Current and Resistance

Current and Current Density

The conventional current is defined as the rate of flow of (positive) charge through any cross-sectional area is called electric current. If a net charge q passes through any cross-section of the conductor in time t, then current i is given by,

$i = q / t$

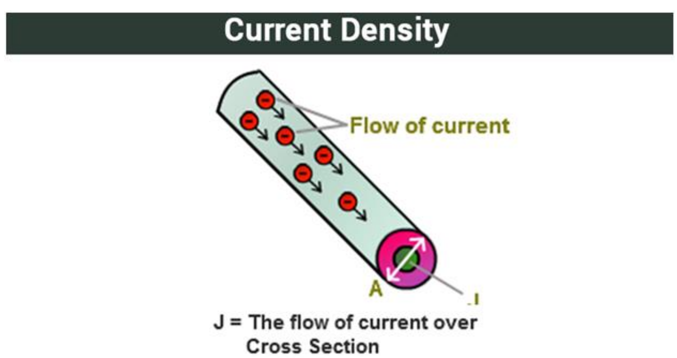
The MKS unit of charge is Ampere that is,

1 Ampere = 1 Coul / 1 Second

The current given by above equation is the constant for the time. If the rate of flow of charge with time is not constant, then i varies with time and is given by, $i = dq / dt$

Current Density

The current density at a point in a conductor carrying current is defined as the current per unit area of cross-section of the conductor the area being taken in a direction normal to the current.



Current density **j** is a macroscopic quantity. It is a vector and is characteristic of a point inside a conductor rather than of the conductor as a whole. If the current is distributed uniformly across a conductor of cross-sectional area A, the magnitude of the current density for all points on that cross-section is

J = i / A

Or i = j A

The MKS unit of current density is **ampere per square meter**

Electrical Resistance or Resistance:

The **electrical resistance** of an electrical conductor is the opposition to the passage of an electric current through that conductor. Normally, it is denoted by R. The SI unit of electrical resistance is the ohm (Ω), All materials show some resistance, except for superconductors, which have a resistance of zero.

The resistance R of an object is defined as the ratio of voltage across it V to current through it I. Mathematically, $R=\frac{V}{I}$

Resistance can be expressed in ohms

1 ohm = 1 Volt / 1 ampere

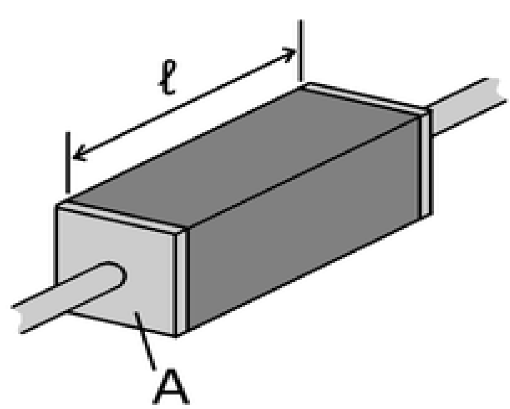
The ohm is defined as the resistance of a conductor in which a potential difference of 1 Volt is developed when a current of 1 ampere flows through it.

Resistivity/Specific Resistance/ Volume Resistivity:

Specific electrical resistance is a property of a material; it quantifies how strongly the material opposes the flow of electric current. A low resistivity indicates a material that readily allows the movement of electric charge. The SI unit of electrical resistivity or specific resistance is the ohm·metre (Ω·m). It is commonly represented by the ρ (rho).

Mathematically, electrical resistivity ρ is defined as,

$$\rho = R\frac{A}{\ell},$$



The quantity ρ is called 'Specific Resistance' or resistivity. Thus, the specific resistance is the resistance offered by a conductor of unit length and unit cross-section that is A = 1 cm^2, and l = 1 cm. Its unit is ohm-meter. Conductivity is the reciprocal of resistivity. Its unit is mho.

Conductivity:

Electrical conductivity or **specific conductance** is the reciprocal of electrical resistivity, measures a material's ability to conduct an electric current.

It is commonly represented by the Greek letter σ, but κ (kappa) (especially in electrical engineering) or γ (gamma) are also occasionally used. Its SI unit is Siemens per meter(S/m)
Mathematically, σ can be defined as

Conductivity is the inverse of resistivity**.**

$$\sigma = \frac{1}{\rho}$$

Ohm's Law:

Ohm's law states that the potential difference between the ends of a conductor varies directly as the current flowing in it, provided the temperature does not change and the physical state of the conductor remains the same.
If V is the potential difference between two ends of a conductor AB and I is the current flow in it, then

I ∞ V

I = GV (where, G is the proportionality constant called conductivity)

$I = \frac{1}{R}V$

IR = V

V = IR

Where, R is known as resistance and it is inverse of conductivity.

