}$$

or in component form, the symmetrical components are

$$\begin{aligned} I_a^0 &= \tfrac{1}{3}(I_a + I_b + I_c) \\ I_a^1 &= \tfrac{1}{3}(I_a + aI_b + a^2 I_c) \\ I_a^2 &= \tfrac{1}{3}(I_a + a^2 I_b + aI_c) \end{aligned} \tag{10.15}$$

From (10.15), we note that the zero-sequence component of current is equal to one-third of the sum of the phase currents. Therefore, when the phase currents sum to zero, e.g., in a three-phase system with ungrounded neutral, the zero-sequence current cannot exist. If the neutral of the power system is grounded, zero-sequence current flows between the neutral and the ground.

Similar expressions exist for voltages. Thus the unbalanced phase voltages in terms of the symmetrical components voltages are

$$\begin{aligned} V_a &= V_a^0 + V_a^1 + V_a^2 \\ V_b &= V_a^0 + a^2 V_a^1 + a V_a^2 \\ V_c &= V_a^0 + a V_a^1 + a^2 V_a^2 \end{aligned} \tag{10.16}$$

or in matrix notation

$$\mathbf{V}^{abc} = \mathbf{A}\ \mathbf{V}_a^{012} \tag{10.17}$$

The symmetrical components in terms of the unbalanced voltages are

$$\begin{aligned} V_a^0 &= \tfrac{1}{3}(V_a + V_b + V_c) \\ V_a^1 &= \tfrac{1}{3}(V_a + aV_b + a^2 V_c) \\ V_a^2 &= \tfrac{1}{3}(V_a + a^2 V_b + aV_c) \end{aligned} \tag{10.18}$$

or in matrix notation

$$\mathbf{V}_a^{012} = \mathbf{A}^{-1}\ \mathbf{V}^{abc} \tag{10.19}$$

The apparent power may also be expressed in terms of the symmetrical components. The three-phase complex power is

$$S_{(3\phi)} = \mathbf{V}^{abc^T} \mathbf{I}^{abc*} \tag{10.20}$$

Substituting (10.9) and (10.17) in (10.20), we obtain

$$\begin{aligned} S_{(3\phi)} &= \left(\mathbf{A}\mathbf{V}_a^{012}\right)^T \left(\mathbf{A}\mathbf{I}_a^{012}\right)^* \\ &= \mathbf{V}_a^{012^T} \mathbf{A}^T \mathbf{A}^* \mathbf{I}_a^{012*} \end{aligned} \tag{10.21}$$

Since $\mathbf{A}^T = \mathbf{A}$, then from (10.13), $\mathbf{A}^T\mathbf{A}^* = 3$, and the complex power becomes

$$\begin{aligned} S_{(3\phi)} &= 3\left(\mathbf{V}^{012^T}\mathbf{I}^{012*}\right) \\ &= 3V_a^0 I_a^{0*} + 3V_a^1 I_a^{1*} + 3V_a^2 I_a^{2*} \end{aligned} \tag{10.22}$$

Equation (10.22) shows that the total unbalanced power can be obtained from the sum of the symmetrical component powers. Often the subscript a of the symmetrical components are omitted, e.g., I^0, I^1, and I^2 are understood to refer to phase a.

Transformation from phase quantities to symmetrical components in *MATLAB* is very easy. Once the symmetrical components transformation matrix **A** is defined, its inverse is found using the *MATLAB* function **inv**. However, for quick calculations and graphical demonstration, the following functions are developed for symmetrical components analysis.

sctm The symmetrical components transformation matrix **A** is defined in this script file. Typing **sctm** defines **A**.

phasor(F) This function makes plots of phasors. The variable F may be expressed in an $n \times 1$ array in rectangular complex form or as an $n \times 2$ matrix. In the latter case, the first column is the phasor magnitude and the second column is its phase angle in degree.

F_{012} = **abc2sc**(F_{abc}) This function returns the symmetrical components of a set of unbalanced phasors in rectangular form. F_{abc} may be expressed in a 3×1 array in rectangular complex form or as a 3×2 matrix. In the latter case, the first column is the phasor magnitude and the second column is its phase angle in degree for a, b, and c phases. In addition, the function produces a plot of the unbalanced phasors and its symmetrical components.

F_{abc} = **sc2abc**(F_{012}) This function returns the unbalanced phasor in rectangular form when symmetrical components are specified. F_{012} may be expressed in a 3×1 array in rectangular complex form or as a 3×2 matrix. In the latter case, the first column is the phasor magnitude and the second column is its phase angle in degree for the zero-, positive-, and negative-sequence components, respectively. In addition, the function produces a plot of the unbalanced phasors and its symmetrical components.

Z_{012} = **zabc2sc**(Z_{abc}) This function transforms the phase impedance matrix to the sequence impedance matrix, given by (10.30).

F_p = **rec2pol**(F_r) This function converts the rectangular phasor F_r into polar form F_p.

F_r = **pol2rec**(F_p) This function converts the polar phasor F_p into rectangular form F_r.

Example 10.1

Obtain the symmetrical components of a set of unbalanced currents $I_a = 1.6\angle 25°$, $I_b = 1.0\angle 180°$, and $I_c = 0.9\angle 132°$.

The commands

```
Iabc = [1.6    25
        1.0   180
        0.9   132];
I012 = abc2sc(Iabc);  % Symmetrical components of phase a
I012p= rec2pol(I012)           % Rectangular to polar form
```

result in

```
I012P =
          0.4512    96.4529
          0.9435    -0.0550
          0.6024    22.3157
```

and the plots of the phasors are shown in Figure 10.2.

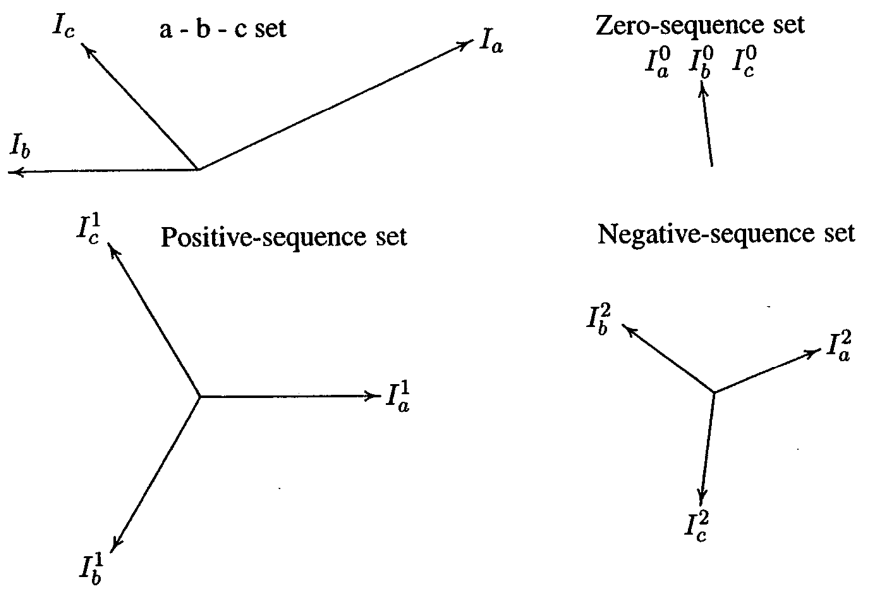

FIGURE 10.2
Resolution of unbalanced phasors into symmetrical components.

Example 10.2

The symmetrical components of a set of unbalanced three-phase voltages are $V_a^0 = 0.6\angle 90°$, $V_a^1 = 1.0\angle 30°$, and $V_a^2 = 0.8\angle -30°$. Obtain the original unbalanced phasors.

The commands

```
V012 = [0.6    90
        1.0    30
        0.8   -30];
Vabc = sc2abc(V012);%Unbalanced phasor to symmetrical comp.
Vabcp= rec2pol(Vabc)             % Rectangular to polar form
```

result in

```
Vabcp =
          1.7088    24.1825
          0.400     90.0000
          1.7088   155.8175
```

and the plots of the phasors are shown in Figure 10.3.

