Electricity and Magnetism

Electricity

Electricity is the flow of charge and it is one of the basic forms of energy. It is associated with electric charge, a property of atomic particles such as electrons and protons. Electric charges can be stationary, as in static electricity, or moving, as in an electric current. Electricity can be generated from many different sources. It can be sent almost instantly over long distances and can also be stored. Moreover, it can be converted efficiently into other forms of energy. Because of this versatility, electricity is an integral part of our modern life. To know the electricity at first we need to know about charges.

Charge

Charge is a fundamental and characteristic property of the elementary particle which makes up matter. This property creates a force on one another. These properties are mainly electrons and protons.

There are two types of electric charges: positive and negative. Positively charged substances are repelled from other positively charged substances, but attracted to negatively charged substances; negatively charged substances are repelled from negative and attracted to positive. An object is negatively charged if it has an excess of electrons, and is otherwise positively charged or uncharged. The SI derived unit of electric charge is the coulomb (C).

Coulomb

A coulomb is defined as the amount of charge that flows through a given cross-section of a wire in one second if there is a steady current of one ampere in the wire.

Symbolically, it is denoted by C.

Conservation of Charge

Conservation of charge means the total net charge of any system is constant. When a glass rod is rubbed with silk, a positive charge appears on the rod and a negative charge of equal magnitude appears on the silk. So that rubbing does not create charge but merely transfers charge from one object to another, disturbing slightly by the electrical neutrality of each. In other words, no new charge is created or destroyed but slightly transferred. The net charge before and after rubbing is same. So, the conservation rule is satisfied and we say that charge is conserved.

Unit of Charge is Coulomb and normally it is denoted by C.

Quantization of Charge

Quantization of charge means the flow of charge in a conductor is not continuous but it is made up of a certain minimum electric charge. The minimum amount of charge is electron charge or electron also the magnitude of proton charge. The magnitude of this charge is 1.6×10^{-19} C.

Experimentally it is established that all free charges are integral multiples of a basic unit of charge denoted by e. Thus q on a body is always given by, $\mathbf{q = n\,e}$

Where, n (1, 2, 3 ……) is any integer, positive or negative. This basic unit of charge is the charge that an electron or proton carries. By convention, the charge on an electron is taken to be negative; therefore charge on an electron is written as –e and that on a proton as +e. The fact that electric charge is always an integral multiple of e is termed as quantization of charge.

Coulomb's law or Coulomb's inverse-square law

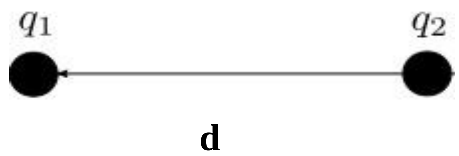
This is a law of physics describing the electrostatic interaction between charged particles. The law was first published in 1785 by French physicist Charles Augustine de Coulomb. **Electrostatics** is a branch of physics that deals with the phenomena and properties of stationary or slow-moving electric charges with no acceleration.

Coulomb's law

"The magnitude of the electrostatic force of interaction between two point charges is directly proportional to the scalar multiplication of the magnitudes of charges and inversely proportional to the square of the distance between them."

The force is along the straight line joining them. If the two charges have the same sign, the electrostatic force between them is repulsive; if they have different sign, the force between them is attractive.

Let us consider, two point charges q_1 and q_2 separated from a distance d. According to Coulomb's law



$$F \propto \frac{q_1 q_2}{d^2}$$

$$F = k \frac{q_1 q_2}{d^2}$$

Where, K is a proportional constant.

$$K = \left(\frac{1}{4\pi \varepsilon_o}\right)$$

Finally,

$$K = \left(\frac{1}{4\pi \varepsilon_o}\right)\frac{q_1 q_2}{d^2}$$

Where, ε_o is the permittivity of the free space and $\varepsilon_o = 8.854\times10^{-12}\ C^2 / N . m^2$

And

$$K = \left(\frac{1}{4\pi\varepsilon o}\right) = 9\times10^{9}\ N . m^2 / C^2$$

Limitations of Coulomb's Law

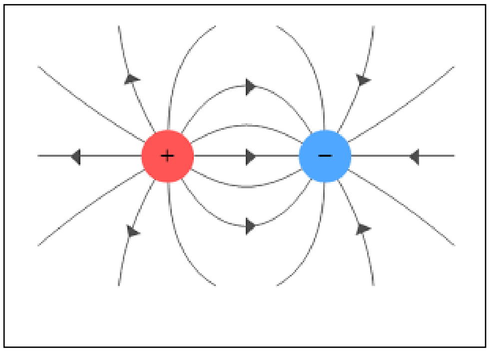
The coulomb's law fails to explain the stability of the nucleus, since, in a nucleus there are several protons, all having the positive charge. According to Coulomb's law they should repel each other. But actually, they do not push themselves apart because nucleus has a stable identity. Hence, here the Coulomb's law fails.

Electric Field

The space surrounding an electric charge within which it is capable of exerting a force on another electric charge is called field.

An **electric field** is generated by electrically charged particles and time-varying magnetic fields.

Figure show an electric field produced by a positive and negative charge.



Electric Field Strength or Intensity

The electric field strength or electric field intensity E at a point is expressed in magnitude and direction by the force per unit charge experienced by a small positive test charge q_o placed at that point.

Mathematically, the electric field strength or electric field intensity Eat the point is defined as

$$E = \frac{F}{q_0}.$$

The value of q_o should be so small that it should not disturb the electric field. In this case, the above equation can be written as

$$E = \lim_{q_0 \to 0} \frac{F}{q_o}$$

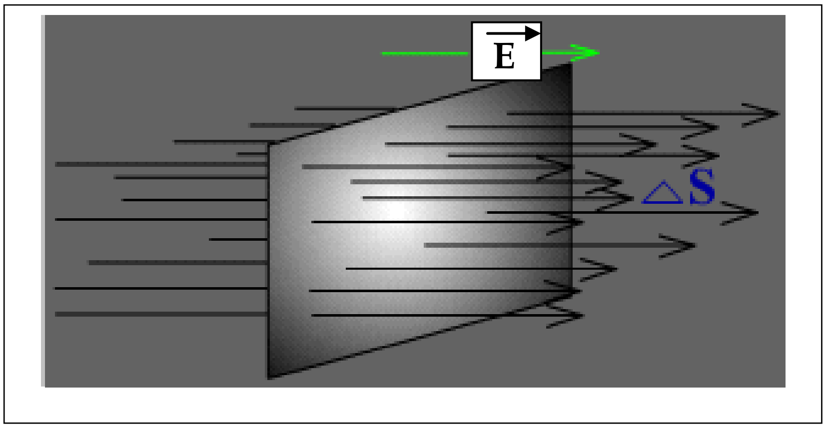
Electric Flux

Electric flux is the rate of flow of the electric field through a given area. Electric flux is proportional to the number of electric field lines going through a virtual surface. If the electric field is uniform, the electric flux passing through a surface of vector area **S** is

$$\Phi_E = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}} = ES\cos\theta,$$

Where, E is the electric field, E is its magnitude, S is the area of the surface, and θ is the angle between the electric field lines and the normal (perpendicular) to S.

Think of air blowing in through a window. How much air comes through the window depends upon the **speed** of the air, the **direction** of the air, and the **area** of the window. We might call this air that comes through the window the "**air flux".**



Gauss's Law

Gauss's law states that the flux of the electric field E through any closed surface is equal to $\frac{1}{\varepsilon_o}$ times the total charge enclosed by the surface.

Mathematically

$\Phi_E = \frac{1}{\varepsilon_o} q$

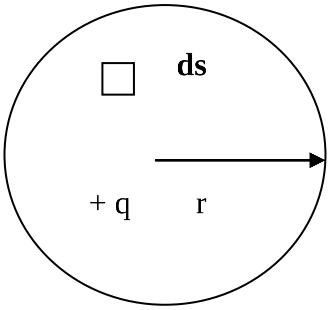


Fig: spherical surface of radius r surrounding a point charge q.

$\Phi_E . \varepsilon_o = q$ ……………………… (1)

This can be written by using integral form

$\Phi_E = \oint \vec{E} . \vec{ds}$

Therefore,

$\varepsilon_o . \oint E \ ds = q$

This is the mathematical expression of Gauss's Law in integral form. Where, **ds** is very small surface.

Coulomb's law from Gauss's law

Let us consider a spherical surface of radius of r, centered on a point charge q, from Gauss's law

$\varepsilon_o . \oint \vec{E} \ \vec{ds} = q$ …………………….(1)

In figure, both **E** and **ds** at any point on the Gaussian surface are directed radially outward. The angle between them is zero. Therefore,

$\vec{E} . \vec{ds} = E. ds . \cos\theta = E . ds . \cos 0^o = E \, ds$

Then from equation no. 1

$\varepsilon_o . \oint E \ ds = q$ …………………….(2)

E is constant for all points in the surface

$\varepsilon_o . E \oint ds = q$ ……………………. (3)

Here, the integral is simple and the area of the sphere, therefore

$\varepsilon_o . E \ (4 \pi r^2) = q$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Let us put a second point charge $\mathbf{q_o}$ at the point at which **E** is calculated. The magnitude of the force that acts on it is

$$F = q_o E \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

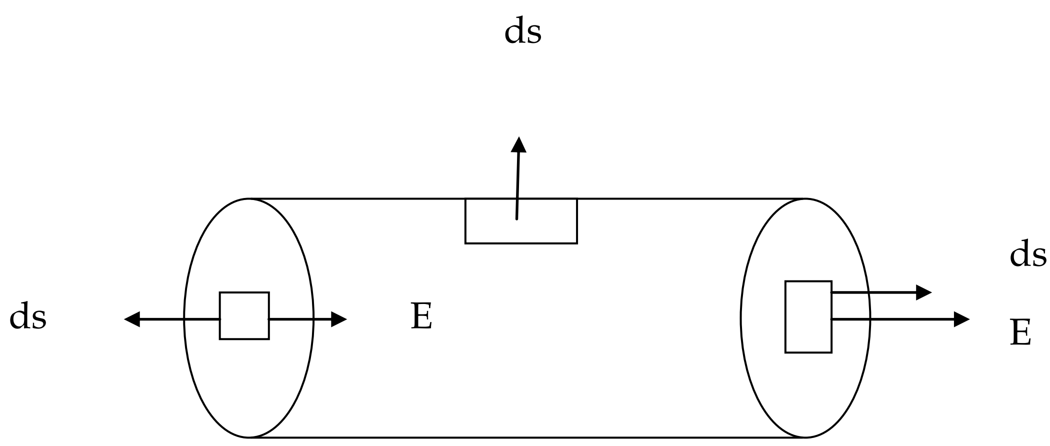
Combining equation number (4) and (5)

$$F = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q\, q_o}{r^2}$$

This is precisely coulomb's law. Thus we have deduced Coulomb's law from Gauss's law.

Application of Gauss's law

- ✓ **Calculate the electric flux for a cylindrical surface immersed in a uniform electric field. The field being parallel to the cylindrical axis.**



The flux can be written as the sum of three terms. The left cylindrical cap, the right cap and the surface.

Thus for left cap the flux

$\Phi_E = \oint \vec{E} \cdot \vec{ds} = \oint E \cdot ds \cdot \cos 180^0 = E \oint ds = - E S$

Where, $S = \pi r^2$

Similarly for the right cap the flux

$\Phi_E = \oint \vec{E} \cdot \vec{ds} = \oint E \cdot ds \cdot \cos 0^0 = E \oint ds = - E S$

Finally, for the cylindrical wall, the flux

$\Phi_E = \oint \vec{E} \cdot \vec{ds} = \oint E \cdot ds \cdot \cos 90^0 = 0$

Thus flux for the closed cylindrical surface is

$\Phi_E = -ES + ES + 0 = 0$

Electric Potential or Electric Field Potential or Electrostatic Potential

The electric potential at a point in an electric field is the work required to bring unit positive electric charge from infinity to the point.

Suppose, we have two points A and B in an electric field. We have a test charge q_o from B to A. If the work done by the agent moving the charge W_{AB}, then

Electric potential difference

A B

$V_A - V_B = \frac{w_{AB}}{q_0}$ ……………………….(1)

V_A **V_B**

Usually, point B is considered to be at infinite distance. In this case V_B is assumed to be zero.

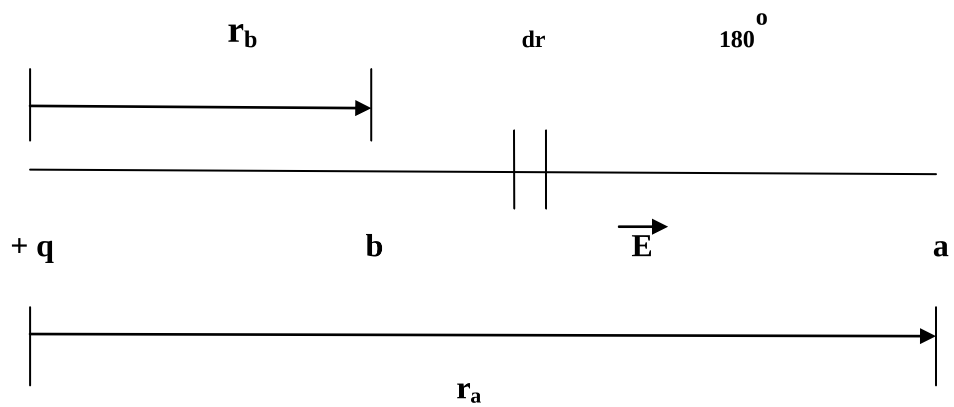
Putting $V_B = 0$ and $V_A = V$, Equation number 1 becomes

$$V = \frac{w}{q_o}$$

This equation gives the general representation of electric potential.

Potential Due to a point charge

Let us consider, two points a and b in an electrostatic field of a single isolated point charge +q .



If a unit positive charge 'q' moves from 'a' to 'b' without acceleration, then the potential difference between 'a' and 'b' is given as

$$V_b - V_a = \int \vec{E} \cdot \vec{dr} = \int E\, dr \cos 180^o$$

But, $\vec{E} \cdot \vec{dr} = E\, dr \cos 180^o = - E\, dr$

$V_b - V_a = - \int E \, . \, dr$ ………………………….. (1)

From equation (1)

$$V_b - V_a = - \int_{r_a}^{r_b} E \, dr \qquad \text{but } E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$\therefore \quad V_b - V_a = - \frac{1}{4\pi\epsilon_0} q \int_{r_a}^{r_b} \frac{1}{r^2} dr$$

$$V_b - V_a = \frac{1}{4\pi\epsilon_0} q \left[\frac{1}{r_b} - \frac{1}{r_a} \right]$$

If the point 'a' is at infinity, then

$$V_b = \frac{1}{4\pi\epsilon_0} \frac{q}{r_b} \quad \text{……………….. (1)}$$

Choosing reference point a to be infinitely distance

$r_a \rightarrow \propto$ and $V_a = 0$

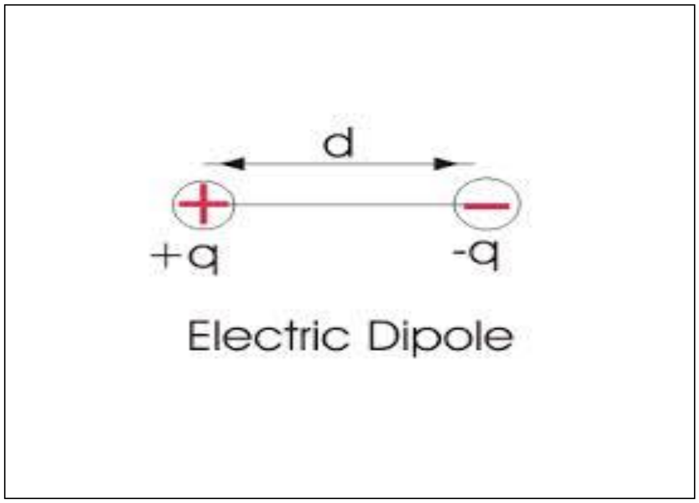
Then equation number (1)

$$V_b = V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

From the above, it is evident that for a given charge 'q', potential depends only on 'r'. Therefore, if the charge is positive, potential is positive and if the charge is negative, potential is negative.

Electric Dipole

If two equal and opposite charges are placed at a short distance, the formation is called electric dipole.



Electric potential at a point due to a dipole

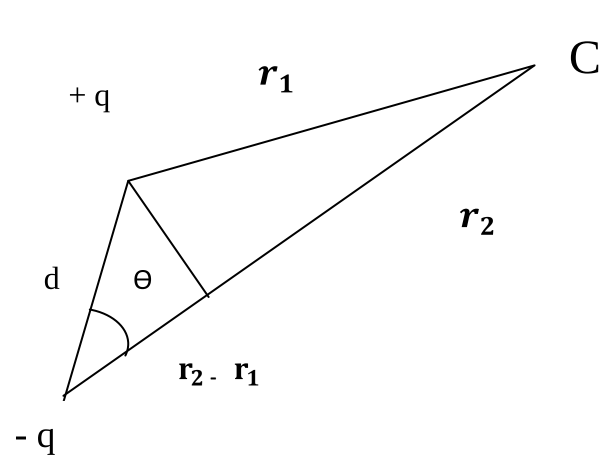
Let two equal and opposite charges +q and –q are placed at a short distance d. P is a point which is at a distance r_1 from +q distance r_2 from –q charge. We need to find the electric potential at P due to the dipole.

