Differential & Integral Calculus

Chapter Wise Problems List

Chapter 1: Limit, Continuity & Differentiability

Theory/ Definitio

Write down the short note about the Limit, Continuity, and Differentiability.

Math: Type 1

1. A function $f(x)$ is defined as follows: $f(x)=\begin{cases} x^2+1 & when \quad x>0 \\ 1 & when \quad x=0 \\ x+1 & when \quad x<0 \end{cases}$, Find the value of $\lim_{x\to 0} f(x)$

2. A function $f(x)$ is defined as follows: $f(x)=\begin{cases} x^2 & when \quad x<1 \\ 2.4 & when \quad x=1 \\ x^2+1 & when \quad x>1 \end{cases}$, Does $\lim_{x\to 1} f(x)$ exist?

3. If $f(x)=\dfrac{1}{1-e^{1/x}}$ then find limits from the left and the right of $x=0$. Does the limit of $f(x)$ at $x=0$ exist?

4. A function $f(x)$ is defined as follows: $f(x)=\begin{cases} e^{-\frac{|x|}{2}} & when \quad -1<x<0 \\ x^2 & when \quad 0\le x<2 \end{cases}$, Discuss the existence of $\lim_{x\to 0} f(x)$.

Math: Type 2

1. A function $f(x)$ is defined as follows: $f(x)=\begin{cases} x^2+1 & when \quad x<0 \\ x & when \quad 0\le x\le 1 \\ 1/x & when \quad x>1 \end{cases}$, Discus the continuity at $x=1$.

2. If $f(x)=\begin{cases} x^2\sin\left(1/x\right) & when \ x\neq 0 \\ 0 & when \ x=0 \end{cases}$ then test the continuity at $x=0$.

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3. A function $f(x)$ is defined as follows:

$$f(x)=\begin{cases} 1 & when \ -\infty<x<0 \\ 1+\sin x & when \ 0\le x<\pi/2 \\ 2+\left(x-\pi/2\right)^2 & when \ \pi/2\le x<\infty \end{cases},$$

Test the continuity at $x=0 \ and \ \pi/2$

Math: Type 3

1. A function $f(x)$ is defined as follows: $f(x)=\begin{cases} x^2+1 & when \ x\le 0 \\ x & when \ 0<x<1 \\ \frac{1}{x} & when \ x\ge 1 \end{cases}$, Discuss the differentiability at $x=0$ and $x=1$.

2. A function $f(x)$ is defined as follows: $f(x)=\begin{cases} 1 & when \ x<0 \\ 1+\sin x & when \ 0\le x<\frac{\pi}{2} \\ 2+\left(x-\pi/2\right)^2 & when \ x\ge\frac{\pi}{2} \end{cases}$,

Discuss the differentiability at $x=0$ and $x=\frac{\pi}{2}$.

Chapter 2: Differentiation

Theory/ Definitio

Write down the short note about the Differential coefficient, Implicit equation and Parametric equation; Statement of Leibnitz Theorem.

Math: Type 1

Find differential coefficient or $\frac{dy}{dx}$ for the following functions **(By using different types of differential formula)**

(1) $y=3x^8-2x^{\frac{5}{4}}-\frac{5}{x}+8$ (2) $y=[\frac{\sqrt{x}-2x}{\sqrt{x}}]$ (3) $y=(x^3-5)(2x+3)$

(4) $y=e^{ax^2+bx+c}$ (5) $y=\sqrt{x^3-2x+5}$ (6) $y=\ln\left(x+\sqrt{x^2+a^2}\right)$