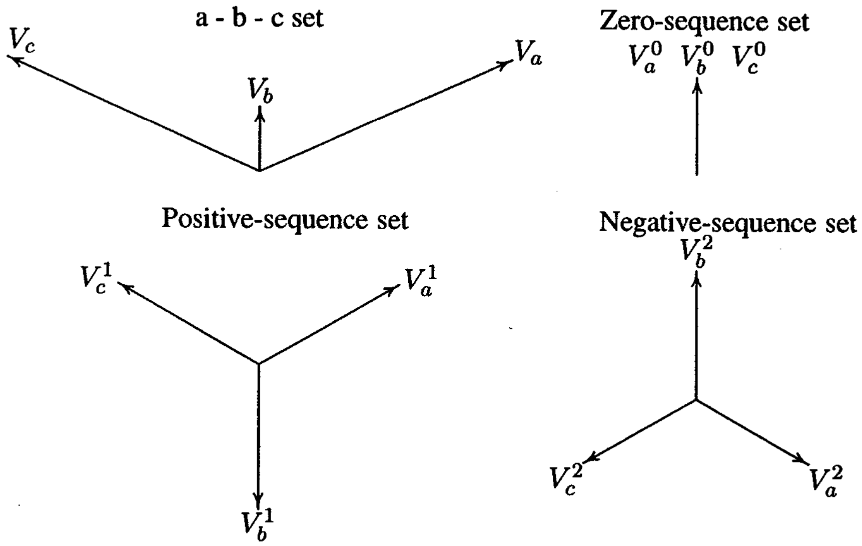

FIGURE 10.3
Transformation of the symmetrical components into phasor components.

10.3 SEQUENCE IMPEDANCES

This is the impedance of an equipment or component to the current of different sequences. The impedance offered to the flow of positive-sequence currents is known as the *positive-sequence impedance* and is denoted by Z^1. The impedance offered to the flow of negative-sequence currents is known as the *negative-sequence impedance*, shown by Z^2. When zero-sequence currents flow, the impedance is called the *zero-sequence impedance*, shown by Z^0. The sequence impedances of transmission lines, generators, and transformers are considered briefly here.

10.3.1 SEQUENCE IMPEDANCES OF Y-CONNECTED LOADS

A three-phase balanced load with self and mutual elements is shown in Figure 10.4. The load neutral is grounded through an impedance Z_n.

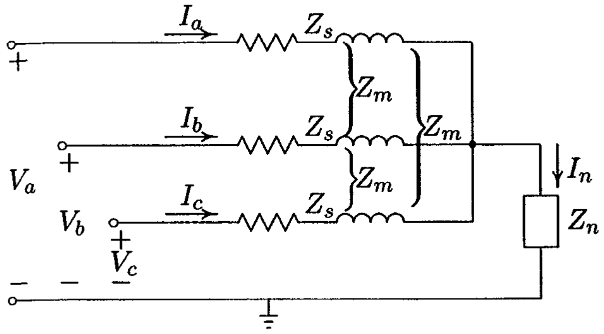

FIGURE 10.4
Balanced Y-connected load.

The line-to-ground voltages are

$$\begin{aligned} V_a &= Z_s I_a + Z_m I_b + Z_m I_c + Z_n I_n \\ V_b &= Z_m I_a + Z_s I_b + Z_m I_c + Z_n I_n \\ V_c &= Z_m I_a + Z_m I_b + Z_s I_c + Z_n I_n \end{aligned} \tag{10.23}$$

From Kirchhoff's current law, we have

$$I_n = I_a + I_b + I_c \tag{10.24}$$

Substituting for I_n from (10.24) into (10.23) and rewriting this equation in matrix form, yields

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_s + Z_n & Z_m + Z_n & Zm + Z_n \\ Z_m + Z_n & Z_s + Z_n & Zm + Z_n \\ Z_m + Z_n & Z_m + Z_n & Zs + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{10.25}$$

or in compact form

$$\mathbf{V}^{abc} = \mathbf{Z}^{abc}\mathbf{I}^{abc} \tag{10.26}$$

where

$$\mathbf{Z}^{abc} = \begin{bmatrix} Z_s + Z_n & Z_m + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_s + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_m + Z_n & Z_s + Z_n \end{bmatrix} \tag{10.27}$$

Writing $\mathbf{V}^{abc}$ and $\mathbf{I}^{abc}$ in terms of their symmetrical components, we get

$$\mathbf{A}\mathbf{V}_a^{012} = \mathbf{Z}^{abc}\mathbf{A}\mathbf{I}_a^{012} \tag{10.28}$$

Multiplying (10.28) by $\mathbf{A}^{-1}$, we get

$$\begin{aligned} \mathbf{V}_a^{012} &= \mathbf{A}^{-1}\mathbf{Z}^{abc}\mathbf{A}\mathbf{I}_a^{012} \\ &= \mathbf{Z}^{012}\mathbf{I}_a^{012} \end{aligned} \tag{10.29}$$

where

$$\mathbf{Z}^{012} = \mathbf{A}^{-1}\mathbf{Z}^{abc}\mathbf{A} \tag{10.30}$$

Substituting for $\mathbf{Z}^{abc}$, $\mathbf{A}$, and $\mathbf{A}^{-1}$ from (10.27), (10.10), and (10.12), we have

$$\mathbf{Z}^{012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_s+Z_n & Z_m+Z_n & Z_m+Z_n \\ Z_m+Z_n & Z_s+Z_n & Z_m+Z_n \\ Z_m+Z_n & Z_m+Z_n & Z_s+Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{10.31}$$

Performing the above multiplications, we get

$$\mathbf{Z}^{012} = \begin{bmatrix} Z_s + 3Z_n + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix} \tag{10.32}$$

When there is no mutual coupling, we set $Z_m = 0$, and the impedance matrix becomes

$$\mathbf{Z}^{012} = \begin{bmatrix} Z_s + 3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} \tag{10.33}$$

The impedance matrix has nonzero elements appearing only on the principal diagonal, and it is a diagonal matrix. Therefore, for a balanced load, the three sequences are independent. That is, currents of each phase sequence will produce voltage drops of the same phase sequence only. This is a very important property, as it permits the analysis of each sequence network on a per phase basis.

10.3.2 SEQUENCE IMPEDANCES OF TRANSMISSION LINES

Transmission line parameters were derived in Chapter 4. For static devices such as transmission lines, the phase sequence has no effect on the impedance, because the voltages and currents encounter the same geometry of the line, irrespective of the sequence. Thus, positive- and negative-sequence impedances are equal, i.e., $Z^1 = Z^2$.

In deriving the line parameters, the effect of ground and shielding conductors were neglected. Zero-sequence currents are in phase and flow through the a,b,c conductors to return through the grounded neutral. The ground or any shielding wire are effectively in the path of zero sequence. Thus, Z^0, which includes the effect of the return path through the ground, is generally different from Z^1 and Z^2. The determination of the zero sequence impedance with the presence of earth neutral wires is quite involved and the interested reader is referred to the Carson's formula [14]. To get an idea of the order of Z^0 we will consider the following simplified configuration. Consider 1-m length of a three-phase line with equilaterally spaced conductors as shown in Figure 10.5. The phase conductors carry zero-sequence (single-phase) currents with return paths through a grounded neutral. The ground surface is approximated to an equivalent fictitious conductor located at the average distance D_n from each of the three phases. Since conductor n carries the return current in opposite direction, we have

$$I_a^0 + I_b^0 + I_c^0 + I_n = 0 \tag{10.34}$$

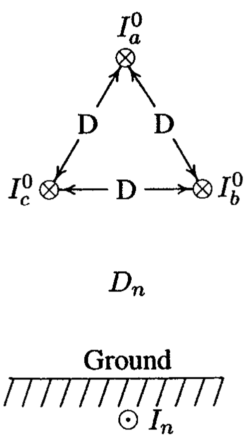

FIGURE 10.5
Zero-sequence current flow with earth return.