From the expression of electric potential at a point of distance r from a charge q is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

So, the electric potential at p due to +q charge

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r1}$$ and for – q charge



$$V_2 = \frac{1}{4\pi\varepsilon_0}\left(-\frac{q}{r2}\right)$$

So, mutual potential at P is

$$V = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r1} - \frac{q}{r2}\right)$$

$$= \frac{1}{4\pi\varepsilon_0} \; q \; \frac{r_2 - r_1}{r_1 r_2}$$

$$= \frac{1}{4\pi\varepsilon_0} \; q. \; \frac{d\cos\theta}{r_1 r_2}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r_1 r_2}$$

If r_1 and r_2 >> d, and $r_1 . r_1 === r^2$

From Figure

$$\cos\theta = \frac{r_2 - r_1}{d}$$

Then

$$r_2 - r_1 = d\cos\theta$$

Electric Dipole Moment,

p = q × d

Then

$$V = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2} \; \ldots\ldots\ldots\ldots.. (1)$$

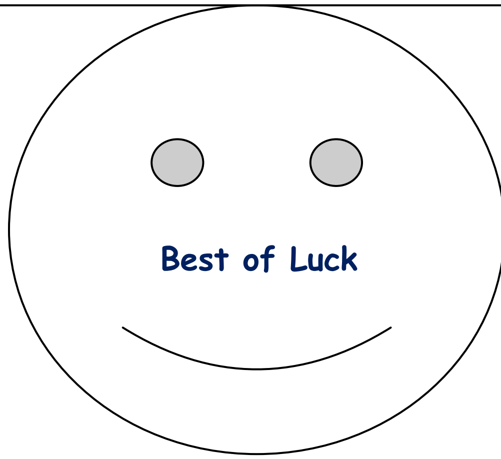
This is an expression for electric potential at a point due to a dipole.

Electric Potential Energy

We define the electric potential energy of a system of point charges as the work required assembling this system of charges by bringing them close together, as in the system from an infinite distance.

However, we assume that initial kinetic energy of the charges is zero at infinity; they are at rest at infinity. Thus, an external work done against the forces between the charges and this external work done is stored in the system as the electrical potential energy of the configuration (or arrangement) of the charges. As the forces between the charges are of two types, attractive for opposite charges and repulsive for similar charges. Therefore, the work done will be positive in the case of like charges and this work done will be negative in the case of unlike (or dissimilar) charges so that, for similar charges potential energy is positive and for dissimilar it is negative.

Take preparation for the First Class Test



Current

The amount of flow of charge per unit time is known as current. If a net charge q passes through any cross section of the conductor in time t, the current is given by

$I = \frac{q}{t}$

The unit of current is Ampere.

1 Ampere = $\frac{1 \text{ coulomb}}{1 \text{ second}}$

Therefore,

If 1 coulomb of charge passes through in any cross section of a conductor by 1 sec, the amount of current is 1 Ampere.

If the electron passes a distance l in time t into a cross sectional area A, then q = nAle, where, n is the number of electron and e is the charge of electron.

Current Density

The ratio of the current to the cross sectional area of the current – carrying conductor is known as current density. Current density J can be written as mathematically

$J = \frac{i}{A}$, where i is the current and A is the cross sectional area. The unit of current density is Ampere / meter ^{2}

Electrical Resistance or Resistance

The **electrical resistance** of an electrical conductor is the opposition to the passage of an electric current through that conductor.

Normally, it is denoted by R. The SI unit of electrical resistance is the ohm (Ω), All materials show some resistance, except for superconductors, which have a resistance of zero.

The resistance R of an object is defined as the ratio of voltage across it V to current through it I. Mathematically,

$$R = \frac{V}{I}$$

Resistivity

Electrical resistivity (also known as **resistivity**, **specific electrical resistance**, or **volume resistivity**) is an intrinsic property that quantifies how strongly a given material opposes the flow of electric current.

A low resistivity indicates a material that readily allows the movement of electric charge. Resistivity is commonly represented by theGreek letter ρ . The SI unit of electrical resistivity is the ohm·metre (Ω·m)

Mathematically, electrical resistivity ρ is defined as

$$\boxed{\rho = R\frac{A}{\ell},}$$

Where, R is the resistance, l is the length of the material and A is the cross-sectional area.

Conductivity

Electrical conductivity or **specific conductance** is the reciprocal of electrical resistivity, and measures a material's ability to conduct an electric current.

It is commonly represented by the Greek letter σ, but κ (kappa) (especially in electrical engineering) or γ (gamma) are also occasionally used. Its SI unit is Siemens per meter(S/m)

Mathematically, σ can be defined as

$$\sigma = \frac{1}{\rho}$$

Conductivity is the inverse of resistivity.

Ohm's Law

Ohm's law states that the potential difference between the ends of a conductor varies directly as the current flowing in it, provided the temperature does not change and the physical state of the conductor remains the same.

If V is the potential difference between two ends of a conductor AB and I is the current flow in it, then

$I \propto V$

$I = GV$ (where, G is the proportionality constant called conductivity.)

$I = \frac{1}{R}V$

$IR = V$

$V = IR$

Where, R is known as resistance and it is inverse of conductivity. That means if the resistance of the conductor increases the conductivity decreases. The flow of current decreases.

Series combination of resistors

Suppose the values of three resistors are respectively R_1, R_2 and R_3. These are connected in such a way as that same current flows through each. This combination of resistors is series combination. The equivalent resistance of the resistors is to be found out.

Let the potential of points A, B, C and D are respectively V_A, V_B, V_C and V_D.

Further let V › V.

Let the potential difference between the two ends of resistors are respectively V_1, V_2 and V_3. So, from the ohm's law, we get

$V_A - V_B = V_1$, $V_B - V_C = V_2$, $V_C - V_D = V_3$

If the potential difference between the two ends of the combination is V, then

$V = V_1 + V_2 + V_3$ ………… (1)

But if the equivalent resistance of the combination is R_s , then from ohm's law

We get, V = iR from equation number (1) we get

$iR_s = iR_1 + iR_2 + iR_3$

$iR_s = i\,(R_1 + R_2 + R_3)$

$R_s = R_1 + R_2 + R_3$

If there are n number of resistors connected in series

$R_s = R_1 + R_2 + R_3 \ldots\ldots\ldots\ldots\ldots + R_n$

Parallel combination of resistors

Suppose, the values of three resistors are respectively R_1 ,R_2 and R_3. One end of each resistor is connected at point A and the other ends of the resistors are connected at B so that same potential difference ($V_A - V_B$) exists between two ends of each resistor; here the potential of points A and B are respectively V_A and V_B.

Here, current i after reaching at point A, gets divided into i_1, i_2 and i_3 flowing through respective resistors R_1, R_2 and R_3 reach at point B and after combining becomes the main current i.

$i = i_1 + i_2 + i_3$ …………………… (1)

From the ohm's law, we get

$i_1 = \frac{V_A - V_B}{R_1}$, $i_2 = \frac{V_A - V_B}{R_2}$ and $i_3 = \frac{V_A - V_B}{R_3}$

If the equivalent resistance of the circuit is R, then from ohm's law

$$i = \frac{V_A - V_B}{R_P}$$

Now, inserting the values of i1, i2 and i3 in equation (1)

$$\frac{V_A - V_B}{R_P} = \frac{V_A - V_B}{R_1} + \frac{V_A - V_B}{R_2} + \frac{V_A - V_B}{R_3}$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

If the number of resistors is n

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{……….} + \frac{1}{R_n}$$

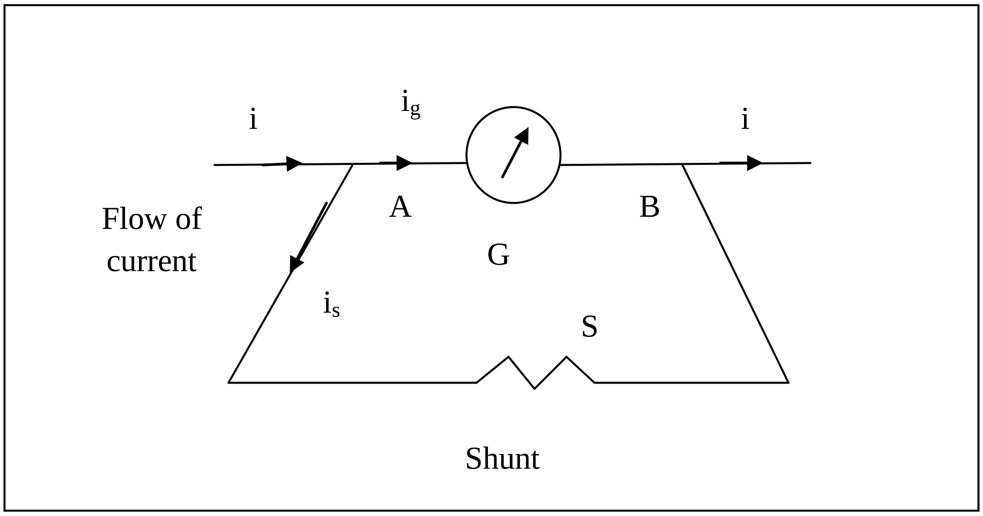
Shunt

Shunt is the practical application of parallel combination of resistances. In many occasions sensitive and sophisticated equipment like galvanometer is used in electric circuits. A low resistance is used parallel to the equipment in order to protect the equipment from damage due to the flow of high current through it that produces excessive heat. So, A low resistance which is used to sensitive and sophisticated equipment so that high current does not flow through it is called a shunt.

Derive the equation of current flowing through the shunt and the galvanometer.

Suppose, a resistance S of small value is connected between two ends A and B, parallel to galvanometer of resistance G. This S is the shunt. Let the principle current in the circuit be i. While reaching at point A, this current will be divided into two parts.

A small portion of the principle current will flow through galvanometer and large current will flow through the shunt. As a result, the galvanometer will not be damaged due to heat produced for large flow of current.



Two currents will meet at B and will form principle current again. Let this current through the galvanometer and the shunt be respectively i_g and i_s. Now, if the potential difference between the points A and B be ($V_A - V_A$) then according to ohm's law, we get

$$i_g = \frac{V_A - V_B}{G} \qquad \text{…………………….. (1)}$$

And

$$i_S = \frac{V_A - V_B}{S} \qquad \text{……………………..(2)}$$

Dividing equation number 2 by 1

$$\frac{is}{ig} = \frac{G}{S}$$

$$i_s = i_g \times \frac{G}{S} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

But, $i = i_g + i_s$ ....………………(4)

Inserting the value of i_s in this equation number 4

$$i_g \times \frac{G}{S} + i_g = i$$

$$i_g \left(\frac{G}{S} + 1\right) = i$$

$$i_g \frac{G+S}{S} = i$$

Then,

$$i_g = \frac{S \times i}{G+S} = \text{Principle current} \times \frac{\text{shunt resistance}}{\text{total resistance}}$$

or

$i = ig \frac{G+S}{S}$: Where, $\frac{G+S}{S}$ is called the power – multiplier of the shunt.

Again, putting the value of i_g in equation number 3, we get

$$i_s = \frac{S \times i}{G+S} \times \frac{G}{S}$$

$$i_s = \frac{G \times i}{G+S}$$

And

$$S = \frac{G \times ig}{i - ig}$$

Drift Speed

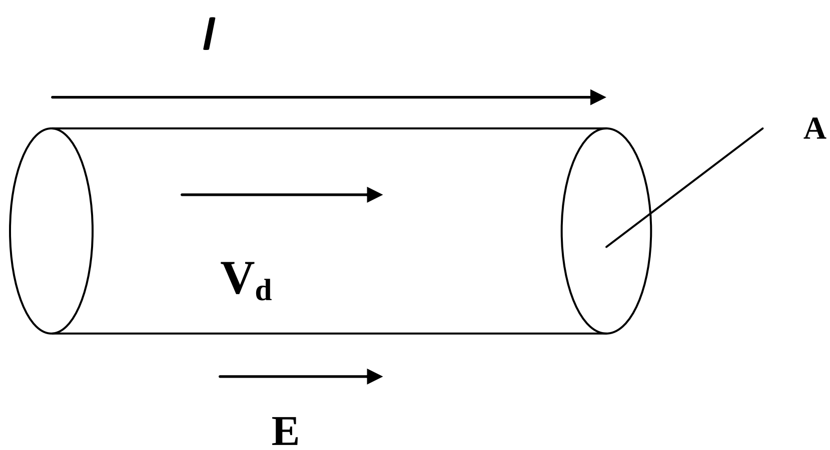
Drift velocity is the average velocity that a particle, such as an electron, attains due to an electric field.

Normally, it is denoted by V_d. When an electric field E is established in a conductor, the charge acquires a drift velocity in the direction of E. The drift velocity V_d is related to the current density (J) by

$V_d = \frac{J}{ne}$

Where, **ne** is the carrier charge density.

Relation between Drift velocity and Current density



Let us consider a section of conductor as shown in Fig. The number of conduction electrons in the wire is **nA*l*.** Where,

n = number of conduction electrons per unit volume

A*l* = volume of the wire

So, the magnitude of the charge

q = (nA*l*) e

This charge passes out the wire, through it's right end in a time t is given by

$t = \frac{l}{Vd}$

Where, V_d = Drift speed of charge

l = Length of the wire

According to the definition of current

$$i = \frac{q}{t}$$

$$= \frac{nAle}{\frac{l}{vd}}$$

$$= (n\ A\ V_d)\ e$$

Again, we know current density

$$J = \frac{i}{A}$$

$$= nV_d e$$

Finally,

$$\mathbf{V_d = \frac{j}{ne}}$$

This is the relationship between Drift velocity and Current density.

Magnetic field

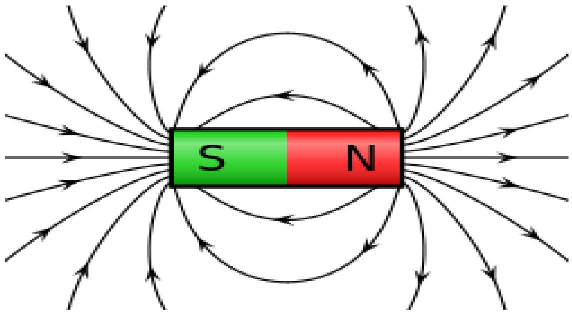
The space around a magnet or a current carrying conductor within which it is capable of attraction or repulsion is known as magnetic field.

Normally, it is denoted by B. the unit of magnetic field is tesla.

1 Tesla = $\frac{1\ N}{Amp-m}$ = 10^4 Gauss

The magnetic field exerts a force on moving charge. Magnetic field is vector quantity.

Magnetic fields are areas where an object exhibits a magnetic influence. The fields affect neighboring objects along things called magnetic field lines. A magnetic object can attract or push away another magnetic object.**Magnetic poles** are the points where the magnetic field lines begin and end.



Magnetic field of an ideal cylindrical magnet. The magnetic field is represented by magnetic field lines

Magnetic Flux

The strength of magnetic field through an area that is the number of lines of magnetic force passing through a surface area is called magnetic flux.

It is denoted by φ_B and mathematically magnetic flux can be defined as

$$\varphi_B = \int \vec{B} \cdot \vec{ds}$$

The unit of magnetic flux is weber (wb). Magnetic flux is a scalar quantity.

Magnetic moment

Magnetic moment is defined as the product of the strength of one pole of a magnet to the magnetic length.

It is denoted by M. It is a vector quantity.

If the strength of one pole of a magnet is m and the magnetic length is 2l, then the magnetic moment M is given by

$$M = m. (2l)$$

Unit of magnetic moment is A-m^2

Magnetic Force

If a positive test charge fired with a velocity V through a point P, and if a magnetic induction B is present at point P, then the magnetic force F can be written as

$$F = q_o(\vec{V} \times \vec{B})$$

$$= q_o V B \sin\theta$$

Where, θ is the angle between V and B.

Lorentz Force

If a charged particle moves through a region where both an electric and magnetic field are present, then the resultant force is given by

$$F = q_o \vec{E} + q_o(\vec{V} \times \vec{B})$$

$$= q_o[\vec{E} + (\vec{V} \times \vec{B})]$$

This resultant force is called Lorentz force.