That means if the resistance of the conductor increases the conductivity decreases. The flow of current decreases.

Series Combination of resistors:

Suppose the values of three resistors are respectively R1, R2 and R3. These are connected in such a way as that same current flows through each. This combination of resistors is series combination. The equivalent resistance of the resistors is to be found out.

Let the potential of points A, B, C and D are respectively VA, VB, VC and VD.

Further let V › V.

Let the potential difference between the two ends of resistors are respectively V1, V2 and V3.

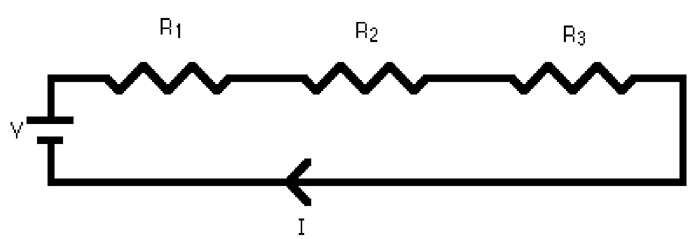
So, from the ohm's law, we get VA - VB = V_1, VB - VC = V_2, VC - VD = V_3

If the potential difference between the two ends of the combination is V,

then $V = V_1 + V_2 + V_3$ ………… (1)

But if the equivalent resistance of the combination is Rs , then from ohm's law

We get, V = iRs from equation number (1)



we get, $iRs = iR_1 + iR_2 + iR_3$

$iRs = i (R_1 + R_2 + R_3)$

$Rs = R_1 + R_2 + R_3$

If there are n number of resistors connected in series

$Rs = R_1 + R_2 + R_3 +$……………..Rn

Parallel Combination of resistors:

Suppose, the values of three resistors are respectively R_1, R_2 and R_3. One end of each resistor is connected at point A and the other ends of the resistors are connected at B so that same potential difference (VA – VB) exists between two ends of each resistor; here the potential of points A and B are respectively VA and VB.

Here, current i after reaching at point A, gets divided into i_1, i_2 and i_3 flowing through respective resistors R_1, R_2 and R_3 reach at point B and after combining becomes the main current $i = i_1 + i_2 + i_3$ …………………… (1)

From the ohm's law, we get

$$i_1 = \frac{V_A - V_B}{R_1}, \quad i_2 = \frac{V_A - V_B}{R_2} \text{ and } i_3 = \frac{V_A - V_B}{R_3}$$

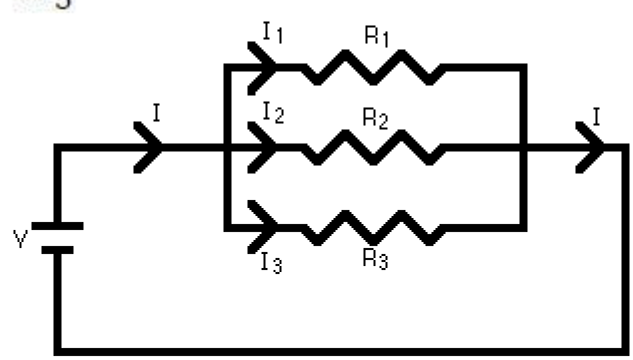
If the equivalent resistance of the circuit is Rp, then from ohm's law

$$i = \frac{V_A - V_B}{R_P}$$

Now, inserting the values of i1, i2 and i3 in equation (1)

$$\frac{V_A - V_B}{R_P} = \frac{V_A - V_B}{R_1} + \frac{V_A - V_B}{R_2} + \frac{V_A - V_B}{R_3}$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