Since $I_a^0 = I_b^0 = I_c^0$, we have

$$I_n = -3I_a^0 \tag{10.35}$$

Utilizing the relation for the flux linkages of a conductor in a group expressed by (4.29), the total flux linkage of phase a conductor is

$$\lambda_{a0} = 2 \times 10^{-7} \left(I_a^0 \ln \frac{1}{r'} + I_b^0 \ln \frac{1}{D} + I_c^0 \ln \frac{1}{D} + I_n \ln \frac{1}{D_n} \right) \tag{10.36}$$

Substituting for I_b^0, I_c^0, and I_n in terms of I_a^0, we get

$$\begin{aligned} \lambda_{a0} &= 2 \times 10^{-7} I_a^0 \left(\ln \frac{1}{r'} + \ln \frac{1}{D} + \ln \frac{1}{D} - 3 \ln \frac{1}{D_n} \right) \\ &= 2 \times 10^{-7} I_a^0 \ln \frac{D_n^3}{r'D^2} \quad \text{Wb/m} \end{aligned} \tag{10.37}$$

Since $L_0 = \lambda_{a0}/I_a^0$, the zero sequence inductance per phase in mH per kilometer length is

$$\begin{aligned} L_0 &= 0.2 \ln \frac{D_n^3}{r'D^2} \\ &= 0.2 \ln \frac{DD_n^3}{r'D^3} \\ &= 0.2 \ln \frac{D}{r'} + 3 \left(0.2 \ln \frac{D_n}{D} \right) \quad \text{mH/Km} \end{aligned} \tag{10.38}$$

The first term above is the same as the positive-sequence inductance given by (4.33). Thus the zero sequence reactance can be expressed as

$$X^0 = X^1 + 3X_n \tag{10.39}$$

where

$$X_n = 2\pi f \left(0.2 \ln \frac{D_n}{D} \right) \quad \text{m}\Omega/\text{km} \tag{10.40}$$

The zero-sequence impedance of the transmission line is more than three times larger than the positive- or negative-sequence impedance.

10.3.3 SEQUENCE IMPEDANCES OF SYNCHRONOUS MACHINE

The inductances of a synchronous machine depend upon the phase order of the sequence current relative to the direction of rotation of the rotor. The positive-sequence generator impedance is the value found when positive-sequence current flows from the action of an imposed positive-sequence set of voltages. We have seen that the generator positive-sequence reactance varies, and in Section 9.2 one of the reactances X_d'', X_d', or X_d was used for the balanced three-phase fault studies.

When negative-sequence currents are impressed in the stator, the net flux in the air gap rotates at opposite direction to that of the rotor. That is, the net flux rotates at twice synchronous speed relative to the rotor. Since the field voltage is associated with the positive-sequence variables, the field winding has no influence. Consequently, only the damper winding produces an effect in the quadrature axis. Hence, there is no distinction between the transient and subtransient reactances in the quadrature axis as there is in the direct axis. The negative-sequence reactance is close to the positive-sequence subtransient reactance, i.e.,

$$X^2 \simeq X_d'' \tag{10.41}$$

Zero-sequence impedance is the impedance offered by the machine to the flow of the zero-sequence current. We recall that a set of zero sequence currents are all identical. Therefore, if the spatial distribution of mmf is assumed sinusoidal, the resultant air-gap flux would be zero, and there is no reactance due to armature reaction. The machine offers a very small reactance due to the leakage flux. Therefore, the zero-sequence reactance is approximated to the leakage reactance, i.e.,

$$X^0 \simeq X_\ell \tag{10.42}$$

10.3.4 SEQUENCE IMPEDANCES OF TRANSFORMER

In Chapter 3 we obtained the per phase equivalent circuit for a three-phase transformer. In power transformers, the core losses and the magnetization current are on the order of 1 percent of the rated value; therefore, the magnetizing branch is neglected. The transformer is modeled with the equivalent series leakage impedance. Since the transformer is a static device, the leakage impedance will not change if the phase sequence is changed. Therefore, the positive- and negative-sequence impedances are the same. Also, if the transformer permits zero-sequence current flow at all, the phase impedance to zero-sequence is equal to the leakage impedance, and we have

$$Z^0 = Z^1 = Z^2 = Z_\ell \tag{10.43}$$

From Section 3.9.1, we recall that in a Y-Δ, or a Δ-Y transformer, the positive-sequence line voltage on HV side leads the corresponding line voltage on the

LV side by 30°. For the negative-sequence voltage the corresponding phase shift is −30°. The equivalent circuit for the zero-sequence impedance depends on the winding connections and also upon whether or not the neutrals are grounded. Figure 10.6 shows some of the more common transformer configurations and their zero-sequence equivalent circuits. We recall that in a transformer, when the core reluctance is neglected, there is an exact mmf balance between the primary and secondary. This means that current can flow in the primary only if there is a current in the secondary. Based on this observation we can check the validity of the zero-sequence circuits by applying a set of zero-sequence voltage to the primary and calculating the resulting currents.

(a) Y-Y connections with both neutrals grounded – We know that the zero sequence current equals the sum of phase currents. Since both neutrals are grounded, there is a path for the zero sequence current to flow in the primary and secondary, and the transformer exhibits the equivalent leakage impedance per phase as shown in Figure 10.6(a).

(b) Y-Y connection with the primary neutral grounded – The primary neutral is grounded, but since the secondary neutral is isolated, the secondary phase current must sum up to zero. This means that the zero-sequence current in the secondary is zero. Consequently, the zero sequence current in the primary is zero, reflecting infinite impedance or an open circuit as shown in Figure 10.6(b).

(c) Y-Δ with grounded neutral – In this configuration the primary currents can flow because there is zero-sequence circulating current in the Δ-connected secondary and a ground return path for the Y-connected primary. Note that no zero-sequence current can leave the Δ terminals, thus there is an isolation between the primary and secondary sides as shown in Figure 10.6(c).

(d) Y-Δ connection with isolated neutral – In this configuration, because the neutral is isolated, zero sequence current cannot flow and the equivalent circuit reflects an infinite impedance or an open as shown in Figure 10.6(d).

(e) Δ-Δ connection – In this configuration zero-sequence currents circulate in the Δ-connected windings, but no currents can leave the Δ terminals, and the equivalent circuit is as shown in Figure 10.6(e).

Notice that the neutral impedance plays an important part in the equivalent circuit. When the neutral is grounded through an impedance Z_n, because $I_n = 3I_0$, in the equivalent circuit the neutral impedance appears as $3Z_n$ in the path of I_0.