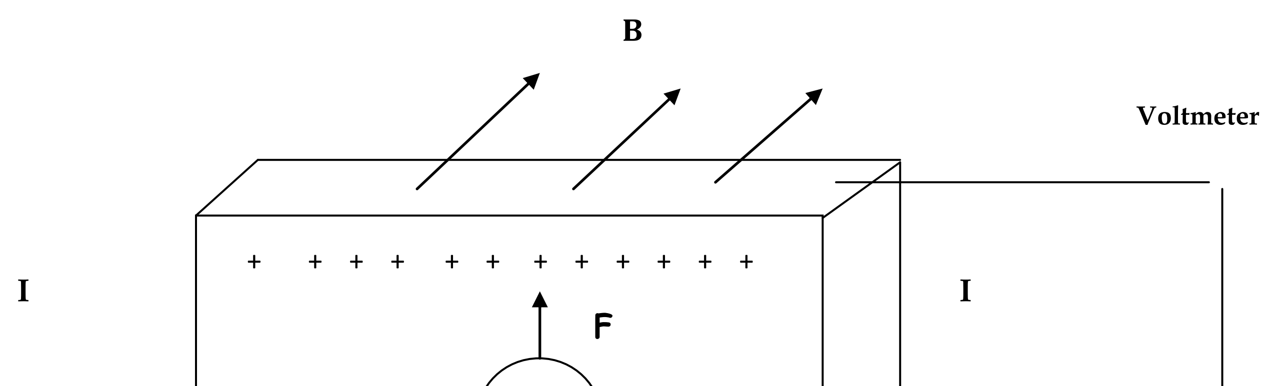
Hall Effect

The Hall Effect was discovered in 1879 by Edwin Herbert Hall while he was working on his doctoral degree at Johns Hopkins University in Baltimore, Maryland.

If a magnetic field is applied perpendicularly to the flow of current, then a potential is created normal to both that current and the magnetic field. This effect is called Hall Effect and the generated potential is called hall potential.

Determination of the number of charges in unit volume

Let us take a thin and wide strip of metallic conductor. Let the current flow along the length of this strip be I. This strip is placed in a uniform magnetic field B.



Let,

q = charge of each carrier

v = velocity of the charge

n = number of charges in unit volume

B = magnetic induction or field

E = electric field generated due to the creation of the hall voltage between the two faces

V_H = Hall Voltage

d = width of the strip

So, the electric field, $E = \frac{V_H}{d}$

Or, $V_H = Ed$ …………… (1)

Electric force acting on each charge,

$F_e = q E$ ……………. (2)

We know, magnetic force acting on each charge

$\vec{F}_m = q\,(\vec{v} \times \vec{B})$

In this experiment, B is perpendicular to v and the direction of $\vec{F}$ is along the normal on the plane containing both $\vec{v}$ and $\vec{B}$.

So,

$F_m = q\,v\,B$

In equilibrium, the magnitudes of electric and magnetic forces will be equal,

$q\,E = q\,v\,B$

$E = v\,B$…………… (3)

Inserting the value of E in equation number (1)

$V_H = v\,B\,d$ ………… (4)

Now, the electric current density

$v = \frac{J}{n\,q}$

From equation number (3)

$E = v\,B$

$E = \frac{J}{n\,q}\,B$

Finally,

$\mathbf{n} = \frac{\mathbf{J\,B}}{\mathbf{q\,E}}$

From this equation the number of charge in unit volume can be determined.

Kirchhoff's Law

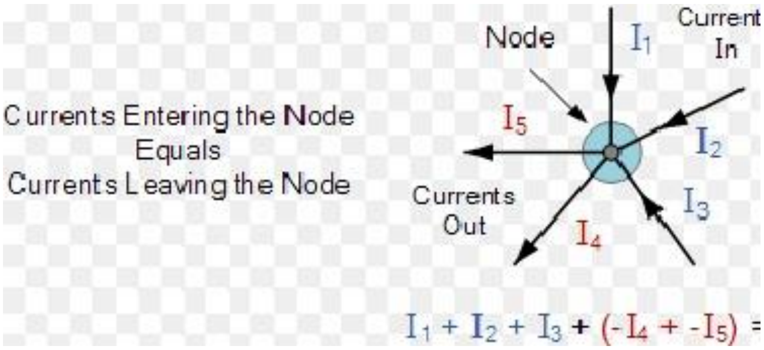
Kirchhoff's current law (First Law)

This law is also called **Kirchhoff's first law**, **Kirchhoff's point rule**, or **Kirchhoff's junction rule** (or nodal rule).

At any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node.

Or equivalently

The algebraic sum of currents in a network of conductors meeting at a point is zero.



According to Kirchhoff's law

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

Kirchhoff's voltage law (Second Law)

This law is also called Kirchhoff's second law, Kirchhoff's loop (or mesh) rule, and Kirchhoff's second rule.

The directed sum of the electrical potential differences (voltage) around any closed network is zero, or:

More simply, the sum of the emfs in any closed loop is equivalent to the sum of the potential drops in that loop, or:

The algebraic sum of the products of the resistances of the conductors and the currents in them in a closed loop is equal to the total emf available in that loop.

According to Kirchhoff's law

$$i_1R_1 + i_2R_2 - i_3R_3 = E_1 - E_2$$

$$i_1R_1 + i_2R_2 + (-i_3R_3) = E_1 + (-E_2)$$

Symbolically, the above equation can be written as $\sum i\ R = 0$

Wheatstone bridge

If four junctions are made due to the formation of a closed loop by connecting four resistors in series and if an electric cell is connected between the two opposite junctions and a galvanometer is connected between the other two opposite junctions then the circuit thus formed is called Wheatstone bridge.

Let us consider that four resistors P, Q, R and S are arranged like tetrahedral ACDF. Wheatstone bridge is formed by connecting a battery B or an electric source, a plug key K and a variable resistor X between the junctions A and D and a galvanometer G between the junctions C and D.

Let the resistance of the galvanometer be G and currents flowing through P, Q, R, S and G are respectively i_1, i_2, i_3, i_4 and i_g.

Now, applying Kirchhoff's first law respectively at points C and F, we get

$i_1 - i_3 - i_g = 0$ or $i_1 = i_3 + i_g$ ………………(1)

$i_2 + i_g - i_4 = 0$ or $i_4 = i_2 + i_g$ …………….. (2)

Again, applying Kirchhoff's second law respectively at closed loops ACFA and CDFC, we get

$i_1P + i_gG - i_2R = 0$ ……………. (3)

$i_3Q - i_4S - i_gG = 0$ ……………..(4)

But, at balanced condition of the bridge, $i_g = 0$

Under this condition, according to equations 1 and 2

$i_1 = i_3$

$i_4 = i_2$

From equation 3 and 4

$i_1P = i_2R$ ………………. (5)

$i_3Q = i_4S$ ……………….(6)

Dividing equation 5 by 6

$$\frac{i_1P}{i_3Q} = \frac{i_2R}{i_4S}$$

$$\frac{P}{Q} = \frac{R}{S}$$

According to this equation of the Wheatstone bridge, if values of any three resistors are known, then the resistance of the fourth resistor can be determined. It is called the Wheatstone bridge principle for the measurement of resistance.

Lorentz Force

If a charged particle moves through a region where both an electric and magnetic field are present, then the resultant force is given by

$$F = q_o \vec{E} + q_o (\vec{V} \times \vec{B})$$

$$= q_o [\vec{E} + (\vec{V} \times \vec{B})]$$

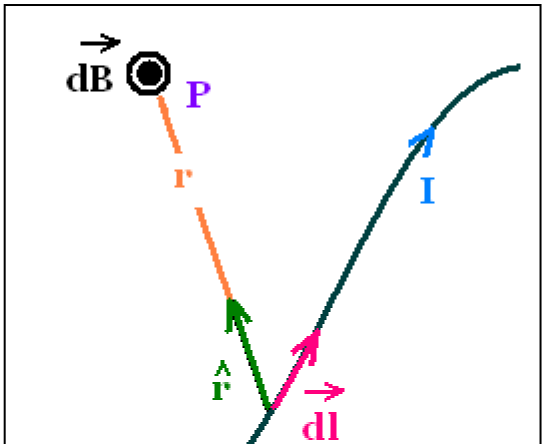
This resultant force is called Lorentz force.

Biot – Savart law

The magnetic field of a point for a long wire is directly proportional to the current, length of the conductor, inversely proportional to the square of the distance between the wire and the point, and proportional to the sign angle between them.

Let, P is the point at which we want to calculate the magnetic field dB associated with the current element.

According to Biot-Savart law, dB can be written as



$\vec{dB} \propto i$

$\vec{dB} \propto dl$

$\rightarrow$

$dB \propto \frac{1}{r2}$

$\vec{dB} \propto \sin\theta$

Now, $\vec{dB} \propto \frac{i\, dl \sin\theta}{r2}$

$\vec{dB} = \frac{\mu o}{4\pi} \frac{i\, dl \sin\theta}{r2}$

Where, $\frac{\mu o}{4\pi}$ is a constant and the value is 10^{-7} N/Amp^2 .

In vector form, the Biot-Savart law may be written as

$$\vec{dB} = \frac{i\,\mu o}{4\pi} \frac{\vec{dl} \times \vec{r}}{r3}$$

Application of Biot-Savart Law

Magnetic induction at a point due to a long straight wire carrying current

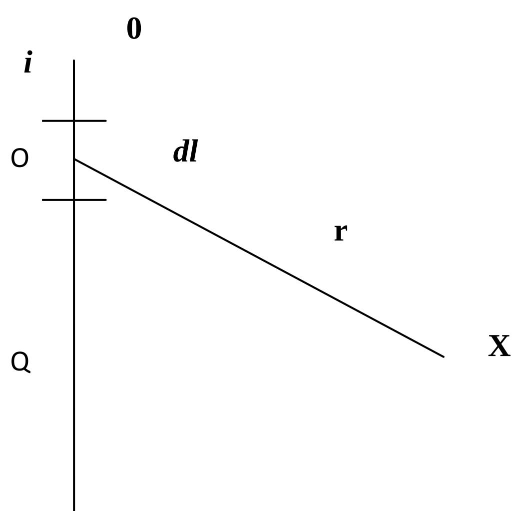
Suppose a long and straight wire and current *i* is flowing through it. Magnetic induction or magnetic field at a point X at a distance r from the wire is to be determined. Let a very small section of that wire be taken. Let its length be dl.

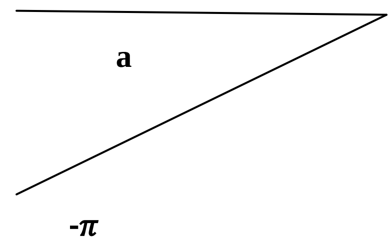
Suppose the distance from the point of that portion to the point X is r and angle Θ. So, applying Biot-Savart's lawwe get the magnetic field

$$dB = \frac{\mu o}{4\pi} \frac{i\, dl \sin\Theta}{r^2} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

Now, from the right-angled triangle ΔOQX, we get

Cosec Θ = $\frac{r}{a}$





or, r = a cosec Θ ………………….. (2)

Again, let OQ = l

Then, l = a cot Θ (cot Θ = $\frac{l}{a}$)

By differentiating, we get

dl = - a cosec²Θ dΘ ………………(3)

Now, inserting the value of r and dl in equation

$$dB = -\frac{\mu o}{4\pi}\frac{i\ a\ cosec^2\theta\ sin\theta\ d\theta}{a^2\ cosec^2\theta}$$

$$= \frac{\mu o\ i}{4\ \pi\ a}\ \sin\Theta\ d\Theta$$

If the wire PQ is infinitely long, then the total magnetic induction at point X due to the whole conductor is

$$B = \int_0^\pi -\frac{\mu o\ i}{4\ \pi\ a}\ \sin\Theta\ d\Theta$$

$$\text{Or, } B = -\frac{\mu o\ i}{4\ \pi\ a}\Big[-\cos\Theta\Big]_0^\pi$$

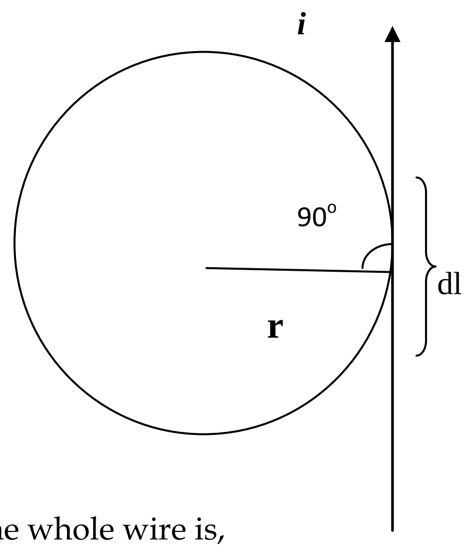
$$\boxed{\text{Finally, } B = \mu o\,\frac{i}{2\ \pi\ a}}$$

Magnetic induction at the centre of the circular coil carrying current

Suppose magnetic field is produced around a infinitely small portion dl meter of a conductor due to the flow of I amp current through it. Let the distance of a point be r meter from the mid-point of this portion of the wire. If this distance makes an angle α with the direction of current, then according to the Biot-Savart's law the magnetic induction or magnetic field at that point is

$$dB = \frac{\mu_0}{4\,\pi} \left(\frac{i\, dl\, \sin\alpha}{r^2} \right) \quad \ldots\ldots\ldots\ldots\ldots\ (1)$$

Now, suppose *i* apm current is flowing through one turn of circular wire of radius of r meter. Magnetic induction at the centre of the coil is to be determined due to the flow of current through the coil.



According to Biot-Savart's law

$$dB = \frac{\mu_0}{4\pi} \left(\frac{i\, dl\, \sin\alpha}{r^2} \right)$$

$$= \frac{\mu_0}{4\pi} \left(\frac{i\, dl\, \sin 90}{r^2} \right)$$

$$= \frac{\mu_0}{4\pi} \left(\frac{i\, dl}{r^2} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Here, $\alpha = 90^o$

So, the total magnetic induction due to the current flowing in the whole wire is,

$$B = \int dB$$

$$= \int \frac{\mu_0}{4\pi} \left(\frac{i\, dl}{r^2} \right)$$

$$= \frac{\mu_0}{4\pi} \left(\frac{i}{r^2} \right) \int dl$$

$$= \frac{\mu_0}{4\pi} \left(\frac{i}{r^2} \right) \times 2\pi r \qquad \text{(Total length of the wire} = 2\pi r)$$

$$B = \mu_0 \frac{i}{2r}$$

If current flows through n turns of coil of radius r, then the magnetic field

$$\boxed{B = \mu_0\, n \frac{i}{2r} \quad \text{Wb.m}^{-2}}$$

Ampere's Law

Ampere's states that, the line integral of magnetic field B around a closed path is equal to μ_o times the current enclosed by the path.

Mathematically,

$$\oint \vec{B}\,.\,\vec{dl} = \mu_o i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

This is Ampere's law.

Capacitor

A capacitor is an electric device consisting of two conductors separated by an insulating or dielectric medium (including air) and carrying equal and opposite charges. The conductors are called plates and may be of any shape.

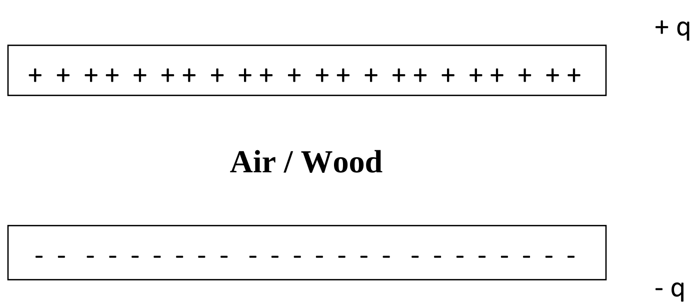


Figure: A symbolic capacitor

A potential difference will produce between the charge carrying conductors. For a fixed pair of conductors, the ratio of charges to potential difference will be a constant. This constant is called capacitance of the capacitor and denoted by C

$\frac{q}{v}$= Constant = C

Types of capacitor

There are three types of generally used capacitors depending on the size and the geometrical shapes of the plates

1. A parallel plate capacitor
2. A cylindrical capacitor
3. A spherical capacitor

Dielectric

Substances that can sustain an electric field but acts as an insulator are called dielectric. If a dielectric placed in an electric field. Induced surfaces charges appear which tend to weaken the original field within the dielectric.

Capacitance of a parallel plate capacitor

Figure shows a parallel plate capacitor, consists of two parallel conducting plates of area A separated by a distance d.

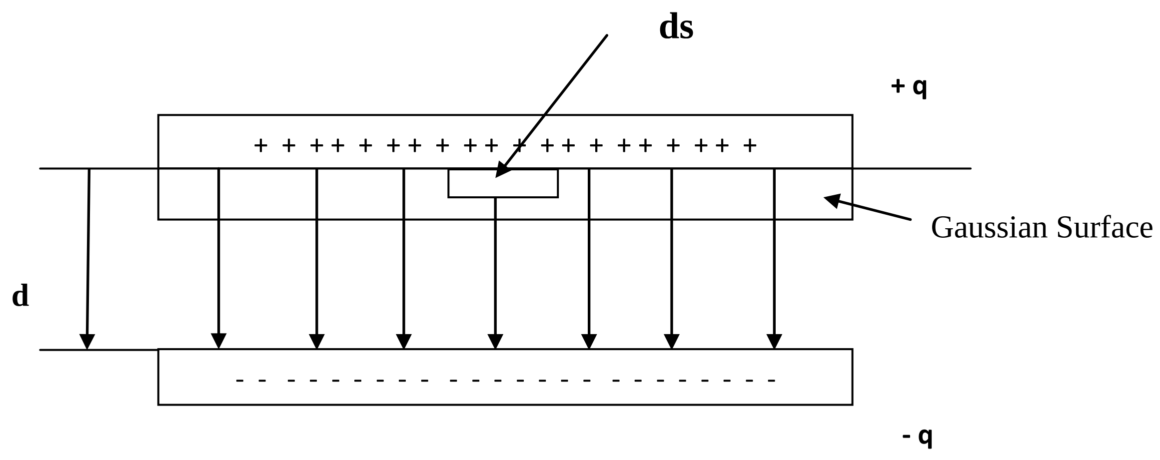


Fig: A parallel plate capacitor

The electric field strength E between the plates will be uniform, which means that the lines of force will be parallel and evenly separated. Taking E to be constant throughout the volume between the plates.