If the number of resistors is n,

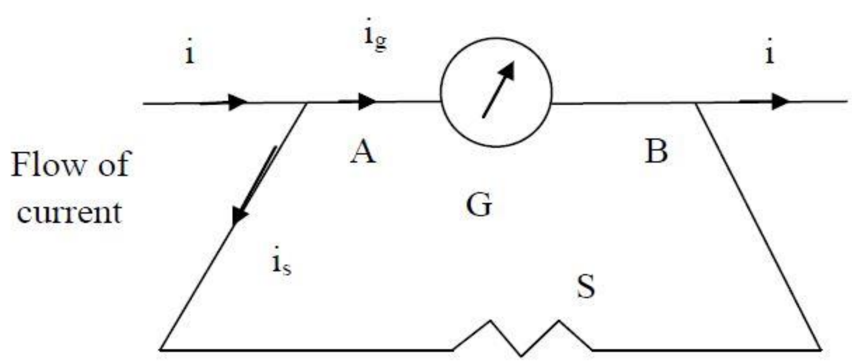
$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \cdots\cdots\cdots\cdots + \frac{1}{R_n}$$

Shunt:

Shunt is the practical application of parallel combination of resistances. In many occasions sensitive and sophisticated equipment like galvanometer is used in electric circuits. A low resistance is used parallel to the equipment in order to protect the equipment from damage due to the flow of high current through it that produces excessive heat. So, a low resistance which is used to sensitive and sophisticated equipment so that high current does not flow through it is called a shunt.

Derive the equation of current flowing through the shunt and the galvanometer:

Suppose, a resistance S of small value is connected between two ends A and B, parallel to galvanometer of resistance G. This S is the shunt. Let the principle current in the circuit be i. While reaching at point A, this current will be divided into two parts. A small portion of the principle current will flow through galvanometer and large current will flow through the shunt. As a result, the galvanometer will not be damaged due to heat produced for large flow of current.



Shunt

Two currents will meet at B and will form principle current again. Let this current through the galvanometer and the shunt be respectively ig and is. Now, if the potential difference between the points A and B be (VA – VA) then according to ohm's law, we get

$$i_g = \frac{V_A - V_B}{G} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (1)$$

And

$$i_S = \frac{V_A - V_B}{S} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Dividing equation number 2 by 1

$$\frac{is}{ig} = \frac{G}{S}$$

$$i_s = i_g \times \frac{G}{S} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3)$$

But, $i = i_g + i_s$ ....(4)

Inserting the value of i_s in this equation number 4

$$i_g \times \frac{G}{S} + i_g = i$$

$$i_g \left(\frac{G}{S} + 1\right) = i$$

$$i_g \frac{G+S}{S} = i$$

Then,

$$i_g = \frac{S \times i}{G+S} = \text{Principle current} \times \frac{\text{shunt resistance}}{\text{total resistance}}$$

or

$$i = ig\ \frac{G+S}{S}$$: Where, $\frac{G+S}{S}$ is called the power – multiplier of the shunt.

Again, putting the value of i_g in equation number 3, we get

$$i_s = \frac{S \times i}{G+S} \times \frac{G}{S}$$

$$i_s = \frac{G \times i}{G+S}$$

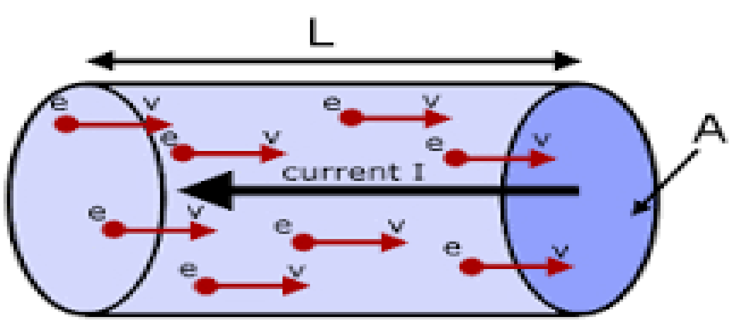
And

$$S = \frac{G \times ig}{i - ig}$$

Drift Velocity

The **drift velocity** is the average velocity that a particle, such as an electron, attains due to an electric field.

Let us consider a conductor AB the two ends of which are connected to a battery (Figure below).



A steady electric field is thus established in the conductor in the direction A to B. The free electrons at the end B experience a force (F = -Ee) from B to A in a direction opposite to that of the field E. The electrons are, therefore, accelerated in this direction. In the process, the electrons collide with each other and with the positive ions in the conductor. At each collision the momentum gained in the direction of the force acting on the charge carrier due to the electric field is lost and the electron is accelerated afresh after each collision. Thus, due to collision, a backward force acts on the electrons. The overall effect of these collisions is that the electrons slowly drift with a constant average drift velocity in the direction of –E.