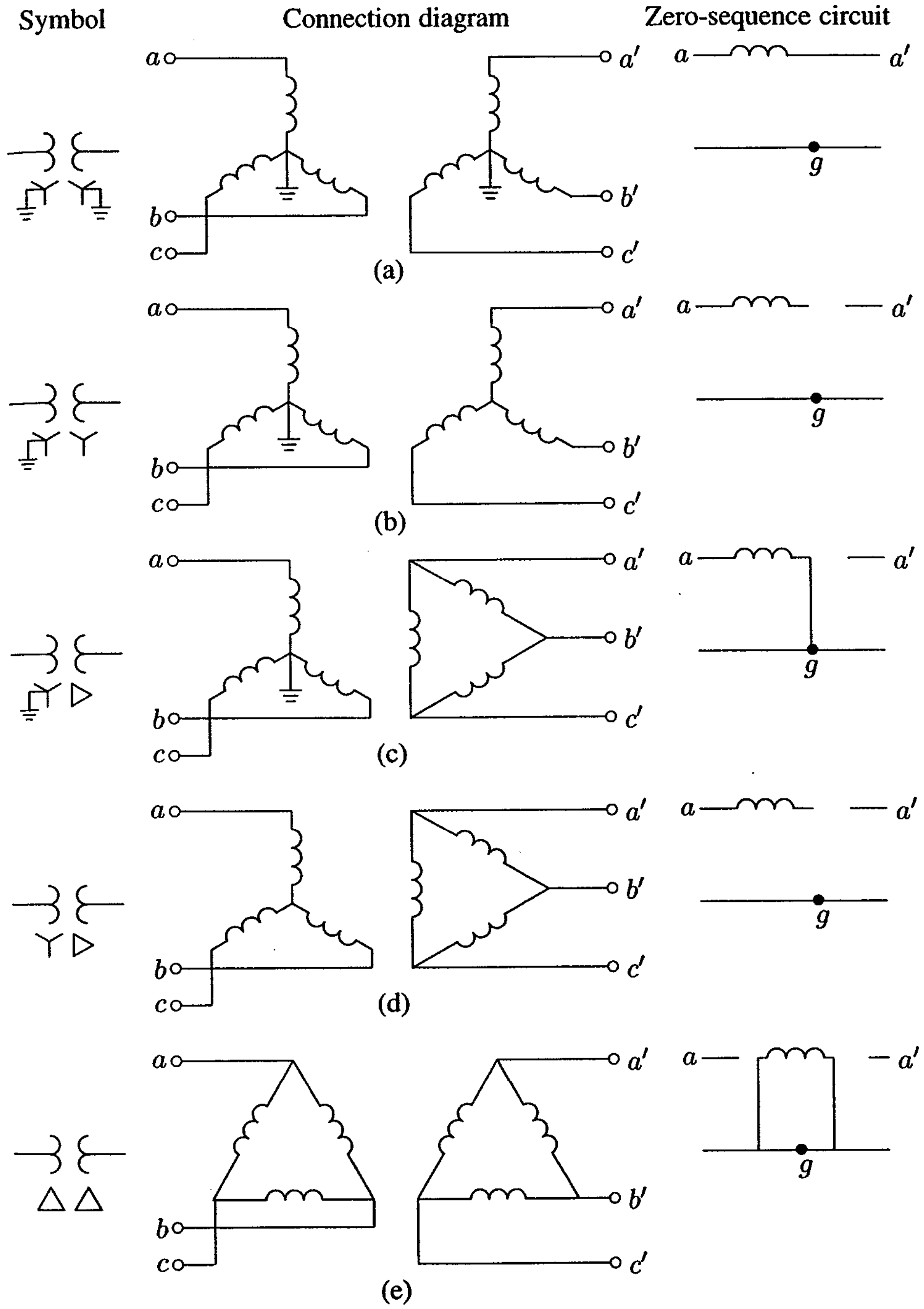

FIGURE 10.6
Transformer zero-sequence equivalent circuits.

Example 10.3

A balanced three-phase voltage of 100-V line-to-neutral is applied to a balanced Y-connected load with ungrounded neutral as shown in Figure 10.7. The three-phase load consists of three mutually-coupled reactances. Each phase has a series reactance of $Z_s = j12\ \Omega$, and the mutual coupling between phases is $Z_m = j4\ \Omega$.

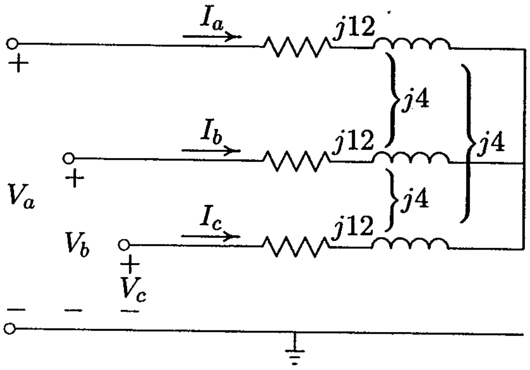

FIGURE 10.7
Circuit for Example 10.3.

(a) Determine the line currents by mesh analysis without using symmetrical components.
(b) Determine the line currents using symmetrical components.

(a) Applying KVL to the two independent mesh equations yields

$$Z_sI_a + Z_mI_b - Z_sI_b - Z_mI_a = V_a - V_b = |V_L|\angle\pi/6$$
$$Z_sI_b + Z_mI_c - Z_sI_c - Z_mI_b = V_b - V_c = |V_L|\angle-\pi/2$$

Also from KCL, we have

$$I_a + I_b + I_c = 0$$

Writing above equations in matrix form, results in

$$\begin{bmatrix} (Z_s - Z_m) & -(Z_s - Z_m) & 0 \\ 0 & (Z_s - Z_m) & -(Z_s - Z_m) \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} |V_L|\angle\pi/6 \\ |V_L|\angle-\pi/2 \\ 0 \end{bmatrix}$$

or in compact form

$$\mathbf{Z}_{mesh}\mathbf{I}^{abc} = \mathbf{V}_{mesh}$$

Solving the above equations results in the line currents

$$\mathbf{I}^{abc} = \mathbf{Z}_{mesh}{}^{-1}\mathbf{V}_{mesh}$$

The following commands

```
% (a) Solution by mesh analysis
Zs=j*12; Zm=j*4; Va = 100; VL=Va*sqrt(3);
Z= [(Zs-Zm)  -(Zs-Zm)       0
       0       (Zs-Zm)  -(Zs-Zm)
       1          1          1   ];
V=[VL*cos(pi/6)+j*VL*sin(pi/6)
   VL*cos(-pi/2)+j*VL*sin(-pi/2)
             0                     ];
Y=inv(Z)
Iabc=Y*V;                      % Line currents (Rectangular form)
Iabcp=[abs(Iabc), angle(Iabc)*180/pi] %  Line currents (Polar)
```

result in

```
     Iabcp =
                12.5     -90.0
                12.5     150.0
                12.5      30.0
```

(b) Using the symmetrical components method, we have

$$\mathbf{V}^{012} = \mathbf{Z}^{012}\mathbf{I}^{012}$$

where

$$\mathbf{V}^{012} = \begin{bmatrix} 0 \\ V_a \\ 0 \end{bmatrix}$$

and from (10.32)

$$\mathbf{Z}^{012} = \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix}$$

for the sequence components of currents, we get

$$\mathbf{I}^{012} = [\mathbf{Z}^{012}]^{-1}\mathbf{V}^{012}$$

We write the following commands

```
% (b) Solution by symmetrical components method
Z012=[Zs+2*Zm   0      0         % Symmetrical components matrix
       0      Zs-Zm   0
       0        0    Zs-Zm];
V012=[0; Va ; 0];     %Symmetrical components of phase voltages
I012=inv(Z012)*V012; %Symmetrical components of line  currents
a=cos(2*pi/3)+j*sin(2*pi/3);
A=[ 1  1  1; 1 a^2  a; 1 a a^2];         % Transformation matrix
Iabc=A*I012;                   % Line currents (Rectangular form)
Iabcp=[abs(Iabc), angle(Iabc)*180/pi]  % Line currents (Polar)
```

which result in

```
Iabcp =
        12.5    -90.0
        12.5    150.0
        12.5     30.0
```

This is the same result as in part (a).

Example 10.4

A three-phase unbalanced source with the following phase-to-neutral voltages

$$\mathbf{V}^{abc} = \begin{bmatrix} 200 & \angle 25^\circ \\ 100 & \angle -155^\circ \\ 80 & \angle 100^\circ \end{bmatrix}$$

is applied to the circuit in Figure 10.4 (page 407). The load series impedance per phase is $Z_s = 8 + j24$ and the mutual impedance between phases is $Z_m = j4$. The load and source neutrals are solidly grounded. Determine

(a) The load sequence impedance matrix $\mathbf{Z}^{012} = \mathbf{A}^{-1}\mathbf{Z}^{abc}\mathbf{A}$.
(b) The symmetrical components of voltage.
(c) The symmetrical components of current.
(d) The load phase currents.
(e) The complex power delivered to the load in terms of symmetrical components, $S_{3\phi} = 3(V_a^0 I_a^{0*} + V_a^1 I_a^{1*} + V_a^2 I_a^{2*})$.
(f) The complex power delivered to the load by summing up the power in each phase, $S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^*$.

We write the following commands

```
Vabc = [200    25
        100  -155
         80   100];
Zabc = [8+j*24     j*4     j*4
            j*4  8+j*24     j*4
           j*4      j*4  8+j*24];
Z012 = zabc2sc(Zabc)      % Symmetrical components of impedance
V012 = abc2sc(Vabc);        % Symmetrical components of voltage
V012p= rec2pol(V012)                % Rectangular to polar form
I012 = inv(Z012)*V012;      % Symmetrical components of current
I012p= rec2pol(I012)                % Rectangular to polar form
Iabc = sc2abc(I012);                           % Phase currents
Iabcp= rec2pol(Iabc)                % Rectangular to polar form
S3ph =3*(V012.')*conj(I012)%Power using symmetrical components
Vabcr = Vabc(:, 1).*(cos(pi/180*Vabc(:, 2)) +...
j*sin(pi/180*Vabc(:, 2)));
S3ph=(Vabcr.')*conj(Iabc)
                          % Power using phase currents and voltages
```

The result is

```
Z012 =
       8.00 + 32.00i   0.00 +  0.00i   0.00 +  0.00i
       0.00 +  0.00i   8.00 + 20.00i   0.00 +  0.00i
       0.00 -  0.00i   0.00 -  0.00i   8.00 + 20.00i

V012p =
        47.7739     57.6268
       112.7841     -0.0331
        61.6231     45.8825

I012p =
         1.4484    -18.3369
         5.2359    -68.2317
         2.8608    -22.3161

Iabcp =
         8.7507    -47.0439
         5.2292    143.2451
         3.0280     39.0675

S3ph =
      9.0471e+002+ 2.3373e+003i

S3ph =
      9.0471e+002+ 2.3373e+003i
```

10.4 SEQUENCE NETWORKS OF A LOADED GENERATOR

Figure 10.8 represents a three-phase synchronous generator with neutral grounded through an impedance Z_n. The generator is supplying a three-phase balanced load.