Let us consider, a Gaussian surface enclosed the charge q on the positive plate. According to Gauss's law

$$\varepsilon_o \int \vec{E} \cdot \vec{ds} = q$$

$$\varepsilon_o \int E \cdot ds = q \qquad \text{(Since, E and ds are parallel)}$$

$$\varepsilon_o E A = q \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Where, A is the area of the Gaussian surface.

The potential difference between the plates is given by

$$V = \int \vec{E} \cdot \vec{dl}$$

$$V = E \int dl \qquad \text{(Since, E and dl are parallel)}$$

$$V = E\ d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Therefore, the capacitance of a parallel plate capacitor is

$$C = \frac{q}{v}$$

$$C = \frac{\varepsilon_o E A}{Ed}$$

$$C = \frac{\varepsilon_o A}{d}$$

This is the required equation for the capacitance of parallel plate type capacitors.

Cylindrical Capacitor

A cylindrical capacitor consists of two coaxial cylinders of radius a and b and length l. We have to find out the capacitance of the device.

Let us construct a Gaussian surface, shown in the figure, having radius r. By using Gaussian Law

$$\varepsilon_o \oint \vec{E} \cdot d\vec{A} = q$$

$$\varepsilon_o \ E\,(2\,\pi\, r\, l) = q$$

$$E = \frac{q}{2\,\pi\,\varepsilon_o\ r\,l} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (1)$$

The potential difference between the plates

$$V = \oint_{+}^{-} \vec{E} \cdot d\vec{r}$$

The angle between E and dr is 0^0.

$$V = \oint_{+}^{-} \frac{q}{2\,\pi\,\varepsilon_o\ r\,l}\ dr$$

$$= \oint_{a}^{b} \frac{q}{2\,\pi\,\varepsilon_o\ r\,l}\ dr$$

$$= \frac{q}{2\,\pi\,\varepsilon_o\ l} \oint_{a}^{b} \frac{1}{r}\, dr$$

$$= \frac{q}{2\,\pi\,\varepsilon_o\ l} \quad \text{In}\,(b/a)$$

Capacitance

$$C = \frac{q}{v}$$

$$\mathbf{C} = \frac{(\ 2\,\pi\,\varepsilon_o\ )\,l}{\ln\left(\frac{b}{a}\right)}$$

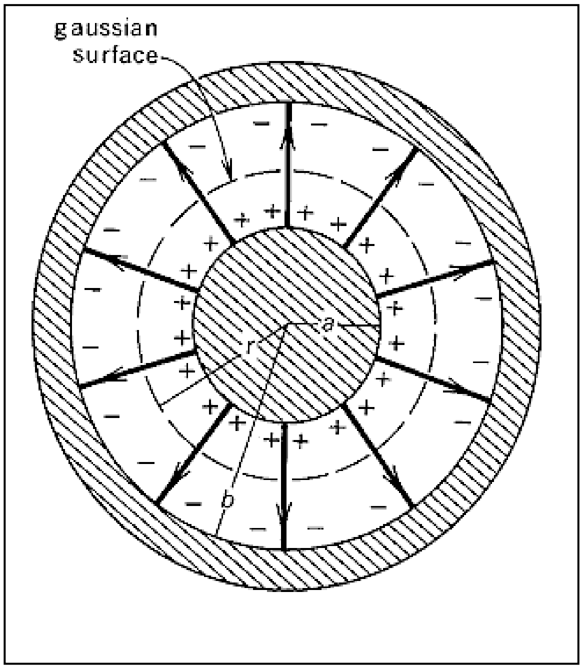


Figure: A cross sectional diagram of a cylindrical capacitor.

This is the required expression.

Spherical capacitor

Let us consider, a central cross section of a capacitor that consist of two concentric spherical shells of radius a and b. Let us draw, a Gaussian Surface of radius r concentric with two shells.

According to Gaussian Law

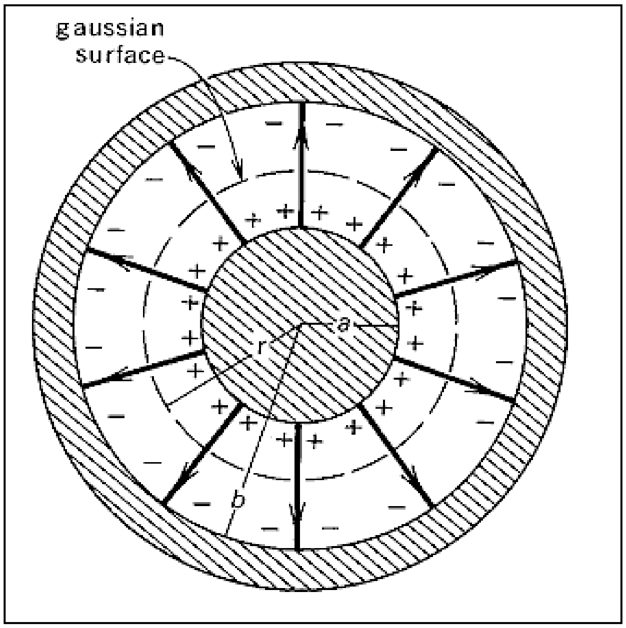


Figure: A cross section of a spherical capacitor

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

$$\varepsilon_0 \ E\,(4\pi r^2) = q$$

$$E = \frac{q}{4\pi r^2 \varepsilon_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The potential difference between the shells

$$V = \oint_{+}^{-} \vec{E} \cdot \vec{dr}$$

The angle between E and dr is 0^0 .

$$V = \oint_a^b \frac{q}{4\pi r^2 \varepsilon_0} dr$$

$$V = \frac{q}{4\pi \varepsilon_0} \oint_a^b \frac{1}{r^2} dr$$

$$V = \frac{q}{4\pi \varepsilon_0} \frac{b-a}{ab} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Therefore, the capacitance

$$C = \frac{q}{v}$$

$$C = 4\pi\varepsilon_o \frac{ab}{b-a}$$

This is the required expression.

Capacitance of series connected capacitors

Figure shows three capacitors are connected in series, the magnitude of charge q on each plate must be the same.

We know

$C = \frac{q}{v}$ or $v = \frac{q}{C}$

Therefore, $V_1 = \frac{q}{c_1}$, $V_2 = \frac{q}{c_2}$ and $V_3 = \frac{q}{c_3}$

$$V = V_1 + V_2 + V_3$$

$$= \frac{q}{c_1} + \frac{q}{c_2} + \frac{q}{c_3}$$

$$= q\left(\frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}\right)$$

Capacitance in combination

$$C = \frac{q}{v} = \frac{q}{q\left(\frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}\right)}$$

$$\frac{1}{C} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}$$

The equivalent series capacitance is always less than the smallest capacitance in the chain.

Capacitance of parallel connected capacitors

Figure shows three capacitors are connected in parallel. We need to find out the single capacitance C is equivalent to this combination.

The potential difference across each capacitor in fig. will be the same because all of the upper plates are connected together with terminal a whereas all of the lower plates with b.

We know

$C = \frac{Q}{V}$

$Q = CV$

Therefore

$Q_1 = C_1 V_1$, $Q_2 = C_2 V_2$ and $Q_3 = C_3 V_3$

Again

$Q = Q_1 + Q_2 + Q_3$

$= (C_1 + C_2 + C_3)\ V$

The equivalent capacitance C is

$C = \frac{Q}{V} = C_1 + C_2 + C_3$

The result can easily be extended to any number of parallel connected capacitor.

Mathematical Problems

1. Let the total positive and the total negative charges in a copper penny be separated to a distance such that their force of attraction is 1.0 lb (4.5 Newton). How far apart must they have?

(Charge of the copper penny is 1.3×10^{5} coulomb)

2. At one time the positive charge in the atom was thought to be distributed uniformly throughout a sphere with a radius of about 1.0×10^{-10} meter that is throughout the entire atom. Calculate the electric field strength at the surface of a gold atom (Z = 79) on this assumption. (Neglect the effect of the electrons.)

3. Calculate the electric field intensity at a point 1m from the charge 100C in air.

4. 20 C charges is given to a sphere of radius of 0.50 m. Find the electric potential at a distance of 0.50 m and 0.8 m from the centre of the sphere.

5. What must the magnitude of an isolated positive point charge be for the electric potential at 10 cm from the charge to be +100 volts?

6. What is the electric potential at the surface of a gold nucleus? The radius is 6.6×10^{-15} m and the atomic number Z=79.

7. What is the potential at the center of the square in Fig

Assume

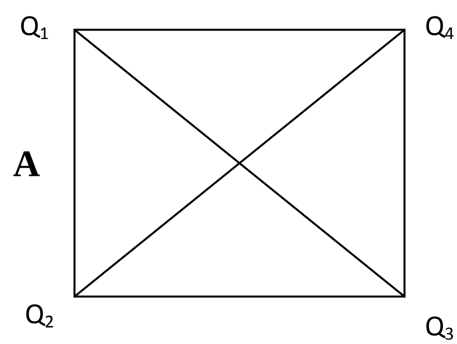
$Q_1 = +1.0\times10^{-8}$ Coulomb

$Q_2 = -2.0\times10^{-8}$ Coulomb

$Q_3 = +3.0\times10^{-8}$ Coulomb

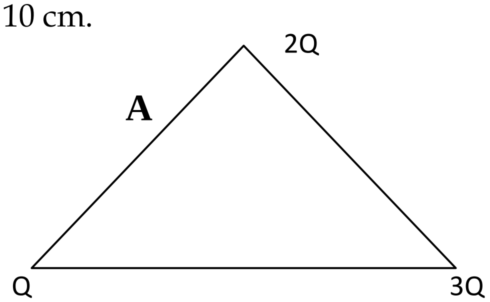
$Q_4 = +2.0\times10^{-8}$ Coulomb

A= 1.0 Meter



8. Two protons in a nucleus of U^{238} are 6.0×10^{-15} meter apart. What is their mutual electric potential energy?

9. Three charges arranged as in Fig. What is their mutual potential energy? Assume that $Q = 1.0\times10^{-7}$ Coulomb and A = 10 cm.



10. What is the magnitude of the electric field strength E such that an electron placed in the field would experience an electric force equal to its weight?

11. A pith ball of mass 0.002 kg is charged with 10^{-4} C. What is the magnitude of the electric field needed to keep the ball at rest in gravitational field?

12. A plastic ball of mass 8.4×10^{-16} kg is kept hanging in an electric field of 2.6×10^{4} N/C. Calculate the charge in the ball. ($g = 10\ ms^{-2}$)

13. The resistance of a wire of length of 0.48 m and diameter of 0.12 mm is 15 ohm. Calculate the specific resistance of the material of the wire.

14. If a wire of resistance 6 ohm is elongated three times, its area of cross-section becomes one third. What will be its final resistance?

15. The resistances of two resistors of the same material are same. If the ratio of the lengths of the two resistors is 4:9, what is the ratio of their diameter?

16. Three resistors of 5 ohm, 10 ohm and 15 ohm are arranged in series and in parallel. Determine the equivalent resistance in both the cases.

17. A series combination of two resistors of 2 and 3 ohms is connected across a cell of electromotive force of 3V. Calculate the potential difference between each resistor.

18. The specific resistance of a wire of diameter of 1 mm is 48×10^{-8} ohm. What length of the wire is needed to make a coil of 100 ohm resistance?

Mathematical Problems

1. Three capacitors have capacitances of 3, 2 and 1 μF respectively. Series combination of second and third capacitors is connected in parallel to the first capacitor. Calculate the equivalent capacitance of the circuit.

2. The area of each plate of a parallel plate capacitors is 1.5 m^2 and distance between the plates in air medium is 0.02 m. calculate the capacitance in micro farad.

3. What is the equivalent capacitance when two capacitors of capacitances 16μF and 22μF are connected in series?

4. Prove that the equivalent capacitance of 4 capacitors of equal values when connected in series is $\frac{1}{16}$ times the equivalent capacitance when connected in parallel.

5. Two parallel plates of capacitors are circular. The radius of each plate is 8×10^{-2} m and the distance between the plates is 2×10^{-3} m. Determine the amount of charge deposited on the plates when potential of 100V is applied in the capacitor.
6. Current of 5A is flowing through a conducting wire of radius of 1×10^{-3} m. Calculate the drift velocity of electron if there are 4 $\times10^{28}$ electrons per meter^3 of the wire. ($e = 1.6\times10^{-19}$ C)
7. A long straight wire is kept fixed in a vertical position and carrier a current in the vertically upward direction. A magnetic field of intensity 5 $\times10^{-5}$ T is available at a point 0.15 m away from the wire. What is the current in the wire?

8. If an electron is moving with velocity of 10^5 ms^{-1} making an angle of 60^0 with a uniform magnetic field of 0.50 T, calculate the force acting on it.

9. What shunt resistance is to be added to a galvanometer of resistance of 20 Ω so that 10 % of total current flows through the galvanometer?

10. A shunt resistance of 20 Ω is connected to a galvanometer of resistance of 100 Ω. The current in the galvanometer is 0.42 A. What is the current in the circuit?

11. The diameter of a circular coil is 31.4 $\times10^{-2}$ m and its number of turns is 400. For what amount of current flow in the coil, the magnetic field at the centre of the coil will be 4×10^{-4} Wb^{-2} .
12. What is the drift velocity for copper wire if the current density is 480 amp/cm^2.

13. Two long straight and parallel wires carry 10 A current in opposite directions. The wires are apart. Calculate the magnetic induction at the point midway between the wires. What would be the induction if the currents were in the same direction?

14. 3 A of current is flowing in a long straight wire. A positive charge, q = 7 μC is moving parallel to the wire at a distance of 0.05 m with a velocity of 250 ms^{-1}. Calculate (a) the magnitude of the magnetic field due to the flow of current in the wire and (b) the force experienced by the charge.

15. Calculate the shunt resistance that is to be added to a galvanometer of resistance of 99 ohm in order to increase the range of the galvanometer 100 times.

16. The resistance of a galvanometer is 100 ohm. What shunt resistance is to be added to it so that 99% of the total current flows through the shunt?

17. A galvanometer of resistance o0f 100 ohm can safely pass 1mA current. What amount of shunt is to be added so that 1A current can be measured?

18. A metallic strip of 0.02 m width is placed perpendicular to the magnetic induction of 6 Wbm^{-2}. The drift velocity of electron in the strip is 4×10^{-3} ms^{-1}. Determine the generated hall voltage.

Alternating Current

Alternating Current

An alternating current is a current that is periodic function of time. In other words, an alternating current is one which passes through a cycle of regular intervals. In an alternating current circuit, we are connected with a steady state current and voltage which are oscillating simultaneously without change in amplitude.

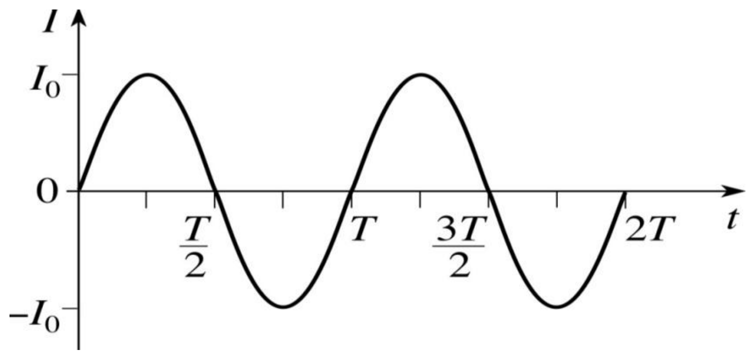
The variation of e.m.f is

$\mathcal{E} = \mathcal{E}_0 \sin \omega t$

And, the corresponding current is

$I = I_0 \sin \omega t$

Where, $\mathcal{E}_0$ and I_0 are the peak value of voltage and current respectively at any instant of time.



There are two methods in general to represent an AC circuit

i) The vector method
ii) The complex number method

The A.C network analysis becomes simple and convenient by the complex number representation.

A complex number can be written as

$Z = x + i\,y$

Where, x is the real part and iy is the imaginary part.

Electromotive force (emf)

Work done to carry a unit positive charge from positive terminal to negative terminal of a cell is called the electromotive force (emf).

Electromotive force is often denoted by $\mathcal{E}$. The unit of the electromotive force of the cell is volt.

Induced emf

If two coils are near each other, a current in one coil will set up a flux through the second coil. If this flux is changed by changing the current, an emf will appear in the second coil according to Faraday's law. This emf is called induced emf.

Mathematically, it can be written as

$$\mathcal{E} = -\frac{d\varphi}{dt}$$

Where, $\frac{d\varphi}{dt}$ is the rate of change of flux. So, the induced emf is equal to the –ve rate of change of flux.