The average velocity with which the charge carriers move under the effect of the electric field is known as drift velocity, the average being macroscopic i.e., taken over a volume large as compared to molecular volume.

Let us consider a section of conductor as shown in Fig. The number of conduction electrons in the wire is nAl.

Where, n = number of conduction electrons per unit volume

Al = volume of the wire

So, the magnitude of the charge $q = (nAl)\,e$

This charge passes out the wire, through it's right end in a time t is given by

$$t = \frac{l}{Vd}$$

Where, V_d = Drift speed of charge

l = Length of the wire

According to the definition of current

$$i = \frac{q}{t}$$

$$= \frac{nAle}{\frac{l}{vd}}$$

$$= (n A V_d) e$$

Again, we know current density

$$J = \frac{i}{A}$$

$$= nV_d e$$

Finally,

$$\mathbf{V_d = \frac{j}{ne}}$$

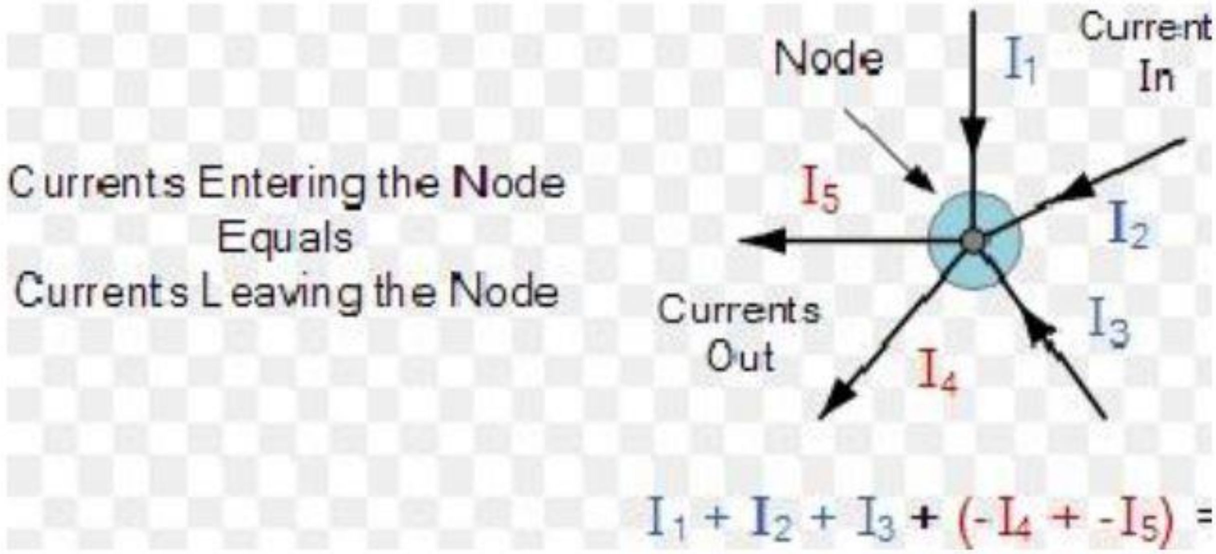
This is the relationship between Drift velocity and Current density

Kirchhoff's Law:

Kirchhoff's current law (First Law)

This law is also called **Kirchhoff's first law**, **Kirchhoff's point rule**, or **Kirchhoff's junction rule** (or nodal rule).
At any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node or

equivalently the algebraic sum of currents in a network of conductors meeting at a point is zero.



According to Kirchhoff's law

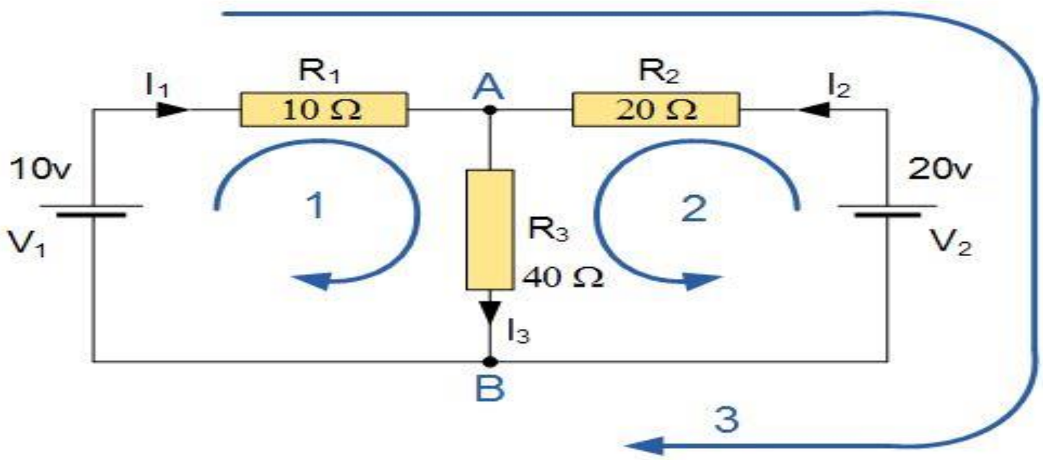
$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