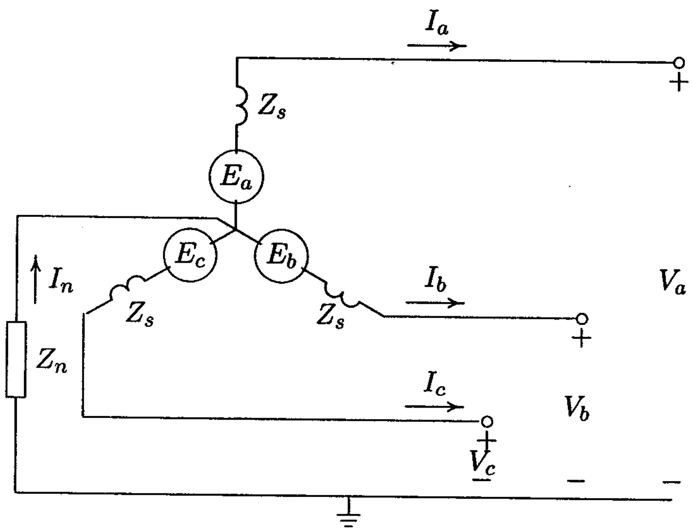

FIGURE 10.8
Three-phase balanced source and impedance.

The synchronous machine generates balanced three-phase internal voltages and is represented as a positive-sequence set of phasors

$$\mathbf{E}^{abc} = \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} E_a \tag{10.44}$$

The machine is supplying a three-phase balanced load. Applying Kirchhoff's voltage law to each phase we obtain

$$\begin{aligned} V_a &= E_a - Z_s I_a - Z_n I_n \\ V_b &= E_b - Z_s I_b - Z_n I_n \\ V_c &= E_c - Z_c I_c - Z_n I_n \end{aligned} \tag{10.45}$$

Substituting for $I_n = I_a + I_b + I_c$, and writing (10.45) in matrix form, we get

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} - \begin{bmatrix} Z_s + Z_n & Z_n & Z_n \\ Z_n & Z_s + Z_n & Z_n \\ Z_n & Z_n & Z_s + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{10.46}$$

or in compact form, we have

$$\mathbf{V}^{abc} = \mathbf{E}^{abc} - \mathbf{Z}^{abc}\mathbf{I}^{abc} \tag{10.47}$$

where $\mathbf{V}^{abc}$ is the phase terminal voltage vector and $\mathbf{I}^{abc}$ is the phase current vector. Transforming the terminal voltages and current phasors into their symmetrical components results in

$$\mathbf{A}\mathbf{V}_a^{012} = \mathbf{A}\mathbf{E}_a^{012} - \mathbf{Z}^{abc}\mathbf{A}\mathbf{I}_a^{012} \tag{10.48}$$

Multiplying (10.48) by $\mathbf{A}^{-1}$, we get

$$\begin{aligned}\mathbf{V}_a^{012} &= \mathbf{E}_a^{012} - \mathbf{A}^{-1}\mathbf{Z}^{abc}\mathbf{A}\mathbf{I}_a^{012} \\ &= \mathbf{E}_a^{012} - \mathbf{Z}^{012}\mathbf{I}_a^{012}\end{aligned} \tag{10.49}$$

where

$$\mathbf{Z}^{012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z_s + Z_n & Z_n & Z_n \\ Z_n & Z_s + Z_n & Z_n \\ Z_n & Z_n & Z_s + Z_n \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{10.50}$$

Performing the above multiplications, we get

$$\mathbf{Z}^{012} = \begin{bmatrix} Z_s + 3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} = \begin{bmatrix} Z^0 & 0 & 0 \\ 0 & Z^1 & 0 \\ 0 & 0 & Z^2 \end{bmatrix} \tag{10.51}$$

Since the generated emf is balanced, there is only positive-sequence voltage, i.e.,

$$\mathbf{E}_a^{012} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} \tag{10.52}$$

Substituting for $\mathbf{E}_a^{012}$ and $\mathbf{Z}^{012}$ in (10.49), we get

$$\begin{bmatrix} V_a^0 \\ V_a^1 \\ V_a^2 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z^0 & 0 & 0 \\ 0 & Z^1 & 0 \\ 0 & 0 & Z^2 \end{bmatrix}\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} \tag{10.53}$$

Since the above equation is very important, we write it in component form, and we get

$$\begin{aligned} V_a^0 &= 0 - Z^0 I_a^0 \\ V_a^1 &= E_a - Z^1 I_a^1 \\ V_a^2 &= 0 - Z^2 I_a^2 \end{aligned} \tag{10.54}$$

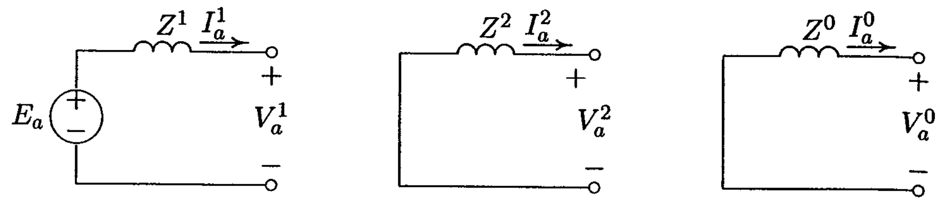

FIGURE 10.9
Sequence networks: (a) Positive-sequence; (b) negative-sequence; (c) zero-sequence.

The three equations given by (10.54) can be represented by the three equivalent sequence networks shown in Figure 10.9.

We make the following important observations.

- The three sequences are independent.
- The positive-sequence network is the same as the one-line diagram used in studying balanced three-phase currents and voltages.
- Only the positive-sequence network has a voltage source. Therefore, the positive-sequence current causes only positive-sequence voltage drops.
- There is no voltage source in the negative- or zero-sequence networks.
- Negative- and zero-sequence currents cause negative- and zero-sequence voltage drops only.
- The neutral of the system is the reference for positive-and negative-sequence networks, but ground is the reference for the zero-sequence networks. Therefore, the zero-sequence current can flow only if the circuit from the system neutrals to ground is complete.
- The grounding impedance is reflected in the zero sequence network as $3Z_n$.
- The three-sequence systems can be solved separately on a per phase basis. The phase currents and voltages can then be determined by superposing their symmetrical components of current and voltage respectively.

We are now ready with mathematical tools to analyze various types of unbalanced faults. First, the fault current is obtained using Thévenin's method and algebraic manipulation of sequence networks. The analysis will then be extended to find the bus voltages and fault current during fault, for different types of faults using the bus impedance matrix.

10.5 SINGLE LINE-TO-GROUND FAULT

Figure 10.10 illustrates a three-phase generator with neutral grounded through impedance Z_n.

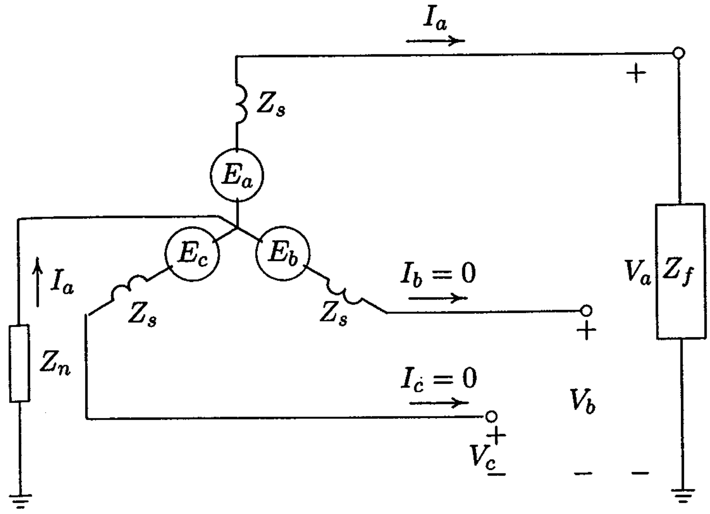

FIGURE 10.10
Line-to-ground fault on phase a.