Faraday's law of induction

Faraday's law of induction states that, the induced emf $\mathcal{E}$ in a circuit is equal to the negative rate at which the flux through the circuit is changing.

Mathematically,

$$\mathcal{E} = -\frac{d\varphi}{dt}$$

The minus sign is an indication of the direction of the induced emf

For a coil having N turns, the induced emf

$$\mathcal{E} = -N\frac{d\varphi}{dt} = -\frac{d(N\varphi)}{dt}$$

Where, N φ_B measures the so-called flux linkage in the device.

Lenz's Law

An induced electromotive force (emf) always gives rise to a current whose magnetic field opposes the original change in magnetic flux.

Lenz's law is shown with the negative sign in Faraday's law of induction:

$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t},$$

This indicates that the induced emf ($\mathcal{E}$) and the change in magnetic flux ($\partial\Phi$B) have opposite signs.

Electromagnetic induction

If an electromotive force is produced in a closed circuit temporarily for a moving magnet or a current carrying circuit, it is called electromagnetic induction.

Self Induction

If the current in a coil is changed, then an induced emf appears in that same coil. This called self induction.

Mutual Induction

If two coils are near each other, a current in one coil is changed. Then an induced emf appears in the second coil. This is called mutual induction.

Self Inductance

Let, in a coil for current I, the passing magnetic flux through the coil is ($N\varphi_B$), then

$N\varphi_B \propto I$

$N\varphi_B = LI$

Where, $N\varphi_B$ is the number of flux linkage (N being the number of turns)

From, Faraday's law, the induced emf

$$\mathcal{E} = -\frac{d(N\varphi\ )}{dt}$$

$$\mathcal{E} = -\frac{d}{dt}(LI)$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

Where, L is a constant called self inductance. That is in a coil, the change in current per second is unity, then the induced emf appears in that coil is called self-inductance or co-efficient of self-induction.

Mutual Inductance

Let, for current I_1 in primary coil, the passing magnetic flux in the secondary coil will be $N_2\varphi_2$

Then

$N_2 \varphi_2 \propto I_1$

$N_2 \varphi_2 = M I_1$……………. (1)

Where, $N_2 \varphi_2$ is the number of flux linkage (N being the number of turns in secondary coil.)

From, Faraday's law, the induced emf in the secondary coil

$$\mathcal{E} = -\frac{d}{dt}(N_2\varphi_2)$$

$$\mathcal{E} = -\frac{d}{dt}(M I_1) \quad \text{using equation number 1.}$$

$$\mathcal{E} = -M\frac{d I1}{dt}$$

Where, M is a proportional constant called mutual inductance. That is, in a primary coil, the change in current per second is unity, then the induce emf in a secondary coil is called mutual inductance.

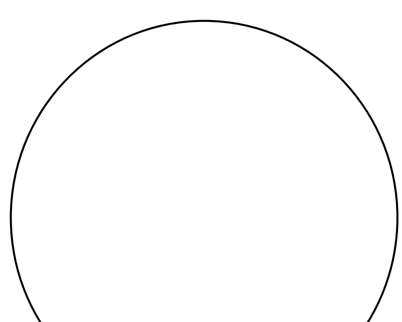
- ### Average value (mean value) of alternating current over one cycle

The value of current at any time t is given by

$I = I_0 \sin \omega t$

The average value of a sinusoidal wave over one complete cycle is given by

$$\bar{I} = \frac{\int_0^T I_0 \sin \omega t \, dt}{\int_0^T dt}$$



$$= \frac{I_0}{\omega} \left(-\cos \omega t\right)_0^T \times \frac{1}{T}$$

Where,

$$\omega = \frac{2\pi}{T}$$

$$= - \frac{I_0}{\omega} \left(\cos \omega t\right)_0^T \times \frac{1}{T}$$

$$= - \frac{I_0\, T}{2\pi} \left(\cos \omega t\right)_0^T \times \frac{1}{T}$$

$$= - \frac{I_0}{2\pi} \left[\cos \frac{2\pi}{T} T - \cos 0\right]$$

$$= - \frac{I_0}{2\pi} \left[\cos 2\pi - \cos 0\right]$$

$$= 0$$

Thus the average value of alternating current over complete cycle is zero.

➢ Average value (mean value) of alternating current during half cycle

The value of current at any time t is given by $I = I_0 \sin \omega t$

The average value of a sinusoidal wave over half cycle is given by

$$\overline{I}_{\text{first}} = \frac{\int_0^{T/2} I_0 \sin \omega t \, dt}{\int_0^{T/2} dt}$$

$$= \frac{I_0}{\omega} \left(-\cos \omega t\right)_0^{T/2} \times \frac{1}{\frac{T}{2}}$$

$$= - \frac{I_0}{\omega} \left(\cos \omega t\right)_0^{T/2} \times \frac{2}{T}$$

Where,

$$\omega = \frac{2\pi}{T}$$

$$= -\frac{2\,I_o}{T\,\frac{2\pi}{T}}\left(\cos\omega t\right)_0^{T/2}$$

$$= -\frac{I_o}{\pi}\left(\cos\frac{2\pi}{T}\frac{T}{2} - \cos 0\right)$$

$$= -\frac{I_o}{\pi}\left[\cos\pi - \cos 0\right]$$

$$= -\frac{I_o}{\pi}\left[-1 - 1\right]$$

$$= 0.636\,I_0$$

The average value for second half cycle is given by

$$\bar{I} = \frac{\int_{T/2}^{T} I_o \sin\omega t\,dt}{\int_{T/2}^{T} dt}$$

$$\bar{I}_{second} = -\,0.636\,I_0$$

The average value of alternating current (Voltage) during the 1st and 2nd half cycles are equal but opposite in sign. i.e. they are alternatively positive and negative,

- ## **Root mean square value of A.C.**

The value of current at any time t is given by

$I = I_0 \sin\omega t$

The average value of I^2

$$\overline{I^2} = \frac{\int_0^T I_o^2 \sin^2 \omega t \, dt}{\int_0^T dt}$$

$2 \sin^2 A = 1 - \cos 2A$

$$= I_o^2 \times \frac{1}{T} \times \int_0^T \sin^2 \omega t \, dt$$

$$= I_o^2 \times \frac{\omega}{2\pi} \times \int_0^T 2 \, \frac{\sin^2 \omega t}{2} \, dt$$

$$= \frac{I_0^2}{2} \times \frac{\omega}{2\pi} \times \int_0^T (1 - \cos 2\omega t) \, dt$$

$$= \frac{I_0^2}{2} \times \frac{\omega}{2\pi} \times \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T$$

$$= \frac{I_0^2}{2} \times \frac{\omega}{2\pi} \times \left[T - \frac{\sin 2.\omega T}{2\omega}\right]$$

$$= \frac{I_0^2}{2} \times \frac{\omega}{2\pi} \times \left[\frac{2\pi}{\omega} - \frac{\sin 2.\omega 2\pi/\omega}{2\omega}\right]$$

$$= \frac{I_0^2}{2} \times \frac{\omega}{2\pi} \times \left(\frac{2\pi}{\omega} - 0\right)$$

$$= \frac{I_0^2}{2}$$

Root mean square value of the alternating current

$$I_{r.m.s} = \sqrt{\frac{I_o^2}{2}}$$

$$= \frac{1}{\sqrt{2}} \times \text{Peak Value}$$

$I_{r.m.s}$ is also called the virtual value. It is represented by I_v

- ***Circuit containing pure inductance only***

Figure shows a circuit containing an inductor of self inductance L with an alternating voltage $\mathcal{E} = \mathcal{E}_0 \sin \omega t$, applied across it. This voltage is the imaginary part of the complex number $\mathcal{E}_0 e^{i\omega t}$.

Thus in complex number representation we can write

$$\mathcal{E} = \mathcal{E}_0 e^{i\omega t} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Let, I be the instantaneous current in the circuit. Further let, $\frac{dI}{dt}$ be the rate of change of current, then the induced emf in the inductor is given by

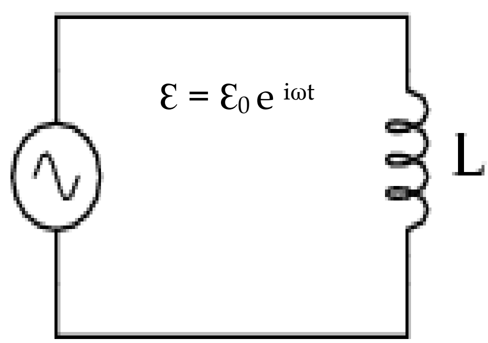
$$\mathcal{E}_{induced} = - L \frac{dI}{dt} \quad \ldots\ldots\ldots\ldots. (2)$$

The negative sign indicates that the induced emf opposes the change of current. To maintain the current flow through the inductor, the applied voltage must be equal and opposite to the induced voltage. Thus the emf equation of the circuit

$$\mathcal{E} = - \mathcal{E}_{induced} = L \frac{dI}{dt}$$

$$\mathcal{E} - L \frac{dI}{dt} = 0$$

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L} = \frac{\mathcal{E}_0 e^{i\omega t}}{L} \quad \ldots\ldots\ldots (3)$$



Integrating equation number 3, we get

$$\int dI = \int \frac{\mathcal{E}_0 e^{i\omega t}}{L} dt$$

$$I = \frac{\mathcal{E}_0}{L} \frac{1}{i\omega} e^{i\omega t}$$

Now,

$$\frac{1}{i} = - i^2 / i = - i = e^{-i\pi/2}$$

Since, $i^2 = -1$ and

$$e^{-i\frac{\pi}{2}} = \cos\frac{\pi}{2} - i \sin\frac{\pi}{2} = 0 - i = - i$$

Therefore, $I = \frac{\varepsilon o}{\omega L} e^{\, i\, (\omega t - \pi/2)}$ ………….. (4)

The quantity ω L is called inductive reactance of the inductance and denoted by X_L

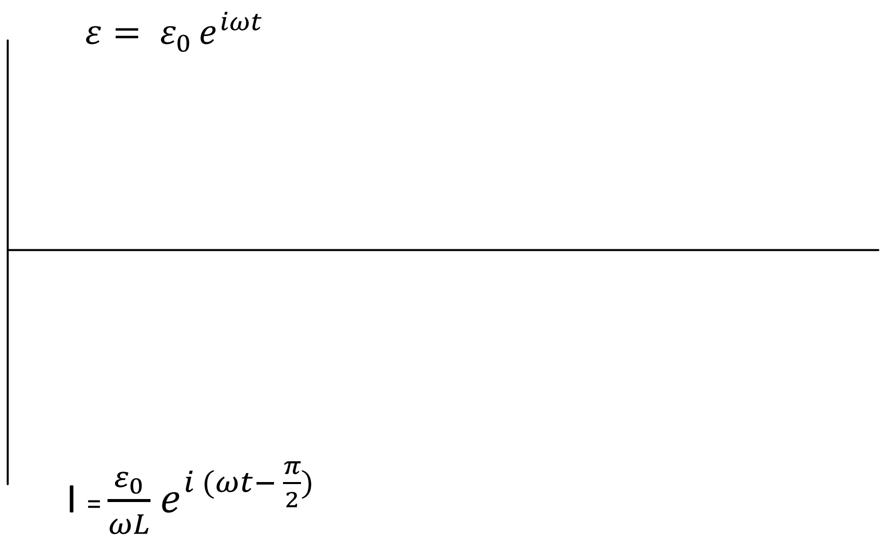
$X_L = \omega L$

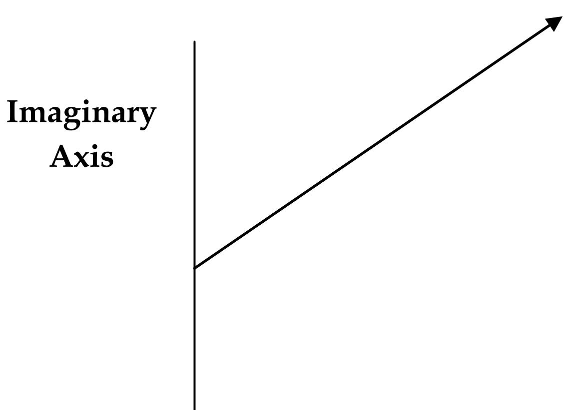
$$I = \frac{\varepsilon_0}{X_L} \; e^{\, i\, (\omega t - \frac{\pi}{2})}$$

$I = I_0 \, e^{\, i\, (\omega t - \pi/2)}$ ……….. (5)

Where, $I_0 = \frac{\varepsilon_0}{X_L}$ is the peak value of the current.

Equation number (5) shows, the current lags behind the voltage by $\frac{\pi}{2}$ (or quarter of cycle) is shown in fig.





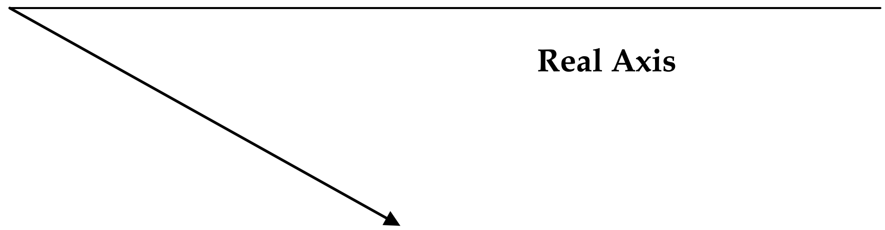


Figure shows the complex number representation

➢ Circuit containing pure capacitor only

Figure shows a circuit containing a capacitor of capacitance C with an alternating current voltage $\mathcal{E} = \mathcal{E}_0 \sin \omega t$ applied across it. This voltage is the imaginary part of the complex number $\mathcal{E}_0 e^{i\omega t}$.

Thus

$\mathcal{E} = \mathcal{E}_0 e^{i\omega t}$ …………… (1)

Capacitance

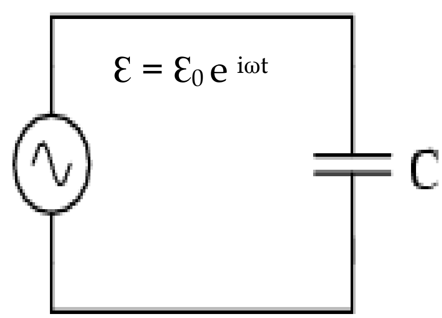
$C = \frac{Q}{V}$

$Q = CV = C\,\mathcal{E} = C\,\mathcal{E}_0 e^{i\omega t}$ …….. (2)

Now,

$$I = \frac{dQ}{dt} = \frac{d}{dt} C\,\mathcal{E}_0 e^{i\omega t} = C\,\mathcal{E}_0 i\omega\, e^{i\omega t}$$

$$I = \frac{\mathcal{E}_0}{\frac{1}{\omega C}}\, i\, e^{i\omega t} \quad \text{…………………. (3)}$$



But,

$$e^{\frac{i\pi}{2}} = \cos\frac{\pi}{2} + i \sin\frac{\pi}{2} = i$$

Here,

$$I = \frac{\varepsilon_0}{\frac{1}{\omega C}} e^{i(\omega t + \frac{\pi}{2})}$$

$$I = \frac{\varepsilon_0}{X_c} e^{i(\omega t + \frac{\pi}{2})}$$

$$I = I_0 e^{i(\omega t + \frac{\pi}{2})} \quad \ldots\ldots\ldots\ldots (4)$$

Where, $X_c = \frac{1}{\omega C}$ = Capacitive resistance, $I_0 = \frac{\varepsilon_0}{X_c}$ is the peak value of the current.

Equation number (5) shows that the current leads the voltage by $\frac{\pi}{2}$ as shown in figure.