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	(7) $y = 4\sin x - \cos x$ (8) $y = \sec^2 x - \tan^2 x$ (9) $y = e^{\sqrt{\cot x}}$ (10) $y = \ln(\sec x + \tan x)$ (11) $y = e^{\sin^{-1} x} + \tan^{-1} x$ (12) $y = \cos^{-1}\left(e^{\cot^{-1} x}\right)$
Math: Type 2	Find differential coefficient or $\frac{dy}{dx}$ for the following functions **(By using** *uv and* u/v **formula)** (1) $y = x^3 \ln x$ (2) $y = xe^x \sin x$ (3) $y = \frac{2x^2+5}{3x-4}$ (4) $y = \frac{\cos x}{1+\sin x}$
Math: Type 3	Find differential coefficient or $\frac{dy}{dx}$ for the following functions **(By using Logarithmic formula)** (1) $y = x^{1+x+x^2}$ (2) $y = (\sin x)^{\ln x}$ (3) $y = x^{\sin x} + (\sin x)^{\cos x}$
Math: Type 4	Find differential coefficient or $\frac{dy}{dx}$ for the following functions **(For Implicit Function)** (1) $xy^3 - 3x^2 = xy + 5$ (2) $e^{xy} + y\ln x = \cos 2x$ (3) $(cosx)^y = (siny)^x$
Math: Type 5	Find differential coefficient or $\frac{dy}{dx}$ for the following functions **(By using Parametric formula)** 1. $x = a(t + \sin t)$, $y = a(1 - \cos t)$ 2. $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$ 3. $x = t - \sqrt{1-t^2}$, $y = e^{\sin^{-1} t}$
Math: Type 6	Find the differential coefficient $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}$ of the following functions. Also, find the value of $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}$ at $x = 2$ this point. (1) $y = \frac{1+x+x^2+x^3+x^4}{x^3}$ (2) $y = (1-x)(1+x)(1+x^2)$ (3) If $y = 3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$ (4) If $y = 3e^{2x} + 2e^{3x}$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

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Math: Type 7	Find differential coefficient or $(\frac{dy}{dx})_n$ *or* y_n for the following functions **(By using Successive Differentiation)** 1. $y = x^n$ 2. $y = e^{ax}$ 3. $y = \cos(ax+b)$ 4. $y = \sin(ax+b)$
Math: Type 8	Find differential coefficient or $(\frac{dy}{dx})_n$ *or* y_n for the following functions **(By using Leibnitz Theorem)** (1) If $y = \tan^{-1} x$ then show that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + (n^2+n)y_n = 0$ (2) If $y = (\sin^{-1} x)^2$ then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0.$

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Chapter 3: Partial Differentiation

Theory/ Definitio	Partial differential coefficient and related notations, symmetric function, homogeneous function, and Euler theorem for homogeneous function.
Math: Type 1	Find the values of u_x, u_y, u_z, u_{xx}, u_{yy}, u_{zz}, u_{xy}, u_{yz}, u_{zx}, u_{xyz}, u_{yyz}, u_{xxz} etc. for the following functions a) $u(x,y,z)=3x^4yz-5y^4z+17z-99xy^6-3\cos z+125$ b) $u(x,y,z)=e^x(\sin xy+\cos z)$ c) If $u=e^{xyz}$ then show that $\frac{\partial^3 u}{\partial x\partial y\partial z}=\left(1+3xyz+x^2y^2z^2\right)e^{xyz}$
Math: Type 2	a) Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=2u$ for $u(x,y,z)=\left(x^2+y^2+z^2\right)$. b) If $u=sin^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right)$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$ c) If $u=\log\left(x^2+y^2\right)$ then show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$. Also verify $\frac{\partial^2 u}{\partial y\partial x}=\frac{\partial^2 u}{\partial x\partial y}$.
Math: Type 3	**Apply the Euler's theorem, solve the problems** a) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$. b) If $u=Sin^{-1}\frac{x^2+y^2}{x+y}$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=tanu$ c) If $u=Sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{4}Sin2u$

Chapter 4: Application of Differentiation

Theory/ Definitio	Write down the short note about the Tangents and Normals; Stationary point, Inflation point, Sadel point, Critical Point, maximum and minimum value of a function.

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Math: Type 1

In Rate of Change: -

Q1. A city's population is modelled as $P(t) = 2t^2 + 10t + 200$ persons (t is the number of years since 2000). What would be the average rate at which the population is changing in 2005?

Q2. The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of an edge is 10 centimeters ?

Math: Type 2

In Tangents and Normal: -

Q1. Consider the curve given by $y = f(x) = x^3 - x + 3$

- **a.** Find the equation of the line tangent to the curve at the point (1,3)
- **b.** Find the line normal to the curve at the point (1,3)

Q2. Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are, **(a)** Parallel to $x-$axis **(b)** Parallel to $y-$axis.