Suppose a line-to-ground fault occurs on phase a through impedance Z_f. Assuming the generator is initially on no-load, the boundary conditions at the fault point are

$$V_a = Z_f I_a \tag{10.55}$$

$$I_b = I_c = 0 \tag{10.56}$$

Substituting for $I_b = I_c = 0$, the symmetrical components of currents from (10.14) are

$$\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} \tag{10.57}$$

From the above equation, we find that

$$I_a^0 = I_a^1 = I_a^2 = \frac{1}{3} I_a \tag{10.58}$$

Phase a voltage in terms of symmetrical components is

$$V_a = V_a^0 + V_a^1 + V_a^2 \tag{10.59}$$

Substituting for V_a^0, V_a^1, and V_a^2 from (10.54) and noting $I_a^0 = I_a^1 = I_a^2$, we get

$$V_a = E_a - (Z^1 + Z^2 + Z^0)I_a^0 \tag{10.60}$$

where $Z^0 = Z_s + 3Z_n$. Substituting for V_a from (10.55), and noting $I_a = 3I_a^0$, we get

$$3Z_f I_a^0 = E_a - (Z^1 + Z^2 + Z^0)I_a^0 \tag{10.61}$$

or

$$I_a^0 = \frac{E_a}{Z^1 + Z^2 + Z^0 + 3Z_f} \tag{10.62}$$

The fault current is

$$I_a = 3I_a^0 = \frac{3E_a}{Z^1 + Z^2 + Z^0 + 3Z_f} \tag{10.63}$$

Substituting for the symmetrical components of currents in (10.54), the symmetrical components of voltage and phase voltages at the point of fault are obtained.

Equations (10.58) and (10.62) can be represented by connecting the sequence networks in series as shown in the equivalent circuit of Figure 10.11. Thus, for line-to-ground faults, the Thévenin impedance to the point of fault is obtained for each sequence network, and the three sequence networks are placed in series. In many practical applications, the positive- and negative-sequence impedances are found to be equal. If the generator neutral is solidly grounded, $Z_n = 0$ and for bolted faults $Z_f = 0$.

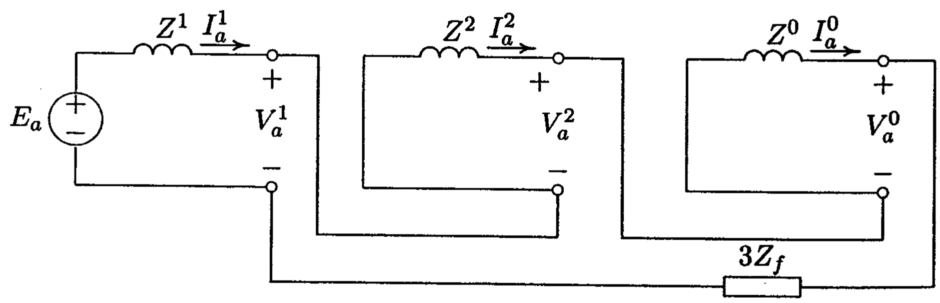

FIGURE 10.11
Sequence network connection for line-to-ground fault.

10.6 LINE-TO-LINE FAULT

Figure 10.12 shows a three-phase generator with a fault through an impedance Z_f between phases b and c. Assuming the generator is initially on no-load, the boundary conditions at the fault point are

$$V_b - V_c = Z_f I_b \tag{10.64}$$

$$I_b + I_c = 0 \tag{10.65}$$

$$I_a = 0 \tag{10.66}$$

Substituting for $I_a = 0$, and $I_c = -I_b$, the symmetrical components of currents from (10.14) are

$$\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} \tag{10.67}$$

From the above equation, we find that

$$I_a^0 = 0 \tag{10.68}$$

$$I_a^1 = \frac{1}{3}(a - a^2)I_b \tag{10.69}$$

$$I_a^2 = \frac{1}{3}(a^2 - a)I_b \tag{10.70}$$

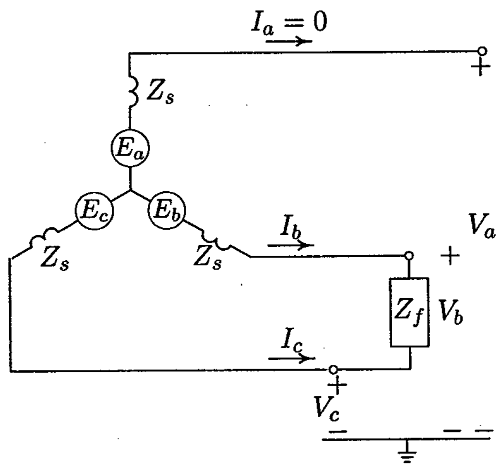

FIGURE 10.12
Line-to-line fault between phase b and c.

Also, from (10.69) and (10.70), we note that

$$I_a^1 = -I_a^2 \tag{10.71}$$

From (10.16), we have

$$\begin{aligned} V_b - V_c &= (a^2 - a)(V_a^1 - V_a^2) \\ &= Z_f I_b \end{aligned} \tag{10.72}$$

Substituting for V_a^1 and V_a^2 from (10.54) and noting $I_a^2 = -I_a^1$, we get

$$(a^2 - a)[E_a - (Z^1 + Z^2)I_a^1] = Z_f I_b \tag{10.73}$$

Substituting for I_b from (10.69), we get

$$E_a - (Z^1 + Z^2)I_a^1 = Z_f \frac{3I_a^1}{(a - a^2)(a^2 - a)} \tag{10.74}$$

Since $(a - a^2)(a^2 - a) = 3$, solving for I_a^1 results in

$$I_a^1 = \frac{E_a}{Z^1 + Z^2 + Z_f} \tag{10.75}$$

The phase currents are

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_a^1 \\ -I_a^1 \end{bmatrix} \tag{10.76}$$

The fault current is

$$I_b = -I_c = (a^2 - a)I_a^1 \tag{10.77}$$

or

$$I_b = -j\sqrt{3}\, I_a^1 \tag{10.78}$$

Substituting for the symmetrical components of currents in (10.54), the symmetrical components of voltage and phase voltages at the point of fault are obtained.

Equations (10.71) and (10.75) can be represented by connecting the positive- and negative-sequence networks in opposition as shown in the equivalent circuit of Figure 10.13. In many practical applications, the positive- and negative-sequence impedances are found to be equal. For a bolted fault, $Z_f = 0$.

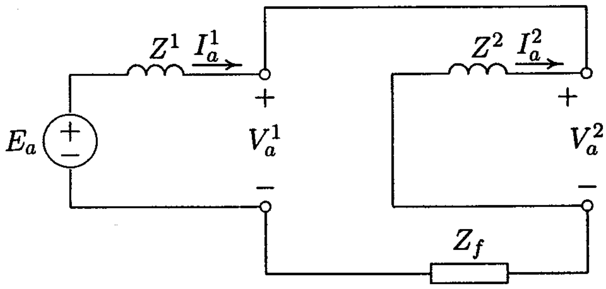

FIGURE 10.13
Sequence network connection for line-to-line fault.