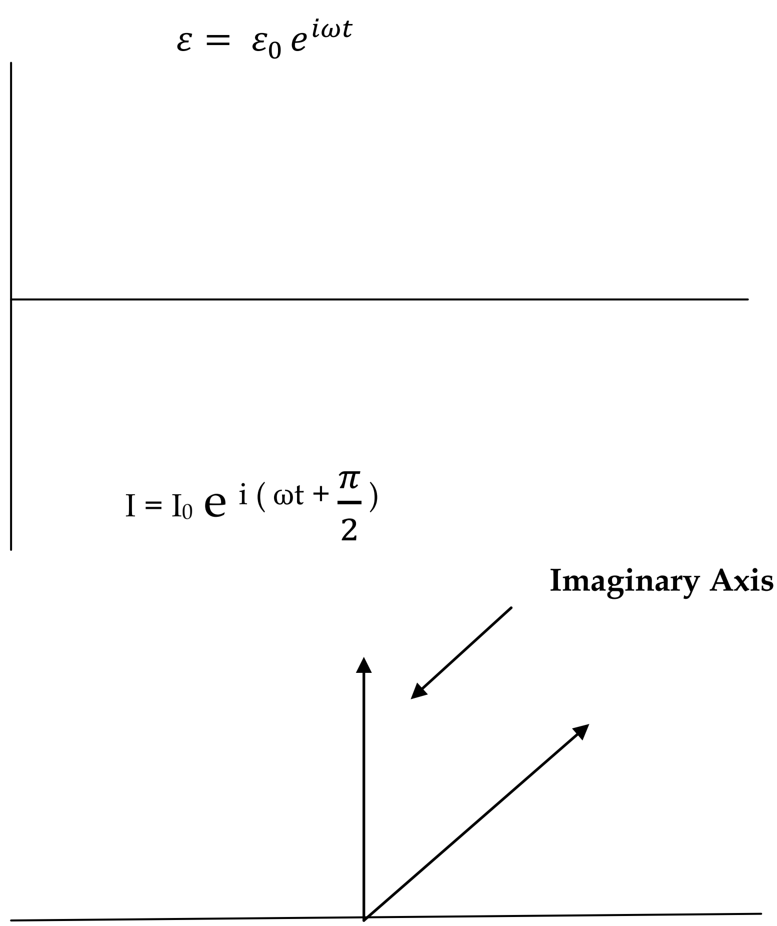
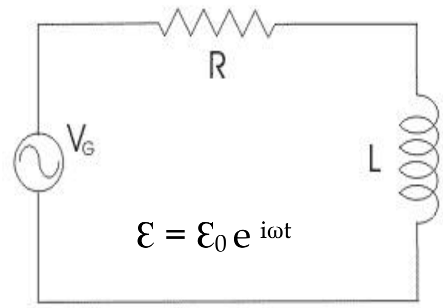


Figure shows the complex number representation

➢ Circuit containing inductor and resistor only

Instantaneous current in the circuit is given by

$$I = \frac{\text{Applied emf}}{\text{Vector impedance of the circuit}}$$



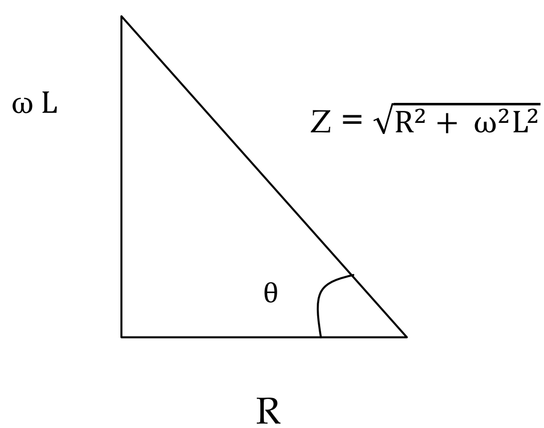
Vector impedance

$Z = R + i\,\omega\,L$

$$|Z| = \sqrt{R^2 + \omega^2 L^2}$$

Thus

$$I = \frac{\varepsilon}{Z} = \frac{\varepsilon_0 e^{i\omega t}}{R + i\,\omega\,L} \;\ldots\ldots\ldots\ldots\; (1)$$



$$\frac{1}{R + i\,\omega\,L} = \frac{(R - i\omega L)}{(R + i\omega L)\,(R - i\omega L)}$$

$$= \frac{(R - i\omega L)}{R^2 + \omega^2 L^2}$$

$$= \frac{R}{R^2 + \omega^2 L^2} - \frac{i\omega L}{R^2 + \omega^2 L^2}$$

From the figure

$$\tan\Theta = \frac{\omega L}{R}$$

$$\cos\Theta = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\sin\Theta = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}$$

Then

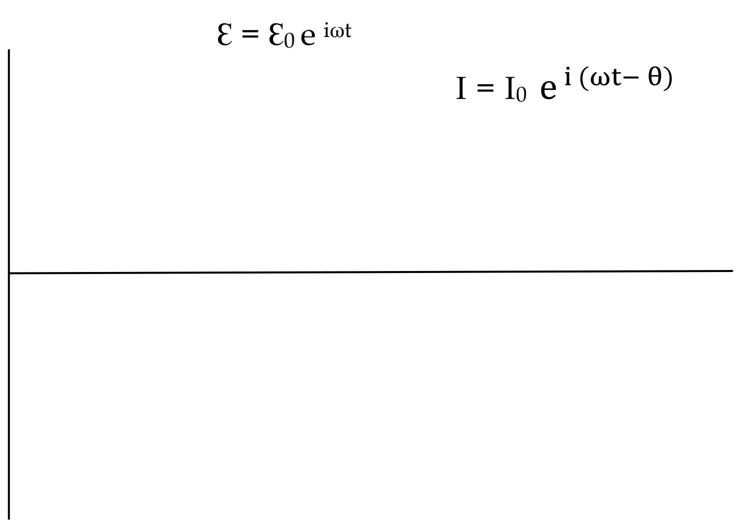
$$\frac{1}{R + i\,\omega\, L} = \frac{1}{\sqrt{R^2 + \omega^2 L^2}}\,(\cos\Theta - i\sin\Theta)$$

Finally, this equation becomes

$$I = \frac{\varepsilon_0\, e^{\,i(\omega t - \theta)}}{\sqrt{R^2 + \omega^2 L^2}} = I_0\, e^{\,i\,(\omega t - \theta)} \quad \text{..........} \ (3)$$

Where, impedance, $Z = \sqrt{R^2 + \omega^2 L^2}$ and $I_0 = \dfrac{\varepsilon_0}{\sqrt{R^2 + \omega^2 L^2}}$

Equation (3) represents the variation of current in the circuit with time as shown in fig. Also indicates that the current lags behind the applied emf in phase by an angle where $\theta = \tan^{-1}\dfrac{\omega L}{R}$



➢ Circuit containing resistor and capacitor only

Figure shows a circuit containing series combination of a resistor and a capacitor C with an alternating emf

$\mathcal{E} = \mathcal{E}_0 e^{i\omega t}$ …………… (1)

Instantaneous current I is given by

$$I = \frac{\text{Applied emf}}{\text{Vector impedance of the circuit}}$$

$$I = \frac{\varepsilon_0 e^{i\omega t}}{Z}$$

$$I = \frac{\varepsilon_0 e^{i\omega t}}{R + \frac{1}{i\omega c}}$$

$$I = \frac{\varepsilon_0 e^{i\omega t}}{R - \frac{i}{\omega c}} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

Where, R is the impedance due to the resistance and $\frac{1}{i\omega c}$ = is the impedance due to the capacitance.

$$\frac{1}{R - \frac{i}{\omega c}} = \frac{R + \frac{i}{\omega c}}{\left(R - \frac{i}{\omega c}\right)\left(R + \frac{i}{\omega c}\right)}$$

$$= \frac{R + \frac{i}{\omega c}}{R^2 + \frac{1}{\omega^2 c^2}}$$

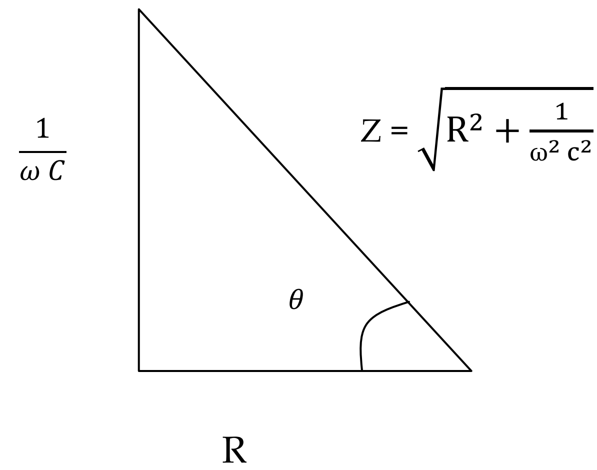
$$= \frac{R}{R^2 + \frac{1}{\omega^2 c^2}} + \frac{\frac{i}{\omega c}}{R^2 + \frac{1}{\omega^2 c^2}}$$

From the figure

$$\tan \Theta = \frac{1}{R\,\omega\,C}$$

$$\cos \Theta = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2\,c^2}}}$$

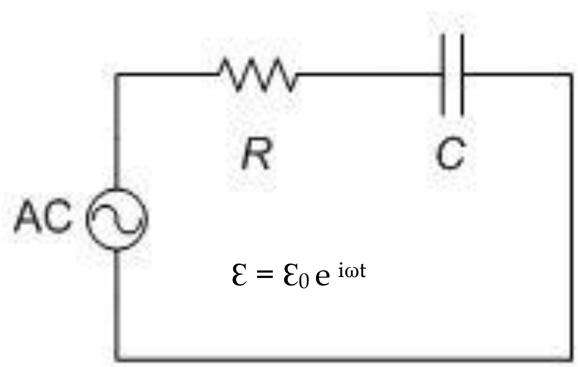
$$\sin \Theta = \frac{\frac{1}{\omega c}}{\sqrt{R^2 + \frac{1}{\omega^2\,c^2}}}$$



Now,

$$\frac{1}{R - \frac{i}{\omega\,c}} = \frac{1}{\sqrt{R^2 + \frac{1}{\omega^2\,c^2}}}\,(\cos \Theta + i \sin \Theta)$$

$$= \frac{1}{\sqrt{R^2 + \frac{1}{\omega^2\,c^2}}}\,e^{i\theta}$$



From equation number (1)

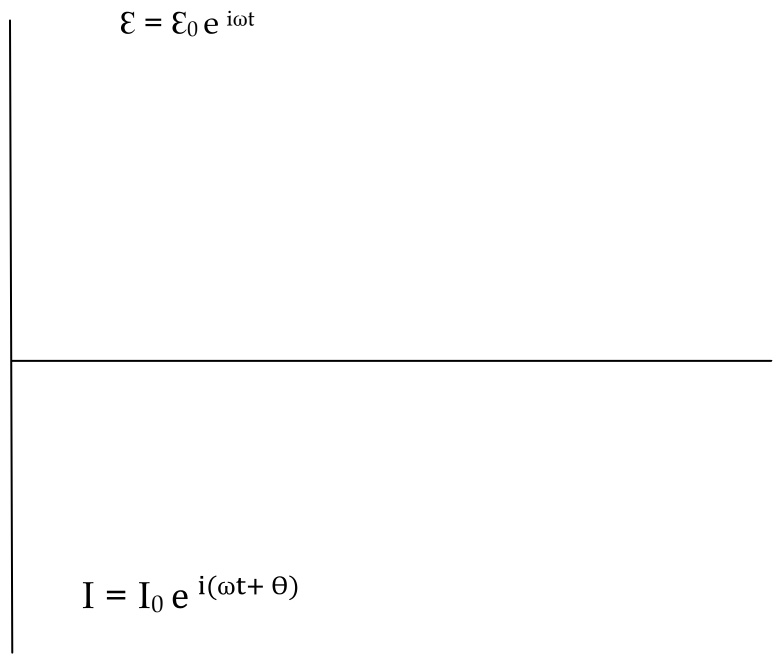
$$I = \frac{\varepsilon_o\, e^{\,i(\omega t + \Theta)}}{\sqrt{R^2 + \frac{1}{\omega^2\,c^2}}}$$

Finally, the equation becomes

$$I = I_0\, e^{\,i(\omega t + \Theta)} \quad \ldots\ldots\ldots\ (2)$$

Where, $I_0 = \frac{\varepsilon_o}{\sqrt{R^2 + \frac{1}{\omega^2\,c^2}}}$

Equation number (2) represents the variation of current with time and shows that current leads the applied voltage in phase by an angle $\theta = \tan^{-1} \frac{1}{R\,\omega\,C}$



➢ ***A.C. Circuit containing inductor, resistor and capacitor***

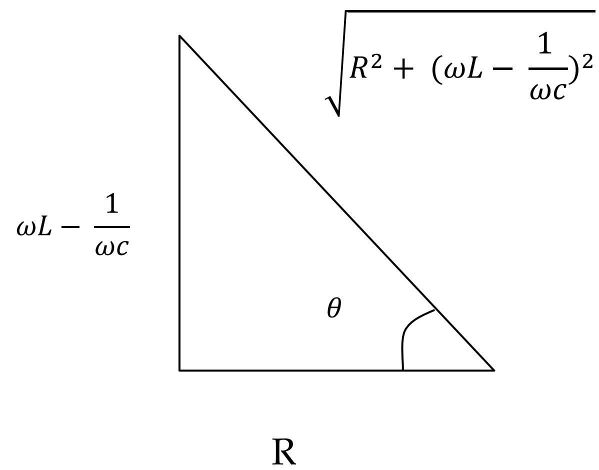
Instantaneous emf

$\mathcal{E} = \mathcal{E}_0 e^{i\omega t}$ …………… (1)

Instantaneous current

$$I = \frac{\varepsilon_0 e^{i\omega t}}{R + (i\omega L + \frac{1}{i\omega c})} = \frac{\varepsilon_0 e^{i\omega t}}{R + (i\omega L - \frac{i}{\omega c})}$$

$$I = \frac{\varepsilon_0\, e^{i\omega t}}{R + i(\omega L - \frac{1}{\omega c})} \quad \text{………… (2)}$$



$$\frac{1}{R + i(\omega L - \frac{1}{\omega c})} = \frac{R - i(\omega L - \frac{1}{\omega c})}{\left[R + i\left(\omega L - \frac{1}{\omega c}\right)\right][R - i\left(\omega L - \frac{1}{\omega c}\right)]}$$

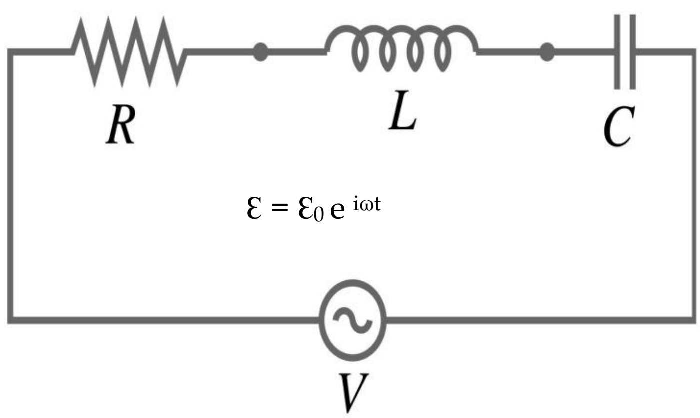
$$= \frac{R - i(\omega L - \frac{1}{\omega c})}{R^2 + (\omega L - \frac{1}{\omega c})^2}$$

From the figure

$$\tan\theta = \frac{\omega L - \frac{1}{\omega c}}{R}$$

$$\cos\theta = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega c})^2}}$$

$$\sin\theta = \frac{\omega L - \frac{1}{\omega c}}{\sqrt{R^2 + (\omega L - \frac{1}{\omega c})^2}}$$



Hence

$$\frac{1}{R + i\left(\omega L - \frac{1}{\omega c}\right)} = \frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}} (\cos \Theta - i \sin \Theta)$$

$$= \frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}} e^{-i\theta}$$

Equation number (2) becomes

$$I = \frac{\varepsilon_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}} e^{i(\omega t - \theta)}$$

Finally, the equation becomes

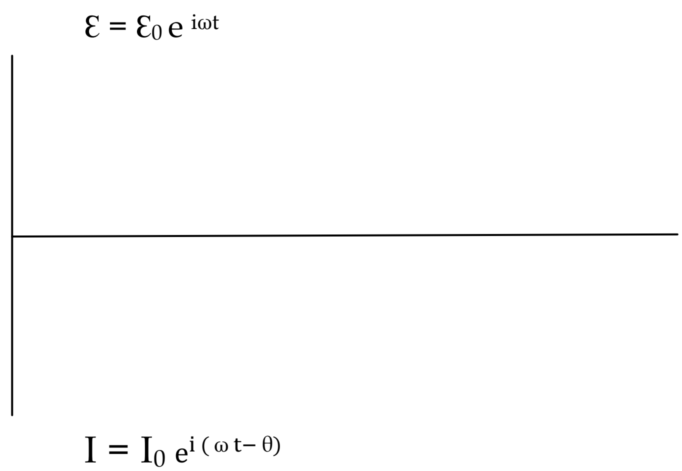
$I = I_0 e^{i(\omega t - \theta)}$ …………… (3)

Where,

$I_0 = \frac{\varepsilon_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}}$, I_0 is the peak value of the following current in the circuit.

Equation shows that, lags the applied voltage in phase by an angle θ given by

$$\theta = \tan^{-1} \frac{\omega L - \frac{1}{\omega c}}{R}$$



Mathematical Problems

1. Magnetic flux of 1.5×10^{-4} Wb is produced when current of 3A flows through a coil of 600 turns. If the current reduces to zero in 0.06 s, then calculate (i) the induced electromotive force in the coil, (ii) self-inductance of the coil and (iii) energy stored in the coil.

2. If the current in the primary coil is changed from 6 A to 1 A in 0.05 s, then electromotive force 5V in induced in the secondary coil. What is the mutual inductance of the coil?

3. A steady current of 2A flow through an inductor of 10 Henry inductance. How can electromotive force of 100V be induced in the coil?

4. Self-inductance of an inductor is 10 Henry. If current decreases from 10 A to 7 A in 6.02×10^{-2} second, find the induced electromotive force in the inductor.

5. The peal value on an alternating current in a circuit is 20A and its frequency is 50 Hz. Find the root mean square value of the current. How long will it take to reach the peak from the zero value?

6. If the equation of an alternating current is, I = 50 sin 628 t, then find (i) peak value (ii) frequency and (iii) root mean square value of the current.

7. A coil has L = 5 henry and resistance of 20 ohm. If a 100 volt emf is applied what energy is stored in the magnetic field after the current has built up to its maximum of $\frac{\varepsilon}{R}$?