Kirchhoff's voltage law (Second Law)

This law is also called Kirchhoff's second law, Kirchhoff's loop (or mesh) rule, and Kirchhoff's second rule.
The directed sum of the electrical potential differences (voltage) around any closed network is zero, or:

More simply, the sum of the emfs in any closed loop is equivalent to the sum of the potential drops in that loop, or:

The algebraic sum of the products of the resistances of the conductors and the currents in them in a closed loop is equal to the total emf available in that loop.



The circuit has 3 branches, 2 nodes (A and B) and 2 independent loops.

Using **Kirchhoffs Current Law, KCL** the equations are given as:

At node A : $I_1 + I_2 = I_3$

At node B : $I_3 = I_1 + I_2$

Using **Kirchhoffs Voltage Law, KVL** the equations are given as:

Loop 1 is given as : $10 = R_1 I_1 + R_3 I_3 = 10I_1 + 40I_3$

Loop 2 is given as : $20 = R_2 I_2 + R_3 I_3 = 20I_2 + 40I_3$

Loop 3 is given as : $10 - 20 = 10I_1 - 20I_2$

As I_3 is the sum of $I_1 + I_2$ we can rewrite the equations as;

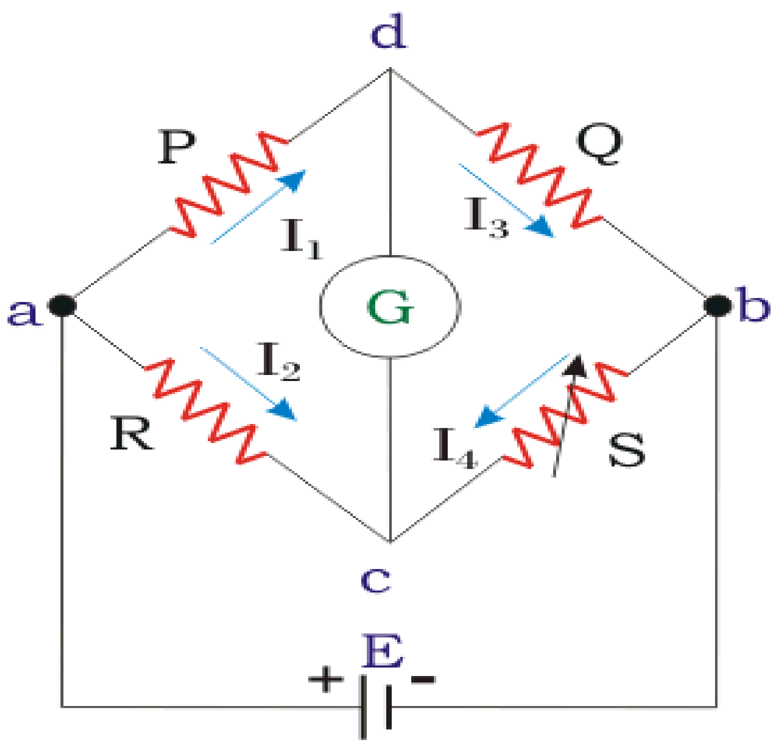
Wheatstone bridge:

If four junctions are made due to the formation of a closed loop by connecting four resistors in series and if an electric cell is connected between the two opposite junctions and a galvanometer is connected between the other two opposite junctions then the circuit thus formed is called Wheatstone bridge. Let us consider that four resistors P, Q, R and S are arranged like tetrahedral **ABCD**. Wheatstone bridge is formed by connecting a battery B or an electric source, a plug key K and a variable resistor X between the junctions A and B and a galvanometer G between the junctions C and D.