10.7 DOUBLE LINE-TO-GROUND FAULT

Figure 10.14 shows a three-phase generator with a fault on phases b and c through an impedance Z_f to ground. Assuming the generator is initially on no-load, the boundary conditions at the fault point are

$$V_b = V_c = Z_f(I_b + I_c) \tag{10.79}$$

$$I_a = I_a^0 + I_a^1 + I_a^2 = 0 \tag{10.80}$$

From (10.16), the phase voltages V_b and V_c are

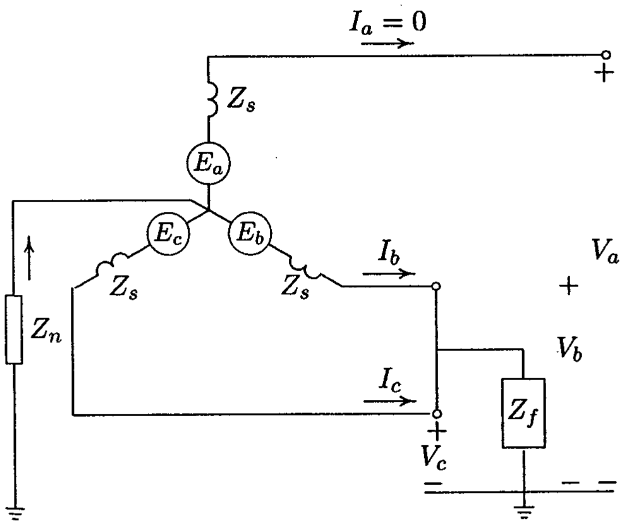

FIGURE 10.14
Double line-to-ground fault.

$$V_b = V_a^0 + a^2V_a^1 + aV_a^2 \tag{10.81}$$
$$V_c = V_a^0 + aV_a^1 + a^2V_a^2 \tag{10.82}$$

Since $V_b = V_c$, from above we note that

$$V_a^1 = V_a^2 \tag{10.83}$$

Substituting for the symmetrical components of currents in (10.79), we get

$$\begin{aligned} V_b &= Z_f(I_a^0 + a^2I_a^1 + aI_a^2 + I_a^0 + aI_a^1 + a^2I_a^2) \\ &= Z_f(2I_a^0 - I_a^1 - I_a^2) \\ &= 3Z_fI_a^0 \end{aligned} \tag{10.84}$$

Substituting for V_b from (10.84) and for V_a^2 from (10.83) into (10.81), we have

$$\begin{aligned} 3Z_fI_a^0 &= V_a^0 + (a^2 + a)V_a^1 \\ &= V_a^0 - V_a^1 \end{aligned} \tag{10.85}$$

Substituting for the symmetrical components of voltage from (10.54) into (10.85) and solving for I_a^0, we get

$$I_a^0 = -\frac{E_a - Z^1I_a^1}{Z^0 + 3Z_f} \tag{10.86}$$

Also, substituting for the symmetrical components of voltage in (10.83), we obtain

$$I_a^2 = -\frac{E_a - Z^1I_a^1}{Z^2} \tag{10.87}$$

Substituting for I_a^0 and I_a^2 into (10.80) and solving for I_a^1, we get

$$I_a^1 = \frac{E_a}{Z^1 + \frac{Z^2(Z^0+3Z_f)}{Z^2+Z^0+3Z_f}} \tag{10.88}$$

Equations (10.86)–(10.88) can be represented by connecting the positive-sequence impedance in series with the parallel combination of the negative-sequence and zero-sequence networks as shown in the equivalent circuit of Figure 10.15. The value of I_a^1 found from (10.88) is substituted in (10.86) and (10.87), and I_a^0 and I_a^2 are found. The phase currents are then found from (10.8). Finally, the fault current is obtained from

$$I_f = I_b + I_c = 3I_a^0 \tag{10.89}$$

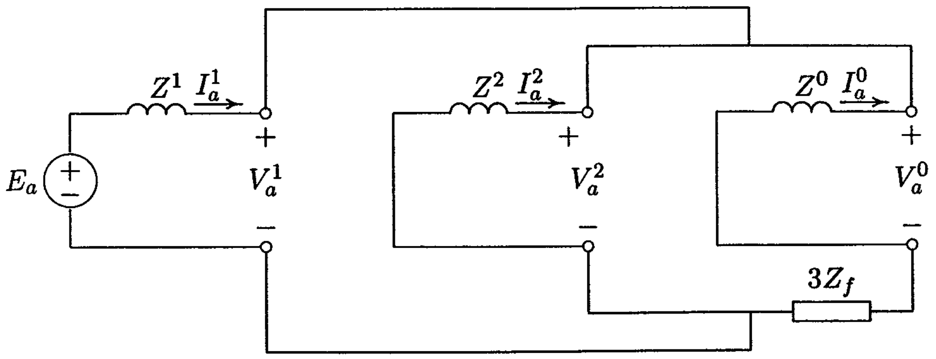

FIGURE 10.15
Sequence network connection for double line-to-ground fault.

Example 10.5

The one-line diagram of a simple power system is shown in Figure 10.16. The neutral of each generator is grounded through a current-limiting reactor of 0.25/3 per unit on a 100-MVA base. The system data expressed in per unit on a common 100-MVA base is tabulated below. The generators are running on no-load at their rated voltage and rated frequency with their emfs in phase.

Determine the fault current for the following faults.

(a) A balanced three-phase fault at bus 3 through a fault impedance $Z_f = j0.1$ per unit.

(b) A single line-to-ground fault at bus 3 through a fault impedance $Z_f = j0.10$ per unit.

(c) A line-to-line fault at bus 3 through a fault impedance $Z_f = j0.1$ per unit.

(d) A double line-to-ground fault at bus 3 through a fault impedance $Z_f = j0.1$ per unit.

Item	Base MVA	Voltage Rating	X^1	X^2	X^0
G_1	100	20 kV	0.15	0.15	0.05
G_2	100	20 kV	0.15	0.15	0.05
T_1	100	20/220 kV	0.10	0.10	0.10
T_2	100	20/220 kV	0.10	0.10	0.10
L_{12}	100	220 kV	0.125	0.125	0.30
L_{13}	100	220 kV	0.15	0.15	0.35
L_{23}	100	220 kV	0.25	0.25	0.7125

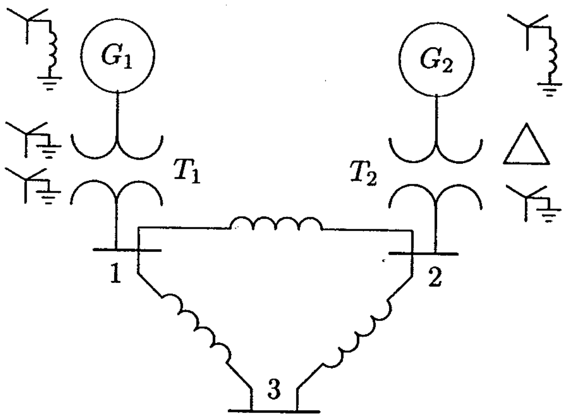

FIGURE 10.16
The one-line diagram for Example 10.5.

The positive-sequence impedance network is shown in Figure 10.17.

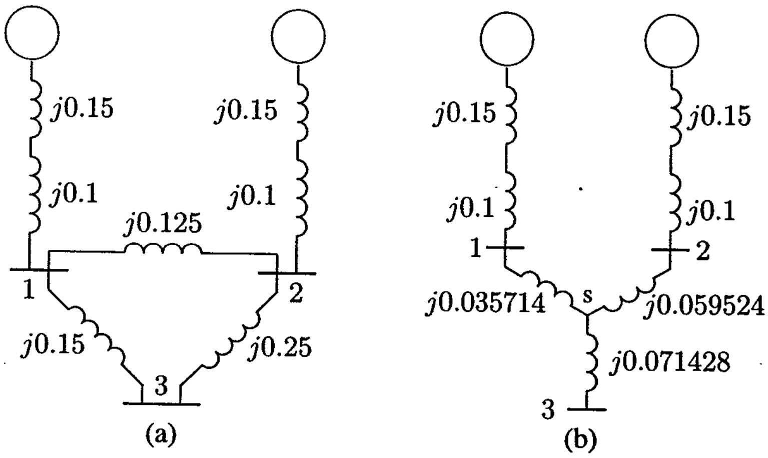

FIGURE 10.17
Positive-sequence impedance diagram for Example 10.5.

To find Thévenin impedance viewed from the faulted bus (bus 3), we convert the delta formed by buses 123 to an equivalent Y as shown in Figure 10.17(b).

$$Z_{1s} = \frac{(j0.125)(j0.15)}{j0.525} = j0.0357143$$

$$Z_{2s} = \frac{(j0.125)(j0.25)}{j0.525} = j0.0595238$$

$$Z_{3s} = \frac{(j0.15)(j0.25)}{j0.525} = j0.0714286$$

Combining the parallel branches, the positive-sequence Thévenin impedance is

$$\begin{aligned} Z_{33}^1 &= \frac{(j0.2857143)(j0.3095238)}{j0.5952381} + j0.0714286 \\ &= j0.1485714 + j0.0714286 = j0.22 \end{aligned}$$

This is shown in Figure 10.18(a).