8. In an LR circuit R = 3.5 ohm, L = 0.1 H. Find the current through the circuit and power factor if a 50 Hz voltage V = 220 volt is applied across the circuit.

Modern Physics

Bohr Atomic Model

In atomic physics, the Rutherford–Bohr model or **Bohr model**, introduced by Niels Bohr in 1913, depicts the atom as a small, positively charged nucleus surrounded by electrons that travel in circular orbits around the nucleus—similar in structure to the solar system, but with attraction provided by electrostatic forces rather than gravity. After the cubic model (1902), the plum-pudding model (1904), the Saturnian model (1904), and the Rutherford model (1911) came the **Rutherford–Bohr model** or just Bohr model for short (1913). The improvement to the Rutherford model is mostly a quantum physical interpretation of it. The Bohr model has been superseded, but the quantum theory remains sound.

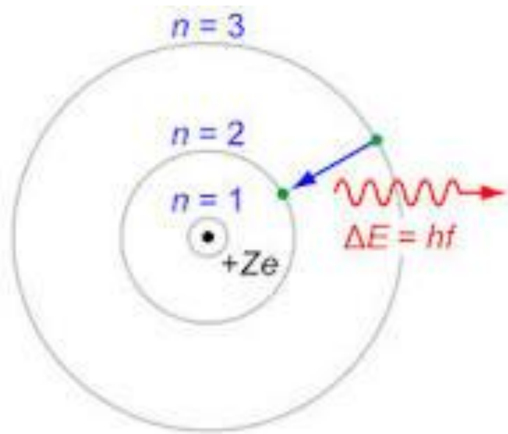
To overcome this difficulty, Niels Bohr proposed, in 1913, what is now called the Bohr model of the atom. He suggested that electrons could only have certain classical motions:

1. Electrons in atoms orbit the nucleus.
2. The electrons can only orbit stably, without radiating, in certain orbits (called by Bohr the "stationary orbits"[5]) at a certain discrete set of distances from the nucleus. These orbits are associated with definite energies and are also called energy shells or energy levels. In these orbits, the electron's acceleration does not result in radiation and energy loss as required by classical electromagnetics. The Bohr model of an atom was based upon Planck's quantum theory of radiation.
3. Electrons can only gain and lose energy by jumping from one allowed orbit to another, absorbing or emitting electromagnetic radiation with a frequency ν determined by the energy difference of the levels according to the Planck relation:

   $$\Delta E = E_2 - E_1 = h\nu \ ,$$

   Where h is Planck's constant. The frequency of the radiation emitted at an orbit of period T is as it would be in classical mechanics; it is the reciprocal of the classical orbit period:

   $$\nu = \frac{1}{T}.$$



The significance of the Bohr model is that the laws of classical mechanics apply to the motion of the electron about the nucleus only when restricted by a quantum rule. Although Rule 3 is not completely well defined for small orbits, because the emission process involves two orbits with two different periods, Bohr could determine the energy spacing between levels using Rule 3 and come to an exactly correct quantum rule: the angular momentum L is restricted to be an integer multiple of a fixed unit:

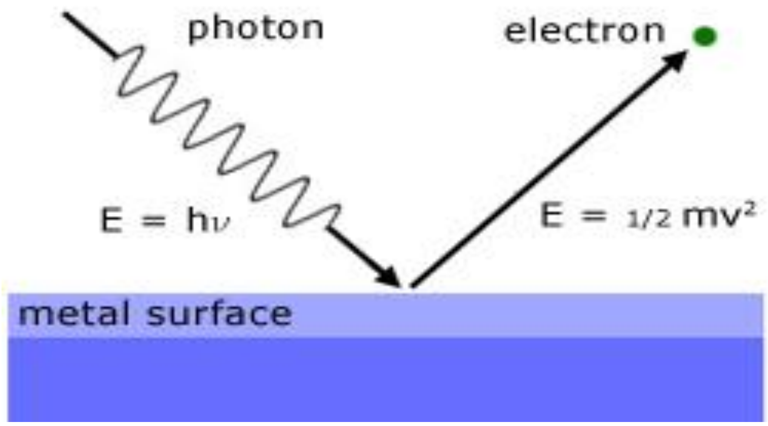
$$L = n\frac{h}{2\pi} = n\hbar$$

Where n = 1, 2, 3, ... is called the principal quantum number, and $\hbar = h/2\pi$. The lowest value of n is 1; this gives a smallest possible orbital radius of 0.0529 nm known as the Bohr radius. Once an electron is in this lowest orbit, it can get no closer to the proton. Starting from the angular momentum quantum rule, Bohr[2] was able to calculate the energies of the allowed orbits of the hydrogen atom and other hydrogen-like atoms and ions.

Photoelectric Effect

When light of appropriate frequency or wavelength is incident on a metal, the electrons are emitted from that metal. This process of emission of electrons is called photoelectric emission and the effect is called photoelectric effect.

The electrons emitted due to the influence of light are called photo electrons and current produced due to the emission is called photoelectric current.



Threshold Frequency

Threshold frequency is defined as the minimum frequency of incident light which can cause photo electric emission i.e.

This frequency is just able to eject electrons without giving them additional energy. It is denoted by v_o

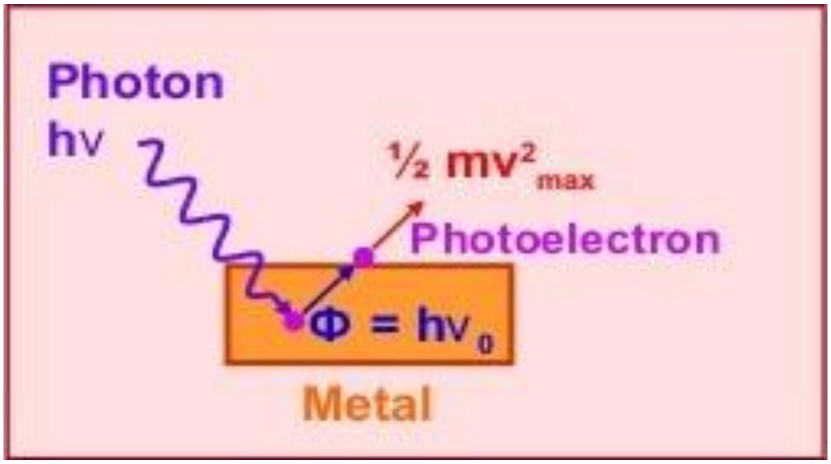
Work Function

Minimum amount of energy which is necessary to start photo electric emission is called Work Function. If the amount of energy of incident radiation is less than the work function of metal, no photo electrons are emitted.

It is denoted by Φ. mathematically, work function of a material is given by, $\Phi = hv_o$ It is a property of material. Different materials have different values of work function

Einstein Photoelectric Equation

Suppose, a photon of energy E (E = h v) in incident on an atom of a metallic sheet. There will be a collision between the photon and electron and this collision will be an elastic collision. Due to this collision, the electron will absorb whole energy of the photon. Since, the electron is bound to the nucleus, so some of the portion of the energy will be used to dissociate the electron from the nucleus; this portion of energy is called work function ($\Phi = h\, v_o$) and the electron will be released with a velocity v along with the rest of the amount of energy. If the mass of the electron is m then the kinetic energy = $\frac{1}{2}mv^2$



So, from the principle of conservation of energy

$E = \frac{1}{2} mv^2 + \Phi$

$h\,v = \frac{1}{2} mv^2 + \Phi$

$\frac{1}{2} mv^2 = hv - hv_o$

$\frac{1}{2} mv^2 = h\,(v - v_o)$

This is called Einstein's photoelectric equation.

Photoelectric Cell

The device, working on photoelectric effect, which can transform light energy into electric energy is called photoelectric cell.

Photoelectric cell of three types

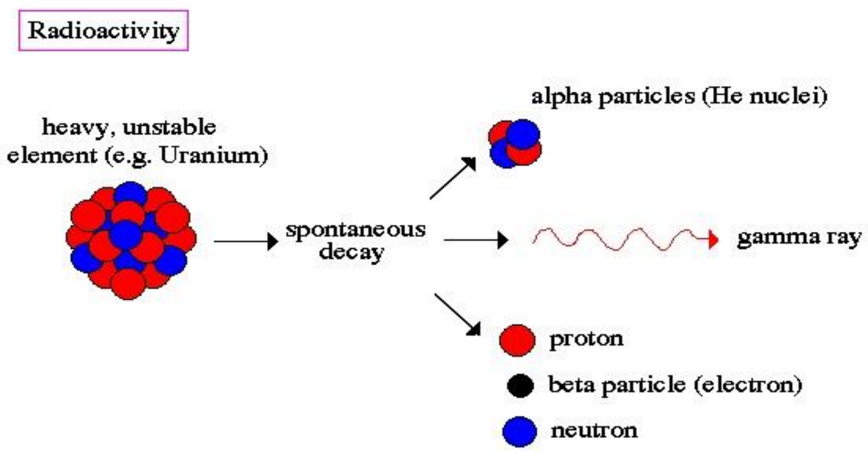
1. Photo-emission cell
2. Photo-voltaic cell
3. Photo-conductive cell

Radioactivity

Radioactive decay, also known as **nuclear decay** or **radioactivity**, is the process by which a nucleus of an unstable atom loses energy by emitting ionizing radiation. A material that spontaneously emits this kind of radiation — which includes the emission of alpha particles, beta particles, gamma rays and conversion electrons — is considered **radioactive**.

Radioactivity is of two types

i) Natural Radioactivity
ii) Artificial Radioactivity



There are two units of radioactivity

a. Curie
b. Becquerel

Some heavier elements in the periodic table exhibited radiation as found in nature is called natural radioactivity.

Applying modern techniques of artificial transmutation of elements have made it possible to produce radioactivity. Such type of radioactivity is known as artificial or induced radioactivity.

Explanation of Alpha, Beta and Gamma particle

Alpha particle

An **alpha particle** consists of two neutrons and two protons ejected from the nucleus of an atom. The alpha particle is identical to the nucleus of a helium atom. Examples of alpha emitters are radium, radon, thorium, and uranium.

Because alpha particles are charged and relatively heavy, they interact intensely with atoms in materials they encounter, giving up their energy over a very short range. In air, their travel distances are limited to no more than a few centimeters. As shown in the following illustration, alpha particles are easily shielded against and can be stopped by a single sheet of paper.

Since alpha particles cannot penetrate the dead layer of the skin, they do not present a hazard from exposure external to the body.

However, due to the very large number of ionizations they produce in a very short distance, alpha emitters can present a serious hazard when they are in close proximity to cells and tissues such as the lung. Special precautions are taken to ensure that alpha emitters are not inhaled, ingested or injected.

Beta Particle

A **beta particle** is an electron emitted from the nucleus of a radioactive atom. Examples of beta emitters commonly used in biological research are: hydrogen-3 (tritium), carbon-14, phosphorus-32, phosphorus-33, and sulfur-35.

Beta particles are much less massive and less charged than alpha particles and interact less intensely with atoms in the materials they pass through, which give them a longer range than alpha particles. Some energetic beta particles, such as those from P-32, will travel up to several meters in air or tens of mm into the skin, while low energy beta particles, such as those from H-3, are not capable of penetrating the dead layer of the skin.

Thin layers of metal or plastic stop beta particles.

Gamma Ray

A **gamma ray** is a packet (or photon) of electromagnetic radiation emitted from the nucleus during radioactive decay and occasionally accompanying the emission of an alpha or beta particle. Gamma rays are identical in nature to other electromagnetic radiations such as light or microwaves but are of much higher energy.

Examples of gamma emitters are cobalt-60, zinc-65, cesium-137, and radium-226.

Like all forms of electromagnetic radiation, gamma rays have no mass or charge and interact less intensively with matter than ionizing particles. Because gamma radiation loses energy slowly, gamma rays are able to travel significant distances.

Depending upon their initial energy, gamma rays can travel tens or hundreds of meters in air.

Gamma radiation is typically shielded using **very dense materials** (the denser the material, the more chance that a gamma ray will interact with atoms in the material) such as lead or other dense metals.

Gamma radiation particularly can present a hazard from exposures external to the body.

Property	*α-rays*	*β-rays*	*γ-rays*
1. Mass	1. 6.67×10^{-27} kg or 4 amu.	1. 9.11×10^{-31} kg	1. Negligible
2. Charge	2. + 2 units	2. – 1 units	2. 0
3. Identity	3. Helium nuclei (He^{2+})	3. Electrons	3. High energy radiations
4. Velocity	4. Nearly $\frac{1}{10}$ th that of light	4. Nearly same as that of light	4. Same as that of light
5. Effect of electric and magnetic fields	5. Deflected towards negative pole	5. Deflected towards positive Pole	5. Not deflected
6. Penetrating power	6. Small	6. Large, 100 times that of α-rays	6. Very large, 10000 times that of α-rays
7. Effect on photographic plate and zinc sulphide	7. Affected more strongly than by β and γ-rays	7. Effect is less than α-rays	7. Least effect

Radioactive decay law

This law states that

"At any moment the number of radioactive atoms that disintegrate in unit time is directly proportional to the number of unchanged radioactive atoms remaining."

If the rate of radioactive disintegration of atoms is $\frac{dN}{dt}$ if N is the number of unchanged atoms at time t, then

$$\frac{dN}{dt} \propto - N$$

$$\frac{dN}{dt} = - \lambda N$$

$$\frac{dN}{N} = - \lambda dt$$

Where, λ is the radioactive decay constant.

$$\int \frac{dN}{N} = - \lambda \int dt$$

$$\text{Log}_e N = - \lambda t + C \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (1)$$

Where, C is the constant of integration.

Suppose, $N = N_0$ at time $t = 0$

Then

$$\text{Log}_e N_0 = C$$

From equation number (1)

$$\text{Log}_e N = - \lambda t + \text{Log}_e N_0$$

$$\text{Log}_e \frac{N}{No} = - \lambda t$$

$$\frac{N}{No} = e^{- \lambda t}$$

$$N = N_0 e^{- \lambda t}$$

This is the law of radioactive decay or disintegration.

Half Life

The half life of a radioactive element is defined as the time during which the number of atoms remaining unchanged becomes half of its initial value.

We have, $N = N_0 e^{-\lambda t}$

By definition, when $t = T_{1/2}$ (i.e. half life)

$N = \frac{1}{2} N_0$

So, $\frac{1}{2} N_0 = N_0 e^{-\lambda T_{1/2}}$

So, $\frac{1}{2} = e^{-\lambda T_{1/2}}$

Thus, $2 = e^{+\lambda T_{1/2}}$ (by inverting both sides)

Now, taking 'logs to base e', known as 'natural logs', symbol $\log_e$ or ln:

$$\ln 2 = \ln e^{+\lambda T_{1/2}}$$

$$= (\lambda T_{1/2}) \ln e \quad \text{(since } \ln e^x = x \ln e\text{)}$$

$$= \lambda T_{1/2} \quad \text{(since } \ln e = 1\text{)}$$

So, $$\boxed{T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.69}{\lambda}}$$

Einstein's mass-energy equivalent law

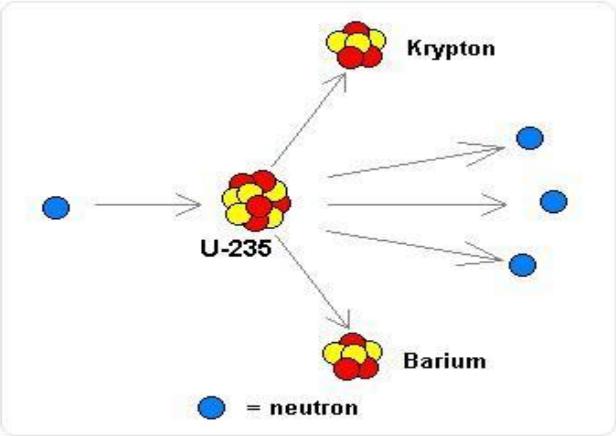
In 1905 famous scientist Albert Einstein showed that matter and energy were actually identical. Matter can be transformed into energy. If a substance of mass m is completely transformed into energy, the amount of energy obtained is

$E = mc^2$, (here c is the speed of light $= 3 \times 10^8$)

This is called the Einstein's mass-energy equivalent law.

Nuclear Fission

Nuclear fission is **nuclear** reaction process in which nucleus, when bombarded with a neutron, splits into smaller parts, often producing free neutrons, and releasing a very large amount of energy.