Math: Type 3

Find the Maximum and Minimum values of the following functions,

Q1. $f(x) = 2x^3 - 3x^2 - 12x$ **Q2.** $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$

Q3. Show that the maximum value of $x^2 \ln\left(\frac{1}{x}\right)$ is $\frac{1}{2e}$.

Q4. A liquid form of antibiotic manufactured by a pharmaceutical firm is sold in bulk at a price 200 tk. Per unit. If the total production cost (in taka) for x units is

$$C(x) = 500000 + 80x + 0.003x^2$$

and if the production capacity of the firm is at most 30,000 units in a specified time, how many units of antibiotic must be manufactured and sold in that time to maximize the profit?

Q5. A factory manufactures ball-pens and all products are sold at Tk. 10 per piece. If for manufacturing x pieces of ball-pens total cost, $c(x) = 30 + 2x + 0.02x^2$.

(a) How many pieces to be produced for maximum income?

(b) What will be the maximum income?

Chapter 5: Various Theorem & its Application

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Theory/ Definition	Statement of Rolle's Theorem, Mean value Theorem & Taylor series Theorem.
Math: Type 1	1. Verify Rolle's Theorem for $f(x) = x^2 - 3x + 2$ in the interval (1,2). 2. Verify Rolle's Theorem for $f(x)$ = (x - 1) (x - 2) (x - 3) in the interval (1, 3). 3. Prove the validity of the Rolle's theorem for the function $f(x) = 1-(x-1)^{\frac{2}{3}}$ in the interval [0,2].
Math: Type 2	1. Ascertain the validity of Mean value theorem for the function $f(x)$ = x (x - 1) (x - 2) on the interval [0, 1]. 2. Find the value of c of the mean value theorem $f(b)$ - $f(a)$ = $f'(c)$ (b - a) if $f(x)$ = (x - 1) (x - 2) (x - 3) on interval (0, 4). 3. Justify the validity of the mean value theorem for the function $f(x)$ = (x - 1) (x - 2) (x - 3) in the interval (0, 4). 4. Find the value of 'c' in the mean value theorem $f(b)$ - $f(a)$ = (b - a) $f'(c)$ if $f(x)$ = x2, a =0, b = 1
Math: Type 3	1. Find the Taylor Series for $f(x) = e^{-x}$ about $x = -4$. 2. Find the Taylor Series for $f(x) = x^3 - 20x + 6$ about $x = 3$. 3. Find the Tylor series for $f(x)$ = $cosx$ about x =3. 4. Find the Tylor series for $f(x)$ = lnx about = 2
Chapter 6: Integration	
Theory/ Definition	Write down the short note about the Integration Graphically.
Math: Type 1	Solve the following integral **(By using normal integration formula)** (1) $\int (5x^3 + \frac{1}{x})dx$ (2) $\int (\frac{1}{2\sqrt{x}} + e^{9x})dx$ (3) $\int \frac{x^2 + 2x - 1}{\sqrt{x}}dx$ (4) $\int (x+2)(x+3)^2\,dx$ (5) $\int \sin^2 x dx$ (6) $\int \tan^2 x dx$ (7) $\int \sqrt{1-\sin 2x}\,dx$ (8) $\int \sqrt{1+\sin x}\,dx$ (9) $\int \frac{dx}{1+\sin x}$

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Math: Type 2

Solve the following integral **(By using Method of Substitution formula)**

(1) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ (2) $\int \frac{(1+x)e^x}{\cos^2(xe^x)} dx$ (3) $\int \frac{e^{m\tan^{-1} x}}{1+x^2} dx$ (4) $\int \frac{dx}{e^x+1}$

Math: Type 3

Solve the following integral **(By using Method of Integration by Parts formula)**

(1) $\int xe^x dx$ (2) $\int x^2 \cos x dx$ (3) $\int \frac{\ln(\ln x)}{x} dx$

Math: Type 4

Integrals of Some Particular Functions

(1) $\int \frac{dx}{x^2-16}$ (2) $\int \frac{dx}{4x^2+4x+5}$ (3) $\int \frac{dx}{\sqrt{x^2-7x+12}}$