(a) Positive-sequence network (b) Negative-sequence network

FIGURE 10.18
Reduction of the positive-sequence Thévenin equivalent network.

Since the negative-sequence impedance of each element is the same as the positive-sequence impedance, we have

$$Z_{33}^2 = Z_{33}^1 = j0.22$$

and the negative-sequence network is as shown in Figure 10.18(b). The equivalent circuit for the zero-sequence network is constructed according to the transformer winding connections of Figure 10.6 and is shown in Figure 10.19.

To find Thévenin impedance viewed from the faulted bus (bus 3), we convert the delta formed by buses 123 to an equivalent Y as shown in Figure 10.19(b).

$$\begin{aligned} Z_{1s} &= \frac{(j0.30)(j0.35)}{j1.3625} = j0.0770642 \\ Z_{2s} &= \frac{(j0.30)(j0.7125)}{j1.3625} = j0.1568807 \\ Z_{3s} &= \frac{(j0.35)(j0.7125)}{j1.3625} = j0.1830257 \end{aligned}$$

Combining the parallel branches, the zero-sequence Thévenin impedance is

$$\begin{aligned} Z_{33}^0 &= \frac{(j0.4770642)(j0.2568807)}{j0.7339449} + j0.1830275 \\ &= j0.1669725 + j0.1830275 = j0.35 \end{aligned}$$

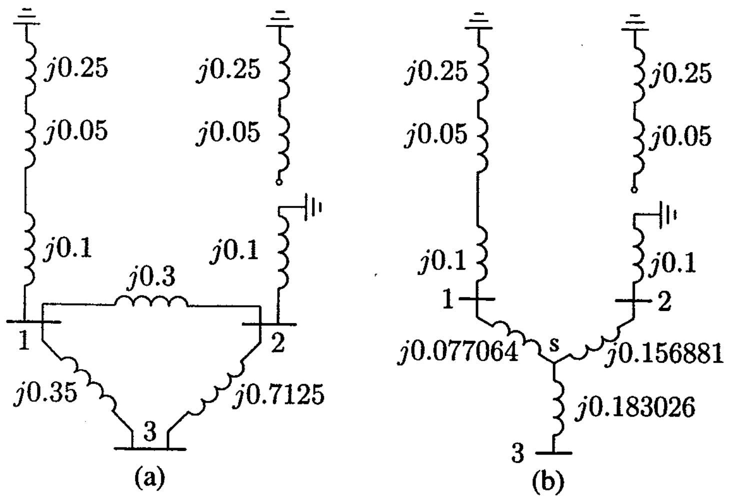

FIGURE 10.19
Zero-sequence impedance diagram for Example 10.5.

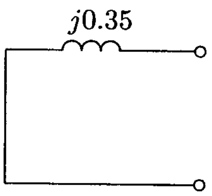

FIGURE 10.20
Zero-sequence network for Example 10.5.

The zero-sequence impedance diagram is shown in Figure 10.20.

(a) Balanced three-phase fault at bus 3.

Assuming the no-load generated emfs are equal to 1.0 per unit, the fault current is

$$I_3^a(F) = \frac{V_{3(0)}^a}{Z_{33}^1 + Z_f} = \frac{1.0}{j0.22 + j0.1} = -j3.125 \quad \text{pu}$$
$$= 820.1\angle{-90°} \quad \text{A}$$

(b) Single line-to-ground fault at bus 3.

From (10.62), the sequence components of the fault current are

$$\begin{aligned} I_3^0 = I_3^1 = I_3^2 &= \frac{V_3^a(0)}{Z_{33}^1 + Z_{33}^2 + Z_{33}^0 + 3Z_f} \\ &= \frac{1.0}{j0.22 + j0.22 + j0.35 + 3(j0.1)} \\ &= -j0.9174 \text{ pu} \end{aligned}$$

The fault current is

$$\begin{bmatrix} I_3^a \\ I_3^b \\ I_3^c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_3^0 \\ I_3^0 \\ I_3^0 \end{bmatrix} = \begin{bmatrix} 3I_3^0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j2.7523 \\ 0 \\ 0 \end{bmatrix} \text{ pu}$$

(c) Line-to line fault at bus 3.

The zero-sequence component of current is zero, i.e.,

$$I_3^0 = 0$$

From (10.75), the positive- and negative-sequence components of the fault current are

$$I_3^1 = -I_3^2 = \frac{V_{3(0)}^a}{Z_{33}^1 + Z_{33}^2 + Z_f} = \frac{1}{j0.22 + j0.22 + j0.1} = -j1.8519 \text{ pu}$$

The fault current is

$$\begin{bmatrix} I_3^a \\ I_3^b \\ I_3^c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j1.8519 \\ j1.8519 \end{bmatrix} = \begin{bmatrix} 0 \\ -3.2075 \\ 3.2075 \end{bmatrix}$$

(d) Double line-to line-fault at bus 3.

From (10.88), the positive-sequence component of the fault current is

$$I_3^1 = \frac{V_{3(0)}^a}{Z_{33}^1 + \frac{Z_{33}^2(Z_{33}^0 + 3Z_f)}{Z_{33}^2 + Z_{33}^0 + 3Z_f}} = \frac{1}{j0.22 + \frac{j0.22(j0.35 + j0.3)}{j0.22 + j0.35 + j.3}} = -j2.6017 \text{ pu}$$

The negative-sequence component of current from (10.87) is

$$I_3^2 = -\frac{V_{3(0)}^a - Z_{33}^1 I_3^1}{Z_{33}^2} = -\frac{1 - (j0.22)(-j2.6017)}{j0.22} = j1.9438 \text{ pu}$$

The zero-sequence component of current from (10.86) is

$$I_3^0 = -\frac{V_{3(0)}^a - Z_{33}^1 I_3^1}{Z_{33}^0 + 3Z_f} = -\frac{1 - (j0.22)(-j2.6017)}{j0.35 + j0.3} = j0.6579 \text{ pu}$$

and the phase currents are

$$\begin{bmatrix} I_3^a \\ I_3^b \\ I_3^c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j0.6579 \\ -j2.6017 \\ j1.9438 \end{bmatrix} = \begin{bmatrix} 0 \\ 4.058\angle 165.93^\circ \\ 4.058\angle 14.07^\circ \end{bmatrix}$$

The fault current is

$$I_3(F) = I_3^b + I_3^c = 1.9732\angle 90^\circ$$

10.8 UNBALANCED FAULT ANALYSIS USING BUS IMPEDANCE MATRIX

We have seen that when the network is balanced, the symmetrical components impedances are diagonal, so that it is possible to calculate $\mathbf{Z}_{bus}$ separately for zero-, positive-, and negative-sequence networks. Also, we have observed that for a fault at bus k, the diagonal element in the k axis of the bus impedance matrix $\mathbf{Z}_{bus}$ is the Thévenin impedance to the point of fault. In order to obtain a solution for the unbalanced faults, the bus impedance matrix for each sequence network is obtained separately, then the sequence impedances $Z^0{}_{kk}$, $Z^1{}_{kk}$, and $Z^2{}_{kk}$ are connected together as described in Figures 10.11, 10.13, and 10.15. The fault formulas for various unbalanced faults is summarized below. In writing the symmetrical components of voltage and currents, the subscript a is left out and the symmetrical components are understood to refer to phase a.

10.8.1 SINGLE LINE-TO-GROUND FAULT USING $\mathbf{Z}_{bus}$

Consider a fault between phase a and ground through an impedance Z_f at bus k as shown in Figure 10.21. The line-to-ground fault requires that positive-, negative-, and zero-sequence networks for phase a be placed in series in order to compute the zero-sequence fault current as given by (10.62). Thus, in general, for a fault at bus k, the symmetrical components of fault current is

$$I_k^0 = I_k^1 = I_k^2 = \frac{V_k(0)}{Z_{kk}^1 + Z_{kk}^2 + Z_{kk}^0 + 3Z_f} \tag{10.90}$$

where Z_{kk}^1, Z_{kk}^2, and Z_{kk}^0 are the diagonal elements in the k axis of the corresponding bus impedance matrix and $V_k(0)$ is the prefault voltage at bus k. The fault phase current is

$$I_k^{abc} = \mathbf{A}I_k^{012} \tag{10.91}$$