Example: When a uranium nucleus is bombarded by high energy neutrons or protons or deuterons, then fission takes place. This reaction can be represented as

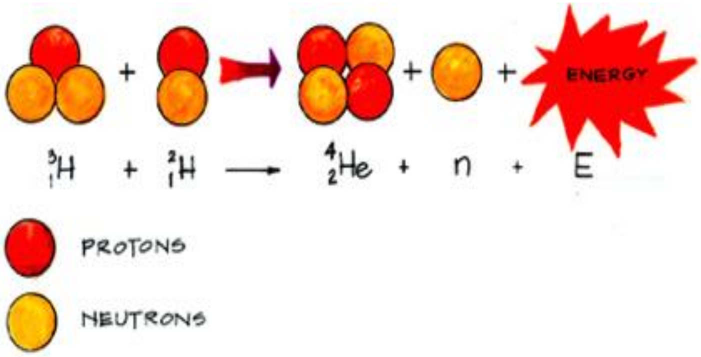
$$^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \longrightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\ {}^{1}_{0}\text{n} + \text{energy}$$

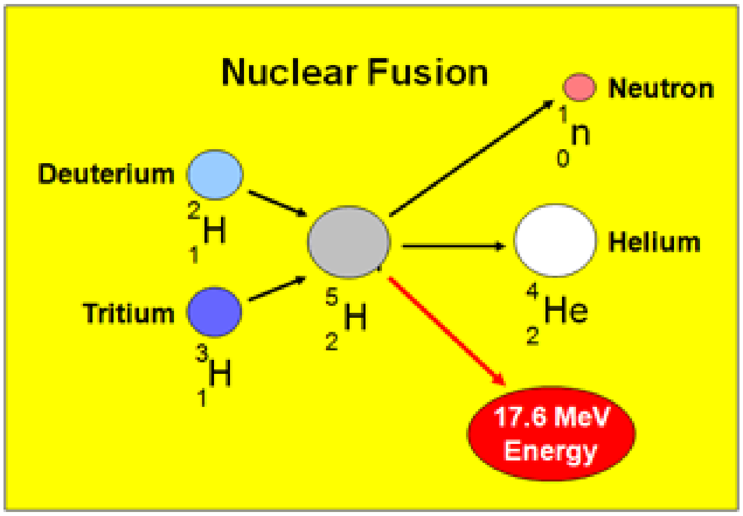
Nuclear Fusion

Nuclear fusion is a nuclear reaction in which two or more atomic nuclei collide at a very high speed and join to form a new type of atomic nucleus.

During this process, matter is not conserved because some of the matter of the fusing nuclei is converted to photons (energy).

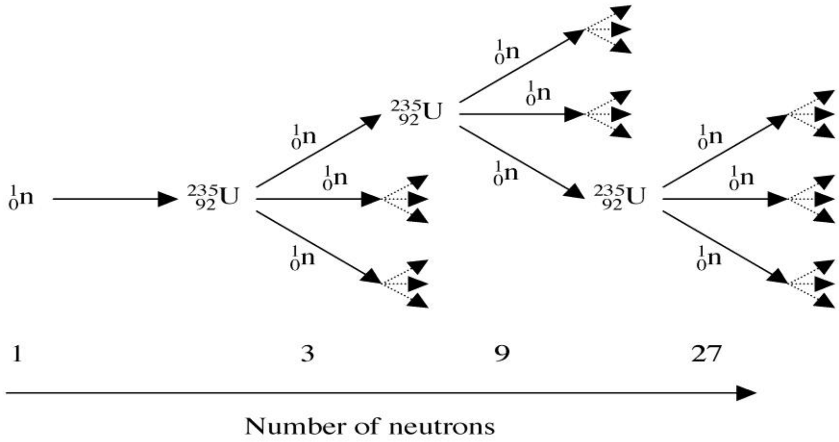
Example:





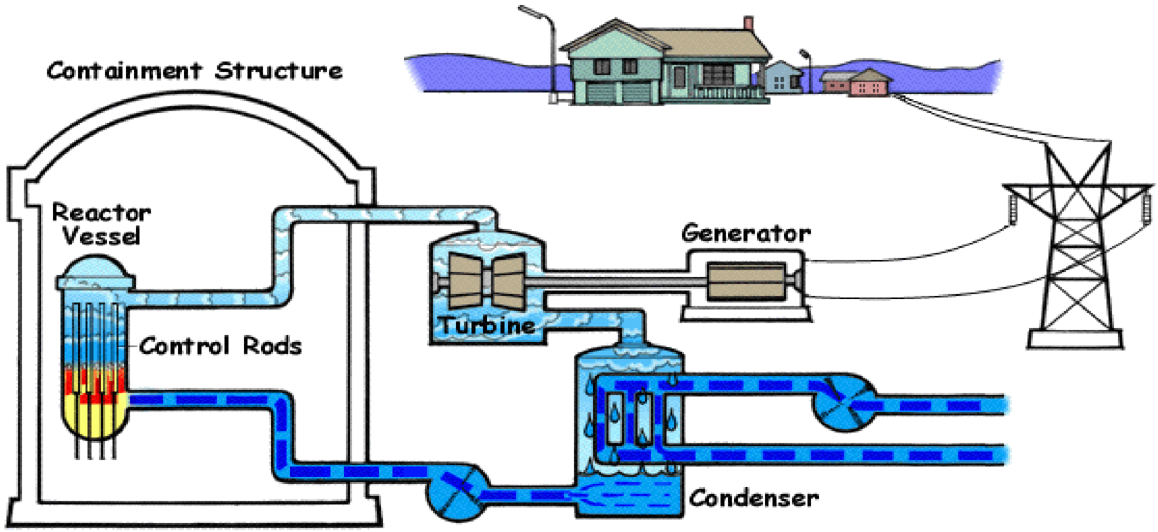
Chain Reaction

Chain reaction is such a process which once started the reaction continues without requiring further energy and a tremendous amount of energy released because of the fission of all nuclei.



Nuclear Reactor

A nuclear reactor is a device to initiate and control a sustained nuclear chain reaction. Nuclear reactors are used at nuclear power plants for electricity generation and in propulsion of ships. Heat from nuclear fission is passed to a working fluid (water or gas), which runs through turbines.



Mass Defect

The mass of an atom's nucleus is usually less than the sum of the individual masses of the constituent protons and neutrons when separated. During the splitting of the nucleus, some of the mass of the nucleus (i.e. some nucleons) gets converted into huge amounts of energy (according to Einstein's equation $E=mc^2$) and thus this mass is removed from the total mass of the original particles, and the mass is missing in the resulting nucleus. This missing mass is known as the mass defect.

Suppose, the actual mass of a nucleus of atomic number A is M, number of protons = Z and number of neutrons = (A-Z). is the masses of each proton and neutron are respectively m_p and m_n

Then the mass defect

$$\triangle M = [Z\, m_p + (A-Z)\, m_n] - M$$

Binding Energy

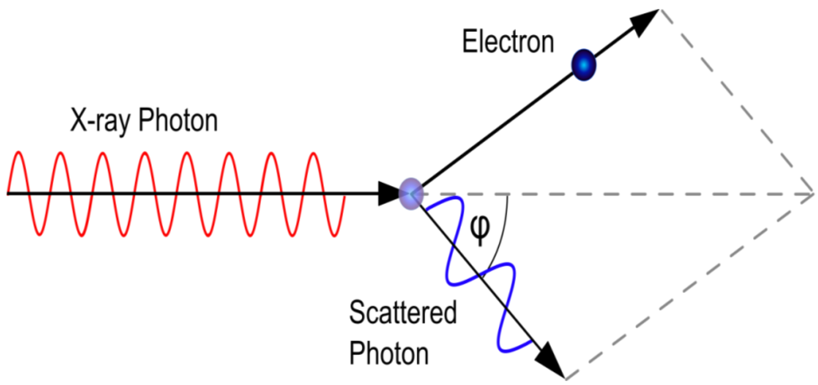
Nuclear binding energy is the energy required to split the nucleus of an atom into its component parts. The component parts are neutrons and protons, which are collectively called nucleons. The binding energy of nuclei is usually a positive number, since most nuclei require net energy to separate them into individual protons and neutrons.

If the mass defect of a nucleus is $\triangle M$, then the binding energy $\triangle E = \triangle M \times c^2$

Compton Effect

The Compton Effect (also called Compton scattering) is the result of a high-energy photon colliding with a target, which releases loosely bound electrons from the outer shell of the atom or molecule. The scattered radiation experiences a wavelength shift that cannot be explained in terms of classical wave theory, thus lending support to Einstein's photon theory.

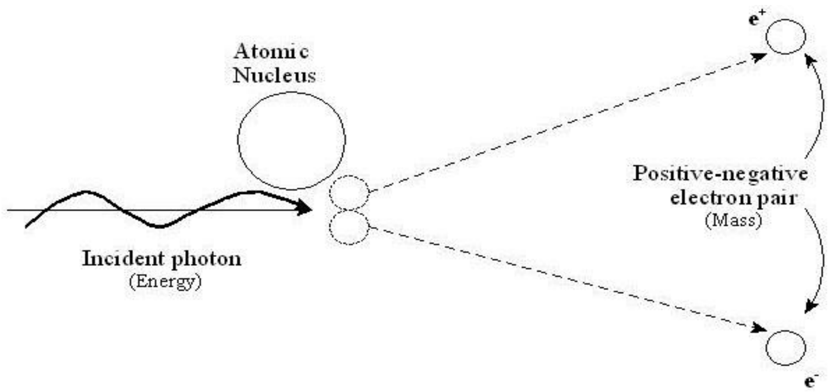
The effect was first demonstrated in 1923 by Arthur Holly Compton (for which he received a 1927 Nobel Prize)



Pair production

Pair production is a phenomenon of nature where energy is converted to mass. In this phenomenon a wave of energy or a photon (a packet of energy) interacts with a heavy nucleus to form an electron - positron pair.

Pair production is the creation of an elementary particle and its antiparticle.



Pair Production - Energy Converstion to Mass

✓ Calculate the energy of one atomic mass unit in MeV

By using Einstein's mass-energy equation, the energy equivalence of 1 a.m.u can be found easily

1 amu = 1.66×10^{-27} Kg

And C = 3×10^{8} ms^{-1}

$E = mc^2 = 1.66 \times 10^{-27} \times (3 \times 10^{8})^2$

$= 14.9 \times 10^{-11}$ Joules

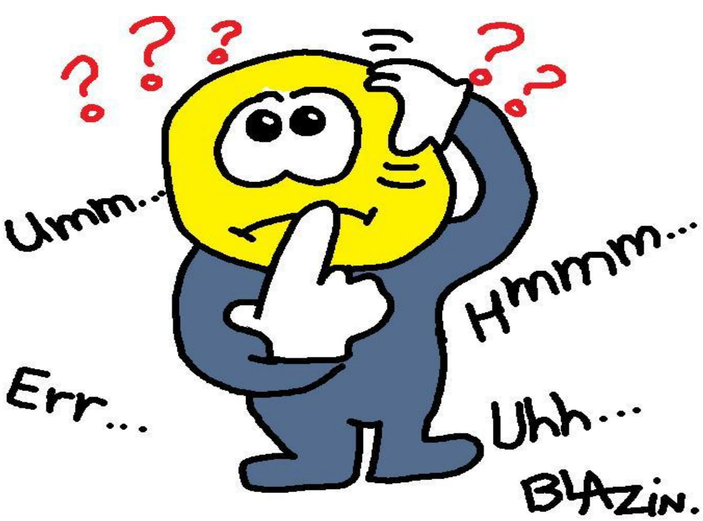
Again

1.6×10^{-19} Joules = 1 eV

14.9×10^{-11} Joules = $\frac{14.9 \times 10^{-11}}{1.6 \times 10^{-19}} = 931.25 \times 10^{6}$ eV

= 931 MeV

✓ Calculate the energy of one atomic mass unit in KeV



Derivation of Einstein's mass-energy relation

We know from the second law of motion that rate of change of momentum is called force. So

$F = \frac{d}{dt}(mv)$

We know from the theory of special relativity that both mass and velocity vary.

$F = \frac{d}{dt}(mv)$

$= m\frac{d}{dt}v + v\frac{d}{dt}m$ ……………. (1)

Suppose, force F creates displacement of dx of a body. So, work done = F. dx. Then the increase in kinetic energy (dE_k) of the body is equal to the work done (F. dx)

$dE_k = F.\,dx$

$= (m\frac{d}{dt}v + v\frac{d}{dt}m).\,dx$

$= m\frac{dx}{dt}dv + v\frac{dx}{dt}dm$

$= mv\,dv + v^2\,dm$ …………. (2)

$$\boxed{\frac{dx}{dt} = v}$$

From the relation of mass and velocity

$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

Squaring both sides, we get

$m^2 = \frac{m_0^2}{1-\frac{v^2}{c^2}}$

$m^2c^2 - m^2v^2 = m_0^2c^2$

or, $m^2c^2 = m^2v^2 + m_0^2c^2$

Differentiating both sides

$2m.\,dm\,c^2 = 2m\,dm\,v^2 + 2v\,dv\,m^2$

$dm\,c^2 = (mv\,dv + v^2\,dm)$ ………… (3)

From (2) and (3)

$dm\,c^2 = dE_K$ ………… (3)

It can be written as

$dE_K \propto dm$

If the object is stationary, then $v = 0$ and K.E = 0. In this condition $m = m_o$, but when the velocity of the body is v, then the mass is m.

$$\int_0^{E_k} dE_k = \int_{m_0}^{m} dm\,c^2$$

$$E_k = c^2 \int_{m_0}^{m} dm$$

$$E_k = c^2\,[m]_{m_0}^{m}$$

$$E_k = c^2\,(m - m_o)$$

$$E_k = m\,c^2 - m_o\,c^2$$

This is the relativistic formula for kinetic energy.

When the body is at rest, the internal energy stored in the body is $m_o\,c^2$. This energy is called rest mass energy

So, the total energy of the body

E = kinetic energy + rest mass energy

$$E = E_k + m_o\,c^2$$

$$E = m\,c^2 - m_o\,c^2 + m_o\,c^2$$

$$E = mc^2$$

This is Einstein's mass – energy relation.

Mathematical Problems

1. Express the energy of a quantum of light of wavelength 4×10^{-7} m in electron volt. ($h = 6.63 \times 10^{-34}$ Js)
2. Calculate the frequency and wavelength of a photon of energy of 100 MeV.
3. Calculate the velocity achieved by an electron at rest when it is passed through a potential difference of 10 kilo volt.
4. An ultraviolet ray of wavelength of 2500 A^0 is incident on a metal surface. If the work function of the metal is 2.3 eV. What is the maximum velocity of the emitted photoelectron?
5. The threshold wavelength of sodium is 6800 A^0 . Calculate its work function.
6. The work function of platinum is 6.31 eV. What is its threshold frequency? ($h = 6.63 \times 10^{-34}$ Js)
7. When radiation of frequency of 4×10^{15} Hz is incident on a metal surface, electron of maximum energy of 3.6×10^{-19} J is emitted. What is the threshold frequency of that metal?
8. The work function of a metal is 1.85 eV. What is its threshold frequency?
9. A metal surface is exposed to light of wavelength of 6000 A^0 . 1.77 eV is required to remove an electron. What is the kinetic energy of the fastest electron? What is the threshold frequency? ($h = 6.63 \times 10^{-34}$ Js and 1eV = 1.6×10^{-19} J)
10. What is the energy of an X-ray photon of wavelength of 2×10^{-10} m?
11. The half life of a radioactive element is 4d. Determine the decay constant of the element.
12. What time will it take to decay 60% of a piece of radon? Half life of radon is 3.82 days.
13. If 10^8 numbers of atoms of radon initially remains in a piece of a substance, then how many atoms will disintegrate in one day? Half life of radon is 4 days.
14. The half life of a radioactive substance is 10 days. After how many days 75% of that substance will be disintegrated?
15. The decay constant of a radioactive substance is 3.75×10^{-3} sec^{-1} . Calculate its half life.
16. A piece of radium is transformed by radioactive emission to $\frac{1}{5}$ th of its initial mass in 4000 years. Calculate the decay constant of radium.

17. If an electron moves with a velocity of 0.99c, what is its mass?

18. A particle is moving with velocity of 0.5c. Determine the ratio of the mass at rest to the moving mass of the particle.

19. The rest mass of an electron is 9.028×10^{-31} Kg. Find its equivalent energy.What will be the value in eV ?

20. The total energy of a moving particle is 2.5 times the stationary energy, what is the speed of the particle?

21. What is the mass of an electron having kinetic energy of 1.6×10^{6} eV ?

22. The mass of a particle is 9.1×10^{-28} Kg. If it is totally converted into energy, how much energy will be obtained? ($c = 3 \times 10^{8}$ ms^{-1})

23. Calculate the mass and speed of an electron having kinetic energy of 1.5×10^{6} eV according to the theory of relativity.

24. Determine the equivalent energy of 1g mass in (i) Joule (ii) MeV.

25. The mass of an object is 8.30×10^{-3} Kg. It is converted totally into energy. How much energy will be generated?

26. Express the equivalent energy of 10 amu mass.

27. Express the equivalent energy of 12 amu mass in (i) eV, (ii) MeV.

28. Determine the momentum, kinetic energy and total energy of a particle moving with velocity $\frac{c}{\sqrt{2}}$.

29. Total energy of a particle is twice its stationary energy. What is the speed of the particle?

30. The mass of a star is $4M_0$. If the star is transformed into a black hole, then what will its Schwarzchild or critical radius? (Mass of the sun, $M_0 = 1.99\times 10^{30}$ Kg)

31. Calculate the binding energy when

    (i) One neutron and one proton combine to form a deuteron.

(ii) Two neutrons and two protons combine to form an Alpha particle.

32. The mass of $_{17}cl^{35}$ is 34.9800 amu. Calculate its binding energy. What is the binding energy per nucleon?
(Mass of $_{0}n^{1}$ = 1.008665 amu and $_{1}H^{1}$ = 1.007825 amu)