Let the resistance of the galvanometer be G and currents flowing through P, Q, R, S and G are respectively i1, i2, i3, i4 and ig. Now, applying Kirchhoff's first law respectively at points C and F, we get

$$i_1 - i_3 - i_g = 0 \text{ or } i_1 = i_3 + i_g \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$i_2 + i_g - i_4 = 0 \text{ or } i_4 = i_2 + i_g \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$



Wheatstone Bridge

Again, applying Kirchhoff's second law respectively at closed loops **ACDA** and **CDBC,** we get

$$i_1P + i_gG - i_2R = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$i_3Q - i_4S - i_gG = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

But, at balanced condition of the bridge, ig = 0

Under this condition, according to equations (1) and (2)

$i_1 = i_3$

$i_4 = i_2$

From equation 3 and 4

$i_1P = i_2R$ ………………. (5)

$i_3Q = i_4S$ ……………….(6)

Dividing equation 5 by 6

$$\frac{i_1 P}{i_3 Q} = \frac{i_2 R}{i_4 S}$$

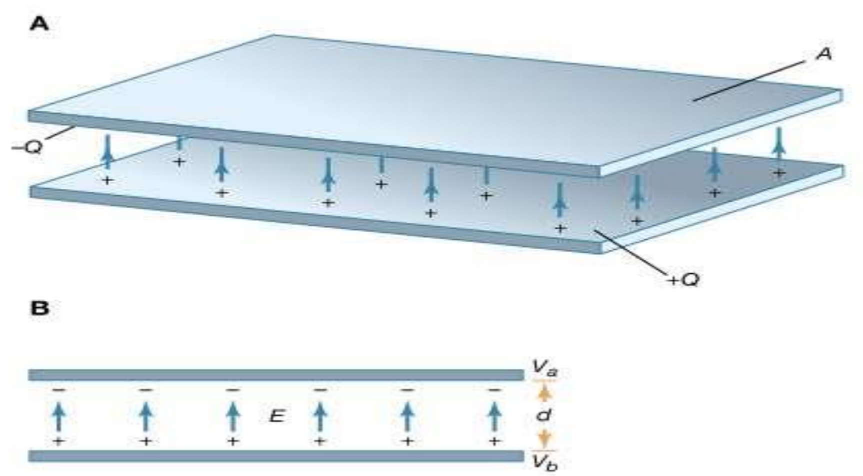
$$\frac{P}{Q} = \frac{R}{S}$$

According to this equation of the Wheatstone bridge, if values of any three resistors are known, then the resistance of the fourth resistor can be determined. It is called the Wheatstone bridge principle for the measurement of resistance.

Capacitor

A capacitor is a device for storing charge. In actual practice, the capacitor is an electrical device consisting of two conductors separated by an insulating or dielectric medium and carrying equal and opposite charges.

The conductors are called plates and may be of any shape. There are some potential differences between the charge carrying conductor



Capacitance

When charge is given to a conductor, its potential rises. So there is a fixed relationship between the two quantities which can be written as

Q ∞ V

Or, Q =CV

Or, C= Q/V

Where c is the constant and is known as the capacity or capacitance of the conductor.

If V =1 then,

C=Q

Thus the capacity of a conductor is the amount of charge required to raise the potential of the conductor.

In electromagnetism and electronics, capacitance is the ability of a body to hold an electrical charge.

Unit of capacitance

The practical and S.I unit of capacitance is Farad

1 Farad = 1 coulomb/ 1 Volt

The sub multiples of the farad,

The microfarad (μF) = 10^{-6} Farad and micro-microfarad (μμF) =10^{-12} Farad

Types of capacitor

According to the shape there are three types of capacitor

1. Parallel plate capacitor
2. Cylindrical capacitor
3. Spherical capacitor

Parallel plate capacitor

A parallel plate capacitor is formed of two thin metallic plates arranged parallel to one another separated by a small distance, contains insulating or dielectric medium inside.

Capacitance of the parallel plate capacitor

The figure shows a parallel plate capacitor where x and y are the two similar thin metallic flat conducting plates and separated by a small distance d. Let A be the surface area of the plate of the capacitor.

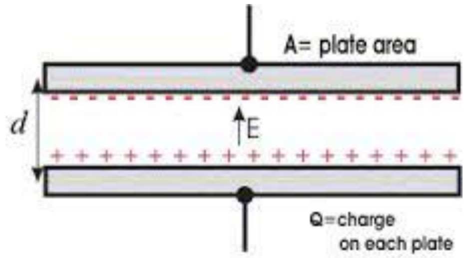


Figure 3

Let each of the plates of the capacitor is charged with a charge which is numerically equal to Q.