Math: Type 5

Solve the following integral **(Definite Integral)**

(1) $\int_{-3}^{1} (6x^2-5x+2) dx$ (2) $\int_{0}^{\log 2} \frac{e^x}{1+e^x} dx$ (3) $\int_{0}^{\frac{\pi}{3}} (2\sin\theta - 5\cos\theta) dx$ (4) $\int_{1}^{2} (x \ln x) dx$

(5) $\int_0^1 \frac{\tan^{-1} x \, dx}{1+x^2}$ (6) $\int_0^{\pi/2} \sin^2\theta \cos^3\theta \, d\theta$ (7) $\int_0^{\frac{\pi}{4}} (\sin^3 2t \cos 2t) dt$

Chapter 7: Multiple Integral

Math: Type 1

Solve the following double integral problems,

a) (a) $\int_2^3\int_1^2 xy^2 \, dy \, dx$ (b) $\int_1^2\int_y^{3y} (3x^2+y^2) dx \, dy$ (c) $\int_1^2\int_x^{e^x} \left(\frac{x}{y}\right) dy \, dx$

Math: Type 2

Solve the following triple integral problems,

(a) $\int_0^1\int_0^{1-x}\int_0^{1-y^2} z \, dz \, dy \, dx$ (b) $\int_{-3}^3\int_0^1\int_1^2 (x+y+z) \, dx \, dy \, dz$ (c) $\int_2^3\int_0^{3y}\int_1^{yz} (2x+y+z) \, dx \, dy \, dz$

Chapter 8: Gamma & Beta Function

Theory/ Definitio

Definition of Gamma, Beta functions, Write the properties of Gamma, Beta functions, Write the difference between Gamma and Beta functions.

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Math: Type 1

Solve the problems by using the property of the Gamma function.

(a) $\Gamma(7)$ (b) $\Gamma\left(\frac{7}{2}\right)$ (c) $\Gamma\left(\frac{10}{3}\right)$ if $\Gamma\left(\frac{1}{3}\right)=\frac{27}{10}$ (d) $\Gamma\left(-\frac{7}{2}\right)$

Q1. Evaluate $\int_0^\infty e^{-x}x^6 dx$, using the property of Gamma function

Q2. Evaluate $\int_0^\infty \frac{25}{7}e^{-x}x^{\frac{7}{3}}dx$ if $\Gamma\left(\frac{1}{3}\right)=\frac{27}{10}$, using the property of Gamma function

Math: Type 2

Apply the properties of Beta function solve the following problems,

(a) $\int_0^{\frac{\pi}{2}} \sin^5\theta\cos^4\theta\, d\theta$ (b) $\int_0^{\frac{\pi}{2}} \sin^9\theta\, d\theta$ (c) $\int_0^{\frac{\pi}{2}} \sin^{\frac{3}{2}}\theta\cos^3\theta\, d\theta$

(d) $\int_0^1 x^4(1-x)^7\, dx$ (e) $\int_0^1 x^{\frac{3}{2}}(1-x)^4\, dx$

Chapter 9: Application of Integration

Math: Type 1

Real life problem:

1. A car starts from rest at $s=3m$ from the origin and has acceleration at time t given by $a=2t-5ms^{-2}$. Find the velocity and displacement of the car at $t=4s$.
2. The electric current (in mA) in a computer circuit as a function of time is $i=0.3-0.2t$. What total charge passes a point in the circuit in $0.050\,s$.

Math: Type 2

Area Related Problem:

1. Find the area of the circle $x^2+y^2=a^2$ is πa^2 square units.
2. Find the area of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab sq. units.
3. Find the area of the region enclosed by the parabola $y^2=4ax$ and $x^2=4ay$.
4. Find the area of the region enclosed by the straight line $y=2x$ and parabola $y^2=4x$

Math: Type 3

Volume Related Problem:

1. Find the volume of the solid that is obtained when the region under the curve $y=\sin x$ over the interval $x=0$ and $x=\pi$ is revolved about the x-axis.
2. Find the volume of the solid that is obtained when the region under the curve $y^2=4ax$ over the interval [1, 2] is revolved about the x-axis.

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