Let E is the field strength inside the capacitor, σ is the charge density of either plate and V is the potential developed between the plates.

Then the capacitance of the plate, $c=\frac{Q}{V}$……………(1)

The surface charge density, $\sigma=\frac{Q}{A}$……………………..(2)

Potential, V=Ed…………………………..(3)

The lines of force passing out through unit area is σ/ϵ_0. The numbers of lines of force are numerically equal to the electrical field strength E.

$$E=\frac{\sigma}{\epsilon 0}$$

$$E=\frac{Q}{\epsilon 0 A}$$

From equation (3)

$V=Ed=\frac{Qd}{\epsilon 0 A}$…………………(4)

Putting equation (4) in equation (1),

$$C=\frac{Q}{V}=\frac{Q}{\frac{Qd}{\epsilon 0 A}}$$

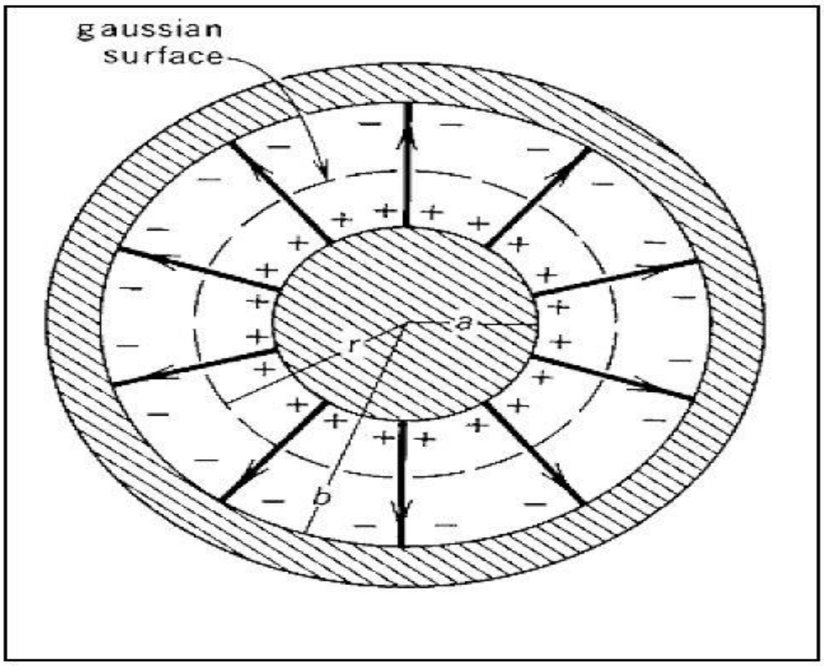
$$C=\frac{\epsilon 0 A}{d}$$

This is the expression for capacitance of a parallel plate capacitor.

Capacitance of the cylindrical capacitor

A cylindrical capacitor consists of two coaxial cylinders of radius a and b and length l. We have to find out the capacitance of the device.

Let us construct a Gaussian surface, shown in the figure, having radius r. By using Gaussian Law



$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

$$\varepsilon_0 \, E\,(2\pi r l) = q$$

$$E = \frac{q}{2\pi\varepsilon_0 \, r l} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The potential difference between the plates

$$V = \oint_{+}^{-} \vec{E} \, . \, \vec{dr}$$

The angle between E and dr is 0^0.

$$V = \oint_{+}^{-} \frac{q}{2 \pi \varepsilon_o \ r \, l} \ dr$$

$$= \oint_{a}^{b} \frac{q}{2 \pi \varepsilon_o \ r \, l} \ dr$$

$$= \frac{q}{2 \pi \varepsilon_o \ l} \oint_{a}^{b} \frac{1}{r} \, dr$$

$$= \frac{q}{2 \pi \varepsilon_o \ l} \quad \text{In } (b/a)$$

Capacitance

$$C = \frac{q}{v}$$

$$\mathbf{C = \frac{(\ 2 \pi \varepsilon_o \ ) \, \mathit{l}}{\ln\left(\frac{b}{a}\right)}}$$

This is the required expression.

Capacitance of series connected capacitors

$$\frac{1}{C} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}$$

Capacitance of parallel connected capacitors

$$C = \frac{Q}{V} = C_1 + C_2 + C_3$$